

**ECONOMIC RESEARCH REPORTS**

ANONYMOUS SEQUENTIAL GAMES

by

Boyan Jovanovic

and

Robert W. Rosenthal

R.R. #86-12

April 1986

**C. V. STARR CENTER  
FOR APPLIED ECONOMICS**



**NEW YORK UNIVERSITY  
FACULTY OF ARTS AND SCIENCE  
DEPARTMENT OF ECONOMICS  
WASHINGTON SQUARE  
NEW YORK, N.Y. 10003**

ANONYMOUS SEQUENTIAL GAMES

by

Boyan Jovanovic\*  
Department of Economics  
New York University  
New York, N.Y. 10003

and

Robert W. Rosenthal\*  
Department of Economics  
State University of New York  
Stony Brook, N.Y. 11794

March 1986

\* Research undertaken while Boyan Jovanovic was a visitor at Stony Brook. Helpful discussions with Rabah Amir and Abraham Neyman are gratefully acknowledged, as is support from National Science Foundation Grants #SES8408788 and #SES8317924 and the C.V. Starr Center for Applied Economics.

## 1. Introduction

In this paper we examine sequential games with a continuum of players possessing the property that individual players have insignificant influences on their opponents. Many applied problems have been studied using specific models of this general sort. Our objective is to lay out a general model which encompasses the applications, to prove existence of an equilibrium, and to prove existence of a steady state in equilibrium for the special case when the sequence of games is appropriately stationary.

An anonymous game is a one-shot noncooperative game with a continuum of players possessing the feature that a player's payoff depends on his opponents' actions only through their distribution; for example, if half the opponents take one action and half take another, the identities of those in each pool make no difference to the player. The pioneering paper on existence of equilibria in such games is [Sc]. Recently [Ma] and [Gr] showed how "distributional strategies" (used also in [H], [HHK] and [MW] in other contexts) can be used to advantage in further work on this subject; and we shall adapt this general approach to our needs.

By a sequential game we mean a process generating a sequence of one-shot, noncooperative games played by the same set of players having the property that the probability distribution governing the identity of the game to be played at any time  $(t+1)$  is a function of (at most)  $t$ , the game played at  $t$ , and the actions chosen at  $t$ . Stochastic games comprise the most widely studied subclass of sequential games. A stochastic game is a sequential game in which the transition function above is stationary and in which the players' evaluations of sequences of payoffs have a stationary structure (e.g.,

discounting with constant discount factor). In the stochastic-games literature, attention focuses mostly on equilibria composed of "policies": stationary, Markovian strategies, where the state of the system at  $t$  is the game played at  $t$ . Because of technical problems, general results on the existence of equilibria in policies for stochastic games have been obtained only when the state space is assumed to be at most countable (e.g., [Fe] and [W]).

In interesting anonymous stochastic games, states must have an individualistic aspect; otherwise, each individual would perceive that his actions do not affect transitions, and he could therefore view each game in the sequence as a one-shot game. Thus, the restriction to a countable state space is a serious limitation. For this reason, in our model we assume an individualistic aspect to the state space and do not seek equilibria in stochastic games which are composed of policies. Instead, we consider equilibria composed of sequences of distributional strategies and therefore do not need to restrict attention to the stationary (stochastic-games) case, except for the result on steady states. The price we pay is that the sequences of distributional strategies have a partially "open-loop" nature and consequently are not "subgame perfect". As we shall argue, however, this is not a significant defect in the world of anonymous games.

In Section 2 we spell out our model, define equilibrium and prove its existence. In addition, we show that in the stationary case there exists an initial state which will be reproduced at every time by a stationary equilibrium. Section 3 is devoted to a summary of some applications of these two results. Section 4 contains remarks on certain aspects of the model and results.

## 2. Model and Results

In order to give meaning to the idea of anonymity in sequential games, it is useful to impose structure on the state space so that individual-specific aspects of states can be distinguished from those aspects that affect the population as a whole. Accordingly, let  $S$  be a compact metric space of individuals' possible states; and let  $M_S$  be the set of Borel probability measures on  $S$ . A typical state from an individual's point of view is an element in  $S \times M_S$ ; thus, each individual is assumed to be aware of his own individual state and the distribution of individual states across the population. Similarly, let  $A$  be a compact, metric space of actions,  $M_A$  the Borel probability measures on  $A$ , and  $M_{SA}$  the Borel probability measures on  $S \times A$ . A typical measure in  $M_{SA}$  contains information on the distribution of states across the population through its marginal on  $S$  and on the distribution of state-action assignments through the conditional measures it induces on  $A$  given the Borel subsets of  $S$ . (It is perhaps helpful to think of elements of  $M_{SA}$  as containing all that is relevant for our purposes from what would be expressed by a measure  $\mu \in M_S$  together with a function from  $S$  to  $M_A$ . See [Ma] and [MW] for more on this interpretation.)

Time is indexed by  $t \in T = \{1, 2, \dots\}$ . At each  $t \in T$ , each individual, knowing his current state  $(s_t, \mu_t) \in S \times M_S$ , selects an action  $a_t \in A_t(s_t, \mu_t)$ , the set of actions feasible at  $t$  given the current state  $(A_t(s_t, \mu_t) \subseteq A, \forall t, s_t, \mu_t)$ . We assume that for each  $t$ ,  $A_t$  is a nonempty-valued, compact-valued, continuous correspondence. (Throughout this paper, continuity of a function of more than one argument means joint continuity.) The distribution of state-action pairs for the entire population at  $t$  is described by an element  $\tau_t \in M_{SA}$  having the property that its marginal on  $S$ ,

denoted  $\tau_{tS}$ , is  $\mu_t$ . In this situation, the individual's immediate reward is described by  $u_t(s_t, a_t, \tau_t)$ . Endowing  $M_{SA}$  (as well as  $M_S$  and  $M_A$ ) with the topology of weak convergence, we assume that each  $u_t: S \times A \times M_{SA} \rightarrow R$  is continuous and that  $\{u_t\}$  is uniformly bounded. Denoting by  $\beta_t$  the amount by which time-t rewards are discounted so that they are comparable to those at (t-1) in each individual's preferences ( $\beta_1 = 1$ ,  $\beta_t < 1 \quad \forall t \geq 2$ ), an individual's evaluation of the reward sequence  $u_1, u_2, \dots$ , is given by  $\sum_{t=1}^{\infty} \beta_1 \dots \beta_t u_t$ . The single-period transition structure at t is similarly described by the continuous function  $F_t: S \times A \times M_{SA} \rightarrow M_S$ . Abusing notation, let  $F_t(\cdot; s_t, a_t, \tau_t)$  denote the probability measure on the individual's state at (t+1) given  $(s_t, a_t, \tau_t)$ . (Note that  $A_t$ ,  $u_t$ ,  $F_t$ , and  $\beta_t$  are the same for all individuals. Certain asymmetries can easily be accommodated within this model by building them into the structure of S. See Section 4.) Let

$$\Psi_t(\cdot; \tau_t) = \int_{S \times A} F_t(\cdot; s, a, \tau_t) \tau_t(ds \times da)$$

be the time-(t+1) population-state distribution when the time-t state-action distribution is  $\tau_t$ ; of course  $\Psi_t(\cdot; \tau_t) \in M_S$ . A significant assumption has been incorporated here: there is no aggregate uncertainty over the population-state distribution  $\mu_{t+1}$  given  $\tau_t$ . We shall exploit this feature below. The description of the anonymous discounted sequential game (ADSG) is completed by the specification of an initial population-state distribution  $\mu_1 \in M_S$ .

Let  $M_{SA}^{\infty}$  denote  $M_{SA} \times M_{SA} \times \dots$ , the set of sequences  $\tau = (\tau_1, \tau_2, \dots)$  composed of elements of  $M_{SA}$ , compact with the product topology, let V denote the set of continuous real-valued functions on  $S \times M_{SA}^{\infty}$ , and let  $V^{\infty}$  denote the space of sequences  $v = (v_1, v_2, \dots)$  of such functions normed by

$$\|v\| = \sup_{t, s, \tau} |v_t(s, \tau)|.$$

Consider

$$(*) \quad v_t(s, \tau) = \max_{a \in A_t(s, \tau_{tS})} [u_t(s, a, \tau_t) + \beta_{t+1} \int v_{t+1}(s', \tau) F_t(ds'; s, a, \tau_t)]$$

Lemma: There exists a unique sequence  $v$  satisfying (\*),  $\forall t \in T$ .

Furthermore,  $v \in V^0$ .

Proof: Write the system defined by (\*) componentwise for all  $t$  as  $v = Qv$ , where  $Q$  is the operator on  $V^0$  defined on the r.h.s. of (\*). We first show that if  $w \in V^0$ , then  $Qw \in V^0$ . Since  $u_t$  and  $F_t$  are continuous, by [B, p. 34, Theorem 5.5], the r.h.s. of (\*) seeks, for any  $w \in V^0$ , the maximum of a continuous function. Since  $A_t$  is continuous and compact-valued, the conditions of Berge's maximum theorem are satisfied; hence,  $Q:V^0 \rightarrow V^0$ . Next,  $Q$  is a contraction operator, since  $\{\beta_t, t \geq 2\}$  is bounded away from one and  $\{u_t\}$  is uniformly bounded. Finally, functions in  $V$  have as their domain a metric - hence normal, topological - space; hence  $V^0$  is a complete metric space [DS, p. 261]. Apply the contraction-mapping fixed-point theorem to complete the proof.||

The notation  $v_t(s, \tau)$  is henceforth reserved for the element of the sequence  $v$  that satisfies (\*) relative to  $\tau$ . We are now in a position to define equilibrium. An element  $\tau \in M_{SA}^0$  is an equilibrium of the ADSE if the following two conditions hold:

1.  $\tau_{1S} = \mu_1$  and  $\tau_{t+1, S}(\cdot) = \Psi_t(\cdot; \tau_t) \quad \forall t \in T$ .
2.  $\tau_t(\{(s, a): u_t(s, a, \tau_t) + \beta_{t+1} \int v_{t+1}(s', \tau) F_t(ds'; s, a, \tau_t) \geq u_t(s, \tilde{a}, \tau_t) + \beta_{t+1} \int v_{t+1}(s', \tau) F_t(ds'; s, \tilde{a}, \tau_t) \quad \forall \tilde{a} \in A_t(s, \tau_{tS})\}) = 1 \quad \forall t \in T$ .

Condition 1 is the consistency condition ensuring that  $\tau$  admits interpretation as a strategy. Condition 2 may be interpreted as requiring that almost all individuals behave optimally at all times. (See [Ma] for the one-shot analogue of this condition and Section 4 for remarks on its interpretation.)

Theorem 1: Every AD SG has an equilibrium.

Proof:  $M_{SA}^{\infty}$  is a convex, compact subset of a locally-convex (metrizable) space. Consider the correspondences from  $M_{SA}^{\infty}$  into itself defined by:

$$C(\tau) = \{\tau' \in M_{SA}^{\infty} : \tau'_{1S} = \mu_1 \text{ and } \tau'_{t+1,S}(\cdot) = \Psi_t(\cdot; \tau_t), \forall t \in T\}$$

$$B(\tau) = \{\tau' \in M_{SA}^{\infty} : \tau'_t(\{(s, a) : u_t(s, a, \tau_t) + \beta_{t+1} \int v_{t+1}(s', \tau) F(ds'; s, a, \tau_t) \\ \geq u_t(s, \tilde{a}, \tau_t) + \beta_{t+1} \int v_{t+1}(s', \tau) F(ds'; s, \tilde{a}, \tau_t) \\ \forall \tilde{a} \in A_t(s_t, \tau_{tS})\}) = 1, \forall t \in T\}$$

Clearly, any fixed point of the correspondence  $\tau \mapsto C(\tau) \cap B(\tau)$  is an equilibrium. We shall show that this correspondence satisfies the hypotheses of the Fan-Glicksberg fixed-point theorem ([Fa],[Gl]). That both B and C, and hence their intersection, are closed-valued, convex-valued, and upper semi-continuous is straightforward: all three characteristics are easily verified for the respective sets of  $\tau'_t$ ,  $\forall t \in T$ . To verify nonemptiness of the intersection, note first that  $C(\tau)$  is nonempty, being defined as a set of sequences with appropriate marginals in  $M_S^{\infty}$ . Thus if  $\hat{\tau} \in C(\tau)$ ,  $\tilde{\tau} \in M_{SA}^{\infty}$ , and the sequences of marginals of  $\tilde{\tau}$  in  $M_S^{\infty}$  agrees with that of  $\hat{\tau}$ , then  $\tilde{\tau} \in C(\tau)$ .

Now, for every  $\tau$  and  $t$ , the mapping from  $S$  to the set of maximizers on the r.h.s. of (\*) is a correspondence from  $S$  into  $A$  which admits a Borel-

measurable selection, say  $h_t: S \rightarrow A$  (see, e.g., [H, p. 54]). Consider the graph of such a selection as the support of a measure in  $M_{SA}$ . In particular, lift the measure  $\hat{\tau}_{tS}$  to this support (i.e., if  $G$  is a Borel subset of  $S$ , define

$$\tilde{\tau}_t(\{(s, a) : s \in G, a = h_t(s)\}) = \hat{\tau}_{tS}(G).$$

The sequence  $\tilde{\tau}$  constructed in this way belongs to both  $C(\tau)$  and  $B(\tau)$ . Q.E.D.

An AD SG is stationary if  $u_t, F_t, \beta_t$ , and  $A_t$  are all constant with respect to  $t$ . (The subscript  $t$  is therefore dropped in what follows.) For stationary games a subject of interest in many applications is the existence of a stationary equilibrium: an equilibrium  $\tau$  such that  $\tau_1 = \tau_2 = \dots$ . In particular, the population-state measure is invariant.

**Theorem 2:** For any stationary AD SG, there is a measure  $\bar{\mu} \in M_S$  such that if  $\mu_1 = \bar{\mu}$ , the game possesses a stationary equilibrium.

**Proof:** Here we identify stationary elements of  $M_{SA}^0$  with elements of  $M_{SA}$ . A typical such element is again denoted  $\tau$  so that (\*) becomes

$$(**) \quad v(s, \tau) = \max_{a \in A(s, \tau_S)} [u(s, a, \tau) + \beta \int v(s', \tau) F(ds'; s, a, \tau)].$$

By analogy with Lemma 1, there is a unique  $v \in V$  satisfying (\*\*).

Let  $C(\tau) = \{\tau' \in M_{SA} : \tau'_S(\cdot) = \Psi(\cdot; \tau)\}$

$$B(\tau) = \{\tau' \in M_{SA} : \tau'(\{(s, a) : u(s, a, \tau) + \beta \int v(s', \tau) F(ds'; s, a, \tau) \\ \geq u(s, \bar{a}, \tau) + \beta \int v(s', \tau) F(ds'; s, \bar{a}, \tau)$$

$$\forall \bar{a} \in A(s, \tau_S)\}) = 1\}.$$

The fixed point of the correspondence  $\tau \mapsto C(\tau) \cap B(\tau)$ , which exists by analogy with the proof of Theorem 1, is the stationary equilibrium with  $\bar{\mu} = \tau_S$ . Q.E.D.

### 3. Applications

We begin with a collection of applications to dynamic economic situations. The idea that one extreme form of economic competition involves individuals ignoring their individual influences on others is of course familiar, as is the mathematical modeling of this in the context of measure spaces. The literature on dynamic versions of such competition is fragmented, however, and our aim here is to show that the model and existence results from Section 2 are of general use. In much of the applications literature, however, existence is taken for granted, and characterization results are sought.

a. Cost-reducing Investment and Spillovers. In each period individual firms produce output and invest in research to reduce future costs of output production. A firm's individual state is a measure of its current technical know-how. Individual transitions may depend on the firm's current state and investment as well as on the distribution of states and actions of the rest of the firms in the industry through spillovers. Immediate rewards for a firm are a function of its state and action as well as the output decisions of the rest of the firms in the industry. [Sp] studies a finite-player version of this model, while [Rom] treats the continuum. In both papers the distribution of actions in the population enters only through its mean.

b. Search. A number of models (eg., [D], [DM], [Mo]) have embedded in them some search externalities: a player's success in search depends in part on how hard

other players are searching. (As an illustration, workers search for employers and vice versa.) Thus the distribution of search activity among other players affects the transition law for his own state. A player's individual state may simply be whether or not he has a partner, or it may also include a description of the quality of his current match. The player's action can be taken to be his search intensity. A discrete-time, continuum-of-players version of the model in [Mo] would be a special case of our structure.

c. Spatial Competition. [JR] analyzes a stochastic-game model of spatial competition in the spirit of [So]. Each firm's individual state is its product location, and the aggregate state is the distribution of firms across locations. The firm's action at any time consists of both its location choice and its investment in information. The payoffs and transitions depend both on the firm's state and action and on the population-state distribution. (Although the general model in [JR] possesses aggregate uncertainty, under certain assumptions both our theorems apply.)

d. Savings Behavior. [L] analyzes a repeated game played by a continuum of agents with random preferences over a single consumption good. Taste shocks are random for an individual, but there is no aggregate uncertainty. Money is the only asset, and the decision of how much of it to hold is the only action. The individual's state consists of his money holding and the realization of his taste shock. The state-transitions are deterministic for the individual's money holding, and purely random for his taste-shock. The aggregate state is the distribution of money across agents. (The money supply is unchanging.)

Interaction among agents stems from the collective influence that their savings behavior has on the contemporaneous price level. [L] proves the existence of a stationary equilibrium in which the price and aggregate state are constant, a result implied by our second theorem. Our first theorem also implies the existence of an equilibrium even when the economy begins with any distribution of money over agents.

e. Convertible Bonds and Warrants. In each period individual owners of convertible securities must decide whether to exercise their conversion options or to wait. If the options are widely held, individual decisions are unimportant to other holders of the firm's securities; but the decisions of population aggregates matter in that they affect the quantity of common-stock shares outstanding. In addition, these aggregate decisions may affect the firm's dividend policy and even the policies it pursues outside of its internal financial considerations. Several authors have studied the strategic implications of these issues, and a model akin to that of Section 2, but with finite horizon and special assumptions about uncertainty, appears in [CR] with an existence result of the nonstationary variety.

f. Noneconomic Applications. Beyond the purely economic applications, several papers (e.g., [RL]) address situations which have the following general form. A large (but finite) set of players is repeatedly partitioned randomly into sets of size 2, each pair to play a symmetric, one-shot, two-player game. The one-shot game is always the same game at each point in (discrete) time; only the identities of the matched opponents change over time. The matchings are assumed to be independent and equally likely at each time. All individuals

desire to maximize the discounted (identical discount factors) sums of their expected payoffs. An individual is not told the identity of his current opponent at any time; only a piece of information about him, termed his reputation. Each individual's reputation is assumed to evolve in a Markovian fashion. The limits of such models are straightforwardly special cases of the stationary model in Section 2. Existence of stationary equilibria in such models is simple: If all players ignore reputations there is a stationary equilibrium in which the players repeatedly play the equilibrium of the one-shot game; but equilibria in which reputations matter are the more interesting objects of study, and non-stationary, nonsymmetric versions in which reputations also affect the nature of the game being played are possible (e.g. [Ros]).

#### 4. Remarks.

a. A typical population strategy in our model is an element  $\tau \in M_{SA}^{\infty}$ . As usual with the distributional approach, strategies for individuals are unspecified; but regular conditional distributions in  $M_A$  are well-defined for each  $t$  and almost every  $s$  and admit interpretations as randomized actions for individuals in state  $s$  at  $t$ . An additional interpretational issue for this model, however, is: what action-distribution results at time  $t$  if  $\mu_t \neq \tau_{ts}$  appears? Such a  $\mu_t$  is inconsistent with play of  $\tau$ , of course, but the point is that this approach fails to extend the familiar requirement that a strategy be well-defined at all of a player's decision nodes in an extensive-form game. In particular, our equilibria fail to satisfy subgame perfectness (ignoring the technical problems that arise when trying to define perfection in our

setting), since they are not defined off the equilibrium path. This is similar to the fact that open-loop type equilibria generally fail to satisfy subgame perfectness (even when well-defined), and our population strategies are open-loop relative to the  $\mu_t$  sequence.

In response to this, note first that if one of our equilibria is extended in any arbitrary way to a full-blown strategy, it remains an equilibrium (though not subgame perfect) in the space of all strategies (however described). Second, take any equilibrium composed of a strategy combination outside the class of strategies we consider. Compute the sequence  $\{\tau_t\}$  induced by that strategy combination. This sequence must be an equilibrium in our sense, since no individual by himself can affect the state-action distributions. Thus, in particular, any subgame-perfect equilibrium, however defined, generates a sequence of state-action distributions which is an equilibrium in our sense.

Finally, we can use Theorem 1 to construct a strategy combination in a richer space which is, in a strong sense, subgame perfect, as follows. First, define a function  $g: TxM_S \rightarrow M_{SA}^o$  which, for each  $(t, \mu)$ , selects one of the Theorem-1 equilibria for the game having initial state-distribution  $\mu$  at time 1 and in which all the time-indexed data of the game are shifted by  $(t-1)$  time units (e.g., the first-period immediate-reward function is  $u_t$ , the second  $u_{t+1}$ , etc.). The strategy combination for the game is defined recursively by: At time  $t$  continue with the time- $t$  component of the element of  $M_{SA}^o$  supposedly in use at time  $(t-1)$  as long as the time- $(t-1)$  component actually was realized by the population at  $(t-1)$ ; otherwise, begin  $g(t, \mu_t)$  where  $\mu_t$  is the observed population-state distribution at  $t$ . Since every subgame begins at a point  $(t, \mu)$  and the currently-in-use element of  $M_{SA}^o$  is expected to

remain in use, each player's calculations a la (\*) are valid and the overall strategy combination is subgame perfect.

b. Uncertainty in our model occurs at the individual, but not the aggregate, level:  $\Psi_t$  is a deterministic function, and in equilibrium each player has perfect foresight about the entire  $r$  sequence. If  $\Psi_t$  were instead a random function (i.e. a function of  $r_t$  and of some aggregate shock), one could not define  $v$  on  $S \times M_{SA}^0$  because aggregate outcomes would no longer be perfectly foreseeable. Therefore our proofs of Theorem 1 and Theorem 2 do not extend to the case of aggregate uncertainty. The extension of these results to the case of aggregate uncertainty appears to be an interesting subject for future research.

c. Although the notation does not stress it, the payoff and transition functions, as well as the feasible set correspondences and discount factors can all differ across players. One simply needs to build  $s$  in two components: the first with the usual interpretation (dictated by the application at hand), and the second, a "fixed effect" indexing the player's "type", which does not change for the duration of the game. The only restriction on this formulation of heterogeneity is that it enter in a continuous way, not a particularly onerous restriction for many applications.

d. Finally, there are two types of convergence questions which one might ask. First, in the stationary model of Theorem 2, if the initial population-state distribution is not  $\bar{\mu}$ , under what conditions does the nonstationary equilibrium  $r$  generate a sequence  $(\mu_t)$  converging to  $\bar{\mu}$ ? Second, is there

some sense in which an equilibrium  $r$  is approached by some disequilibrium dynamic model? To both questions we have no answers.

## References

- [B] Billingsley, P., Convergence of Probability Measures, John Wiley and Sons, New York, 1968.
- [CR] Constantinides, G., and R. Rosenthal, "Strategic Analysis of the Competitive Exercise of Certain Financial Options", Journal of Economic Theory 34 (1984), pp.128-138.
- [D] Diamond, P., "Demand Management in Search Equilibrium", Journal of Political Economy 90 (1982), pp.881-894.
- [DM] Diamond, P. and E. Maskin, "An Equilibrium Analysis of Search and Breach of Contract, I: Steady States", Bell Journal of Economics 10 (1979), pp.282-316.
- [DS] Dunford, N., and J. Schwartz, Linear Operators. Part I: General Theory, Interscience Publishers, New York, 1958.
- [Fa] Fan, K., "Fixed-point and Minimax Theorems in Locally Convex Topological Linear Spaces", Proceedings of the National Academy of Sciences, U.S.A. 38 (1952), pp.121-126.
- [Fe] Federgruen, A., "On N-person Stochastic Games with Denumerable State Space", Advances in Applied Probability 10 (1978), pp.452-471.
- [Gl] Glicksberg, I., "A Further Generalization of the Kakutani Fixed Point Theorem with Application to Nash Equilibrium Points", Proceedings of the American Mathematics Society 3 (1952), pp.170-174.
- [Gr] Green, E., "Continuum and Finite-Player Noncooperative Models of Competition", Econometrica 52 (1984), pp.975-995.
- [H] Hildenbrand, W., Core and Equilibria of a Large Economy, Princeton University Press, Princeton, N.J. 1974.

- [HHK] Hart, S., W. Hildenbrand, and E. Kohlberg, "On Equilibrium Allocations as Distributions on the Commodity Space", Journal of Mathematical Economics 1 (1974), pp. 159-167.
- [JR] Jovanovic, B., and R. Rob, "Demand Driven Innovation and Spatial Competition Over Time", Discussion Paper, New York University, August 1985.
- [L] Lucas, R., "Equilibrium in a Pure Currency Economy", Economic Inquiry 18 (1980), pp. 203-220.
- [Ma] Mas-Colell, A., "On a Theorem of Schmeidler", Journal of Mathematical Economics 13 (1984), pp.201-206.
- [Mo] Mortensen, D. "Property Rights and Efficiency in Mating, Racing and Other Related Games", American Economic Review 72 (1982), pp.968-979.
- [MW] Milgrom, P., and R. Weber, "Distributional Strategies for Games with Incomplete Information", Mathematics of Operations Research 10 (1985), pp.619-632.
- [Rom] Romer, R., "Externalities and Increasing Returns in Dynamic Competitive Analysis", Discussion Paper, University of Rochester, September 1984.
- [Ros] Rosenthal, R., "A Model of Far-Sighted Electoral Competition", Mathematical Social Sciences 2 (1982), pp.289-297.
- [RL] Rosenthal, R., and H. Landau, "A Game-Theoretic Analysis of Bargaining with Reputations", Journal of Mathematical Psychology 20 (1979), pp.233-255.
- [Sc] Schmeidler, D., "Equilibrium Points of Nonatomic Games", Journal of Statistical Physics 7 (1973), pp.295-300.
- [So] Sonnenschein, H., "Price Dynamics Based on the Adjustment of Firms", American Economic Review 72 (1982), pp.1088-1096.

[Sp] Spence, M., "Cost-reduction, Competition and Industry Performance",  
Econometrica 52 (1984), pp.101-123.

[W] Whitt, W., "Representation and Approximation of Non-cooperative  
Sequential Games", SIAM Journal of Control and Optimization 18 (1980),  
pp.33-48.