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QUOTA-INDUCED SALES GAMES

by

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ABSTRACT

This paper considers the impact of the imposition of a quota with some "slack" in it in the sense that either it exceeds the initial level of sales of foreign firms or it is expressed as a fixed percentage of a growing domestic market. If the quota is industry-wide, as opposed to a set of firm-specific quotas, then it will force the foreign firms into a game by making the possibility of increasing sales, i.e., whether the quota is binding, dependent on the sales strategies of the other foreign firms.

The set of Nash equilibria to this game is characterized. Only two equilibria exist, both of which are in some sense symmetric. One of the equilibria, the slow sales growth one, Pareto dominates the other and will be chosen if the foreign firms can collude.

The policy implications of the game are explained.

0. Introduction

Consider the imposition of a quota that allows some room for import growth either because it is set at a level above the current level of imports or because it is expressed as a fixed percentage of a growing domestic market.[1] How does this quota affect the behavior of foreign firms? This paper tries to answer that question under the assumption that foreign firms face some convex costs of increasing their sales, e.g., advertising costs or investment in a distribution system.

The answer depends crucially on whether the quota is imposed at the firm level, i.e., firm-specific quotas, or at the industry level, an industry-wide quota. In the case of firm-specific quotas the firm is faced with choosing an optimal sales trajectory. This optimal choice has the property that the policymaker faces a trade-off between the size of the quota and how fast the foreign firm will expand its sales. This is dealt with in Section 1. If, instead, the policymaker chooses an industry-wide quota the situation facing the foreign firms changes drastically. The optimal policy of any individual foreign firm depends on the sales trajectories of the other foreign firms, not because the latter is affecting prices but because it is affecting the time at which the industry-wide quota will become binding. This means that the foreign firms have been thrust into a sales growth game.

The outcome of this game is obviously critical for the policymaker and so Sections 2 and 3 of the paper characterize the set of Nash equilibria to the game. There exist exactly two equilibria to the game and, if all foreign firms start with the same level of sales, the equilibria are symmetric. In one of these equilibria the firms choose the same sales trajectories that they would have done had the industry quota been allocated equally between the firms in the form of firm-specific quotas. In the other equilibrium the firms grow much faster, how much faster depending positively on the number of foreign firms in the market.

This multiplicity of equilibria presents the policymaker with the problem of predicting which equilibrium his or her policy will give rise to. While this uncertainty cannot be resolved, it is the case that the "slow" equilibrium is preferred by the foreign firms so that if they can collude, then the slow equilibrium will be chosen. Indeed, this may lead the policymaker to encourage such collusion.

The fact that imposing a quota on the industry creates a game between the foreign firms is closely related to the results of Graetz, Reinganum and Wilde (1984). They consider situations in which a regulator or tax authority charged with costly monitoring of compliance with the law faces a budget constraint which prevents him or her from checking every regulated firm or taxpayer. Although this is a considerably more complex problem than the one dealt with here, it has the feature that the enforcer's budget constraint strategically interlinks the taxpayers' choice of actions, e.g., your decision to cheat on your taxes, in equilibrium, affects the enforcer's monitoring policy and so my chances of being audited. Here the quota achieves the same effect although, because information is complete, there is no question of evading or cheating on the quota.

Section 4 concludes the paper with a discussion of the implications of the results for policy.

1. Firm-Specific Quotas

Consider a foreign firm entering the domestic market and faced with a sales quota $q > 0$. This quota is small relative to the market and so the firm can take the market price, $p_t > 0$, as given. To simplify our analysis, and with no loss of generality, we will assume that price is constant over time, $p_t = p$, $\forall$ t. The firm operates under constant costs with an average cost $v > 0$. To ensure that the firm strictly prefers to enter the domestic market we will assume that its per unit profits, $\pi = p - v$, are strictly positive.

In order to increase its sales, the foreign firm must incur some nonproduction costs. These costs can take the form of advertising or setting up a distribution network. For instance, in the case of foreign banks in Canada the banks must set up offices and employ calling officers to generate sales. Assuming diminishing marginal returns to such activities, these costs may be summarized by the function $g(\dot{s}_t)$ where $\dot{s}_t$ is the rate of change of sales at time t and $g(\cdot)$ is a non-negative, twice differentiable, increasing, convex function with $g(0) = g'(0) = 0$. The firm's total profits in any period t can, therefore, be written as

$$\pi s_t - g(\dot{s}_t) \tag{1}$$

where $s_t \geq 0$ is the sales of the firm at t.

The firm will choose a strategy of investing in advertising, distribution, etc. and so a trajectory of sales. As $\pi > 0$, one would expect (correctly) the firm to increase sales monotonically and so eventually to hit its quota constraint $s_t \leq q$. Let the date at which it exhausts its quota be T. From T onwards the firm will cease investment in sales expanding activities, set $s_t = q$, $t \geq T$, and earn πq profits per period from T onwards. With this in mind we can write the firm's present value as

$$V = \int_0^T [\pi s_\tau - g(\dot{s}_\tau)] e^{-\rho\tau} d\tau + \pi q \int_T^\infty e^{-\rho\tau} d\tau \tag{2}$$

where $\rho > 0$ is the firm's discount rate. The firm's problem is to maximize V by choosing a time path of $\dot{s}_t$, i.e., a trajectory of investments in sales increasing activities, subject to

$$s_0 = 0, \; 0 \leq s_t \leq q \; \forall \; t \tag{3}$$

In order to get a closed-form solution to this problem assume that $g(\cdot)$ is a quadratic and so $g(\cdot) = c(\dot{s}_t)^2$, $c > 0$. See Section 3 for a treatment of the more general problem. The firm's objective functional becomes

$$V = \int_0^T [\pi s_\tau - c(\dot{s}_\tau)^2] e^{-\rho\tau} d\tau + \pi q \int_T^\infty e^{-\rho\tau} d\tau \tag{4}$$

which it must maximize subject to (3).

The Euler equation for this problem[2] is

$$e^{-\rho t}[\pi - \rho 2c\dot{s}_t + 2c\ddot{s}_t] = 0 \tag{5}$$

which, upon integrating gives

$$\dot{s}_t = k_1 e^{\rho t} + \frac{\pi}{2c\rho} \tag{6}$$

and

$$s_t = \frac{k_1 e^{\rho t}}{\rho} + \frac{\pi t}{2c\rho} + k_2 \tag{7}$$

where k_1 and k_2 are constants of integration. Using the initial condition $s_0 = 0$ and (7) gives

$$k_1\rho^{-1} + k_2 = 0 \tag{8}$$

The transversality condition that must hold at T (Kamien and Schwartz, 1981, p. 68) reduces to

$$c(\dot{s}_T)^2 = 0 \tag{9}$$

which immediately implies that $\dot{s}_T = 0$. Evaluating (8) at T therefore gives

$$k_1 = -\frac{\pi e^{-\rho T}}{2c\rho} < 0 \tag{10}$$

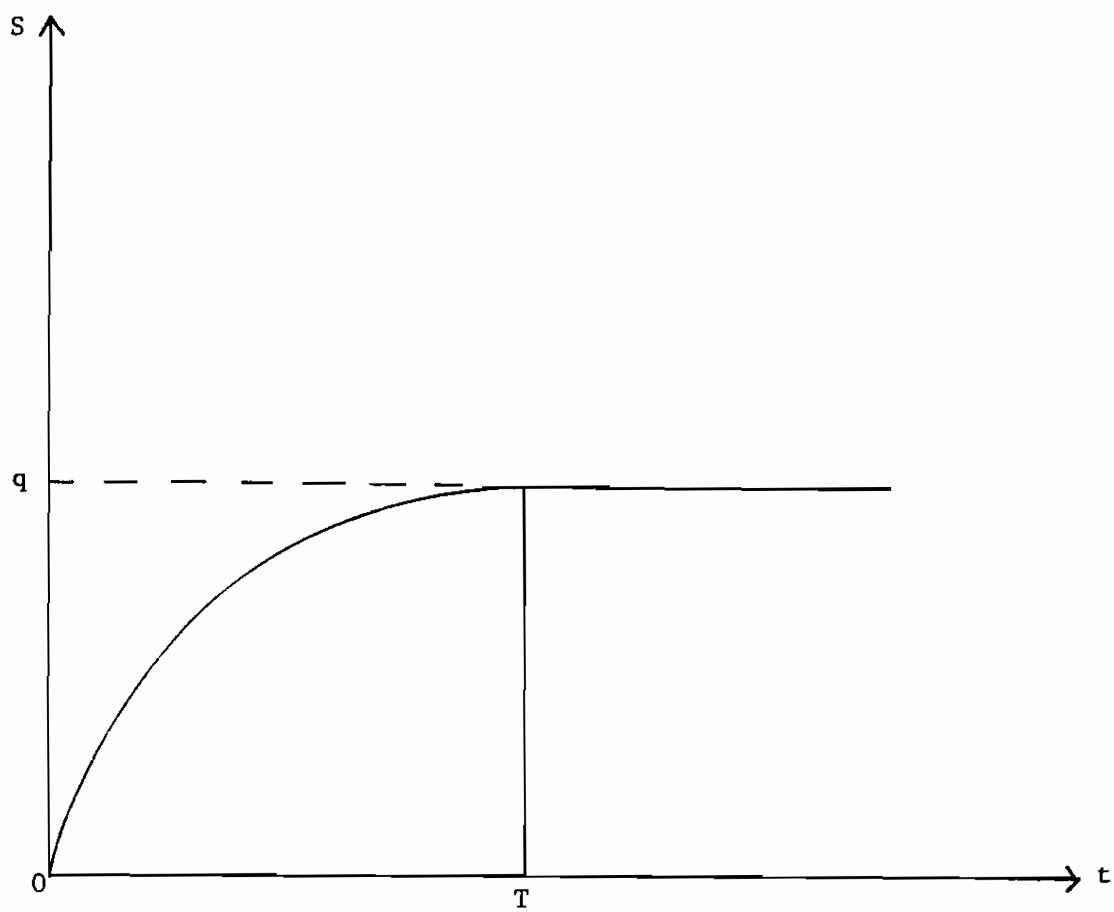
Substituting this into (6) shows that $\dot{s}_t \geq 0$, and that $\ddot{s}_t \leq 0$, $\forall$ t.

These results enable us to characterize the optimal trajectory of sales which is shown in Figure 1. Note that the firm's most rapid growth of sales, and so its highest investment in sales expanding activities, occur at the time it first enters the market. These activities and so the rate of growth of sales then decelerate until the firm smoothly reaches its quota constraint. To show that T is finite and to check that the constraint $0 \leq s_t$ is not being violated, it is necessary to solve for k_2 from (8) and (10). Substituting this into (7) gives the trajectory of sales,

$$s_t = \frac{\pi}{2c\rho^2} \cdot \frac{(1 - e^{\rho t})}{e^{\rho T}} + \frac{\pi t}{2c\rho} \geq 0 \tag{11}$$

As $s_T = q$ we can derive the following implicit expression for T:

Figure 1



$$T = \frac{2qc\rho}{\pi} - \frac{(1 - e^{\rho T})}{e^{\rho T}} \rho^{-1} \tag{12}$$

which can readily be shown to have a unique, finite root.[3]

Notice that, as one would expect, the larger are per unit profits, π, the faster the firm will expand and the quicker it will hit its quota constraint. Conversely, the higher is the cost of expanding sales, c, the slower will it expand and the longer it will take to reach its quota. More interestingly, raising the quota, q, lengthens the time before the quota is reached and, by lowering k_1, lowers the rate of growth of sales. Thus the higher the quota, the longer it will take the firm to reach any given level of sales. This means that the policymaker is faced with a trade-off in the choice of the quota. The higher is q the larger is the ultimate penetration of the domestic market by foreign firms. However, choosing a high q slows the rate of growth of sales by such firms. Without specifying the policymaker's objective function we obviously cannot say where along this trade-off he or she will choose to locate q but certainly if he or she has a high discount rate this would encourage the policymaker to choose a high q.

If the quotas are firm-specific, then the foreign firm's problem is purely an optimization problem and in particular it need not consider what other foreign firms are doing. If, instead, the quota is industry-wide the foreign firm is faced with a very different environment. Even if the growth of sales by foreign firms does not affect the domestic price path, as we will assume for simplicity, the foreign firm must still take into account the sales strategies of the other foreign firms because that will determine, along with the firm's own sales strategy, when the industry-wide sales quota will be exhausted. In short, the imposition of an industry-wide quota will thrust the firm into a gaming situation. This game is studied in the next section.

2. Industry-wide Quotas

Assume that there are two foreign firms that will enter the domestic market each of which has the technology of the firm of the previous section. These firms are faced by an industry-wide quota, $Q > 0$, on sales by foreign firms. When this quota is reached each foreign firm is not allowed to increase its sales. In this regulatory setting the firms are now forced into a game where their strategies are the time paths of investments in sales expanding activities or, equivalently, sales. The game ends as soon as the sum of their sales, at a point in time, equals Q. Denoting the sales of the two firms by superscripts 1 and 2, the payoff to firm $i = 1,2$ from adopting a particular strategy $\{s_t^i\}$, $t \geq 0$ is just

$$V^i = \int_0^T [\pi s_\tau^i - c(\dot{s}_\tau^i)^2] e^{-\rho\tau} d\tau + \pi[Q - s_T^j] \int_T^\infty e^{-\rho\tau} d\tau \tag{13}$$

where $j \neq i$ and T is the first time at which $s_t^1 + s_t^2 = Q$.

It is clearly important for the policymaker to know what, if any, equilibria there might be to this game so that he or she can predict the rate at which foreign sales will grow and so we will characterize the set of Nash (pure strategy) equilibria to the game. Consider firm 1. Firm 2's strategy only affects firm 1's payoff through its impact on T and so s_T^1. In particular, given firm 2's strategy, firm 1's optimal strategy must fulfill the Euler equation, (5), in the previous section. However, the firm's transversality condition is altered because the impact of marginally raising $\dot{s}_t^1$ at all points is not only to bring the end of the game forward as before, but also to raise the per period payoff once the game has ended by grabbing a larger share of Q. Firm 1's transversality condition is

$$\pi s_T^1 + c(\dot{s}_T^1)^2 - \pi[Q - s_T^2] - \pi\dot{s}_T^2\rho^{-1} = 0 \tag{14}$$

Notice that T is the same for both firms and that at T, $(Q - s_T^2) = s_T^1$ and so (14) can be simplified to

$$c(\dot{s}_T^1)^2 - \pi\dot{s}_T^2\rho^{-1} = 0 \tag{15}$$

Comparing this with the transversality condition of the previous section, (9), we see that an additional term has been added which reflects the impact of the other firm's strategy on the first firm's per period payoff at the end of the game.

Firm 2 has a transversality condition identical to (15) except that the superscripts are transposed,

$$c(\dot{s}_T^2)^2 - \pi\dot{s}_T^1\rho^{-1} = 0 \tag{16}$$

A pair of Nash equilibrium strategies must, therefore, fulfill both (15) and (16). The set of all pure Nash equilibria consists of that subset of the set of strategies that fulfill (15) and (16) which in addition fulfill (5) for each firm and for which $s_t^i \geq 0$, $\forall\, t \geq 0$, $i = 1,2$.

From the fact that Nash equilibria must fulfill (15) and (16) we obtain immediately the following results.

<u>Proposition 1</u>. There exist at most two Nash equilibria to the above game both of which are symmetric.

<u>Proof</u>: (i) By inspection, $\dot{s}_T^1 = \dot{s}_T^2 = 0$ solves (15) and (16).

(ii) Assume $s_T^1, s_T^2 > 0$. Solving (15) for $\dot{s}_T^2$ and substituting into (16) gives

$$c^3(\dot{s}_T^1)^3\rho^3 - \pi^3 = 0$$

which has a unique root $\dot{s}_T^1 = \pi/c\rho$. Substituting this into (15) or (16) gives $\dot{s}_T^1 = \dot{s}_T^2$.

(iii) For each value of s_T^i, $i = 1,2$, there is a unique path $\{s_t^i\}$, $t \geq 0$, which satisfies the Euler equation and $s_0^i = 0$.

Q.E.D.

<u>Proposition 2</u>. All the Nash equilibria result in the firms sharing the market equally, $s_T^1 = s_T^2 = Q/2$.

<u>Proof</u>: Follows immediately from Proposition 1. Q.E.D.

In order to complete our characterization of the set of Nash equilibria we must check that the two sales trajectories which fulfill Proposition 2 do not involve violating $s_t \geq 0$, $t \geq 0$. This is done below.

<u>Proposition 3</u>. There exist two Nash equilibria to the above game both of which are characterized by the following strategies for $i = 1,2$ and $t \geq 0$:

(i) $$s_t^i = \frac{\pi}{2c\rho^2} \cdot \frac{(1 - e^{\rho t})}{e^{\rho T}} + \frac{\pi t}{2c\rho} \qquad (\dot{s}_T^i = 0)$$

(ii) $$s_t^i = \frac{\pi}{2c\rho^2} \cdot \frac{(e^{\rho t} - 1)}{e^{\rho T}} + \frac{\pi t}{2c\rho} \qquad (s_T^i = \pi/c\rho)$$

and where T in each case is given by $s_T^1 + s_T^2 = Q$.

<u>Proof</u>: (i) In the model of the previous section set $q = Q/2$. The solution was shown to be (11), which is just (i) above.

(ii) The Euler equation (5) must hold and so (6) and (7) must also hold. Evaluating (6) at T and using $\dot{s}_T^i = \pi/c\rho$ gives

$$k_1 = \frac{\pi}{2c\rho} e^{-\rho T} > 0$$

thus $\dot{s}_t^i > 0$, $t \geq 0$, which, given $s_0^i = 0$, gives $s_t^i \geq 0$, $t \geq 0$. Substituting for k_1 in (7) and evaluating (7) at $t = 0$ gives

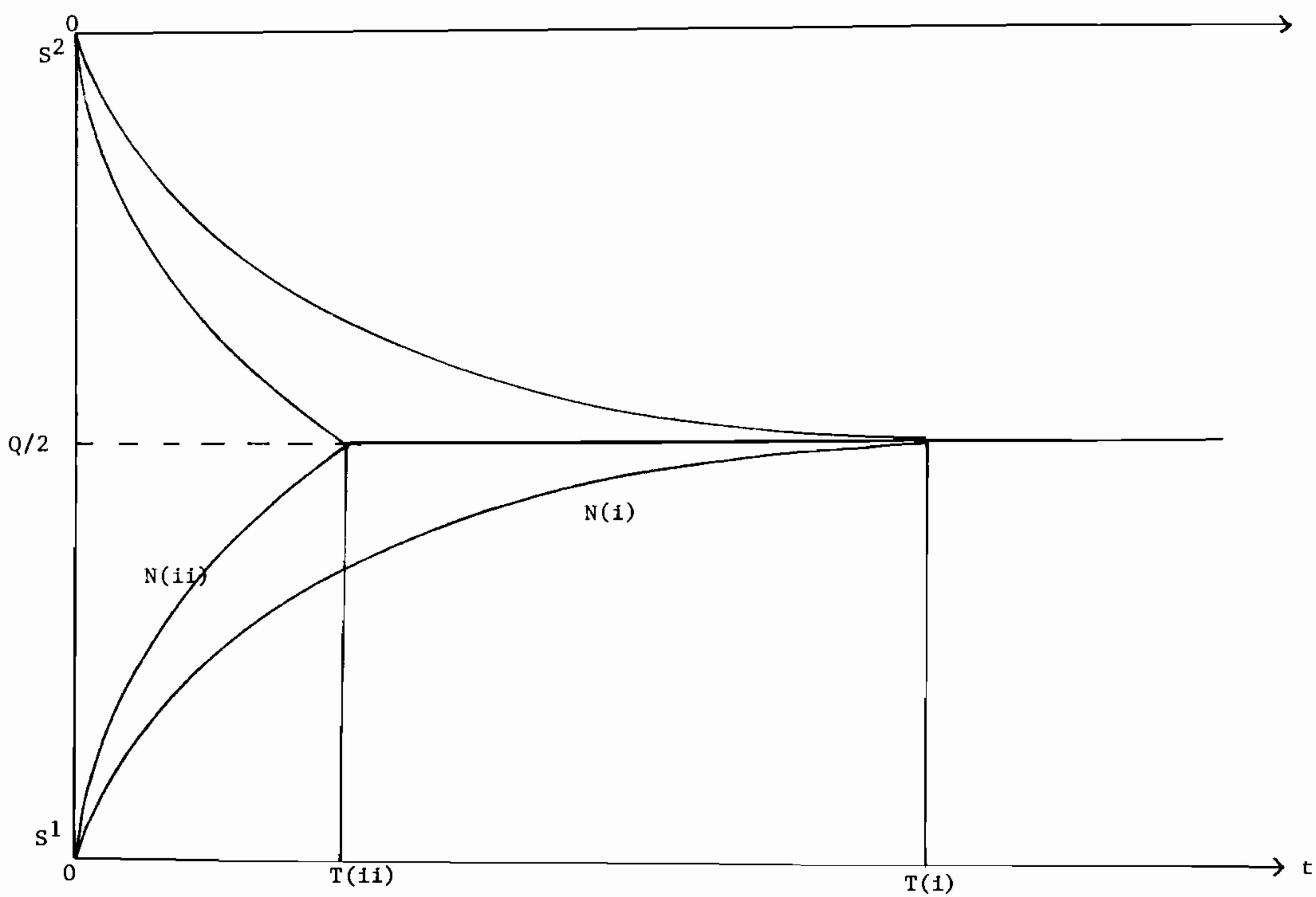
$$k_2 = - \frac{\pi}{2c\rho^2} e^{-T}$$

Substituting this into (7) gives (ii) above. Q.E.D.

The possible Nash equilibria are shown in Figure 2. It is easy to show that the Nash equilibrium in which $\dot{s}_T^i > 0$, which we denote by N(ii), results in the quota being reached, and so the game ending, more quickly than in the Nash equilibrium corresponding to $\dot{s}_T = 0$ (denoted N(i)). Thus in N(ii), the sales of the foreign firms grow more rapidly, at least initially, than in N(i) and they are higher along N(ii) at each point in time prior to T(i).

Before discussing the policy implications of these results in Section 4 we first turn to the question of generalizing these results beyond the specific case used in this section.

Figure 2



3. Generalizations

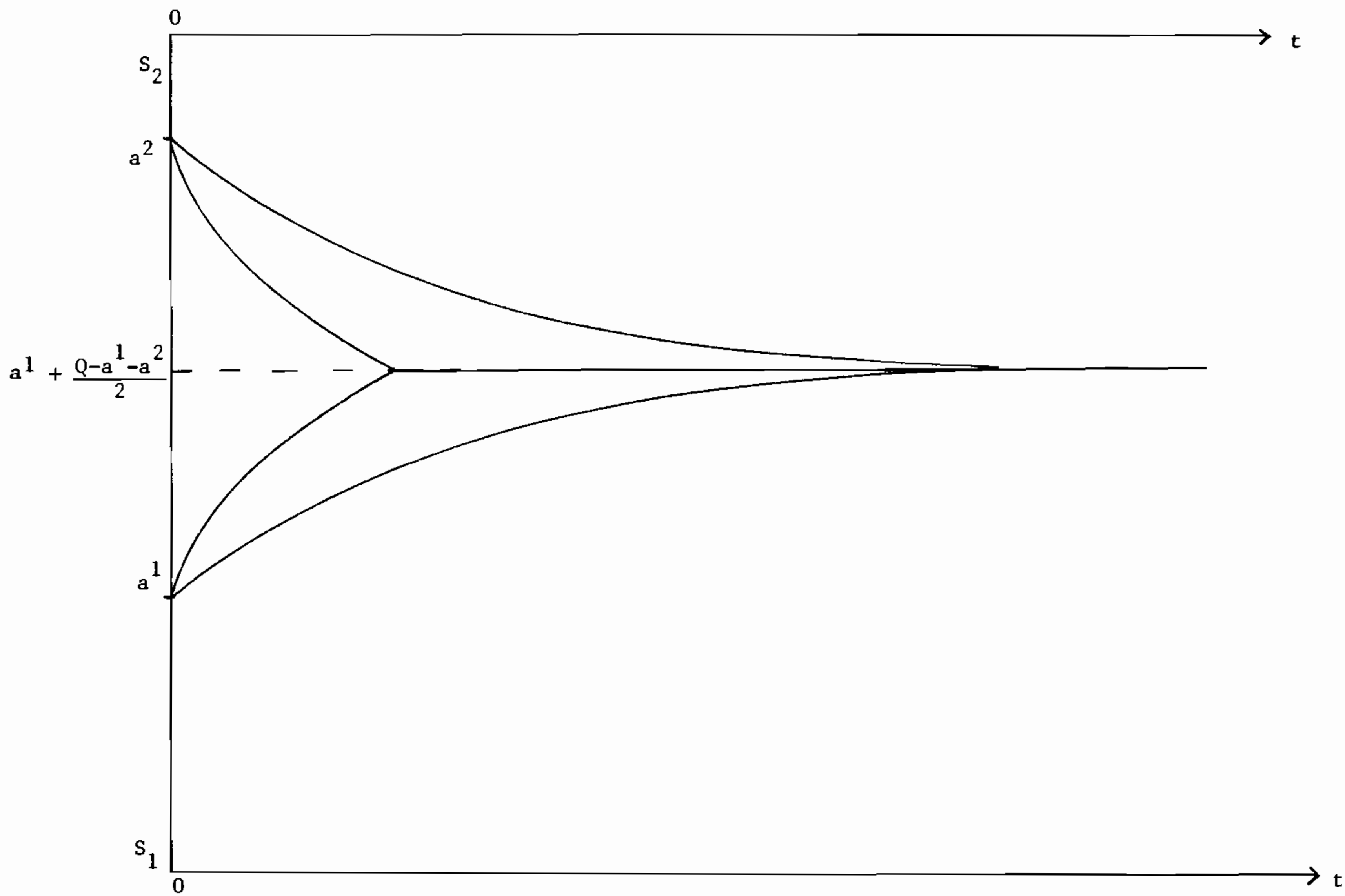
In this section we will ask whether the results of the previous section can be extended in three directions, namely, to situations in which the firms start with nonzero sales at the time the quota is imposed, to where the number of firms exceeds 2, and to the case of the more general cost function $g(s_t)$. The first generalization is straightforward. Let the firms have sales a^1, $a^2 > 0$ at the time the quota is announced and let $Q > a^1 + a^2$. Transform this game by defining sales to be $\hat{s}_t^i = s_t^i - a^i$, $i = 1,2$ and the quota to be $\hat{Q} = Q - a^1 - a^2$. This transformed game is simply the game of the previous section and so all the preceding propositions carry over for the transformed game. In terms of the actual game this means that there exist two Nash equilibria. These equilibria are not, however, symmetric. If $\{s_t^1\}^N$, $t \geq 0$, is firm 1's Nash equilibrium strategy, then firm 2's will be $\{s_t^1 + a^2 - a^1\}$, $t \geq 0$. Thus the symmetric equilibria hold only for the game in which the firms play for the excess of the quota over their joint initial sales. This is shown in Figure 3.

All of the results of the previous section carry over to the case of many firms in a direct fashion. Increasing the number of firms leaves each firm's Euler equation unaltered but changes the transversality condition to, for $i = 1,\ldots,n+1$

$$c(s_T^i)^2 - \pi\rho^{-1} \sum_{\substack{j=1 \\ j\neq i}}^{n+1} \dot{s}_T^j = 0 \tag{17}$$

It is easy to show that the only solutions of (17) are of the form $s_T^i = s_T^j \;\forall\; i,j$ and the $s_T^i = s_T^j = 0$ is a solution independent of n. There exists one other solution given by

Figure 3



$$c(\dot{s}_T)^2 - \pi\rho^{-1}n\dot{s}_T = 0$$

$$\dot{s}_T = \frac{\pi n}{c\rho} \quad (18)$$

The interesting feature of the extension to $n + 1 > 2$ firms is how the two Nash equilibria vary with n. This behavior is summarized in the following proposition.

Proposition 4. Consider two games being played that are the same as the game of the previous section except that in game one $n_1 + 1 > 2$ firms are playing and in game two $n_2 + 1$, $n_2 > n_1$, firms are playing. Then

(i) For period t such that t occurs before either game ends, then s_t^i of game two is less than s_t^i for game one if we compare Nash equilibria with $\dot{s}_T^i = 0$, $\forall$ i.

(ii) For period t such that t occurs before either game ends, then s_t^i of game two is greater than s_t^i for game one if we compare Nash equilibria with $\dot{s}_T^i > 0$, $\forall$ i.

Proof: (i) In the case where $\dot{s}_T^i = 0$, $\forall$ i, the solutions for k_1 and k_2 are independent of n and thus the trajectory of sales remains of the same form as Proposition 3(i). However, the terminal value of sales $s_T^i = Q(1 + n)^{-1}$ is declining in n and so T also declines with n. From Proposition 3(i) this implies that s_t^i, $t < T$ is declining in n.

(ii) Following the same procedure as in the last section to solve for $\{s_t^i\}$ yields

$$s_t^i = \frac{\pi(n - 1/2)}{2\rho^2 c} \cdot \frac{e^{\rho t} - 1}{e^{\rho T}} + \frac{\pi t}{2c\rho}$$

T can also be shown to be declining in n and so s_t^i, $t < T$, is increasing in n.

Q.E.D.

In the Nash equilibrium with $\dot{s}_T^i = 0$ at T the rate at which the industry is reaching its quota, zero, is independent of n. This means that as n increases a firm experiences no change in its marginal incentives at T and so consequently the trajectory traced out by the Euler equation as we move back in time remains the same. Raising n does reduce s_T^i and so the bottom or earlier part of the trajectory is cut off as n is raised. Because the trajectory is concave this means that along the new, shortened trajectory sales will increase more slowly.

In contrast, when $s_T^i > 0$ the rate at which the industry approaches its quota at T increases proportionately with n. This means that at T the marginal incentive to a firm to accelerate its rate of growth of sales increases. This acceleration at T feeds back through the Euler equation with the result that at all points in time the firm tries to expand sales faster.

The generalization to the more general cost function is more complicated. In this case the Euler equation becomes

$$\pi - \rho g'(\dot{s}_t) + \ddot{s}_t g''(\dot{s}_t) = 0$$

for which a closed form solution has not been found. In view of this we will assume that a solution to it exists given $s_0 = 0$ and $\dot{s}_T$ solves the appropriate transversality condition. This latter condition is, i = 1,2; j ≠ i,

$$g(\dot{s}_T^i) + \dot{s}_T^i g'(\dot{s}_T^i) - \pi \dot{s}_T^j \rho^{-1} = 0 \tag{18}$$

Assuming that a unique solution to the Euler equation exists then the following result generalizes the propositions of the previous section.

Proposition 5. (i) All Nash equilibria to the game are symmetric.

(ii) There exist two such equilibria.

Proof: (i) Proof by contradiction is immediate from system (17).

(ii) Given (i) we can restrict ourselves to cases where $\dot{s}_T^i = \dot{s}_T^j$. In view of this rewrite (17) as

$$g(\dot{s}_T) + \dot{s}_T g'(\dot{s}_T) = \pi \dot{s}_T \rho^{-1} \tag{19}$$

Because $g(0) = g'(0) = 0$, $\dot{s}_T^i = \dot{s}_T^j$ and their corresponding trajectories constitute a Nash equilibrium. Both sides of (18) are continuous in $\dot{s}_T$. In the neighborhood of $\dot{s}_T = 0$ the derivative of the left hand side of (18) is less than that of the right hand side and so close to $\dot{s}_T = 0$, $g(\dot{s}_T) + \dot{s}_T g'(\dot{s}_T) < \pi \dot{s}_T \rho^{-1}$. However, in the limit as $\dot{s}_T \to \infty$, because $g(\cdot)$ is strictly convex, $g(\dot{s}_T) + \dot{s}_T g'(\dot{s}_T) > \pi \dot{s}_T \rho^{-1}$ and so at least one additional Nash equilibrium exists. To prove that only one such additional Nash equilibrium exists consider the slope of the two sides of (18) at a point $\dot{s}_T > 0$. These slopes are $2g'(\dot{s}_T) + \dot{s}_T g''(\dot{s}_T)$ for the LHS and $g'(\dot{s}_T) + g(\dot{s}_T)/\dot{s}_T$ for the RHS. Thus the LHS can only cut the RHS from below. Q.E.D.

It seems, then, that provided unique solutions to the Euler equation exist, the existence of two symmetric Nash equilibria does generalize from our quadratic cost example. In view of this, it is reasonable to consider the policy implications of these results.

4. Policy Implications and Conclusion

Quotas are, of course, imposed to protect domestic industry. However, policymakers often wish to allow some foreign competition either because of foreign political pressure, pressure from domestic consumers or because of the efficiency gains to be had from additional competition. For these reasons quotas are not set equal to zero and are sometimes set greater than existing foreign sales in the domestic market. Notice that whenever a quota is set in terms of a percentage of domestic sales, provided the domestic market is growing, this means that there is room for import growth and the games of the previous sections will be played. The results of the previous sections have considerable importance for a policymaker planning to impose quotas.

The most fundamental point, and one that does not depend on whether the quota is industry-wide or firm-specific, is that the excess of the quota over current sales is inversely related to the rate of growth of foreign sales. This means that the policymaker must trade off the ultimate size of foreign penetration of the domestic market against the speed with which foreign firms will raise their sales, e.g., one could make a magnanimous bow towards free trade and choose a large quota and in the short run also reap the benefits of a relatively slow growth of imports but only at the cost of a higher long-run share of imports in the domestic market.

Sections 2 and 3 show that if the policymaker uses an industry-wide quota he or she will be faced with an additional prediction problem because the game that the foreign firms are forced into has two Nash equilibria. It is easy to check that if the foreign firms can collude they will choose the "slow growth" equilibrium ($\dot{s}_T = 0$). Thus, if collusion is present, the policymaker can predict which Nash will result. Indeed, if the policymaker has a preference for the slow growth equilibrium one way of choosing it is to encourage collusion among the foreign firms. A second issue that the policymaker must deal with is the number of foreign firms that will enter the market. The greater the number of firms, the faster imports will hit the quota under either equilibrium, but in the case of the "fast growth" equilibrium ($\dot{s}_T > 0$), raising the number of firms very markedly accelerates the growth of sales. This implies that a policymaker may well wish to restrict the number of firms for any given industry-wide quota in order to fine tune the rate of growth of imports.

Finally, there is the choice between using firm-specific and industry-wide quotas. For any given number of foreign firms, firm-specific quotas will minimize the rate of growth of imports, though if the quotas are symmetric, in the sense that the gap between each firm's quota and its initial level of sales is the same, then the slow growth Nash equilibrium with the same aggregate quota will give the same result. Of course, the firm-specific quota does give the policymaker the additional ability to choose a distribution of firm-specific sales which is not possible with the industry-wide quota.

One caveat is in order about policy and that concerns dynamic inconsistency. It has been assumed that the policymaker commits him or herself to a quota. To the extent that this commitment cannot be made binding then dynamic consistency problems will arise. As an example, the policymaker could encourage low rates of growth of imports by announcing a high quota and then at a later date cut the quota to avoid extensive market penetration by foreign firms. Alternatively, if the foreign firms believe that every time the quota is reached political pressure can be applied that will result in the quota being raised, as it indeed has been in the Canadian banking case, then the quota will cease to have an impact on sales growth rates.

Footnotes

1. An example of this is the Canadian Bank Act of 1980 that restricted foreign banks in Canada from taking more than 8% (now raised to 16%) of the market for bank loans. In 1980 foreign banks' loans were under 8% of the Canadian market.

2. The Legendre condition for a maximum is met.

3. For T in $[0,\infty)$ the right hand side of (12) is a continuous, monotonically increasing concave function of T that is positive at $T = 0$ and goes to a finite limit.

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APPENDIX: NOTES FOR THE REFEREE

Section 1.

Define $F(s,t) = [\pi s_t - c(\dot{s}_t)^2]e^{-\rho t}$

and $G(s,t) = \pi q \int_T^\infty e^{-\rho t}dt$

then, $F_s = \pi e^{-\rho t}$, $F_{\dot{s}} = -2c\dot{s}_t e^{-\rho t}$, $F_{\dot{s}s} = 0$

$F_{\dot{s}\dot{s}} = -2ce^{-\rho t}$, $F_{\dot{s}t} = \rho 2c\dot{s}_t e^{-\rho t}$, $G_T = -\pi q e^{-\rho T}$

Legendre: $F_{\dot{s}\dot{s}} < 0$, $-2ce^{-\rho t} < 0$, $\forall\ t$

Euler: $F_s - F_{\dot{s}t} - F_{\dot{s}s}\dot{s} - F_{\dot{s}\dot{s}}\ddot{s} = 0$, $\forall\ t$

$\pi e^{-\rho t} - \rho 2c\dot{s}_t e^{-\rho t} + 2ce^{-\rho t}\ddot{s}_t = 0$, $\forall\ t$

which gives (5).

Transversality: $F - \dot{s}F_{\dot{s}} + G_t = 0$ at T

$[\pi s_T - c(\dot{s}_T)^2]e^{-\rho T} + \dot{s}_T^2 2ce^{-\rho T} - \pi q e^{-\rho T} = 0$

which, setting $s_T = q$ and multiplying by $e^{\rho T}$, gives (9).

(12): The derivative of the RHS is $e^{-\rho T} > 0$ and the second derivative is negative. The limit of the RHS as $T \to \infty$ is

$0 < 2qc\rho/\pi + \rho^{-1} < \infty$.

Section 2

(14): $G(s^1,T)$ now becomes

$$G(s^1,T) = \pi[Q - s_T^2]\int_T^\infty e^{-\rho t}dt$$

and so G_T becomes: $-\pi[Q - s_T^2]e^{-\rho T} - \pi\dot{s}_T^2\int_T^\infty e^{-\rho t}dt$

Multiplying through by $e^{\rho T}$ and evaluating the integral gives (14).

Section 3

(17): The proof that only symmetric solutions to the system (17) exist is as follows. Assume the contrary. Then there exist at least two firms, say a and b, such that $\dot{s}_T^a \neq \dot{s}_T^b$. Without loss of generality assume $\dot{s}_T^a > \dot{s}_T^b$. Define

$$D = \sum_{\substack{j=1 \\ j\neq a,b}}^{n+1} s_T^j$$

We can write the transversality conditions of firms a and b as

$$c(\dot{s}_T^a)^2 - \pi\rho^{-1}\dot{s}_T^b - \pi\rho^{-1}D = 0$$

$$c(\dot{s}_T^b)^2 - \pi\rho^{-1}\dot{s}_T^a - \pi\rho^{-1}D = 0$$

from which follows $\dot{s}_T^a = \dot{s}_T^b$, a contradiction.

Proposition 5(i): The contradiction argument follows the lines of that above.