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ABSTRACT

This paper compares monopsonistic and competitive search equilibria. While the competitive equilibrium is efficient, the monopsonistic equilibrium is not--there is too much search, and the employment rate is too low.

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Introduction

This paper compares monopsonistic and competitive search equilibria. While the competitive equilibrium is efficient, the monopsonistic equilibrium is not--there is too much search, and the employment rate is too low, as Phelps (1972) claimed.

The meaning of competitive search equilibrium is unclear. We associate the term with an assumption made by Azariadis (1975) and others: firms can make credible commitments even though they might prefer to abrogate these ex post. A catastrophic loss in reputation precludes firms from exploiting their ex post monopsony power. The same ability to pre-commit underlies Arrow-Debreu contingent claims markets.

This link between the ability to commit and the absence of ex post monopsony power is the key to our distinction between competitive and monopsonistic equilibrium. The former presumes a costless commitment technology, the latter rules out commitment entirely because of enforceability or communication problems. The relative empirical relevance of the two kinds of equilibria therefore hinges on the ease of commitment and communication in the labor market.

The monopsonistic equilibrium we consider is one in which firms make take-it-or-leave-it wage offers to job applicants. The firm's monopsony power is a consequence of the implicit cost of search--a worker who rejects an offer must wait a period to generate another one. However, the firm does not know the applicant's reservation wage, and its monopsony power is not complete.[1]

This wage-setting mechanism results in pairwise inefficient bargains, but this does not necessarily imply inefficiency of the equilibrium allocation. If all firms possess monopsony power, then it is not obvious that an applicant's incentive to reject one monopsonist's offer in favor of sampling from another monopsonist is excessive. When private and social payoffs do not coincide, a privately pairwise inefficient mechanism could well be socially optimal. This is merely another way of saying that efficiency questions must be posed in a general equilibrium setting.

2. The Model

A firm consists of a single machine, and a machine, when operating, employs one worker and produces an output of z. Whether its machine is operating or not, the firm incurs a cost k per period. Like all flow values in the model, this fixed cost is accounted in end-of-period terms. Workers and firms are of measure zero. The measure of all workers taken together is normalized at one, while the measure of all firms is determined endogenously by a free entry condition. Workers and firms live forever and discount the future at the rate δ.

An unemployed worker can sample at most one vacancy per period, and each vacant firm can contact at most one unemployed worker per period. Contact requires the payment of a fee of ϕ by the firm to the worker (ϕ may be negative), and we assume that ϕ is competitively determined.[2] This means that the number of vacancies can never exceed the number of unemployed workers in equilibrium since if such were the case, a firm without a worker could attract a job applicant with probability one via a slight increase in its contact fee. At the same time, free entry by firms ensures that the number of unemployed can never exceed the number of vacancies. In equilibrium the value of entry must be non-negative, so an excess of unemployed relative to vacancies would mean a strictly positive return to entry, as firms could get workers at less favorable terms. Thus, the assumption of a competitively determined contact fee together with free entry ensure an equal number of unemployed workers and vacancies in equilibrium, and free entry determines the measure of all firms taken together at one. Since contacts are rationed by price, ϕ, no congestion externalities of search arise.

Once the worker receives ϕ, he inspects the job. If the match is formed, he enjoys a utility of $w + \theta$ per period, where w is the wage and θ the non-pecuniary component associated with the job. We assume that θ is a match-specific random variable with mean zero and a symmetric distribution $G(\theta)$. The density function is $g(\theta)$, and we assume $g(\theta) > 0$ for all θ. In the equilibria we consider all firms will offer the same wage so search will be motivated solely by the desire for a better θ. Finally, any match that is formed ends with an exogenously given probability of s at the end of each period--a device that yields the existence of unemployment in the steady state.

The unemployed worker (employed workers cannot search) accepts a job offer iff the value of doing so exceeds the value of remaining unemployed. Let U be the value of unemployment. If all firms pay the same w and ϕ, then

(1) $$U = \frac{\phi}{1 + \delta} + \int \max \left[\frac{w + \theta + sU}{\delta + s}, \frac{U}{1 + \delta} \right] dG(\theta).$$

The unemployed worker will receive ϕ to contact a vacant firm. If he accepts that firm's offer, then he receives a value of $\frac{w + \theta + sU}{\delta + s}$ (the computation of the acceptance value is illustrated in Figure 1); whereas if he rejects the offer, he receives a value of $U/(1 + \delta)$, i.e., the value of unemployment deferred one period. An unemployed worker thus accepts a job offering $w + \theta$ iff $\frac{w + \theta + sU}{\delta + s} > \frac{U}{1 + \delta}$. Letting $\alpha = \frac{1 - s}{1 + \delta}$, the probability that a randomly drawn θ will lead to a formed match is

(2) $$p = 1 - G(\alpha\delta U - w).$$

The optimal sequential search strategy for an unemployed worker can equivalently be expressed in terms of a per-period reservation utility, r. A worker accepts a job offer iff $w + \theta > r$, where r is such that the value of accepting a job offering r equals the value of rejecting that job. The value of rejecting a job offering r is $U/(1 + \delta)$, whereas the value of accepting that job is $(r + sU)/(\delta + s)$. Equating rejection and acceptance values gives $r = \alpha\delta U$. This implies

(3) $$p = 1 - G(r - w)$$

and, substituting into (1),

(4) $$r = \alpha\left[\int_{r-w}^{\infty} (w + \theta)dG(\theta) + rG(r - w) + (1 - \alpha)\phi\right].$$

[Figure 1 goes here]

3. Competitive Contractual Equilibrium

We begin with a competitive contractual approach to wage determination. Firms are assumed to bid for job applicants by offering ϕ and w as a pair. After receiving ϕ, the worker inspects the firm and observes θ. Once the worker is at the firm's doorstep, the firm would prefer to take advantage of his partial ex post immobility by reneging on its wage offer; however, we assume that it is precluded from doing so by reputational considerations.

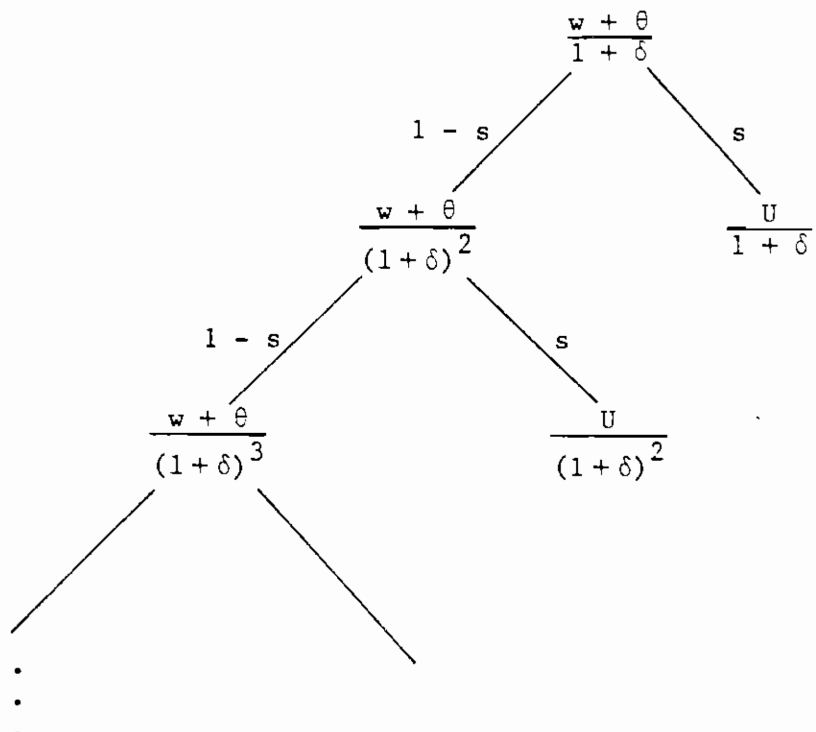

Figure 1. Value of accepting a job offering w + θ per period.

This is a contractual approach to wage determination (in the spirit of Azariadis [1975]) in that firms can pre-commit to wage offers they would prefer to abrogate ex post. It is a competitive approach in that unemployed workers can costlessly find the (ϕ,w) offer they prefer so that the (ϕ,w) pair plays the role of a market-clearing price.

A firm with a vacancy chooses (ϕ,w) to maximize the value of its empty slot. Treating U as given, the probability that a worker will accept w once he has observed θ is given by (2). (The contact fee has already changed hands by the time θ is observed and hence does not influence the acceptance probability.) Let $\hat{\pi}$ denote the present value of the slot in the future should it become vacant by termination of the match or due to the applicant's rejection of the offer. Then the value of the vacant slot as a function of the offered (ϕ,w) contract is

$$(5) \qquad \pi(\phi,w;U,\hat{\pi}) = \frac{-\phi}{1+\delta} + [1 - G(\alpha\delta U - w)]\,\frac{z-w-k+s\hat{\pi}}{\delta+s} + G(\alpha\delta U - w)\,\frac{\hat{\pi}-k}{1+\delta}$$

since the value to the firm should the worker accept the offer is $\frac{z-w-k+s\hat{\pi}}{\delta+s}$, while the value should the worker reject the offer is $\frac{\hat{\pi}-k}{1+\delta}$.

The firm's choice of (ϕ,w) is made subject to the constraint that the value to a worker of applying to that firm be at least as great as U, the present value of applying elsewhere. Still taking U as given, the value of a (ϕ,w) contract to a job applicant is

$$(6) \qquad Q(\phi,w;U) = \frac{\phi}{1+\delta} + \int \max\left[\frac{w+\theta+sU}{\delta+s}, \frac{U}{1+\delta}\right]dG(\theta).$$

Thus, the firm chooses (ϕ,w) to maximize $\pi(\phi,w;U,\hat{\pi})$ subject to

$$(7) \qquad Q(\phi,w;U) \geq U.$$

If (7) did not hold, the firm could not get any workers.

Definition: A competitive contractual equilibrium consists of four scalars ϕ, w, U and $\hat{\pi}$ such that

(i) $\pi(\phi,w;U,\hat{\pi})$ is at a maximum over (ϕ,w) subject to the constraint (7).

(ii) $\pi(\phi,w;U,\hat{\pi}) = 0$,

(iii) $\hat{\pi} = 0$, and

(iv) $Q(\phi,w;U) = U \geq 0$.

We remark that (ii) and (iii) follow from a free entry, and (iv) ensures that workers will participate in the labor market.

Theorem 1: If $z \geq k$, there exists a unique competitive contractual equilibrium in which

(8) $$w = z - k\alpha$$

(9) $$\phi = - k.$$

Proof: Define the Lagrangean

$$L = \pi(\phi,w;U,\hat{\pi}) + \lambda[Q(\phi,w;U) - U]$$

The necessary conditions for a maximum are

(10) $$Q(\phi,w;U) = U$$

and that the following two conditions are met:

(11) $$\frac{\partial L}{\partial \phi} = \frac{-1}{1+\delta} + \frac{\lambda}{1+\delta} = 0$$

(12) $$\frac{\partial L}{\partial w} = g(\cdot)\left\{\frac{z-w-k+s\hat{\pi}}{\delta+s} - \frac{\hat{\pi}-k}{1+\delta}\right\} - \frac{(1-G(\cdot))}{\delta+s} + \frac{\lambda(1-G(\cdot))}{\delta+s} = 0.$$

Substituting zero for $\hat{\pi}$ in (12), using (11) to eliminate λ from (12), and noting that $g(\cdot) > 0$ everywhere, leads to the conclusion that (8) holds. We observe in (5) that if (8) and (9) hold, then (ii) holds if (iii) holds. It remains to show that (iv) also holds. Substituting from (8) and (9) into (6) we see that condition (iv) is a contraction and that a unique U therefore exists. To ensure that U is non-negative, substitution from (8) and (9) into (1) shows that we must prove that $-\frac{k}{1+\delta} + \int \max[\frac{z+\theta-k\alpha}{\delta+s}, \frac{U}{1+\delta}]dG(\cdot) \geq 0$. Using Jensen's Inequality together with our assumption that $E(\theta) = 0$, a sufficient condition for $U \geq 0$ is $\max[\frac{z-k\alpha}{\delta+s}, \frac{U}{1+\delta}] \geq \frac{k}{1+\delta}$. This inequality is in turn implied by $\frac{z-k\alpha}{\delta+s} \geq \frac{k}{1+\delta}$, i.e., by $z \geq k$. Q.E.D.

An efficient equilibrium is one in which $U = Q(\phi,w)$ is maximum subject to the free-entry constraint that $\pi(\phi,w) = 0$. That is, the incentives to search given by ϕ and w are such that each worker's expected lifetime utility is maximum given the free-entry constraint. Since the Lagrangean $L^* = Q(\phi,w) + \zeta\pi(\phi,w)$ is also maximized by $\phi = -k$ and $w = z - k\alpha$ the competitive contractual equilibrium is efficient.[3]

4. Monopsonistic Equilibrium

This section characterizes the equilibrium in which firms exploit their ex post local monopsony power. The unemployed worker receives the competitively determined ϕ for contacting the firm. Once he has observed θ, the firm makes him a take-it-or-leave-it wage offer of w. The firm makes this offer in ignorance of θ, but it possesses monopsony power insofar as the worker is willing to accept a somewhat inferior wage in order to avoid missing a period's worth of utility. Once ϕ has changed hands, the firm's ex post problem is to

(13) $$\max_{w}\{\pi(\phi,w;U,\hat{\pi}) + \frac{\phi}{1+\delta}\},$$

which is the same as maximizing $\pi(\phi,w;U,\hat{\pi})$ over w. This maximization is unconstrained--since the worker is already there, the ex ante utility constraint (7) does not apply.

Since firms are of measure zero, each faces the same U; thus each firm chooses the wage to maximize the same expression in (13). If this expression is strictly concave in w, each firm will choose the same wage, i.e., no asymmetric equilibria (equilibria involving wage dispersion) can exist. The following definition of equilibrium therefore includes only symmetric equilibria.

Definition: A monopsonistic equilibrium consists of four scalars ϕ, w, U and $\hat{\pi}$ such that

M(i) $\pi(\phi,w;U,\hat{\pi})$ is at a maximum over w

M(ii) $\pi(\phi,w;U,\hat{\pi}) = 0$

M(iii) $\hat{\pi} = 0$, and

M(iv) $Q(\phi,w;U) = U \geq 0$.

Let $f(\theta) = [1 - G(\theta)]/g(\theta)$ be the inverse hazard. We then have

Theorem 2. If $f' < 1$ everywhere, and if $z \geq (2 - \alpha)k$, a unique monopsonistic equilibrium exists, with

(14) $$w = z - k\alpha - f(\alpha\delta U - w),$$

and

(15) $$\phi = \frac{[1 - G]f}{1 - \alpha} - k.$$

Proof: If $f' < 1$ everywhere, then (14) is equivalent to M(i) when M(iii) holds too, as can be seen by differentiating in (13) and evaluating at $\hat{\pi} = 0$. If, in addition, M(ii) holds, then (15) follows. It remains for us to prove M(iv). From Section 2, $r = \alpha\delta U$, so M(iv) follows if one can show the existence of a unique $r \geq 0$. Using (15) to eliminate ϕ from (4) yields

$$(16) \qquad r = \alpha\{\int_{r-w}^{\infty} (w + \theta)dG(\theta) + rG(r - w) - (1 - \alpha)k + [1 - G(r - w)]f(r - w)\}$$

Let $\gamma = r - w$, and subtract w from both sides of the above equation to generate two equations in γ and w:

$$\gamma = -(1 - \alpha)w + \alpha\{\int_{\gamma}^{\infty} \theta dG(\theta) + \gamma G(\gamma) - (1 - \alpha)k + [1 - G(\gamma)]f(\gamma)\}$$

$$w = z - f(\gamma) - k\alpha.$$

Eliminating w,

$$(17) \qquad \gamma = (1 - \alpha)[f(\gamma) + k\alpha - z] + \alpha\{\int_{\gamma}^{\infty} \theta dG(\theta) + \gamma G(\gamma) - (1 - \alpha)k + [1 - G(\gamma)]f(\gamma)\} \equiv \psi(\gamma).$$

Next,

$$\psi'(\gamma) = (1 - \alpha)f'(\gamma) + \alpha[G(\gamma) - g(\gamma)f(\gamma) + f'(\gamma)[1 - G(\gamma)]].$$

Using $f(\gamma)g(\gamma) = 1 - G(\gamma)$ and $f'(\gamma) < 1$, $\psi'(\gamma) < 1 - \alpha[1 - G(\gamma)] < 1$; i.e., there is a unique γ solving (17) and hence a unique r solving (16). To show that $r \geq 0$, we use M(ii) along with the observation that a wage offer of $w = r$ is feasible, though in general not optimal, so that $\pi(\phi,r;r/\alpha\delta,0) < 0$ because M(ii) and M(iii) hold. The symmetry of G around zero implies that

When w=r, $1-G(r-w) = 1/2$, so that

$$0 > \pi(\phi, r; r/\alpha\delta, 0) = \frac{-\phi - k}{1 + \delta} + \frac{1}{2(\delta + s)} [z - r - k\alpha].$$

Suppose $r < 0$. This leads to a contradiction because $0 > \frac{-\phi - k}{1 + \delta} +$ $+ \frac{1}{2(\delta + s)} [z - k\alpha]$, i.e., $\phi > -k + (z - k\alpha)/2(1 - \alpha)$. But $z \geq (2 - \alpha)k$ then implies $\phi > 0$, contradicting $r < 0$ since with $\phi > 0$ the worker can guarantee himself at least ϕ/δ (i.e., $U \geq \phi/\delta$) by collecting the contact fees but refusing all jobs. Q.E.D.[4]

Having proven the existence of a unique monopsony equilibrium, we now formally establish its inefficiency.

Proposition: The monopsonistic equilibrium involves excessive search relative to the efficient level implied by the competitive equilibrium.

Proof: The proof consists of showing that the acceptance probability in the monopsony equilibrium is less than the corresponding acceptance probability in the efficient equilibrium. Let r^* be the reservation utility in the efficient equilibrium, and let p^* be the efficient acceptance probability. The corresponding unstarred quantities refer to the reservation utility and the acceptance probability in the monopsony equilibrium. Using equation (3) we have

$$p^* = 1 - G[r^* - z + k\alpha],$$

where r^* is given by (4) with $w = z - k\alpha$ and $\phi = k$, and

$$p = 1 - G[r - z + f(r - w) + k\alpha],$$

where r is given by (4) with $w = z - f(r - w) - k\alpha$ and $\phi = -k + \frac{(1 - G(r - w))f(r - w)}{1 - \alpha}$.

Letting $\gamma^* = r^* - z + k\alpha$ and $\gamma = r - z + f(\gamma) + k\alpha$, we want to show $\gamma > \gamma^*$. From (17) we have

$$\gamma = (1-\alpha)[f(\gamma) + k\alpha - z] + \alpha\{\int_{\gamma}^{\infty} \theta dG(\theta) + \gamma G(\gamma) - (1-\alpha)k + [1 - G(\gamma)]f(\lambda)\}$$

The corresponding expression for γ^* is

$$\gamma^* = (1-\alpha)[k\alpha - z] + \alpha\{\int_{\gamma^*}^{\infty} \theta dG(\theta) + \gamma^* G(\gamma^*) - (1-\alpha)k\}.$$

Thus,

$$\gamma - \gamma^* = f(\gamma)[1 - \alpha G(\gamma)] + \alpha[\int_{\gamma}^{\gamma^*} \theta dG(\theta) + \gamma G(\gamma) - \gamma^* G(\gamma^*)]$$

$$= f(\gamma)[1 - \alpha G(\gamma)] - \alpha\int_{\gamma}^{\gamma^*} G(\theta) d\theta.$$

Now suppose $\gamma \leq \gamma^*$. Then

$$\gamma - \gamma^* > f(\gamma)[1 - \alpha G(\gamma)] - \alpha G(\gamma)(\gamma - \gamma^*), \text{ i.e.,}$$

$$\gamma - \gamma^* > f(\gamma).$$

But $f(\gamma) > 0$, implying a contradiction, i.e., $\gamma > \gamma^*$. Q.E.D.

Footnotes

1. When the firm is imperfectly informed about the applicant's reservation wage, the best possible bargaining mechanism from the firm's point of view is one in which it makes a take-it-or-leave-it wage offer. In particular, this mechanism is at least as good as plausible alternatives in which the firm makes a sequence of revocable offers in an attempt to elicit the applicant's reservation wage. See Samuelson [1984] for a proof. This result requires risk neutrality on the part of both parties to the bargain and that there be no costs to the bargaining process itself.

2. The empirical counterpart of ϕ is the fee collected by employment agencies from employers for assisting in the search process. More loosely, ϕ might be interpreted as a first-period probationary wage.

3. Alternatively, one can establish the efficiency of this equilibrium by arguing that the efficient level of search occurs when the wage equals the expected marginal product per period. (The non-pecuniary component plays no explicit role in the determination of the optimal wage since the private and social values of information about θ are equal by assumption.) The expected marginal product of a formed match equals the present value of the output expected over the duration of the match ($= z/(\delta + s)$) less the present value of avoidable fixed costs. This latter is computed as the present value of the fixed costs expected over the duration of the match ($= k/(\delta + s)$) less the fixed cost that has already been sunk prior to the decision on whether or not the match is to be formed ($= k/(1 + \delta)$); i.e., $k\alpha/(\delta + s) = k[(\delta + s)^{-1} - (1 + \delta)^{-1}]$. Thus, once again, the efficient wage is given in (8), while (9) follows from the zero profits constraint.

4. In equilibrium the firm chooses w to

$$\max_w [1 - G(r - w)](z - w - k\alpha).$$

If the worker accepts, the firm knows that $\theta > r - w$. If the firm is unable to commit to a constant wage over the duration of the match--and the inability to commit is what underlies the monopsonist construct--, then one might expect the firm to take advantage of this information by offering a lower wage in subsequent periods. However, such is not the case, at least in equilibria in which workers expect firms to offer a constant wage over time. To see this, let $\tilde{w}$ denote the initial wage offer. Given that $\tilde{w}$ was accepted, the conditional probability that a subsequent, lower offer w will also be accepted is

$$\Pr[\theta > r - w \,|\, \theta > r - \tilde{w}] = \Pr[\theta > r - w]/\Pr[\theta > r - \tilde{w}].$$

The firm thus chooses w to

$$\max_w \frac{[1 - G(r - w)]}{[1 - G(r - \tilde{w})]} (z - w - k\alpha);$$

i.e., excepting the irrelevant scale factor $\{1 - G(r - \tilde{w})\}^{-1}$, the firm's decision problem is unchanged.

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