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MANAGEMENT ERROR AND THE OPTIMAL
STRUCTURE OF THE FIRM:
A DECISION - THEORETIC APPROACH

by

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Management Error and the Optimal Structure of the Firm:

A Decision - Theoretic Approach

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0: INTRODUCTION

One can think of the managers of a firm as having two conceptually separate functions. One of these functions could be called bureaucratic and involves coordination and record keeping, e.g. ensuring that components arrive on-site at the right time, that the payroll is met on time and that contracts with customers are recorded and monitored. The other function is more strategic than bureaucratic and involves decision making such as whether to launch a new product, whether to invest in new plant, or whether to make an acquisition or a divestiture and so on. It is this latter function that this paper will deal with and so the management of the firm will be modeled as a decision-making organization. Given this view of management one would like to know how the owner(s) of a firm would structure management; how many managers would be hired, will the managers be organized hierarchically, what decision-making procedures and rules will be used? This paper attempts to provide some answers to these questions.

Critical to our analysis of the structure of management is the assumption that managers are fallible and thus make honest decision errors.¹ More specifically, we study the following problem: A firm is presented with a sequence of projects each of which may either be profitable or not. Unfortunately, the owner(s) of the firm is not able to distinguish ex ante whether a particular project is profitable or not. The owners therefore hire managers who evaluate the profitability of the proposed project. Although each manager's opinion is informative it is not perfectly so, i.e. each manager is fallible. Plainly, because managers' project evaluations are

¹ See Sah and Stiglitz (1984, 1985) for an analysis of some of the implications of fallibility.

potentially error ridden the owners of the firm must decide upon the number of managers who should review the project and on how the opinions of the managers should be aggregated into a final decision about the project.

This approach to the optimal size and structure of management organizations has several interesting features. First of all it treats the managers as a group or organization rather than the individual manager as the decision-making unit. Indeed, the optimal decision rule will usually require several managers to review the project and will specify how the resulting opinions should be aggregated into a decision. Secondly, organizing management so as to make optimal decisions gives, in a very natural way, both a theory of the optimal size of management and of its hierarchical organization. Finally, this approach can potentially tie the size and organization of management directly to parameters of the decision-making technology, the product market and the market for managers. Thus, for example, the decision-theoretic approach permits us to analyse the impact of monopolizing the output market on the structure of management. It also enables us to study the impact of a parametric rise in the price of managers on the size, configuration and number of the management teams that will compete in the product market. This integration of market and firm structure is both novel and, perhaps, the greatest benefit of following the decision-theoretic approach adopted in this paper.

The main conclusions of our analysis can be briefly summarized as follows. First, we find that an optimally designed management organization uses managers sequentially and that these managers form a (stochastic) pyramidal structure. Secondly, the opinions of the managers concerning a proposed project are optimally aggregated through, in the case studied, a form of qualified majority voting where the majorities needed for acceptance and

rejection may differ. Thirdly, we find that, although the response of these optimally chosen majorities to parameter changes are frequently ambiguous, they in general change as one would expect, e.g. an increase in the wage paid to managers reduces these majorities and thereby the expected number of managers that will be used to evaluate the project. On the other hand, an improvement in the quality of screens increases these majorities. However, because the precision of the screens has increased these majorities may be reached faster and so the expected number of screens used falls.

Fourthly, when the firms, viewed as consisting of management teams, are embedded in a market for managers in which the supply curve is upward sloped, these changes in parameters effect both the number and size of the optimal management organizations in that market. Thus, for instance, a rise in the payoff from doing a good project will raise the number of firms in the industry and lower the equilibrium size of the management organization. Finally, if the industry is monopolized the monopolist will use fewer management organizations but each organization may be either smaller or larger than the firms in the competitive industry. In addition, even when each screening organization is smaller and so less efficient than the corresponding competitive firm, the monopolist may still make more efficient decisions because it will not, unlike the competitive industry, resample previously rejected projects.

Before proceeding with our analysis, we should point out some limitations of the model developed here. First, we abstract from the standard incentive problems that form the core of the principal-agent approach to the internal structure of the firm. In recent papers, Sah and Stiglitz (1984, 1985) and Arrow (1985) provided cogent reasons for the analytic approach adopted here. We are not suggesting that incentive issues can be ignored in firm

organization. Indeed, we regard our analysis as complementary to, rather than a substitute for, the principal-agent approach. However, error making is at least as common as principal-agent types of problems and has consequences for the owners of firms that are potentially at least as great as those stemming from the possibly incorrect incentives for the managers.

Second, we do not specify the source of the projects that are evaluated by the managers. Thus we abstract from the important managerial or entrepreneurial function of finding a source of such projects. Our focus here is on the evaluative function performed by managers once the projects have been generated.

Third, our view of how these evaluations are performed is also quite simple. For example, we abstract from the possibility that the decision procedure may involve individual managers meeting in committee to "hash-out" the merits of a project. Such procedures seem to occur when different managers have different areas of expertise. We abstract from this potentially interesting complication.

Finally, even though the management structures derived in this paper are (stochastically) pyramidal, they are not hierarchical in the sense that managers at higher levels perform different functions from those at lower levels. However, using the approach developed here, meaningful hierarchies can be generated endogenously if, for example, managers differ in their evaluative skills. (See Bull and Ordovery, 1985).

Despite these limitations, the analysis developed below sheds important light on the role of decision-theoretic concerns in structuring organizations. It also points the way to further analytically tractable extensions of our simple model.

The paper is organized as follows. Section 1 models the problem facing

an individual entrepreneur who has to choose a management size and structure and then goes on to carry out some comparative static experiments on that problem. The following section embeds the firm in a competitive industry and enables us to show the link between firm and market structure. Section 3 considers how a monopoly would organize its management. There we also compare a monopolized and a competitive industry. Section 4 contains some concluding comments.

1: THE FIRM'S OPTIMAL MANAGEMENT ORGANIZATION

A: Preliminaries

An entrepreneur has a proposed investment project that has been drawn randomly from a population of projects containing two types of projects, good and bad. The proportion of good projects in the population is π^0 and is known to the entrepreneur. If the entrepreneur decides to accept and carry out the project his payoff will be $P > 0$ if the project is good and $-H$, $H > 0$ if it is bad. If the entrepreneur rejects the project then the payoffs are $-J$ if it is good and K if it is bad where $J, K \geq 0$. Given these payoffs the entrepreneur has an incentive to examine the project before deciding to accept or reject it. The entrepreneur can hire "screens" to use the terminology of Sah and Stiglitz (1984). These screens can be thought of as managers. The screens are identical and can be hired at a cost $c > 0$ to provide opinions as to the type of the project. These opinions have a probability $p \in (1/2, 1)$ of being correct. Thus the screens' opinions are informative but subject to error. The problem faced by the entrepreneur is, then, to choose how many screens or managers to hire and what acceptance/rejection decision rule to adopt.

The entrepreneur's problem is the choice of an optimal statistical

experiment. Each screen's opinion is a realization of a Bernoulli trial with a parameter $\theta \in (p, 1-p)$. We will assume that the screens form their opinions independently and so the opinions of $n \in \{0, 1, 2, \dots\}$ screens form n realizations from n identical and independent Bernoulli trials. From these n observations the entrepreneur can estimate whether the parameter of those trials was p , i.e. the project is good, or $1-p$, i.e. the project is bad. While more observations would reduce the probability of making a false inference about θ , the marginal cost of additional observations (managers) is positive and so the entrepreneur is faced by a true optimization problem.

It can be shown² that the Bayes sequential decision problem facing the entrepreneur has as a solution a procedure which consists of a (Bayes) decision rule and a stopping rule. The decision rule is particularly simple. Let π^n denote the entrepreneur's posterior probability that the proposed project is good after receiving n opinions. The optimal decision rule if he makes an accept/reject decision at that point is simply to choose,

$$\text{Max } (\pi^n P - (1-\pi^n)H, -\pi^n J + (1-\pi^n)K) \quad (1.1)$$

Denote the expected payoff from following this decision rule at n by $\rho_0(\pi^n)$.

This is called the Bayes decision risk of making an immediate decision at n .³

Notice that it only depends on n through the effect of n on the posterior.

² See, for instance, Berger (1980) Corollary 7.1.

³ The statistical decision literature always expresses payoffs in terms of losses rather than gains. Hence, the Bayes decision risk is the minimized (across decision rules) expected loss of making a decision at n . We have cast everything in terms of gains to make the exposition congruent with the way problems are usually discussed in economics. So for us the Bayes decision risk is the maximized expected payoff from making a decision at n .

The stopping rule is more complex. Let $\rho^{\oplus}(\pi^n)$ be defined by,

$$\rho^{\oplus}(\pi^n) = \text{Max} \{ \rho_0(\pi^n), \rho^*(\pi^n) \}$$

where $\rho^*(\pi^n)$ is the minimum Bayes risk (maximum expected payoff) across decision procedures which take at least one more observation. The optimal stopping rule can be expressed as, stop sampling at the first n such that,

$$\rho_0(\pi^n) = \rho^{\oplus}(\pi^n)$$

This rule, because of its dynamic programming formulation, is often very difficult to solve for explicitly. Fortunately, for the problem analysed here the rule is tractable. Figure 1.1 plots $\rho_0(\pi^n)$ and $\rho^*(\pi^n)$. The latter can be shown to be convex, continuous and to take on values of $K-c$ and $P-c$ at $\pi^n=0$ and 1 respectively.⁴ If there exists an interval of π , (π', π'') , in which $\rho^*(\pi^n)$ is above $\rho_0(\pi^n)$ and π^0 lies in it, then the entrepreneur will hire screens. Moreover, the entrepreneur will continue to hire screens (sample) until the first time his posterior probability that the project is good leaves (π', π'') . His optimal decision rule implies that if his posterior crosses $\pi'(\pi'')$ he will reject (accept) the project.

The Bayes sequential procedure for the entrepreneur is, then, the stopping rule described above together with the decision rule given in equation (1.1)⁵ In this case the optimal procedure can be formulated quite simply as a sequential probability ratio test (SPRT) of the following form.

Let z_i be the log of the likelihood ratio of the i th. screen⁶, i.e $z_i \equiv$

⁴ See Berger, 1980, p.332, Lemma 1.

⁵ For a formal proof of this see Theorem 8, Berger (1980).

⁶ Let x_i denote the opinion of the screen. The likelihood

$\ln[(1-p)/p] \equiv -z$, if the opinion is that the project is bad and $z_i \equiv \ln[p/(1-p)] \equiv z$ otherwise. Let $n > 0$ be the number of screens that have given opinions and let $Z(n)$ be the sum over the n screens of z_i . The SPRT is,

if $Z(n) \geq b$ stop sampling and reject the project
 if $Z(n) \leq -a$ stop sampling and accept the project
 otherwise hire another screen.

In this procedure,

$$-a = \ln[(1-\pi')\pi^0/\pi'(1-\pi^0)]$$

$$b = \ln[(1-\pi')\pi^0/\pi'(1-\pi^0)].$$

Thus the decision rule is nothing more than a qualified majority voting system though one in which the majorities needed to accept and reject a proposed project will usually be different.⁷ In Figure 1.2, we plot two hypothetical paths of opinions, one leading to rejection of the project and the other to acceptance.

The preceding analysis has already produced two important findings about the optimal management organization. First, we have seen that the project will be reviewed sequentially and so the management team will be structured hierarchically in the sense that the project proposal will be evaluated by one manager and then passed up to another manager and so on until a decision is made. Notice that the stopping boundaries $-a$ and b may be greater in absolute value than z . Let the minimum of the absolute values of the two boundaries be $k_0 z^B$. If $k_0 > 1$, then the entrepreneur can let k_0 screens initially evaluate the

ratio of θ_1 to θ_0 given the i th. screen's opinion is $L = \frac{f(x_i|\theta_1)}{f(x_i|\theta_0)}$.

⁷ An important empirical question not pursued here is whether, in fact, firms use different majorities for acceptance and rejection of projects.

⁸ It is easy to show in this case that the optimally chosen boundaries will be integer multiples of z .

project simultaneously without violating the Bayes procedure because at least k_0 evaluations will have to be carried out before it becomes optimal to make a decision. In this case the team is structured in a hierarchical pyramid with a base size k_0 . The size of the second tier will depend on the outcome of these first k_0 screenings. The log likelihood ratio after these screenings is $Z(k_0)$. Denoting the minimum of the absolute values of $[-a-Z(k_0)]$ and $[b-Z(k_0)]$ by k_1 we see that the size of the second tier of the management team will be equal to k_1 , i.e. the minimum number of additional evaluations needed before it could possibly be optimal to decide on the project. This process can then be repeated to get the size of the subsequent tiers. Thus, both the number of tiers in the organization and their size are stochastic. Of course, this characterization of the internal structure does not carry over to cases when the entrepreneur must precommit to the structure and number of screens.

The second fact about the optimal team is that no single manager is the decision maker except in the degenerate case where $b=z$ and $-a=-z$. Each manager provides an opinion and the organization provides the decision. The decision rule aggregates the individual managers' opinions via a simple voting mechanism that uses a qualified majority rule though the precise nature of the qualifications depend on whether the decision is to accept or reject the policy.

B: Optimization

We examine next the precise dependency of the expected size of the optimal management team and the accuracy of its decisions on the parameters of the entrepreneur's problem. Formally, the entrepreneur's problem is to choose a and b so as to maximize his expected payoff, $r(\pi, d)$. Thus,⁹

⁹ To ease notation, from now on we will denote the proportion

$$\begin{aligned} \text{Max}_{a,b} r(\pi, d) = & \pi [P - \beta(\theta_0)(P+J) - cE_0N] + \\ & (1-\pi)[-H + \beta(\theta_1)(H+K) - cE_1N] \end{aligned} \quad (1.2)$$

where $\beta(\theta)$ is the probability of rejecting the project conditional on the value of θ and E_0N and E_1N are, respectively, the expected number of screenings prior to a decision being made conditional on θ_0 or θ_1 . More precisely it can be shown¹⁰ that the β and EN terms are equal to

$$\beta(0) = \frac{e^a - 1}{e^{a+b} - 1} \quad (1.3a)$$

$$\beta(1) = \frac{e^b(e^a - 1)}{e^{a+b} - 1} \quad (1.3b)$$

$$E_0N = \frac{-1}{z(2p-1)} [-a + (a+b)\beta(0)] \quad (1.3c)$$

$$E_1N = \frac{1}{z(2p-1)} [-a + (a+b)\beta(1)] \quad (1.3d)$$

Equations (1.3a) and (1.3b) show that $\beta(1) > \beta(0)$ which means that if the firm follows the Bayes procedure the probability that it will reject a bad project (accept a good project) is greater than the probability that it will reject a good project (accept a bad project). The probabilities of rejection depend on both a and b in ways that are intuitively sensible. If a (the

of the population of the projects that are good by π rather than
 10. See Berger (1980), pp. 341-342.

majority needed for acceptance) is raised the probability of rejecting either kind of project rises while raising b (the majority needed for rejection) raises the probability of accepting either kind of project.

The expressions for the expected number of screenings, (1.3c) and (1.3d) are, unfortunately less transparent. Certainly raising a or b or both will raise the expected number of screens used no matter whether the project is good or bad. However, ranking the relative sizes of E_0N and E_1N is difficult except in the case where $a=b$. In this case $E_0N = E_1N$. When a is close to b then it can be shown that if $b > a$, a good project will be expected to be screened more often than a bad project while if $b < a$ this is reversed.

Turning now to the necessary conditions for the optimum choice of the stopping bounds we have

$$r_a = -\pi(P+J)\beta_a(0) + (1-\pi)(K+H)\beta_a(1) - \quad (1.4a)$$

$$\frac{c\pi}{z(2p-1)} [-1 + \beta(0) + (a+b)\beta_a(0)] +$$

$$\frac{c(1-\pi)}{z(2p-1)} [-1 + \beta(1) + (a+b)\beta_a(1)] \leq 0$$

$$r_b = -\pi(P+J)\beta_b(0) + (1-\pi)(K+H)\beta_b(1) - \quad (1.4b)$$

$$\frac{c\pi}{z(2p-1)} [\beta(0) + (a+b)\beta_b(0)] + \frac{c(1-\pi)}{z(2p-1)} [\beta(1) + (a+b)\beta_b(1)] \leq 0$$

$$ar_a = br_b = 0; \quad a, b \geq 0 \quad (1.4c)$$

In Appendix 1 we collect all the relevant partial derivatives. Regarding r_a , we note that the only positive term in the whole expression is $(1-\pi)(K+H)\beta_a(1)$. Increasing a raises the expected number of screens that will be hired and so expected screening costs. It also lowers the probability of

TABLE 1.1

COMPARATIVE STATICS

	$r_{ab} > 0$	$r_{ab} < 0$
da/dc	-	- -/+ or
db/dc	-	- +/-
$da/d(P+J)$?	-
$db/d(P+J)$?	+
$da/d(H+K)$?	+
$db/d(H+K)$?	-
da/dp	+	+ +/- or
db/dp	+	+ -/+
$da/d\pi$?	-
$db/d\pi$?	+

accepting a good project. The only benefit from raising a is that it raises the probability of rejecting a bad project. Thus, unless the cost of accepting a bad project relative to rejecting it, $(K+H)$, is substantial, the optimal a will be zero and no screening will occur.

With respect to r_b , the only positive term is $-\pi(P+J)\beta_b(0)$. Again, increasing b raises expected screening costs and raises the probability of accepting a bad project. Its only positive effect on expected profits is through lowering the probability of rejecting a good project and so unless the cost of rejecting a good project relative to accepting it, $(P+J)$, is high, b will be set to zero and no screening will take place.

C: Comparative Statics

There is, of course, no hope for solving (1.4a) and (1.4b) explicitly for the optimal bounds a and b . However, additional insight into the problem can be obtained by analyzing the comparative statics effects of changes in some of the underlying parameters. Such comparative statics exercises are also valuable because they produce testable hypotheses about the responses of management organizations to changes in the environment in which they operate. It turns out, unfortunately, that at the level of generality used in this section there are only a few unambiguous predictions that can be made.

The comparative statics results for the firm are presented in Table 1.1 below. (See Appendix 1 for the calculations). Table 1.1 demonstrates the importance of the cross partial of the expected profit function in the comparative static calculations. It is therefore worthwhile exploring why r_{ab} can be of either sign. r_{ab} can be written as

$$r_{ab} = -\pi(P+J)\beta_{ab}(0) + (1-\pi)(K+H)\beta_{ab}(1) - c\pi E_{ab,0}N - c(1-\pi)E_{ab,1}N \quad (1.5)$$

From (1.5) we see that the sign of r_{ab} depends, in part, on the signs of the cross partials of the rejection probabilities on the one hand and those of the screen demands, E_0N and E_1N , on the other. We can view the entrepreneur as employing a management organization as a technology or production function that use a and b as "inputs" and produces two "outputs" or products. These are accuracy in accepting good projects, $[1-\beta(0)]$, and accuracy in rejecting bad projects, $\beta(1)$. A little calculation shows that $\beta_{ab}(0) < 0$ and $\beta_{ab}(1) > 0$. Thus a and b are complements in the production of both outputs. Expected revenue is linear in the two outputs. However, unlike the traditional theory of the firm, costs, or here expected costs, are not linear in the two inputs. In fact it can be shown that $E_{ab,0}N > 0$ and $E_{ab,1}N > 0$. Thus the two "inputs" are also complements in the expected cost function. Heuristically then, one can say that r_{ab} is more likely to be positive(negative) the higher(lower) is the degree of complementarity between a and b in producing accuracy relative to their complementarity in the expected cost function.

From Table 1.1 we see that the values of r_{ab} that give ambiguous signs for the comparative statics differ across the set of parameters. Notice that when c and p are changed, the parameter change has no effect on the relative impacts of marginal changes in a or b on either the accuracy of the acceptance and rejection decisions nor on their relative marginal impacts on the expected cost of screening. Thus these parameter changes give no direct incentives to substitute a for b or vice versa. The direct impact of these parameter changes is just to change the marginal cost of accuracy. In the case of a

rise in c which raises the expected cost of accuracy at the margin, the entrepreneur wishes to respond by lowering the expected number of screens used. If he attempts to do this by lowering $b(a)$, then, with $r_{ab} > 0$ this will lower the marginal product of $a(b)$ providing an additional reason to lower $a(b)$. If $r_{ab} < 0$, then lowering $b(a)$ might raise the marginal product of $a(b)$ by enough to actually induce a rise in $a(b)$. Note, however, that both a and b will not rise. In addition, as c rises the demand for screens falls even if $r_{ab} < 0$.

Similar reasoning applies to p . An increase in p represents neutral technological progress in decision making and the entrepreneur will increase the accuracy of the decisions by raising a or b . If $r_{ab} > 0$ then a rise in b will induce a rise in a and vice versa. However, if r_{ab} is sufficiently negative it may be optimal to reduce one (but not both) of the bounds.

Unlike c or p , a change in π does change the relative marginal impacts of a and b on both accuracy and the expected number of screens that will be used. As a rise in π raises the proportion of good projects, it shifts weight onto the costs of erroneously rejecting good projects and the expected cost of screening good projects. These costs can both be reduced by lowering a and raising b and provided $r_{ab} < 0$, these changes will reinforce each other.

Changes in $(P+J)$ and $(H+K)$ clearly change the relative costs of type one and type two errors and so the marginal impacts of a and b on the program. Note that these changes do not make screening more or less costly and so do not give rise directly to a change in the marginal cost of both types of accuracy. Their direct effect is simply to induce a substitution between a and b . Raising $(P+J)$ raises the relative value of accuracy in accepting good projects and so gives rise to an incentive to lower a and raise b . Provided $r_{ab} < 0$, these changes will reinforce each other by systematically driving up

the marginal product of b and lowering that of a . If on the other hand $r_{ab} > 0$, then one might get 'perverse' results. A similar argument applies to the comparative statics for $(H+K)$.

D: Some Special Cases

In order to sharpen our comparative statics results we postulate a rather special case of project screening abilities. In particular, assume that the screens are able to detect a good project with certainty but can detect a bad project only with some probability p which lies in the open interval $(0,1)$. Thus, in this case the errors of the screens are one-sided. The screens only commit Type II errors. The reader can easily work out the polar case when the screens commit only Type I errors.

With this information structure the Bayes procedure is to choose a maximum number of screens, n , and to have the screens review the project sequentially. The optimal stopping rule is to stop the first time a report of 'bad' is received or at n if no bad reports have been received. If screening stopped because of a bad report then the optimal decision is to reject the project while if screening stopped without a bad report the optimal decision is to accept the project.¹¹ The advantage of this screening technology from our point of view is that it reduces the choice variables of the entrepreneur to one. In terms of the more general model of the previous section this change in the screening technology means that each screen has been given a veto power and so no majority is needed to reject a project while the optimal acceptance majority, a , is now just n .

¹¹ If n was chosen so that the optimal decision at n was to reject then clearly the proposed project will be rejected with certainty and this procedure would be dominated by never screening and rejecting all projects.

Given this new screening technology the probability of rejecting a good project, $\beta(0)$, is zero and so the expected profits of the entrepreneur become (c.f. equation 1.2),

$$r(n) = \pi[P - cn] + (1-\pi)(-H + (K+H)[1 - (1-p)^n] - cE_1N) \quad (1.6)$$

where $[1-(1-p)^n]$ is $\beta(1)$, the probability of rejecting a bad project. The entrepreneur's problem is to maximize (1.6) by the choice of a nonnegative integer n . A little calculation shows that this problem is concave and that a necessary condition for it to be optimal to screen at all is that $[(1-\pi)p(H+K) - c]$ be positive. If this condition does not hold then the expected benefit from screening once, $p(H+K)$, does not outweigh the cost, c , and so $n=0$ dominates $n=1$. Given the concavity of the problem this means that $n=0$ dominates $n>0$ and so the entrepreneur will not hire screens.

Denote $r(n+1)-r(n)$ by $\Delta(n)$. Then,¹²

$$\Delta(n) = -\pi c + (1-\pi)(H+K)p(1-p)^n - (1-\pi)c(1-p)^n \quad (1.7)$$

The optimal choice for the entrepreneur will be one of the two adjacent integers to n^* , the value of n for which $\Delta(n)=0$. At this point the marginal increase in expected screening costs (the sum of the terms in c in (1.7)) just equals the expected marginal reduction in the probability of erroneously accepting a bad project times the cost of such an error. Most of the comparative statics for this problem are straightforward. (See the Appendix 1

¹²

$$E_1N = p \sum_{i=1}^n i(1-p)^{i-1} + n(1-p)^n$$

Table 1.2

Effect of Parameter Changes on the Optimal Number of Screens

Type of Errors Made by Screens

Parameter	Type II	Type I
P	0	+
J	0	+
H	+	0
K	+	0
c	-	-
π	-	-
ρ	+	+

for details). Raising π or c lowers $\Delta(n)$ and so results in a fall in n^* while a rise in $(H+K)$, by raising the marginal benefit from screening, raises $\Delta(n)$ and so raises n^* . A rise in the accuracy of the screens, p , lowers the expected marginal screening costs should the project drawn happen to be bad by increasing the probability that such a project will be detected early on in the screening process. This causes n^* to be raised. These comparative statics as well as those for the case in which screens make only Type I errors are summarized in Table 1.2.

We turn now to the task of embedding these management organizations in a market equilibrium setting.

2: FIRM STRUCTURE AND MARKET STRUCTURE

As noted in the introduction, one of the advantages of the approach to optimal management organizations in this paper is that management organization is intimately connected to market parameters and the performance of management organizations, in turn, effects the long-run structure of the market as measured by such standard indices as the concentration ratio. In contrast, the principal-agent approach to firm organization has yet to connect firm structure and the nature of intra-firm relationships to such product market features as the number of firms.¹³ There are many ways of tying the firm's problem of organizing its internal structure to market parameters. For

¹³ A partial exception to this is Willig (1985) though even in this paper the direction of causation is from exogenously given product market conditions to management performance with no feedback from the latter to the structure of the product market. See, however, Fershtman and Judd (1984) and Vickers (1985). See also Nalebuff and Stiglitz (1983) who analyse market competition as a vehicle for assessing management performance.

example, the payoff to carrying out good projects could be made to depend on the number carried out by the industry. Or the costs of carrying out any type of project could be increasing for the industry in the number of projects being carried out. Alternatively the cost of hiring managers could be endogenized.

Ideally we would like to analyse the links between firm and market structure for the general model of the Section 1(b). Unfortunately at that level of generality no qualitative results are available. This is not surprising given the comparative statics results derived in the previous section. Each firm's expected demand for managers or screens (denoted by s) is just $\pi E_0 N + (1-\pi) E_1 N$. At any given cost of screens this means that the impact of a change in some arbitrary parameter x [other than p or π] on the firm's expected demand for screens, i.e. the expected size of its management organization, is given by,

$$\frac{ds}{dx} = \pi \left[\frac{\partial E_0 N}{\partial a} \frac{da}{dx} + \frac{\partial E_0 N}{\partial b} \frac{db}{dx} \right] + (1-\pi) \left[\frac{\partial E_1 N}{\partial a} \frac{da}{dx} + \frac{\partial E_1 N}{\partial b} \frac{db}{dx} \right]$$

Only in cases where the signs of da/dx and db/dx are the same can we hope to sign ds/dx . A glance at Table 1.1 containing the comparative statics results for the general case shows that only for the parameter p , the precision of individual screens, when $r_{ab} > 0$ did this hold. Unfortunately, p also enters directly into the expressions for $E_0 N$ and $E_1 N$ which greatly complicates ds/dp and prevents us from obtaining qualitative results.

The cause of these ambiguities is that the entrepreneur has two choice variables corresponding to the majorities necessary for acceptance and rejection of the project. As we would expect from production theory, in cases

where there are more than one variable factors, very little can be said about how these choices vary with parameter changes unless the technology is made quite specific. In our case we cannot arbitrarily pick a decision-making technology for the entrepreneur. However, we are at liberty to specify the abilities of the screens. This we have done in Section 1(c). The discussion that follows maintains the assumption that screens only make Type I or Type II errors but not both.

As mentioned earlier, there are two natural ways to close the models in Section 1 which will result in simultaneous determination of the structure of the firm and the number of management teams in operation. One way is to focus on the final product market and to assume that payoffs to accepted projects decline in the number of projects accepted. Alternatively, one can focus on the input market and assume that the supply function of screens to the industry is upwardsloping. In either case, the equilibrium number of management teams will be determined by a zero profit condition as long as there is free entry. We deal with each method of closing the model in turn.

A: Market Equilibrium: Preliminaries

In this subsection, we describe the equilibrium structure and number of firms or management teams when the return from projects depends on the number of projects accepted and the cost of screens depends on the number hired. In principle, one could make the returns on accepting good and bad projects independent functions of the number of good projects accepted and the number of bad projects accepted. However, to keep the analysis manageable we will restrict ourselves to the case in which the payoff to accepting a good project is declining in the number of good projects accepted only and the payoff to bad projects is independent of the number of either kind of project accepted.

Let there be a continuum of firms or entrepreneurs that could potentially operate in the industry and let q represent the measure of firms that are actually active in the industry. We will assume throughout that parameter values are such that $q < 1$, that is that there is always a fringe of potential entrants. Similarly, let there be a continuum of screens able to work in the industry and let the ratio of the potential firms in the industry to the potential screens be $\alpha < 1$ and let α be sufficiently low that for the parameter values under consideration there will always be unused screens. The measure of screens willing to work at a price of c is ℓ . The supply of managers or screens is a non-negatively sloped function of the price of screens, c , denoted by $\ell = \vartheta(c)$. The expected demand for screens by a representative firm or the size of a firm's management hierarchy, s , is given by,

$$s = \pi n^* + (1-\pi) \left[p \sum_{i=1}^{n^*} (1-p)^{i-1} + n^* (1-p)^{n^*} \right] \quad (2.1)$$

Given the continuum of firms, the actual demand for screens by the industry, S , is just $\alpha s q < 1$.

Let there be continua of both types of projects and the parameters of the model be such that the supply of neither kind of project is exhausted. We assume that each management team can examine one project per period and so the measure of good projects accepted each period is proportional to the measure of teams in operation times the probability of accepting a good project. Without loss of generality we can set the factor of proportionality to one. Denote the measure of good projects accepted by v .

The inverse "demand" function which relates the net benefit of good projects to the number accepted is denoted by $D(v; z)$ where z is a shift

parameter. $D_v < 0$ and $D_z > 0$. The expression for v and its equilibrium level depends on the screening technology:

$$v = \begin{cases} q\pi & \text{if Type II errors only} \\ q\pi[1-(1-p)^n] & \text{if Type I errors only} \end{cases} \quad (2.2)$$

The conditions for market equilibrium are,

$$P^* = D(v^*; z) \quad (2.3a)$$

$$\alpha s q = \phi(c) \quad (2.3b)$$

and

$$r(n^*(P^*, c^*; .)) = 0 \quad (2.3c)$$

The first of these conditions is that supply must equal demand for good projects and the second is that supply and demand for screens must be equal. The third states that in an equilibrium with free entry, competitive firms must earn zero expected profits.

B: Market Equilibrium with Decreasing Payoffs.

Here we investigate the structure of market equilibrium when the supply of screens is perfectly elastic. Our main interest is in the effects of changes in the underlying parameters on the equilibrium size of the management teams and on the equilibrium number of active firms (teams) in the industry.

Figure 2.1

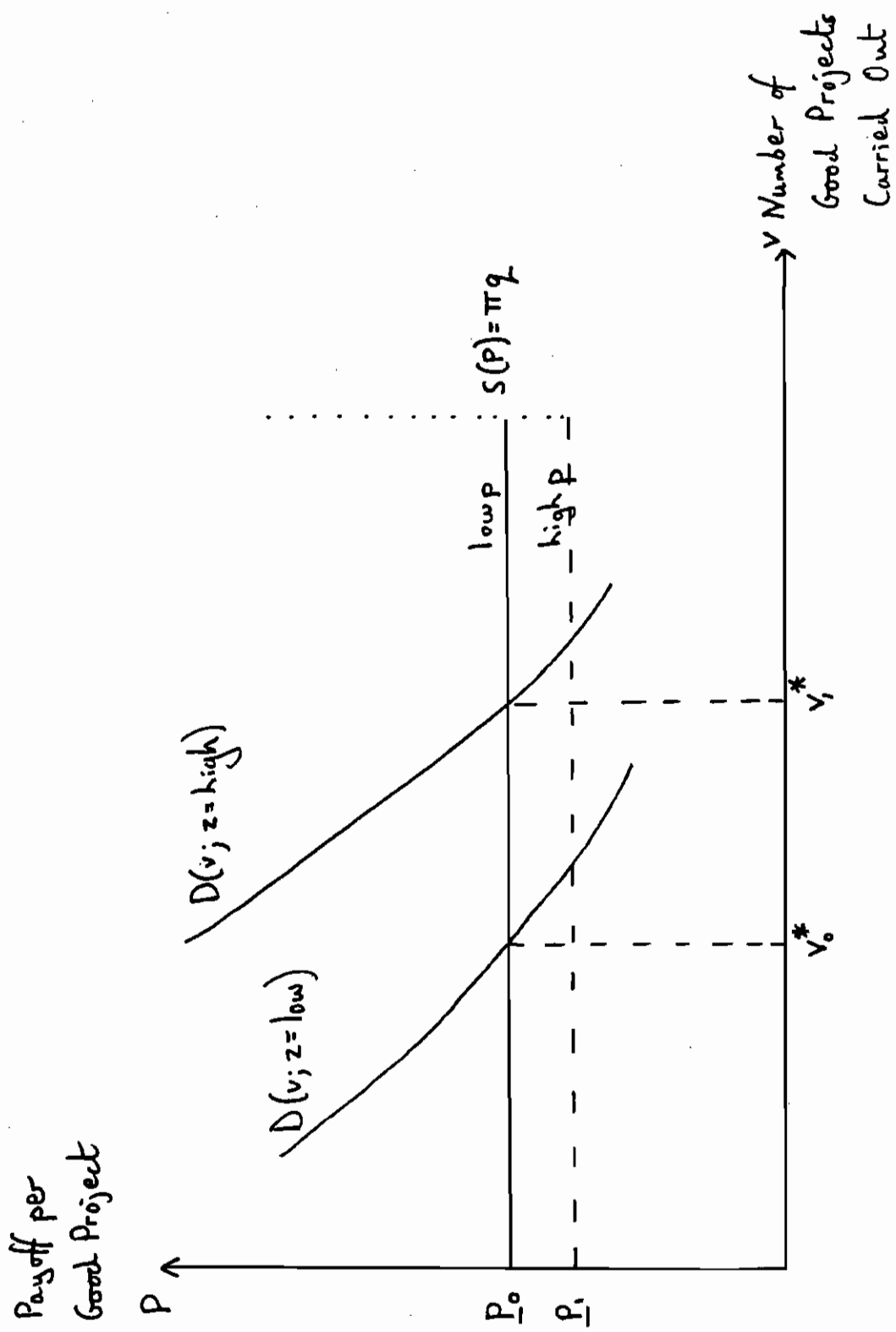


Table 2.1

Impact of Parameter Changes on Competitive Equilibrium Firm Size and Number of Firms when the Payoff to a Good Profit is Endogenous.

	s^*	q^*
p	?	+
π	?	≥ 0 as $(\varepsilon_{\sigma_{\pi}/\sigma_p}) - 1 \geq 0$
z	0	+

In the case when $v = q\pi$, the comparative statics are particularly simple. The relevant comparative statics effects are given in Table 2.1. A diagrammatic analysis is provided in Figure 2.1. There, \underline{P} is the minimum payoff from accepting a good project that will yield a firm non-negative expected profits at the optimal choice of n . Thus for $P < \underline{P}$, the supply of good projects is zero. For $P > \underline{P}$ the supply of good projects will be bounded only by the supply of screens which we have assumed to be sufficiently large never to be reached. Thus the only equilibrium value for P is \underline{P} . An increase in p , screening precision, makes screening cheaper, i.e. the cost of information falls. This causes each firm to raise the maximum number of screens it is willing to use, n^* . The firm's expected demand for screens may or may not rise as a result of this. On one hand if the project is good the firm will use more screens but on the other hand it will detect bad projects earlier on average. Which effect dominates depends on the specific values of the parameters. The increase in p will definitely raise expected profits and so will shift \underline{P} down and v^* up. Since π stays constant this means that q^* must also increase, i.e. there is entry into the market. An increase in z , that is a shift up in the demand function, leaves the supply function unaffected and so results in a rise in q^* but leaves the size of the management team unaffected.

The effect of a change in π is more complicated. From the zero-profit condition we have

$$dP^*/d\pi = -r_{\pi}/r_P.$$

Hence,

$$q + \pi dq/d\pi = -(r_{\pi}/r_p)[dD^{-1}(\cdot)/dv]^{-1}$$

$$= q(\sigma_{\pi}/\sigma_p)\epsilon$$

where σ_i , ($i=\pi, P$) is the elasticity of the expected profit function with respect to i and ϵ is the elasticity of demand for good projects. Hence,

$$(dq/d\pi)(\pi/q) = \epsilon(\sigma_{\pi}/\sigma_p) - 1.$$

This is more likely to be positive, and hence entry more likely, if the demand function is highly elastic, for example.

Calculations of the effects of a change in a parameter other than p on the optimum size of the screening unit are fairly straightforward because a change in P^* induced by the parameter change has no impact on n^* when screens make only Type II errors.

C: Market Equilibrium with an Increasing Cost of Screens

To isolate the consequences of an imperfectly elastic supply of screens, we assume that the demand for good projects is perfectly elastic. We are interested in how the competitive equilibrium values of q and s , q^* and s^* , respond to parameter changes, i.e. changes in P , H , K , p and π . Because the zero expected profit condition (2.3c) must hold in equilibrium, we can establish immediately the impact of a parameter change on the equilibrium level of c . It can be shown that expected profits are increasing in P , K , π and p independently of whether the screens make only Type I or Type II errors. Thus, to maintain zero expected profits, c^* will increase with P , K , π and p . In the case where screens make Type I errors profits are decreasing in J and

Table 2.2

Elasticities of the Firm's Expected Demand for Screens

<u>Elasticity</u>	<u>Type of Error Made by Screens</u>	
	<u>I</u>	<u>II</u>
σ_{sc}	-	-
σ_{sP}	+	0
σ_{sJ}	+	0
σ_{sH}	0	+
σ_{sK}	0	+
$\sigma_{s\bar{I}}$?	?
σ_{sp}	?	?

independent of H . Conversely, when the screens make Type II errors expected profits are independent of J and decreasing in H . Thus if the screens make Type I (Type II) errors c^* will move inversely with (be independent of) P and be independent of (move inversely with) H . Denote the elasticity of c^* with respect to a parameter x by μ_{cx} .

Knowing the signs of μ_{cx} enables us to determine the behavior of the equilibrium firm size, s^* . Denote the elasticity of s^* with respect to i , $i=P, K, H, J, c$, by e_{si} . Then it can be shown that,

$$e_{si} = \sigma_{si} + \sigma_{sc}\mu_{ci} \quad (2.4)$$

where σ_{si} is the elasticity of the firm's expected demand for screens, s , with respect to i . Note that $\sigma_{sc} < 0$ for both types of screens. The sign of σ_{si} can be found from dn^*/di and ds/dn . These elasticities are shown in Table 2.2. Consider the elasticities in the case where screens make Type II errors. These signs follow immediately from the comparative statics of Section 1(c) with the exception of the ambiguous sign of $\sigma_{s\pi}$. Although $dn^*/d\pi < 0$, π enters the expression for s directly and in an offsetting manner with the result that the sign of $\sigma_{s\pi}$ is indeterminate.

Combining the signs of σ_{si} , σ_{sc} and μ_{ci} gives the sign of e_{si} . Given that $\sigma_{sc} < 0$, only when σ_{si} and μ_{ci} have opposite signs will we be able to obtain a definite sign for e_{si} . Thus, for example, when screens make Type II errors a rise in H will directly raise n^* ($\partial n^*/\partial H > 0$) and lower expected profits. The latter implies that c^* must fall to bring expected profits up to zero which in turn will further increase n^* . Thus n^* will rise and with it s^* . The results for the sign of e_{si} are shown in Table 2.3.

Given the impact of the parameter changes on s^* we can derive the impact

Table 2.3

Effects of Parameter Changes on Equilibrium Firm Size (s^*) and The Equilibrium Size of the Industry (q^*) When Payoffs are Exogenous

Parameter	Errors made by screens			
	Type I		Type II	
	s^*	q^*	s^*	q^*
P	?	?	-	+
J	+	-	0	0
K	-	+	?	?
H	0	0	+	-
p	?	?	?	?
Π	?	?	?	?

on q^* by differentiating (2.3b). Letting $\eta > 0$ denote the elasticity of the supply curve of screens and ϵ_{qi} be the elasticity of the measure of firms in the industry at equilibrium with respect to parameter i , then (2.3b) implies

$$\epsilon_{qi} = \eta \mu_{ci} + e_{si} \quad (2.5)$$

When μ_{ci} and e_{si} have the same sign we can state definitely the impact of the parameter on the number of firms in the industry. These results are shown in Table 2.3. When the signs of these two elasticities are not the same then qualitative results are not available. However, (2.5) shows that the larger is the elasticity of the supply of screens, η , the more important will be the impact of μ_{ci} . The intuition for this is as follows. If for a particular parameter, x , $e_{sx} > 0$, that is the firm size rises with x , then at the original q^* the demand for screens will rise and with it c^* . The size of the rise in c^* will obviously depend on the elasticity of the supply function of screens. If the increase in x raises expected profits at the original c^* then we know that at the new equilibrium c^* must be higher. However, if η is small, the rise in s^* may already have driven c above its new equilibrium value and so there will have to be exit from the industry to achieve the new equilibrium. Alternatively, if the supply curve of screens is highly elastic, the rise in s^* will have had a minimal effect on c and so there will have to be entry in order to drive c up and expected profits down to zero, i.e. entry will occur when $\mu_{cx} > 0$.

The results in Table 2.3 show that in all cases in which qualitative results are available, entry will be accompanied by a decline in the optimal size of the management organization. Take the case of screens that make Type II errors. These results suggest that, for instance, a sudden increase in the

demand for the output that will come from doing a good project (e.g. discovering an oil well) will, as one would expect, lead to entry into the industry in which such projects can be found (prospecting) but will also result in a smaller equilibrium size of the management hierarchies in the industry. Conversely, a rise in H will cause exit from the industry and a rise in equilibrium management size.

Our model seems to be consistent with observed behavior. Again assume that screens make Type II errors. Consider a firm that is about to launch a new product. The reception of a new product is, in this industry, hard to predict and extreme in the sense that either there will be a very large demand for the product or zero. Women's fashion clothes or a new brand of beer might be examples. Now consider two different cost environments. In one there are large fixed costs of production but these fixed costs are not sunk. This means that while the profits if the product is successful (the project is 'good') are large (P is big), the losses from an unsuccessful product will be small (H small) because the fixed costs, e.g. the lease on a factory, can be avoided. The other cost environment is one in which the fixed costs are sunk costs, e.g. specialized capital equipment, specialized R & D, large-scale and long-lived advertising. Although this leaves P unaltered it raises H , the "downside" risk. The model in this paper predicts that the industry equilibrium under the second cost environment will have management hierarchies larger than those in the industry equilibrium under the first cost environment. This seems to be consistent with what we observe, e.g. the small management hierarchies in the women's fashion industry compared with much larger hierarchies in the consumer goods industries. A corollary of this prediction is that decisions on product launches will take longer in the large sunk cost industries which also seems to be correct though, of course, many

other factors are also involved.

Finally, we might note that the analysis above also tells us how management performance will change between industry equilibria. Given the ability of the screens, the appropriate measure of the technical quality of the performance of the management hierarchies is the probability that they will reject a bad project, i.e the size of $\beta(1)$.¹⁴ The derivative of $\beta(1)$ with respect to n is positive and so the impact of a parameter change on the industry equilibrium level of management performance has the same sign as the impact of the parameter change on the equilibrium level of n^* and so, except for changes in p , s^* . From the results for s^* we can see that a rise in H raises the industry equilibrium performance of firms' managements while a rise in P lowers management performance. Changes in the remaining parameters have ambiguous effects on equilibrium management performance.

3: THE MONOPOLY FIRM

The previous section showed how an industry's structure and the management organizations used were jointly determined by the decision making technology available and the payoffs facing the firms. In this section we explore the impact of a lack of competition between firms on the size and performance of the management hierarchies used. We do this by replacing the set of potential firms by one firm that can set up many establishments. The screens make only Type II errors and the supply of these screens is perfectly elastic at a cost $\bar{c} > 0$. The payoff to doing a good project is endogenous and

¹⁴ In the case of screens that make Type I errors the appropriate measure is the probability that the firm will carry out a good project, $[1 - \beta(0)]$. This is increasing in s^* .

given by $P = D(v; z)$ as in the previous section where $v = \pi q$. Absorbing π into the functional form, P can be expressed more compactly as $f(q)$ where $f'(q) < 0$.

Using the notation of the previous section we know that the competitive equilibrium levels of q , s and n , denoted by q^C , s^C and n^C respectively, satisfy

$$\Delta(n) = 0 \quad (3.1)$$

$$r(n) = 0 \quad (3.2)$$

The monopolist maximizes expected profits, $qr(n)$, and so his first order conditions are,

$$q\Delta(n) = 0 \quad (3.3)$$

$$r(n) + \pi f'(q)q = 0 \quad (3.4)$$

Denote the monopolist's optimal choices by q^m , s^m and n^m . Given that P does not enter $\Delta(n)$, we see immediately from (3.1) and (3.3) that $n^m = n^C$ and so $s^m = s^C$. Denote by \bar{s} the optimal value $s^C = s^C$. Using the fact that $n^m = n^C$ in (3.2) and (3.4) shows that $q^m < q^C$. Thus in the case where the payoff to good projects is endogenous but the cost of screens exogenous, the effect of monopolizing the market is just to cut back on the number of screening establishments but to leave their size, and so their decision making efficiency, unaltered.¹⁵ It can be shown, however, that minor alterations to

¹⁵ The same result occurs when the firm is a monopsonist in the market for screens and faces exogenous payoffs to projects. See Appendix 2 for details.

how the payoff of projects is endogenized can lead to variations in the size and so efficiency of the monopolist's screening establishments. For instance, assume that the monopolist faces an increasing convex cost of carrying out projects because of a less than perfectly elastic supply curve of a factor of production. This means that the net payoff of accepting either a good or a bad project is decreasing in the number of projects accepted. Then in this case the monopolist will choose a team of screens for each establishment that is larger than those chosen by firms in the competitive equilibrium. See Appendix 2 for details.

We now consider in more detail the case where the payoff to a good project is endogenously determined. We maintain the assumption that this payoff depends only on the number of good projects that have been accepted. In addition, assume that the inverse supply curve of screens is linear with a slope $b \geq 0$ and let it pivot around the competitive equilibrium demand for screens when $b=0$, $(\bar{c}, q^c \bar{s})$. Thus the inverse supply curve of screens is

$$c = q^c \bar{s} + b(qs - q^c \bar{s}) \quad (3.5)$$

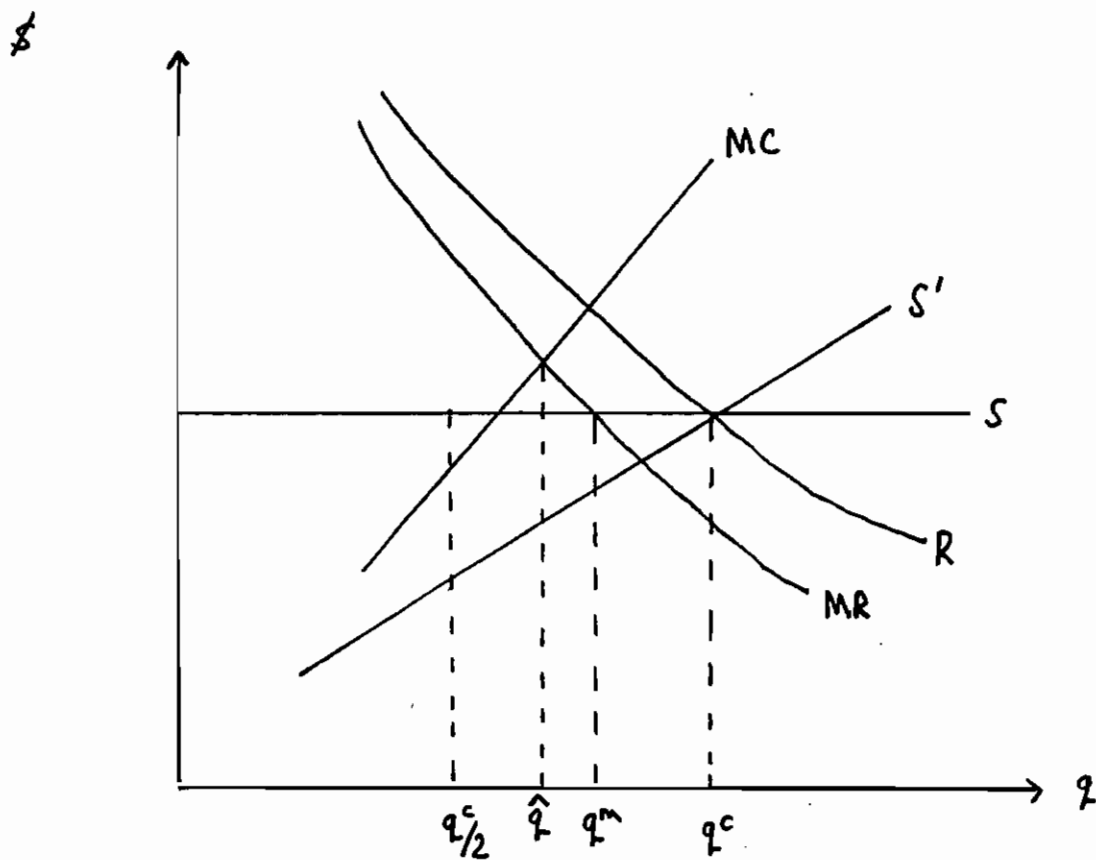
The competitive equilibrium is invariant with respect to b as can be seen from (3.1) and (3.2) but the same is not true of the monopolist's choices. His first order conditions are now

$$q[\Delta(n) - sqb\partial s/\partial n] = 0 \quad (3.6)$$

$$r(n) + \pi f'(q)q - s^2 bq = 0 \quad (3.7)$$

Totally differentiating (3.6) and (3.7) and evaluating the resulting

Figure 3.1



differentials at $b=0$, and so $q^m \bar{s}$, gives

$$\frac{dq}{db} = \frac{\bar{s}^2(2q^m - q^c)}{\pi(f'(q^m)[1+f'(q^m)] + f''(q^m)q^m)} \quad (3.8)$$

$$\frac{dn}{db} = \frac{q^m \bar{s} \partial s / \partial n (2q^m - q^c)}{\Delta^2(n)} \quad (3.9)$$

By the concavity of the monopolist's problem the denominators of (3.8) and (3.9) are negative and so both dq/db and dn/db take the sign of $(2q^m - q^c)$.

The fact that whether monopolization raises or lowers the size of the management team should hinge on the sign of $(2q^m - q^c)$ at first looks strange but is, in fact, to be expected. The intuition behind this result is captured in Figure 3.1. Hold s fixed at \bar{s} . The price of screens then becomes a function of q alone and so the horizontal axis measures q . In the case where $b=0$, the cost of a screen is \bar{c} and the expected cost of a screening establishment is $\bar{c}\bar{s}$ and is independent of q . This is shown by curve S . The expected revenue of such an establishment is $\pi f(q) - A(\bar{s})$ where $A(\bar{s})$ is $(1-\pi)(-H + (H+K)[1-(1-p)^n])$. Curve R on the Figure represents such a revenue function. At the competitive equilibrium the expected cost of a screening establishment must equal its expected revenue, i.e. $S=R$. In contrast the monopolist sets marginal revenue equal to marginal cost which when $b=0$ gives $MR=\partial R/\partial q=S$. When b becomes positive, and so the cost function becomes S' for instance, then the marginal cost function (MC), because of the linearity of the cost function, crosses S at $q^c/2$. The monopolist, if he were restricted to keep $s=\bar{s}$, would choose $q=\hat{q}$, at the point where $MR=MC$. Say this intersection occurred at $S=MC$ then $\hat{q}=q^m$ and $q^m=q^c/2$. Moreover, at that constrained

equilibrium the shadow cost of a screen to the monopolist is just \bar{c} and so even when the monopolist is free to choose s he will in fact choose $s=\bar{s}$ and so $q=q^m$. Alternatively, as is the case in Figure 3.1, $MR=MC$ might occur at a value of q less than q^m . Note that this implies that q^m is greater than $q^c/2$, i.e. where $MC=S$. In this case at \hat{q} the shadow cost of a screen to the monopolist is greater than \bar{c} and so when the monopolist is free to choose any s he will choose an s less than \bar{s} .

Whether q^m is greater than or less than q^c depends on the concavity or convexity of $f(q)$. (See Appendix 2 for details). If $f''(q)=0$, that is $f(q)$ is linear then $q^m=q^c$ and so $dq/db=dn/db=0$. In terms of Figure 3.1, in this case R is linear and so MR cuts S at $q^c/2$ and so $q^m=q^c/2$. Alternatively if $f''(q)<0$, then $q^m<2q^c$ and the optimally chosen s will be less than \bar{s} . So in this case the monopolization of the market when $b>0$ not only lowers the number of establishments and so the number of projects reviewed per period but it also reduces the size of the management teams in those establishments. This implies that the decision making efficiency of the monopolist's establishments will be less than those of the competitive firms. Thus along with the usual welfare loss from monopolization associated with reduced output, in this case there is another distortion caused by an excessive proportion of accepted bad projects.

The most interesting case, is perhaps that when $f''(q)>0$. In this case $q^m>q^c/2$ and so dq/db and $dn/db >0$. Here monopolization leads to the usual reduction in the number of establishments and so output. However, it also leads to an increase in the size of management teams. Moreover, this increase in team size will raise the proportion of accepted projects that are good, i.e. the monopolist's establishments will make fewer errors than the competitive firms.

There is an additional aspect of monopolization of the market that is peculiar to this decision-theoretic approach to the internal structure of the firm. So far in the analysis the composition of the pool of projects, the π , has been treated as exogenous. A more complete analysis would allow for the fact that the management structures adopted by the firms affects the number of projects screened and the composition of the projects withdrawn from, and returned to, the pool.¹⁶ With the screening technology of the previous section, while all good projects that are screened will be accepted and so withdrawn from the pool, only some of the bad projects screened will be accepted and withdrawn. Those bad projects that are screened and rejected are returned to the pool and may be sampled again by another firm.¹⁷

Such multiple sampling of projects is a social waste. If projects that had been rejected once could be costlessly labeled as 'used' then this waste could be avoided. However, such labeling is costly and in a competitive industry it is hard to see who has the incentive to carry it out. Consider, for example, the project being a patent for a product that has not yet been fully developed. A firm leases the patent, carries out the necessary R & D and concludes that the patented product cannot be converted into a marketable commodity. Neither the firm that leased the patent nor the patent owner have

¹⁶ In fact, one of the driving forces behind the Sah-Stiglitz analysis of comparative economic systems is the assumption that in a competitive economy rejected projects have a second chance of being reviewed. Such a second chance is, of course, a social waste unless firms adopt suboptimal rules for accepting or rejecting projects.

¹⁷ Similar concerns arise in the case of the general screening technology of Section 1. In that case good projects can also be returned to the pool. However, the mix of the returned projects will be worse than that in the pool. As the firm will set different values of a and b , and so have a different composition of rejected projects, if it is a monopolist rather than a competitive firm, market structure will effect the quality of the pool of projects.

an incentive to reveal the fact that the patent has been 'rejected' before should a new firm wish to try out the patent. A monopolist, however, has a direct incentive to keep track of which projects its establishments have reviewed and rejected. Consequently, the pool of projects that it samples from will not be polluted by such rejected projects.¹⁸ Thus the inefficiencies caused by the lack of project reputations in a competitive market can be avoided in a monopolized market and this efficiency gain must be set against the traditional welfare losses caused by monopolies.

4: CONCLUSIONS

This paper examines the internal structure of firms from a decision-theoretic viewpoint. Firms employ managers to make strategic decisions such as investment decisions, new product decisions, acquisitions etc.. The firm sets up a management organization and this organization rather than the individual manager makes the decisions. The firm must choose both the size of the management organization and the decision rule it will follow. Decisions will optimally be made by a sequential review of the project at hand and the optimal rule for deciding to accept or reject the project is a qualified majority voting system where the majorities needed for acceptance and rejection will ususally differ.

The decision-theoretic approach leads to a number of testable predictions pertaining to the optimal structure of management as a function of various parameters of the model. For example, our analysis predicts that, cet. par.,

¹⁸ Alternatively, the "project" could be a worker or manager who is being considered for a particular task. Neither the firm considering employing the person nor the person himself has any incentive to reveal the negative outcome of the test.

industries in which the qualities of projects are easier to detect will use smaller management organizations than other industries and will contain a larger number of such organizations. Thus the size and structure of management organizations is affected by screening technology and this in turn effects the market structure in terms of the number of firms. The optimal structure of management and the market equilibrium structure are, therefore, interdependent.

Of course, the link running from market competition to internal efficiency of the firm has long been recognized through the concept of Liebenstein's X-inefficiency. The decision-theoretic structure shows in addition that the structure of the product market is also dependent on the internal structure of the firm. Unfortunately, this important, if not unexpected, insight cannot be pursued very far in a context in which the sequential probability ratio test (SPRT) is the optimal decision procedure because the SPRT rapidly becomes analytically intractable. Hence, the next step is to deal with environments in which the optimal decision procedure is somewhat simpler, e.g. a fixed sample size. Furthermore, firms do appear to precommit to a certain maximum number of managers in the short-run which suggests an environment that generates a fixed sample size as an optimal procedure.

The benefit of such a move would be to provide more clear cut comparative statics results and also to enable the firm to be embeded into more complex markets such as oligopolies. In particular, this move would allow us to investigate some of the welfare issues raised by Sah and Stiglitz (1984) concerning the relative merits of more or less hierarchical decision making. It is crucial in such comparisons that one only compare optimally organized decision making organizations so that one can distinguish between the effects

of hierarchical decision making per se and suboptimal decision making organizations.

Perhaps the most important future task is to merge this analysis of the implications of error prone decision makers with principal-agent considerations. This promises to be a complex undertaking for several reasons. Perhaps the prime complication is that the management organization as a whole produces decisions and so one runs into the usual problems of joint production in a principal-agent setting. These are further complicated by the fact that the firm wishes the managers to provide independent opinions. This means that tying an individual manager's payment to his opinion and the outcome of projects will have the unfortunate consequence of encouraging him to use other managers' opinions in forming his own. The resulting bandwagon effects considerably reduce the information content of the opinions and so the quality of the organization's decisions. How to keep the information content of opinions high, i.e. making managers behave noncooperatively, while providing the correct incentives to be accurate seems to be a difficult problem.

REFERENCES

- Arrow, Kenneth, "Informational Structure of the Firm", American Economic Review, 75(2), May 1985, 303-307.
- Berger, James O., Statistical Decision Theory, Springer-Verlag: New York, 1980.
- Bull, Clive and Janusz A. Ordover, "Who's on Top: Endogenous Managerial Hierarchies", New York University, December 1985, mimeo.
- Fershtman, Chaim and Kenneth L. Judd, "Equilibrium Incentives in Oligopoly," Center for Mathematical Studies in Economics and Management Science, Northwestern University, Working Paper #642, December 1984.
- Nalebuff, Barry J. and Joseph E. Stiglitz, "Information, Competition and Markets", American Economic Review, 73, May 1983, 278-283.
- Sah, Raaj K. and Joseph E. Stiglitz, "The Architecture of Economic Systems: Hierarchies and Polyarchies", NBER Working Paper No. 1334, April 1984.
- , "Human Fallibility and Economic Organization", American Economic Review, 75(2), May 1985, 303-307.
- Vickers, John, "Delegation and the Theory of the Firm", Economic Journal, Conference Papers Supplement, 95, 1985.
- Willig, Robert D., "Corporate Governance and Product Market Structure", mimeograph, Princeton University, 1985.

APPENDIX 1

1: Partial derivatives relevant to 1.4a and 1.4b

$$\beta_a(0) = [1-\beta(0)]e^{-a}/(e^b-e^{-a}) > 0; \quad \beta_a(1) = -[1-\beta(1)]e^{-a}/(e^{-b}-e^a) > 0$$

$$\beta_b(0) = -e^{-b}\beta(0)/(e^b-e^{-a}) < 0; \quad \beta_b(1) = \beta(1)e^{-b}/(e^{-b}-e^a) < 0$$

$$\frac{\partial E_0 N}{\partial a} = \frac{[1-\beta(0)](1+a+b-e^{a+b})}{z(2p-1)(e^{a+b}-1)} > 0; \quad \frac{\partial E_1 N}{\partial a} = \frac{-[1-\beta(1)](1+e^{a+b}[a+b-1])}{z(2p-1)(e^{a+b}-1)} > 0$$

$$\frac{\partial E_0 N}{\partial b} = \frac{\beta(0)(e^{a+b}[1-a-b]-1)}{z(2p-1)(e^{a+b}-1)} > 0; \quad \frac{\partial E_1 N}{\partial b} = \frac{-\beta(1)([1+a+b]-e^{a+b})}{z(2p-1)(1-e^{a+b})} > 0$$

2: Comparative statics for the general case

First we need to sign the second partials.

$$\beta_{aa}(0) = \frac{-[1-\beta(0)](1+e^{a+b})}{(e^{a+b}-1)^2} < 0, \quad \beta_{bb}(0) = \frac{\beta(0)}{(e^{a+b}-1)^2} > 0$$

$$\beta_{bb}(1) = \frac{\beta_b(1)[1+e^{a+b}]}{1-e^{a+b}} > 0, \quad \beta_{aa}(1) = \frac{\beta_a(1)[e^{a+b}+1]}{1-e^{a+b}} < 0$$

$$\beta_{ab}(0) = \frac{e^{a+b}[2\beta(0)-1]}{(e^{a+b}-1)^2} < 0, \quad \beta_{ab}(1) = \frac{e^{a-b}[2\beta(1)-1]}{(1-e^{a+b})^2} > 0$$

$$\frac{\partial E_0 N}{\partial ab} = \frac{-1}{z(2p-1)} \left\{ \frac{\beta_a(0)[e^{a+b}(1-a-b)-1]}{e^{a+b}-1} + \frac{\beta(0)e^{a+b}[1+a+b-e^{a+b}]}{(e^{a+b}-1)^2} \right\} > 0$$

$$\frac{\partial E_1 N}{\partial ab} = \frac{1}{z(2p-1)} \left\{ \frac{\beta_a(1)[1+a+b-e^{a+b}]}{1-e^{a+b}} + \frac{\beta(1)(1-[1-a-b]e^{a+b})}{(e^{a+b}-1)^2} \right\} > 0$$

From the SOC we know that the determinant of the Hessian, $(r_{aa}r_{bb}-r_{ab}^2)$, is positive and so

sign $da/dc = \text{sign } -r_{ac}r_{bb}+r_{bc}r_{ab}$ which is negative if $r_{ab}>0$.

sign $db/dc = \text{sign } -r_{aa}r_{bc} + r_{ac}r_{ab}$ which is negative if $r_{ab}>0$.

sign $da/d(P+J) = \text{sign } -r_{bb}r_{a(P+J)} + r_{ab}r_{b(P+J)}$ which is negative if $r_{ab}<0$.

sign $db/d(P+J) = \text{sign } -r_{aa}r_{b(P+J)} + r_{a(P+J)}r_{ab}$ which is positive if $r_{ab}<0$.

sign $da/d(H+K) = \text{sign } -r_{a(H+K)}r_{bb} + r_{ab}r_{b(H+K)}$ which is positive if $r_{ab}<0$.

sign $db/d(H+K) = \text{sign } -r_{b(H+K)}r_{aa} + r_{a(H+K)}r_{ab}$ which is negative if $r_{ab}<0$.

sign $da/d\pi = \text{sign } -r_{a\pi}r_{bb} + r_{b\pi}r_{ab}$ which is negative if $r_{ab}<0$.

sign $db/d\pi = \text{sign } -r_{b\pi}r_{aa} + r_{a\pi}r_{ab}$ which is positive if $r_{ab}<0$.

sign $da/dp = \text{sign } -r_{ap}r_{bb} + r_{bp}r_{ab}$ which is positive if $r_{ab}>0$.

sign $db/dp = \text{sign } -r_{bp}r_{aa} + r_{ap}r_{ab}$ which is positive if $r_{ab}>0$.

3: Comparative statics for the firm using one-sided screens

A: Type II errors

(1.7) is derived as follows:

$$\Delta(n) = -\pi c + (1-\pi)(H+K)\{- (1-p)^{n+1} + n(1-p)^n\} \\ -c(1-\pi)\{p(n+1)(1-p)^n + (n+1)(1-p)^{n+1} - n(1-p)^n\}$$

$$\Delta(n) = -\pi c + (1-\pi)(H+K)(1-p)^n(1-1+p) \\ -c(1-\pi)\{- (1-p)n(1-p)^n + p(1-p)^n + (n+1)(1-p)^{n+1}\}$$

$$\Delta(n) = -\pi c + (1-\pi)(H+K)p(1-p)^n - c(1-\pi)\{-n(1-p)^{n+1} + p(1-p)^n + (n+1)(1-p)^{n+1}\}$$

$$\Delta(n) = -\pi c + (1-\pi)(H+K)p(1-p)^n - c(1-\pi)\{(1-p)^{n+1} + p(1-p)^n\} = (1.7)$$

First differencing (1.7) gives

$$\Delta^2(n) = -(1-\pi)p^2(1-p)^n(H+K) + (1-\pi)p(1-p)^nc$$

which takes the sign of $c-p(H+K)$ but this must be negative for any screening to be optimal.

$$\text{sign } dn/d\pi = \text{sign } \partial\Delta/\partial\pi = (1-p)^n[c-p(H+K)-(1-p)^{-n}]<0$$

$$\text{sign } dn/dH = \text{sign } \partial\Delta/\partial H = p(1-\pi)(1-p)^n > 0$$

$$\text{sign } dn/dK = \text{sign } dn/dH.$$

$$\text{sign } dn/dc = \text{sign } \partial\Delta/\partial c = -\pi(1-\pi)(1-p)^n < 0$$

$\text{sign } dn/dp = \text{sign } \partial\Delta/\partial p = \text{sign } -pn(H+K) + (1-p)(H+K) + cn$ which is positive because $c > p(H+K)$ in order for screening to be optimal.

$$dn/dP = dn/dJ = 0.$$

B: Type I errors

In this case

$$\Delta(n) = \pi p(1-p)^n (P+J) - (1-\pi)c - \pi c(1-p)^n$$

All the comparative statics are the same in this case with the exceptions that $dn/dH = dn/dK = 0$ and both dn/dP and dn/dJ are positive.

APPENDIX 2

1: Pure Monopsony

Here the payoffs are exogenous and so the monopsonist's first order conditions are

$$\text{wrt. } n: \quad q\Delta(n) - q^2 s \alpha \phi'(\alpha q s) \partial s / \partial n = 0 \quad (\text{A3.1})$$

$$\text{wrt. } q: \quad r(n) - q s \alpha \phi'(\alpha q s) = 0 \quad (\text{A3.2})$$

Substituting (A3.2) into (A3.1) gives

$$q[\Delta(n) - r(n) \partial s / \partial n] = 0 \quad (\text{A3.3})$$

At the competitive equilibrium values of n and q (A3.3) is zero while (A3.2) is negative provided $\phi'(\alpha q s) > 0$. Thus the monopsonist chooses the competitive n and a q less than the competitive value of q .

2: Increasing convex cost of doing projects

The cost of doing any kind of project is increasing and convex in the number of projects done, that is in $q[\pi + (1-\pi)(1-p)^n]$. This means that the payoff to doing a good project is given by $P = f(A)$ where $A \equiv q[\pi + (1-\pi)(1-p)^n]$ and $f(A)$ is increasing and concave in A . Similarly the negative of the payoff to doing a bad project is given by $H = g(A)$ which is increasing and convex in A .

Using the notation of the previous section we know that the competitive equilibrium levels of q , s and n , denoted by q^c , s^c and n^c respectively, satisfy

$$(1-\pi)(H+K)p(1-p)^n - \bar{c} \partial s / \partial n = 0 \quad (\text{A3.4})$$

$$r(n) = 0 \quad (\text{A3.5})$$

The monopolist maximizes expected profits, $qr(n)$, and so his first order conditions are,

$$q[\pi f_n - (1-\pi)(1-p)^n g_n + (1-\pi)(H+K)p(1-p)^n - \bar{c} \partial s / \partial n] = 0 \quad (A3.6)$$

$$r(n) + q\pi f_q - q(1-\pi)(1-p)^n g_q = 0 \quad (A3.7)$$

Evaluating (A3.6) and (A3.7) at q^C, n^C gives

$$\pi f_n - (1-\pi)(1-p)^n g_n > 0 \quad (A3.8)$$

$$\pi f_q - (1-\pi)(1-p)^n g_q < 0 \quad (A3.9)$$

Denoting the monopolist's optimal choices by q^m and n^m we see from (A3.8) and (A3.9) together with the concavity and convexity of, respectively, $f(\cdot)$ and $g(\cdot)$ that $q^m < q^C$ and $n^m > n^C$.

3: Relationship between monopolist's and competitive q 's

When $b=0$, (3.2) can be written as

$$\pi f(q^C) - A(\bar{s}) = 0, \quad A(\bar{s}) > 0 \quad (A3.10)$$

Similarly, (3.4) becomes

$$\pi f(q^m) - A(\bar{s}) + \pi f'(q^m)q^m = 0 \quad (A3.11)$$

and so

$$f(q^C) = f(q^m) + f'(q^m)q^m \quad (A3.12)$$

If we take a second order Taylor approximation around $f(q^m)$ we see that

$$f(q^C) \approx f(q^m) + f'(q^m)(q^C - q^m) + f''(q^m)(q^C - q^m)^2/2 \quad (A3.13)$$

Comparing (A3.13) with (A3.12) and recalling that $q^C > q^m$ we see that

$$\begin{aligned} \text{if } f''(q^m) = 0, & \text{ then } q^m = q^C/2 \\ \text{if } f''(q^m) > 0, & \text{ then } q^m < q^C/2 \\ \text{if } f''(q^m) < 0, & \text{ then } q^m > q^C/2 \end{aligned}$$