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0: INTRODUCTION

This paper merges the two dominant and, so far, distinct approaches to labor turnover. One approach has been to model turnover as a response to the changing composition of product demands across firms. See Lucas and Prescott (1974). As demand for a firm's product rises relative to those of other firms, it raises its wages and attracts workers from the other firms. Notice that this re-allocation of labor will occur even if workers are homogenous in their productivities and even in the absense of uncertainty. The alternative approach, job-matching, relies instead on uncertainty and worker heterogeneity to generate turnover. See Jovanovic (1979). In these models a worker's productivity will depend upon which firm he is employed by. The worker, therefore, searches for firms with which he is better matched and so over his career will move between jobs as better matches are found.

It is clear that both reasons for turnover are operating at once in the economy. Much of the movement of labor out of the steel industry and the north-east and into the service sector and the south-west is not due to a massive and highly correlated (across workers) change in match qualities. However, much of the turnover found in panel data, for instance that of new entrants to the laborforce [Topel and Ward (1985)], is presumably not due to changes in the composition of product demand across firms. Thus, to explain actually observed turnover, a model must allow for both causes. As will be shown later in the paper, explicitly including both causes in a model is especially important because they interact in rather complex ways. For instance, workers may well transfer to a job that has a lower match but whose output is currently in high demand. Thus many of the usual results in the matching literature on wage as function of tenure require modification. Similarly, recognition of the presence of matching means that even with complete information a firm or sector will face an upward sloping supply curve of labor.

A second advantage of including both sources of turnover in a model is that it enables one to merge, to some extent, the theories being used to explain labor market behavior at the macro and micro levels. As Heckman (1984) has pointed out, many models of macro labor market behavior (time series data), e.g. Kydland and Prescott (1982), Long and Plosser (1983), largely because they are representative agent models, cannot explain observed behavior at the micro level (panel data). For instance, such models cannot shed light on the unemployment series, or, indeed, even the existence of unemployment. Similarly, models designed to explain panel data are not successful at explaining macro time series data. This is obviously an undesirable situation which this paper goes some way to remedying by introducing relative demand shocks into a matching model.

The paper is organized as follows. The next section presents the model and studies the impact of a mean preserving spread of either the distribution of match quality or relative labor demands on welfare. Section 2 looks at the impact of introducing relative demand shocks on the behavior of wages with tenure. Section 3 looks at the empirical implications of this model. A likelihood function is derived for wage changes and the probability of separation given tenure. We also note the implications for wage-tenure regression analysis. Section 4 shows that including relative demand shocks into a matching model will cause average wages paid by a firm to respond only slowly and in an autocorrelated way to product price changes even when each individual worker's wage is equal to his value marginal product and when the relative demand shocks are iid over time. Finally, section 5 contains some concluding comments.

1: THE MODEL

All workers are risk neutral, have the same value of leisure, u, the same discount factor, β, $(0<\beta<1)$ and the same constant hazard of death $(1-\gamma)$, $0 < (1-\gamma) < 1$. Thus in each period the probability that a worker will live to the next period is γ. Workers differ only in their match parameters, θ, which are their productivities at a given firm. For an individual worker these are distributed according to the cumulative distribution $G(\theta)$ with support $[\underline{\theta},\bar{\theta}]$, $0\leq\underline{\theta}<\bar{\theta}<\infty$, across firms. The draws from $G(\theta)$ are iid across workers. A worker's match with a firm is time invariant. The wage paid to a worker in any period is just his value marginal product at the firm that period, $p_t\theta$. Prices across firms follow a Markov process with a steady state cumulative distribution $F^*(p)$ and transition probability function $F(p_{t+1};p_t)$ with support $[\underline{p},\bar{p}]$, $0\leq\underline{p}<\bar{p}<\infty$.

Workers can, whether employed or unemployed, sample only one firm each period and the cost of this sampling is, for simplicity, set at zero. The sampling is random and reveals both p_t and θ. These offers cannot be recalled. Given these constraints it is clear that all workers will search each period and an unemployed worker will take a job if $p_t\theta > u$. As we shall see presently, the absence of recall implies that some jobs for which $p_t\theta < u$ will also be acceptable. The acceptance strategy of an employed worker is more complex. Define λ as the present value of quitting a job and becoming unemployed. The worker's value function can then be defined as

$$v(p_t,\theta) = p_t\theta + \gamma\beta\int_{\underline{p}}^{\bar{p}}\int_{\underline{p}}^{\bar{p}}\int_{\underline{\theta}}^{\bar{\theta}}\text{Max}\{\lambda, v(p_{t+1},\theta), v(p',\theta')\}$$

$$dF(p_{t+1};p_t)dF^*(p')dG(\theta') \qquad (1)$$

Where $F^*(p)$ and $F(p_{t+1};p_t)$ are related by[1]

$$F^*(p) = \int_{\underline{p}}^{\bar{p}} F(p,s)dF^*(s) \qquad (2)$$

From (1) the value of quitting can be found to be

$$\lambda = u + \gamma\beta\int \text{Max } \{\lambda, v(p,\theta)\}dF^*(p)dG(\theta) \qquad (3)$$

From (1) and (3) it follows that when $p_t\theta=u$, $v-\lambda$ is in general strictly positive, which means that certain jobs for which $p_t\theta<u$ will also be acceptable. Thus the worker may choose to take a job that has a high permanent component to the wage (high θ) even if the current wage is low because of a (transitory) low product price. This result is due, of course,

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1 To ease notation multiple integral signs and the limits of integration will only be made explicit where there is a risk of confusion.

to the absence of recall of past offers.

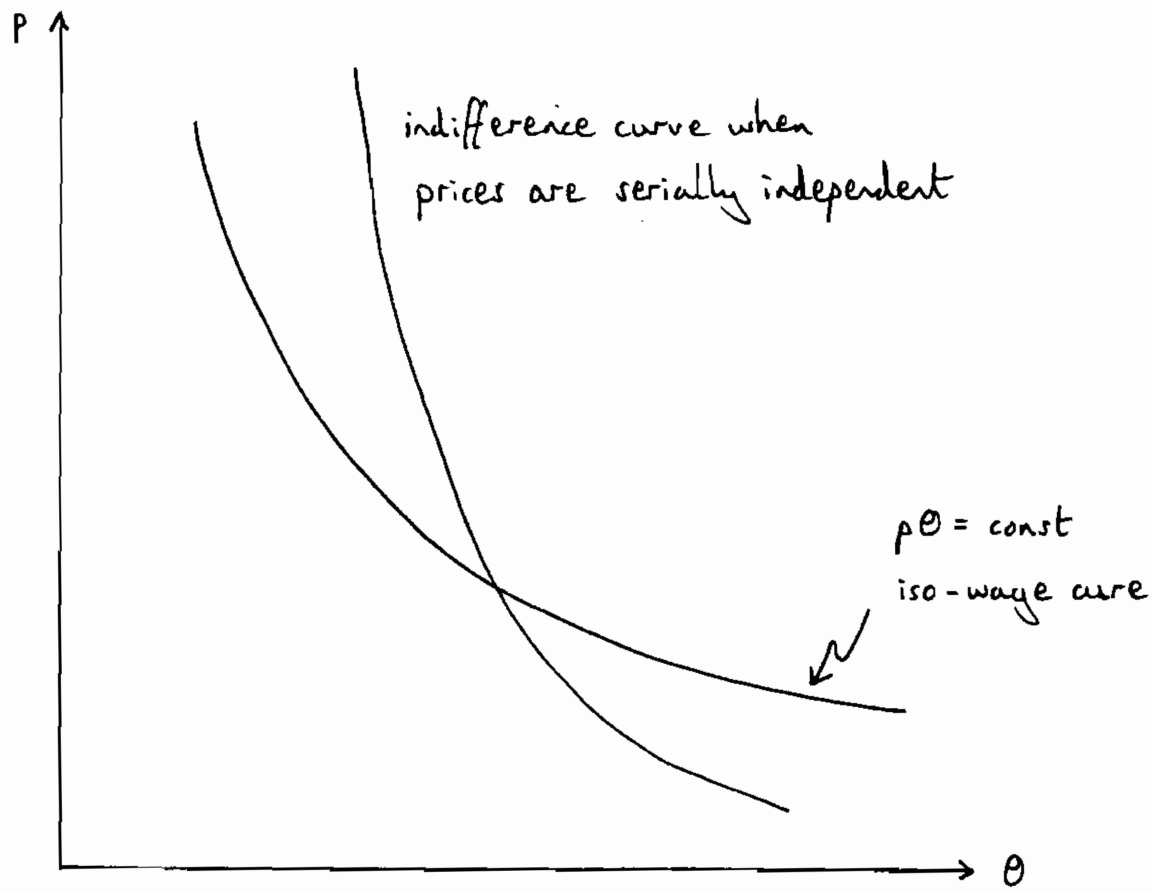
To gain some understanding of (3) it is best to consider two extreme cases. First, consider the case in which the price at a firm is perfectly autocorrelated. In this case, because both the quality of the match and the price are time invariant, the worker will care only about the wage offered and not about its composition in terms of p and θ. Thus the worker's value function reduces to a function of $p\theta$. In this case we get a standard on-the-job search model and the worker will accept any and all jobs that offers a higher wage than his current one (or u if he is unemployed). In θ-p space, the worker's indifference curves coincide with the iso-wage curves which are themselves rectangular hyperbole. (See Figure 1).

The second special case is that in which prices are serially independent. Notice that in this case p_t does not appear under the integral signs in (1) and so the value function reduces to

$$v(p_t, \theta) = p_t\theta + \zeta(\theta) \qquad (4)$$

where $\zeta(\theta)$ is some differentiable, increasing function of θ alone. (4) shows that, unlike the previous case, the worker is no longer indifferent to the composition of the wage offer. In particular, because the match component is permanent while the price component is transitory, the former will have a larger

Figure 1



p
θ
indifference curve when
prices are serially independent
pθ = const
iso-wage curve

impact on the worker's acceptance decision than the latter. This is shown by the worker's indifference curves in Figure 1 being much steeper than the iso-wage curves when the price is serially independent.[2]

The traditional matching model can be regarded as a special case of the model described above in which $\underline{p}=\bar{p}$. In that case there are no relative labor demand shocks affecting turnover and the only reason to re-allocate labor is in order to improve the match quality. In this environment the workers' opportunity set will be a horizontal line at height $\underline{p}=\bar{p}$ in θ-p space. Conversely, a composition of demand model is a special case in which $\underline{\theta}=\bar{\theta}$. In this case there are no productivity reasons for turnover and only movements in relative labor demand drive turnover. Here, the workers' opportunity set will be a vertical line at $\underline{\theta}=\bar{\theta}$. In the general case dealt with in this paper, both reasons for turnover are operative and this results in a two dimensional opportunity set.

To gain a better understanding of the impact of non-degenerate distributions of both matches and relative prices on labor allocation, let us consider two simple

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[2] In the case of time invariant prices $dp/d\theta = -p/\theta$ while, from (4), in the case of serially independent prices $dp/d\theta = -(p+\zeta')/\theta$. The convexity of the indifference curves in this latter case follows from the convexity of $v(p,\theta)$ in θ which implies that $\zeta''>0$.

depictions of the labor market.[3] Consider first the impact of diversity of matches on the intersectoral flows of labor. Assume for simplicity that there are two firms, 1 and 2, and no uncertainty. Workers prefer firm 1 over firm 2 iff $w_1 > w_2$ or, alternatively, if $w_i = p_i \theta_i$, i=1,2, they will prefer firm 1 iff $p_1 \theta_1 / p_2 - \theta_2 > 0$. If θ_1 and θ_2 are normally and independently distributed with a common mean, $\mu > 0$, and variance, σ^2, then the fraction of workers prefering firm 1 is

$$1 - F\left[\frac{[1-(p_1/p_2)]\mu}{[1+(p_1/p_2)^2]^{1/2}\sigma} \right] \equiv s(p_1/p_2, \sigma) \qquad (5)$$

where F[.] is the standard cumulative normal function. Three of these supply functions are shown in Figure 2. Notice that they become less elastic as the variance of the distribution of matches increases and that when the match distribution is degenerate ($\sigma^2=0$) the supply curve is horizontal. Thus the amount of labor reallocated between sectors in response to a shift in relative demands is reduced by matching.

An exactly analogous analysis holds for the impact of relative price dispersion on the arbitrage of match differentials. Assume that workers are matched with sectors (1

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3 This analysis relies heavily on Roy (1951).

and 2) rather than firms and that firms in each sector have product prices which are normally and independently distributed with the same mean and variance in each sector. In this case the supply curve of workers to sector 1 is given by (5) with the p's replaced by θ's. These supply curves will look exactly like those in Figure 2 with the axes appropriately relabelled. Thus price dispersion will reduce the amount of labor reallocation in response to changes in sectoral match quality (technology).

If both the distributions of match quality and relative demands are important for turnover, a relevant question to ask is how welfare in the economy would change if these distributions changed. That is, how do the returns to search and turnover change? Here we restrict the changes in distributions to be mean preserving spreads. These have the advantage of leaving the expected value of output on a random worker-firm assignment constant.[4] So any change in welfare must come about only from an increase in the returns to search and turnover. The results are summarized in the following two propositions.

<u>Proposition 1</u>

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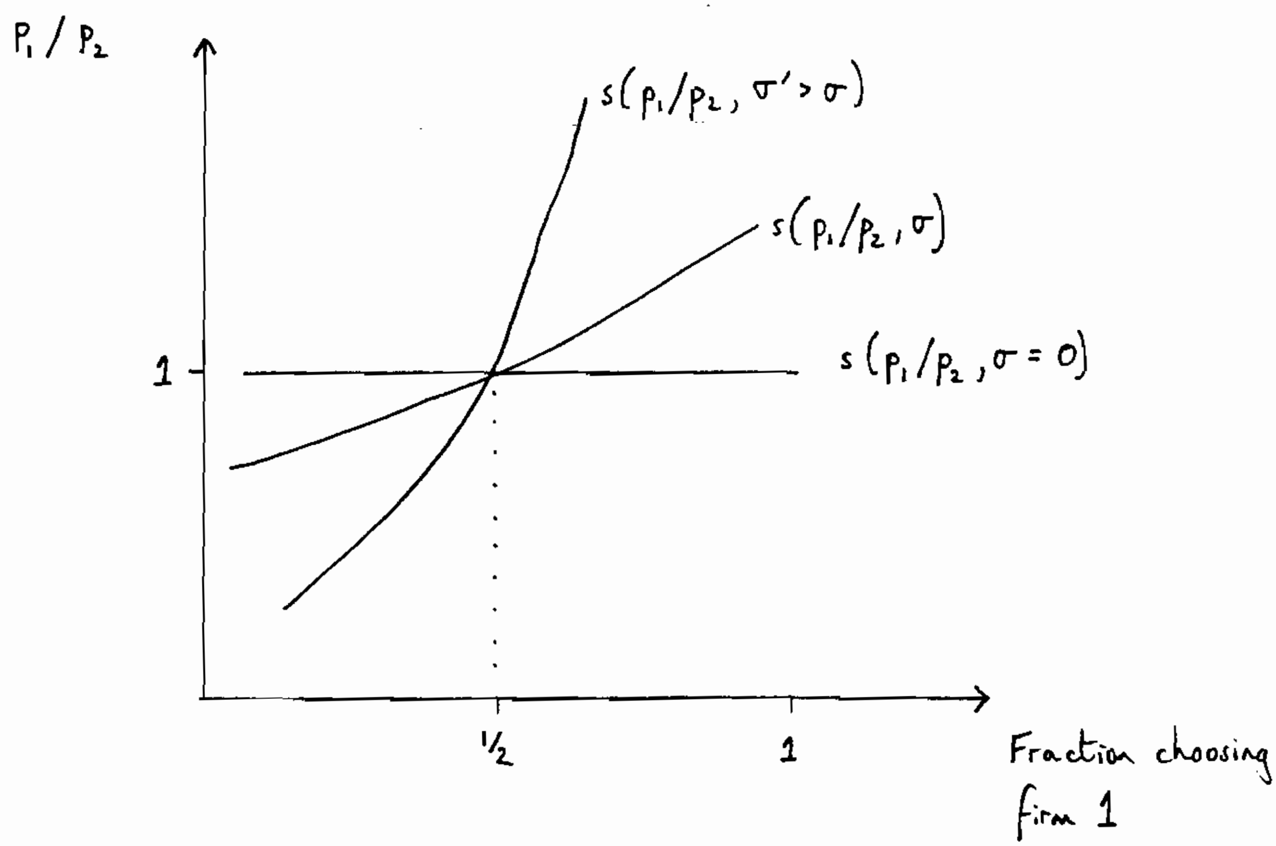
4 Random assignment of workers to firms implies that $Ep\theta = EpE\theta$.

Figure 2



A mean preserving spread of $G(\theta)$ increases both λ and $v(.,.)$.

Proof See Appendix.

Thus as the quality of matches becomes more variable, the welfare of both employed and unemployed workers increases. This is due to the fact that the employed worker can always stay at his current job at the current θ while an unemployed worker can stay unemployed and receive u. In other words, both hold put options and the value of those options rises with the dispersion of the underlying distribution.

Now define $F_1(p';p)$ to be a mean preserving spread of $F_2(p';p)$ if, for each p (fixed), $F_1(p';p)$ is a mean preserving spread of $F_2(p';p)$.

Proposition 2

If $F_1(p';p)$ is a mean preserving spread of $F_2(p';p)$ and if $F_i^*(p)$ (i=1,2) are the corresponding steady state distributions, and if $F_1^*(p)$ is a mean preserving spread of $F_2^*(p)$, then the mean preserving spread leads to an increase in both λ and $v(.,.)$.

Proof See Appendix.[5]

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[5] Note that a mean preserving spread on the transition probabilities does not by itself guarantee a mean preserving spread of the steady state distributions. For instance, if the

The rationale for this result is the usual option argument used in the search literature namely that the option to search rather than accept a job rises in value with the dispersion of the distribution of offers. Notice that this result goes some way to providing the theoretical underpinnings for Lilien's (1982) empirical model. Here we have seen that an appropriately defined mean preserving spread on the distribution of the relative demands for labor will raise the value of quitting into unemployment, λ, and so the level of unemployment in the steady state. Note, however, that the result requires that the transition probabilities undergo a mean preserving spread and that it implies that welfare increases. We turn now to the economic implications of the model.

2: WAGES AND TENURE

The model yields a new explanation for why wages grow with tenure. The explanation is due neither to learning about the quality of the match (Jovanovic, 1979) nor to selection out

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transition probabilities were such that $F(p;p') = F(p';p)$ for all p and p', then the steady state distributions would be invariant with respect to mean preserving spreads of the transition probabilities. See Parzen (1962), p.255.

through on-the-job search (Jovanovic, 1984), but rather to the presence of a time-varying product price. In matching models that ignore price variability across firms and in which the quality of the match is revealed at hiring, a new hire's wage is expected neither to grow nor fall with tenure. The average wage of workers hired at a given date is, however, expected to grow with tenure because on-the-job search by these workers will result in those whose matches are poorest leaving the firm. This selection process will drive up the average wage of a tenure cohort as tenure lengthens.

In this model, even if there were no on-the-job search, there would be a selection process at work that would drive up average wage for a tenure cohort. Over time a given cohort will encounter lower product prices and so lower wages and these will induce those workers with the lowest quality matches to quit and become unemployed. However, there is also a countervailing force at work. Here, in contrast to the usual matching model, a worker's wage is expected to fall with tenure. Consider the set of all potential new hires who, if hired by the firm under consideration, would have a match of θ_i. Let the firm's price at the time of hiring be p_0. The proportion of this set of workers that would accept an offer from the firm of $w_0=p_0\theta_i$ is clearly increasing in p_0 and so the expected wage of a new hire with match θ_i, $\bar{w}_0=\bar{p}_0\theta_i$, is greater than θ_i times the expected price [the mean of $F^*(p)$],

μ. That is the average price at hiring is $\bar{p}_O > \mu$. If the distribution of prices is stationary, then the expected price as the tenure of these new hires lengthens goes to μ and so their expected wage goes to $\mu\theta_i < \bar{p}_O\theta_i$. Thus, absent any selection effect, the average wage of a tenure cohort will fall because workers will typically be hired at times when business, and so value marginal product, is unusually good.[6]

In order to contrast the forces at work here from those in a typical matching model we will, for the rest of the section, assume that there is no on-the-job search and so workers that leave the firm become unemployed. Without explicit functional forms there is little that can be said about how average wage for a tenure cohort will behave as a function of tenure when tenure is relatively short. The rate at which an individual worker's wage is expected to decline is inversely related to the degree of autocorrelation in the firm's price process, i.e. the degree of persistance in business conditions. Using the notation of the previous sections, the average wage of workers at the time of hiring if the price is p_O is

$$p_O\int_{\theta(p_O)}^{\infty}\theta d\hat{G}(\theta;p_O) \tag{6}$$

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[6] This is consistent with the empirical findings of Adams (1985, Table II, p.42). Using panel data he found that industry growth rates had a significant positive impact on wages.

where $\hat{\theta}(p_0)$ is the minimum quality match that will give a wage $(p_0\theta)$ that will induce the worker not to quit and $\hat{G}(\theta;p_0)$ is the truncated distribution of θ with truncation point $\hat{\theta}(p_0)$. If we assume that the price process is AR(1) with autocorrelation coefficient ρ and mean μ, then in the absence of selection the expected average wage for this cohort of workers in the period after hiring (tenure=1) is

$$Ep_1 | p_0 \int_{\hat{\theta}(p_0)}^{\infty} \theta d\hat{G}(\theta;p_0) = [(1-\rho)\mu + \rho p_0] \int_{\hat{\theta}(p_0)}^{\infty} \theta d\hat{G}(\theta;p_0) \qquad (7)$$

Subtracting (6) from (7) gives the expected change in the average wage of this cohort as they go from new hires to having one period of tenure. Notice that this change will obviously depend upon the wage at hiring and so to get the ex ante expected change for new hires we must integrate over the price price distribution. This gives

$$(1-\rho)\int_{\underline{p}}^{\overline{p}} (\mu-p) [\int_{\hat{\theta}(p)}^{\infty} \theta d\hat{G}(\theta;p)] dF^*(p) \quad < 0 \qquad (8)$$

The sign of the expected change follows because more new hires will join the firm at high prices than at low prices (the term in square brackets is increasing in p) and so these high prices receive more weight than the low prices. However, as we can see from (6) and (7), it is precisely prices above the mean that lead to an expected fall in average wage. The role of the degree of autocorrelation in prices is clear from (8). The lower the autocorrelation coefficient, ρ, the larger is the expected fall in average wage and so the more likely it is that this effect will outweigh the selection effect. Conversely, the higher the degree of autocorrelation in prices the more likely it is that the selection process will dominate and the tenure cohort's average wage will rise with tenure. Note that in the extreme case of perfectly autocorrelated prices, each individual worker's wage is independent of tenure and so expression (8) is zero and only the selection effect is at work. However, this selection effect does not come from on-the-job search as in the usual matching models but rather from quits induced by low product (and so value marginal product) prices.

In the limit as tenure increases the selection effect will dominate the behavior of wages as a function of tenure. This follows immediately from the stationarity of the price process. As tenure goes to infinity, the average wage of workers that tenure cohort will go to

$$\mu\int_{\hat{\theta}(\underline{p})}^{\infty}\theta d\hat{G}(\theta;\underline{p}) \equiv \underline{w}$$

Thus selection ensures that the only workers that will remain with the firm in the limit are those that are so well-matched that they will not leave the firm even when their value marginal product is as low as it can be. In contrast, in the absense of selection the wage of the same cohort would, in the limit, go to

$$\mu\int_{\hat{\theta}(p_0)}^{\infty}\theta d\hat{G}(\theta;p_0) < \underline{w}$$

Thus regression to the mean in the price cannot dominate the selection process in the long-run as far as determining the behavior of wages as a function of tenure.

The next section deals with the implications of this theory for empirical work on wage-tenure profiles.

3: IMPLICATIONS FOR EMPIRICAL WORK

There are two empirical approaches to estimating the relationship between wages and tenure on panel data namely estimating the likelihood function of future wages conditional on tenure and estimating some form of wage regression. We will first develop the appropriate likelihood functions and then develop some of the implications for wage regressions. In both cases, the major problem raised by the theory for empirical work is that many wage changes and job changes may be caused by changes in relative product prices. This force is omitted in existing empirical work. The major difficulty with including it explicitly in empirical studies is that observations on these prices are not included in panel data nor is it clear how they could be obtained from other sources and matched to the panel data. Thus empirical studies must either be designed to avoid any bias caused by this omitted variable and/or must use a proxy for the impact of changes in relative prices. When the bias cannot be avoided caution must be used in interpreting the time series behavior of residuals in wage equations and the presence of sectoral dummies as indicators of the behavior of match qualities.

Likelihood Function

To bring out clearly the role of relative price movements, assume that there is no on-the-job search. Let w, p and τ represent, respectively, current wage, product price and tenure. We wish to know the likelihood functions over the wage next period for this worker should he stay at the firm and over the probability that he will leave his job. This is straight forward _if_ one has data on the triple (w,p,τ) but all panel datasets of which we are aware only have (at best) data on (w,τ). Thus to be empirically relevant the likelihood function must be conditioned only on wage and tenure. We now develop such a function under the assumption that prices are iid over time.

Conditional on θ, next period's wage (w') is $\theta p'$, where p' is next period's price, and so the probabiltiy distribution of w' conditional on θ is just $F^*(w'/\theta)$. The data on tenure gives us information indirectly about θ in the following way. For any p there is a minimum level of match, $\hat{\theta}(p)$, such that workers with matches equal to or above $\hat{\theta}(p)$ will stay at the firm. This "reservation match" is decreasing in p. Since the worker has stayed τ periods we know that $\theta > p_{t-j}$, $j=1,..,\tau-1$.[7] However, because $\hat{\theta}(p)$ is decreasing in p this amounts to $\theta > \hat{\theta}(p^*_{\tau-1})$ where $p^*_{\tau-1}$ is defined as $\text{Min}\{p_{t-j}: j=1,..,\tau-1\}$. So the tenure data gives us information about θ by telling us the size of the sample of prices and so the distribution of the minimum price that the worker has faced. However, we have more information on θ than that in the length of tenure because we also have the worker's current wage.

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7 Note that this is the minimum _past_ price, i.e. it excludes the current price.

Consider first the information contained in the length of tenure. The cumulative probability distribution of $p^*_{\tau-1}$ is

$$H(p^*;\tau-1) = 1 - [1 - F^*(p^*)]^{\tau-1}$$

Thus, conditional on $p^*_{\tau-1}$, the cumulative distribution of θ is just

$$\hat{G}(\theta;p^*_{\tau-1}) \equiv \begin{cases} \dfrac{F^*(\theta) - F^*[\hat{\theta}(p^*_{\tau-1})]}{1 - F^*[\hat{\theta}(p^*_{\tau-1})]} & \text{for } \theta \geq \hat{\theta}(p^*_{\tau-1}) \\ 0 & \text{otherwise} \end{cases}$$

This gives the distribution of θ amongst those workers who stayed with the firm for the τ-1 periods prior to the current one, conditional upon $p^*_{\tau-1}$. Integrating out over $p^*_{\tau-1}$ gives the distribution of θ for this group conditional only on the observable tenure,

$$G^*(\theta|\tau-1) = \int \hat{G}(\theta;z)dH(z;\tau)$$

We now need to incorporate the information that the workers stayed with the firm in the current period at a wage w. Conditional on θ, the probability of this event is just the probability that $p = w/\theta$ <u>and</u> that $\theta > \hat{\theta}(p)$. But

$$\Pr.[\, p = w/\theta \text{ and } p > \hat{\theta}(p)]$$

$$= \quad \Pr.[p=w/\theta \mid p>\hat{\theta}^{-1}(\theta)]\Pr.[p>\hat{\theta}^{-1}(\theta)]$$

$$= \begin{cases} f^*(w/\theta)[1 - F^*(\hat{\theta}^{-1}(\theta))] & \text{if } w/\theta \geq \hat{\theta}^{-1}(\theta) \\ 0 & \text{otherwise} \end{cases}$$

$$\equiv q(w|\theta)$$

Thus q(.) gives the probability density of staying at wage w during the current period conditional on θ.

We can now combine the two pieces on information. Let $\Psi(w|\tau)$ be the density of current wages among all current workers with tenure τ. Then

$$\Psi(w|\tau) = \int q(w|\theta)dG^*(\theta|\tau-1) \qquad (9)$$

By the rule of conditional probability,

$$q(w|\theta)g^*(\theta|\tau-1) = \phi_\tau(\theta|w)\Psi(w|\tau) \qquad (10)$$

where $g^*(.)$ is the probability denstity function corresponding to $G^*(.)$ and $\phi_\tau(\theta|w)$ is the density of θ among workers with tenure τ conditional on the current wage. Combining (9) and (10) gives

$$\phi_\tau(\theta|w) = \frac{q(w|\theta)g^*(\theta|\tau-1)}{\int q(w|z)dG^*(z|\tau-1)} \tag{11}$$

(11) leads directly to the two conditional likelihoods that we are looking for. Conditional on θ, the probability that a worker will quit next period is just the probability that $p'<\hat{\theta}(\theta)$. Thus the likelihood of quitting next period is

$$\int F[\hat{\theta}^{-1}(\theta)]d\Phi_\tau(\theta|w)$$

where $\Phi(.)$ is the cumulative of $\phi(.)$. The probability that a worker with tenure τ will receive an acceptable wage offer w' next period, conditional on θ, is just the probability that the price next period will be w'/θ which is $f^*(w'/\theta)$ [where $f^*(.)$ is the density of $F^*(.)$] if $\theta > \hat{\theta}(w'/\theta)$ and zero otherwise. Thus, the joint likelihood of acceptance and w' is

$$\int f^*(w'/\theta)I[\theta-\hat{\theta}(w'/\theta)]d\Phi_\tau(\theta|w)$$

where $I(y) = $ 1 for $y>0$
0 otherwise

Wage Regressions

The most popular empirical approach to the wage-tenure relationship is to use a wage regression. Suppose x_t^i is a vector of the observable characteristics of worker i at time t. Then writing productivity as $p_t \theta_t^i f(x_t^i) \varepsilon_t^i \equiv w_t^i$, the following regression is implied where j indexes the employer,

$$\ln w_t^{ij} = \ln f(x_t^i) + u_t^{ij} \tag{12}$$

The unobservable component, u_t^{ij}, can be decomposed into,

$$u_t^{ij} = \ln p_t^j + \ln \theta^{ij} + \ln \varepsilon_t^{ij}. \tag{13}$$

The term ε_t^{ij} in (13) is assumed to be iid. $\ln \theta^{ij}$ is the match-specific fixed effect which is time invariant, although if the worker changes employers, he obtains a new draw of θ. The component $\ln p_t^j$ is employer-specific and <u>not</u> time invariant. In our theoretical development we assumed p_t^j to be a first order Markov process in anticipation of strong, positive serial correlation as could be captured, for instance, by

$$\ln p_{t+1}^{j} = \alpha \ln p_t^j + \eta_t^j, \quad (\alpha \in [0,1))$$

with η_t^j an iid process. So while θ is a permanent component of u_t^{ij}, p_t^j is less so depending on the value of α. In particular, at the extremes when α equals 0 or 1, $\ln p_t^j$ becomes distinguishable from ε_t^{ij} and $\ln \theta^{ij}$, respectively, only by the fact that it is fixed across all workers at the same firm. As panel data does not usually include several workers from the same firm, empirically, in these extreme cases, these different effects cannot be distinguished.

In the more reasonable case in which $0<\alpha<1$, one would expect the unobservable component of wage to exhibit autocorrelation exactly as has been found in the data by, for instance, MaCurdy (1982). An alternative way in which p_t^j may show up in the data is in industry dummies. How good a proxy such dummies are likely to be depends on two factors: the extent to which the product in the SIC code is homogenous and the degree of autocorrelation in the price process. In the extreme case in which both the product is homogenous, so that all firms in the industry have the same price, and $\alpha=1$, the industry dummy will pick up all of the unobserved heterogeneity caused by variation in the p_t^j across sectors. Unfortunately, however, neither limiting case is likely to hold in practice. First, each sector is subject to demand shocks and technology shocks which lead to variations of p_t^j over time. Second, it is well known [See Stigler and Kindahl, 1970] that the degree of product heterogeneity which exists even in the narrowest SIC code is usually vast. Sectoral dummies are not, therefore, likely to control fully for the unobserved variation in the p_t^i.

4: DOWNWARD STICKINESS AND AUTOCORRELATION IN THE AVERAGE WAGE

One advantage of incorporating the demand side into a model of turnover is that one can derive implications for the responsiveness of wages to demand shocks. In the usual matching models demand shocks effect wages only if they have a technological impact on the quality of matches. Thus simple shifts in relative prices which shift the value marginal product of all workers at a firm are not considered. Introducing stochastic relative product prices, as has been done in this paper, provides interesting dynamics for the average wage paid by a firm. In particular, because of the selection process induced by price changes, the average wage paid by a firm will respond asymmetrically to price changes and will exhibit downward stickiness even though each individual worker's wage is always equal to his or her value marginal product. Moreover, even when the relative price process is iid, the average wage will not be iid. The average wage will show a very complex autocorrelated structure because the average product of a given tenure cohort depends on the minimum price that has been realized during that tenure. Thus the average product, and so the average wage, of the current workforce depends in a complex way on current and all previous product prices. It is this same dependence of average product on price that causes the downward wage stickiness.

To see clearly how selection caused by price changes can cause downward stickiness in the average wage we will assume that there is no on-the-job search. Let the price in the current period be p_0 and consider the average wage paid to workers with tenure $\tau>0$,

$$p_0\int_{\hat{\theta}(p_\tau^*)}^{\infty}\theta d\hat{G}(\theta;p_\tau^*) \equiv \bar{w}_0 \qquad (13)$$

Note that p_τ^*, the minimum price that has been realized over the length of these workers' tenure, is less than or equal to p_0. Clearly, if the price in the following period, p_1, is the same as p_0 then there will be no change in the average wage paid to this tenure cohort. If the price rises by some amount $k>0$, then p_τ^* will not change, no workers in this tenure cohort will leave the firm and so the average wage paid will rise in proportion to the increase in price. Alternatively, the change in the average wage will be

$$\bar{w}_1-\bar{w}_0 = k\int_{\hat{\theta}(p^*_\tau)}^{\infty} \theta d\hat{G}(\theta;p^*_\tau) \tag{14}$$

Now consider what would happen to the average wage if, instead, the price had fallen by $k>0$. In this case $\bar{w}_1$ is given by

$$(p_0-k)\int_{\hat{\theta}(p^*_\tau)}^{\infty} \theta d\hat{G}(\theta;p^*_\tau) \tag{15}$$

Notice that this only implies that the fall in the average wage is the same size as (2) if $p_0-k \geq p^*_\tau$. In this case no workers of tenure τ will leave the firm as a result of the decline in wages. However, if p_0-k is the lowest price that has occured since these workers joined the firm, i.e. $p^*_{\tau+1}$ is p_0-k, then some workers will quit or be laid off. As these workers will have the lowest quality matches (lowest marginal products) the average wage will drop less than in proportion to the fall in price. Thus the average wage paid to this tenure cohort will exhibit some downward wage stickiness once the price drops below p^*_τ.[8]

Because this effect is at work on all tenure cohorts

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[8] Stockman (1983) finds that compositional changes in the laborforce over the business cycle dampen the observed sensitivity of the average manufacturing wage to business cycle conditions. Notice that the compositional effect in this paper yields an asymmetric response of average wage to demand shocks.

within the firm it follows that the average wage paid by the firm to all its workers will exhibit this type of downward stickiness. Moreover, because p^*_τ is (weakly) decreasing as a function of τ, the average wage of those cohorts with least tenure will show the greatest degree of downward wage stickiness in the sense that the shorter tenure is, the less price will have to drop before some of the workers of that tenure leave the firm. Notice that none of this wage sluggishness is due to non-maximizing behavior nor does it indicate any kind of inefficiency.

The selection process that generates the downward stickiness determines an impulse response function from product prices to the average wage. The average product, $AP(p^*_\tau)$, of a given tenure cohort depends on the minimum price that cohort has suffered during their tenure with the firm,

$$AP(p^*_\tau) \equiv \int_{\hat{\theta}(p^*_\tau)}^{\infty} \theta d\hat{G}(\theta;p^*_\tau) \qquad (16)$$

Defining $\mu(\tau,p^*_\tau)$ as the measure of tenure cohort τ in the firm's labor force,[9] then the average wage paid by the firm at date t, w_t, is just

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[9] The direct dependence of the measure on τ comes from the exponential death rate of workers.

$$w_t = p_t . \sum_{\tau=0}^{\infty} AP(p_\tau^*) . \mu(\tau, p_\tau^*) \qquad (17)$$

While this impulse response function shows clearly that an iid price process will not generate an iid average wage process, it is perhaps less clear that the function can give some very unusual dynamics.[10] To illustrate this consider the following case. Up until time t the price facing the firm had been constant at p_0. Thus for all tenure cohorts in the firm prior to t, $p_\tau^*=p_0$. At t the price fell to p_1 and at t+1 it returned to p_0 where it stays forever. The response path of the average wage to this price path is shown in Figure 3a. Notice that it is decidedly nonmonotonic. The rationale for this response path is as follows. At t when the price dropped the average wage fell less than proportionately because some workers, those with low productivities, left the firm. This is shown in Figure 3b by the average wage falling along the curve to w_t rather than along the ray from the origin through (p_0, w_{t-1}). In t+1 the average wage of those workers with positive tenure rises in proportion to the price. Diagrammatically, it rises along the ray through (p_1, w_t) to w'. However, the firm will also hire some new workers. These new hires will have a lower average product than the workers with

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10 Altonji and Shakotko(1985) and Raisian(1983) provide empirical evidence on the dynamic responses of wages.

Figure 3 a

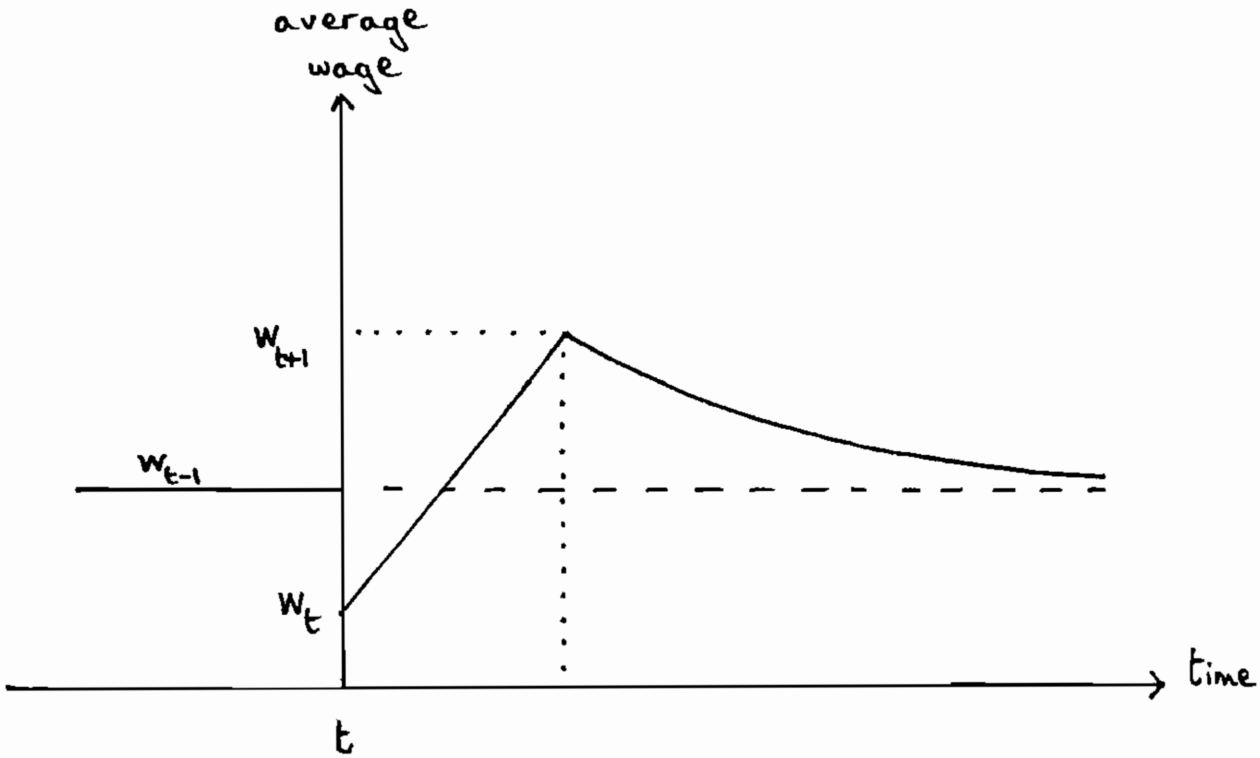
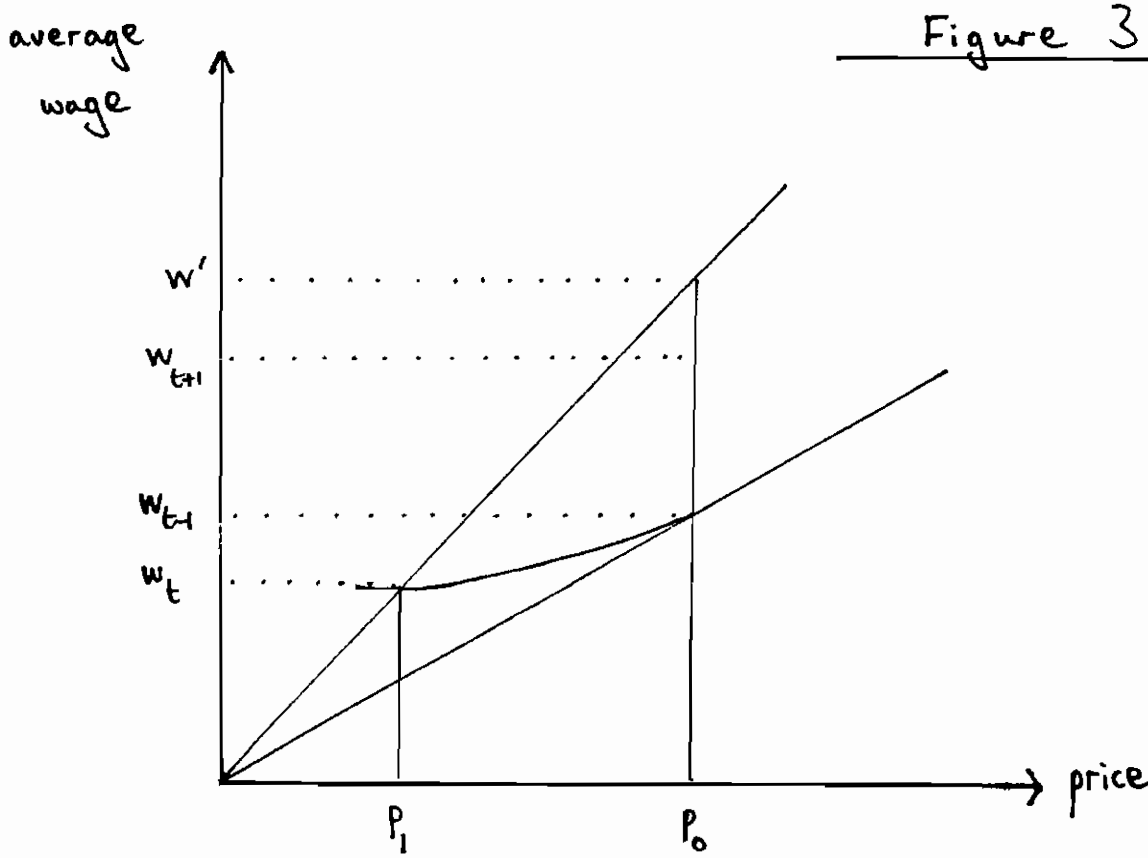


Figure 3 b



positive tenure[11] because for the new hires $p^*=p_0$ while for all the other workers $p^*=p_1$. Thus the addition of the new hires will push the firm's average wage below w′ but will leave it above w_{t-1}. As time progresses, the old, higher quality workers will die out and be replaced by new workers. As the measure of workers who have suffered p_1 goes to zero, the average wage converges from above to w_{t-1} once more.

This simple example of the complexity of the impulse response function of average wage to a relative price shock should act as a warning that simply observing that the process governing wages is more sluggish (e.g. a higher AR process) than that governing prices does not necessarily mean that individual wages respond sluggishly to demand shocks, for example, because of implicit contracts.

5: CONCLUSION

This paper has shown that it is important to merge both existing approaches to turnover. This is not only desireable at a theoretical level but is also important at the empirical level. In particular, empirical studies carried out on panel

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11 The new hires' average product will be exactly the same as the average product of the initial workforce prior to the price drop.

data which adopt a purely matching approach to turnover are liable to be misspecified and so lead to erroneous inferences simply because the role of movements in product prices in determining wages, and so turnover, will have been omitted. Furthermore, the results of empirical studies of wage behavior, especially wage behavior in response to demand shocks, that use average wage paid by a firm or sector can only be interpreted correctly in a model with matching.

In addition, we have uncovered a new explanation for upward sloping wage-tenure profiles (Section 2) and for the persistent effect of price shocks on wages (Section 4). Finally, we have shown (Section 3) that the model can, at least under certain additional assumptions, be implemented empirically.

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Appendix

Proof of Proposition 1

Let $f(p_t,\theta) = \text{Max}\ (\lambda, v)$. Thus

$$\lambda = u + \gamma\beta\iint f(\cdot)\ dF^*(p)d\ G(\theta) \tag{A.1}$$

The $\text{Max}\ \{\lambda,\ v(p_{t+1},\theta),\ v(p',\theta')\} = \text{Max}\ [f(p_{t+1},\theta),\ f(p',\theta')]$ and so (1) can be written as

$$v(p_t,\theta) = p_t\theta + \gamma\beta\iiint \text{Max}\{f(p_{t+1},\theta), f(p',\theta')\}dF(p_{t+1},p_t)dF^*(p')dG(\theta') \tag{A.2}$$

(A.1) and (A.2) imply

$$f(p_t,\theta) = \text{Max}\ [(A.1),(A.2)] \equiv T_G f \tag{A.3}$$

But T_G preserves convexity in θ, i.e., if f_0 is convex in θ, then $T_G f_0$ also is. By induction, $T_G^{(n)} f_0$ is also convex for all n and so

$$f = \lim_{n\to\infty} T_G^{(n)} f_0$$

is convex in θ.

Next note the following fact:

If: (a) $f_1(x)$ and $f_2(x)$ are convex functions and $f_1(x) \geqq f_2(x)$

(b) $G_1(x)$ and $G_2(x)$ are distributions s.t. $G_1(x)$ is a mean preserving spread (MPS) of $G_2(x)$,

then $\int f_1 dG_1(\theta) \geqq \int f_2 dG_2(\theta)$

Now let f_1 and f_2 be the solutions to (A.3) under G_1 and G_2. We wish to prove that $f_1 \geqq f_2$. But

$$f_i = \lim_{n\to\infty} T_{G_i}^{(n)} f_0 \quad (i = 1,2) \quad \text{for } \underline{\text{any}}\ f_0$$

(Recall that T_G is a contraction mapping on a complete metric space). Moreover, T_{G_i} preserves convexity in θ and is monotone. This, plus the above fact, proves that for all n, $T_{G_1}^{(n)} f_0 \geqq T_{G_2}^{(n)} f_0$. This implies that

$$f_1(p,\theta) \geqq f_2(p,\theta) \qquad \forall\theta \tag{A.4}$$

Letting v_i and λ_i be the corresponding solutions to (A.1) and (A.2) leads to Proposition 1.

Proof of Proposition 2

Same as for Proposition 2 with the roles of θ and p reversed.