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MICRO SHOCKS AND AGGREGATE RISKS

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Abstract

The paper argues in favor of a "micro" shock explanation of aggregate risk. Shocks are independent over agents, and equilibria are always unique. It is shown that any amount of aggregate risk can be generated by games in which shocks to players are independent. This is encouraging if one's objective is to dispense with aggregate shocks. Explicit examples are given. Implications are drawn for factor-analytic methods of extracting aggregate shocks.

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1. Introduction

This paper is concerned with the sources of aggregate risk. Aggregate risk evidently does exist--per capita values of aggregates are quite unpredictable. Thus far, the sources of this risk have been modeled in two ways: with intrinsic aggregate shocks (e.g., Kydland and Prescott, 1982) and with extrinsic aggregate shocks ("sunspots") (Cass and Shell, 1981). This paper puts forth a "micro" shock explanation of aggregate risk. In contrast to the intrinsic aggregate shock approach, the shocks are independent over agents. In contrast to the sunspot approach, equilibria are unique, and no single shock need be of overriding importance.

In section 2 we show that it's quite easy to construct examples in which the endogenous data do not obey the law of large numbers even though the exogenous data do. In section 3 we present a general result: Any distribution of decisions over agents (regardless of the degree of correlation in the distribution) can be generated by games which have unique equilibria (theorem 1), where the shocks to players' payoff functions are restricted to be independent. In other words, the restriction that shocks be independent over agents does not of itself provide any restrictions on the observed outcomes. This is a positive result if one wishes to dispense with aggregate shocks in modeling the movement of aggregates. The elimination of aggregate shocks does not restrict the sorts of phenomena that one can explain as arising from the maximizing behavior of agents.

Section 4 presents an example which shows, as does example 1, that to get aggregate randomness in large economies, a strong form of strategic complementarity must be present. But a general result of this sort is not available at present.

Section 5 shows how the observed data may make it appear as though one or more aggregate shocks are driving the system, even if no aggregate shocks are present in reality. These arguments follow directly from the results of section 2. Finally, section 6 contains some concluding remarks.

2. Example

This introductory example, similar to Diamond (1982) and Mortensen (1982), shows that it is fairly easy to come up with examples of games in which the exogenous data are random and obey the strong law of large numbers, but the endogenous data do not obey the law. In an n-player game, a vector of player-specific, independent shocks $\varepsilon_1,\ldots,\varepsilon_n$ is realized, and a Nash equilibrium to the game of complete information is computed. Then we allow n to get large and study the properties of the limiting distribution of the endogenous variables.

In an economy of n agents, each agent can specialize to a variable extent in one of two activities, A and B. Let $x_i \in R$ be the i'th agent's degree of specialization in A. A value of $x_i = 0$ means that agent i has not expended any effort in specializing in either A or B. If x_i is positive and large, he is highly specialized in A; if it is negative and large in absolute value, he is highly specialized in B. If agent i teams up with agent j, their joint payoff is $2x_ix_j$, which they share equally. Equal and high specialization in the same activity yields high returns, while specialization in opposite directions yields low returns.

The cost of specializing is $\frac{1}{2}x^2 + \varepsilon x + k$. The parameter ε denotes the player's relative suitability to A or B. If $\varepsilon < 0$, he is better suited to A, while if $\varepsilon > 0$, he is better suited for B. The ε_i are zero-mean, independent samples from the distribution $G(\varepsilon)$. The parameter k is the same across players and ensures that costs are positive for all (x,ε) pairs, and this fixed cost is incurred no matter what value of x is chosen.

A player must produce with the first person he meets, and meetings are random. Not everyone succeeds in meeting someone else within the relevant period of time. Players that do not meet another player collect $-\frac{1}{2}x^2 - \varepsilon x - k$ as their payoff. Let q_n be the expected fraction of players that will secure a meeting. The randomness in the meeting process allows us to cast this as an anonymous game. Being risk-neutral, player i's payoff function depends only on his own action, on the mean of the actions of the other players, and on n:

$$q_n x_i \frac{1}{n-1} \sum_{j \neq i} x_j - \frac{1}{2} x_i^2 - \varepsilon_i x_i - k.$$

The first order condition for a maximum yields the reaction functions for $i = 1,\ldots,n$:

$$x_i = q_n \frac{1}{n-1} \sum_{j \neq i} x_j - \varepsilon_i. \tag{1}$$

Summing over i and dividing by n yields

$$\frac{1}{n} \sum_i x_i = \frac{-1}{\sqrt{n}\,(1 - q_n)} \left(\frac{1}{\sqrt{n}} \sum_i \varepsilon_i\right).$$

It is not implausible that $1 - q_n$ is of order at least $1/\sqrt{n}$; for instance, an increasing density of searchers on an island of a fixed size could produce this result. Consider first the case in which $1 - q_n \to 0$ exactly at the rate $1/\sqrt{n}$. Then $n^{-1}\Sigma x_i \to \theta$, a random variable that has a non-degenerate limiting (and normal, by the central limit theorem) distribution with variance $\sigma_\varepsilon^2 > 0$. From (1) we also find that, for each i,

$$x_i \to \theta - \varepsilon_i, \tag{2}$$

so that even in the limit it will appear as though each x were composed of an aggregate shock θ and an idiosyncratic component ε (a "one factor model"), even though the model has no aggregate shocks. We return to this point again in section 5. Finally, (2) shows that production per worker ("GNP per capita") tends towards θ^2, and it too is a non-degenerate random variable.

If $1 - q_n \to 0$ at a rate that is faster than $1/\sqrt{n}$ (say as fast as $(1 - \hat{q})^n \to 0$ for some $\hat{q} \in (0,1)$, as would be reasonable in some explicit models of the search process), then aggregate randomness again exists in the limit, except that θ has infinite variance. Note finally that for each fixed n-vector ε, equilibrium is unique because the reaction functions satisfy a contraction property. This argument is made more carefully in the next section.

3. The Main Result

Consider an n-player game in which X, a closed subset of R^m, is the set of actions that each player can take. Let x_{-i} denote the $(n-1)$-vector of actions of all players but the i'th. Let $R_i(x_{-i}) \subseteq X$ be the set (possibly a single point) of optimal reactions of player i when the remaining players play x_{-i}.

Lemma 1. Given any collection of closed-valued reaction correspondences (R_i), there exists a set of payoff functions $u_i : X^n \to R$ $(i = 1,\ldots,n)$ generating them.

Proof: Let ρ be a metric on X, and for any $z \in X$ and any closed subset A of X, let $\delta(z,A) = \min_{x \in A} \rho(z,x)$. Now set $u_i(x_i,x_{-i}) = -\delta[x_i,R_i(x_{-i})]$. Then clearly $\{x_i \in X | u_i(x_i,x_{-i}) \geq u_i(X,x_{-i})\} = R_i(x_{-i})$. Q.E.D.

The payoff functions u_i are not uniquely determined by the R_i. From now on, we shall therefore talk only about reaction functions, not payoff functions, when describing a game. The restriction in the lemma to closed-valued R_i will be of no relevance in the development below.

Let $\varepsilon_1,\ldots,\varepsilon_n$ be the shocks to the n players' reaction functions; they are independent and uniformly distributed on the unit interval. The ε_i are "micro" shocks because they are independent. The reaction functions (or correspondences) will, for a fixed realization of the ε-vector, be written as

$$\phi_i(x_{-i},\varepsilon_i) \qquad i = 1,\ldots,n \tag{3}$$

These may be thought of as the n structural equations of the model.

For given $\varepsilon \in [0,1]^n$, a Nash Equilibrium is a solution for $x \in X^n$ to the system

$$x_i \in \phi(x_{-i}, \varepsilon_i) \qquad i = 1,\ldots,n. \tag{4}$$

Let $h(\varepsilon) \subseteq X^n$ be the set of equilibria of the game. Thus h is the equilibrium correspondence, or the reduced form correspondence, relating the endogenous variables x to the exogenous variables ε.

Let λ be Lebesgue measure on $[0,1]^n$, and B the collection of Borel subsets of $[0,1]^n$. Since X is complete and separable, so is X^n in the product topology. Therefore the probability space $([0,1]^n, B, \lambda)$ is rich enough to generate any cumulative distribution function F(x) on X^n [Hildenbrand (1974), p. 50]. That is, there are many measurable functions $h : [0,1]^n \rightarrow X^n$ for which

$$F(x) = \lambda(h^{-1}(-\infty, x]), \text{ all } x \in X^n. \tag{5}$$

That is, there are many measurable reduced forms giving rise to any distribution of the observables.

Not all reduced forms h that generate F are of equal interest. Suppose, for instance, that F is a symmetric distribution. That is, for any permutation ξ of x, $F(\xi x) = F(x)$. If the agents are ex-ante symmetric in the sense that ϕ_i in (3) does not depend on i, then the reduced form corresponding to the family of symmetric equilibria would be symmetric; i.e., it would satisfy

$$h(\xi\varepsilon) = \xi h(\varepsilon) \tag{6}$$

for any permutation ξ of the ε vector. Such reduced forms give each player an equal weight, and both of the examples presented in this paper involve reduced forms that satisfy (6).

<u>Theorem</u>. For each reduced form h, there is a family of games such that for each ε, $h(\varepsilon)$ solves eq. (4). Furthermore, each of the games in the family has a unique equilibrium.

<u>Proof</u>: Let $B(x_{-i},\varepsilon_i) = \{\varepsilon_{-i} \in [0,1]^{n-1} | h_{-i}(\varepsilon_1,\ldots,\varepsilon_i,\ldots,\varepsilon_n) = x_{-i}\}$, where $h_{-i}(\varepsilon)$ is the $(n-1)$-vector of coordinates of h excluding the i'th. Let Γ be the graph of h (a subset of $X^n \times [0,1]^n$). Throughout this proof, we shall write statements such as $(x_{-i},\varepsilon_i) \in \Gamma$ to mean "there exist x_i,ε_{-i} such that the point $(x,\varepsilon) \in \Gamma$ when x_{-i},ε_i are specified." Then $B(x_{-i},\varepsilon_i)$ is nonempty if and only if $(x_{-i},\varepsilon_i) \in \Gamma$. Now let

$$\hat{\phi}_i(x_{-i},\varepsilon_i) = \{x_i \in X | x_i = h_i(\varepsilon), \text{ and } \varepsilon_{-i} \in B(x_{-i},\varepsilon_i)\}.$$

Now define the collection of functions $\tilde{\phi}_i : X^{n-1} \times [0,1] \to X$ such that the system of equations $x_i = \tilde{\phi}_i(x_{-i},\varepsilon_i)$ $(i = 1,\ldots,n)$ has no solution anywhere on X^n (this can always be done, of course). Now let

$$\phi_i(x_{-i},\varepsilon_i) = \begin{cases} \hat{\phi}_i(x_{-i},\varepsilon_i) & \text{for } (x_{-i},\varepsilon_i) \in \Gamma, \\ \tilde{\phi}(x_{-i},\varepsilon_i) & \text{for } (x_{-i},\varepsilon_i) \notin \Gamma. \end{cases}$$

By construction, if an equilibrium exists, it must be a point $x \in \Gamma$. Then since $h(\varepsilon)$ is a singleton, at most one equilibrium exists, i.e., at most one solution to (4) exists when the ϕ_i assume the form above. But it is immediate from the construction of $\hat{\phi}_i$ that $\hat{\phi}_i[h(\varepsilon),\varepsilon_i] = h_i(\varepsilon)$ for all i, and the claim follows. Q.E.D.

The theorem asserts that any reduced form h can be generated by a family of games each of which is a complete information game, and each of which has a unique equilibrium. Since any F(x) can be generated by an h in the sense of eq. (5), the theorem shows that restricting attention to stochastically independent agents imposes, in general, no restrictions on the distribution of observed outcomes. Since we may insist (when this is appropriate) that h satisfy (6), the result does not depend on any one player being of special importance relative to other players. And since each game has a unique equilibrium, aggregate randomness is in no sense generated by the "sunspot"-type mechanisms of shifting the economy across different equilibria.

The theorem unfortunately also shows that a wide variety of collections of reaction functions (i.e., structures) can give rise to a given reduced form. Therefore the identification problem that is usual in econometric work continues to be present even if one insists, as we do, that disturbances to agents' payoff functions be independent. On the other hand, it should come as a relief that the elimination of unexplained aggregate shocks in no way reduces the number of phenomena that one can explain as arising from maximizing behavior on the part of agents in the economy.

4. Example 2

We now derive in explicit form a family of games that generate a multivariate normal or log-normal distribution of an arbitrarily large number of symmetric random variables. Since normality is a common assumption in empirical work, the example shows that the "independent shocks" approach lends itself readily to implementation.

Let x_i be the observed action (or its logarithm) of player i. Suppose that we wish to generate the system of equations

$$x_i = \theta + v_i \qquad i = 1,\ldots,n,$$

where $\theta \sim N(\bar{\theta},\sigma_\theta^2)$ is an "aggregate shock," and where the $v_i \sim N(0,\sigma_v^2)$ are i.i.d. "idiosyncratic shocks" which also are independent of θ. We shall generate an observationally equivalent system without using aggregate shocks. Moreover, each of the games in the family will have a unique equilibrium.

An equivalent representation is the following. Let $x \sim N(\mu,\Sigma)$, where

$$\mu = \begin{pmatrix} \bar{\theta} \\ \bar{\theta} \\ \cdot \\ \cdot \\ \cdot \\ \bar{\theta} \end{pmatrix} \quad \text{and} \quad \Sigma = \begin{bmatrix} \sigma^2 & \rho^2\sigma^2 & \cdots & \rho^2\sigma^2 \\ \rho^2\sigma^2 & \sigma^2 & & \\ \vdots & & \ddots & \\ \rho^2\sigma^2 & & & \sigma^2 \end{bmatrix}$$

and where $\sigma^2 = \sigma_\theta^2 + \sigma_v^2$ and $\rho^2\sigma^2 = \sigma_\theta^2$. Here μ is an $(n \times 1)$ vector, and Σ an $(n \times n)$ matrix. Note that $N(\mu,\Sigma)$ plays the role of $F(x)$ in this special case.

Let $\varepsilon_i \sim N(0,\sigma_\varepsilon^2)$ be a collection of n i.i.d. variables. These will be the micro shocks. They have the same interpretation as the ε's in the last section except that it is now convenient to assume them to have a distribution that is normal, not uniform. Let the reduced form be linear:

$$h(\varepsilon) = A + B\varepsilon,$$

where

$$A = \begin{pmatrix} \alpha \\ \alpha \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \alpha \end{pmatrix} \text{ and } B = \begin{bmatrix} \beta+\gamma & \beta & \cdot & \cdot & \cdot & \beta \\ \beta & \beta+\gamma & & & & \\ & & \cdot & & & \\ & & & \cdot & & \\ & & & & \cdot & \\ & & & & & \cdot \\ \beta & & & & & \beta+\gamma \end{bmatrix}$$

Again, A is an $(n \times 1)$ vector, B a $(n \times n)$ matrix. Note that h is symmetric in the sense that it satisfies (6).

Since the mean vectors and the covariance matrices of h and x must be equal,

$$\alpha = \bar{\theta} \ , \tag{8}$$

and

$$\sigma_\varepsilon^2 BB' = \textstyle\sum \tag{9}$$

Eq. (9) contains two separate restrictions; these are gotten by computing the diagonal and off-diagonal elements respectively of $\sigma_\varepsilon^2 BB'$:

$$\sigma_\varepsilon^2[(\beta + \gamma)^2 + (n - 1)\beta^2] = \sigma^2, \tag{9)'$$

and

$$\sigma_\varepsilon^2[2\beta(\beta + \gamma) + (n - 2)\beta^2] = \rho^2\sigma^2. \tag{9)''$$

These two restrictions imply (see the appendix) the following solutions for β and γ:

$$\beta = \frac{\sigma}{n\sigma_\varepsilon}\left[(1-\rho^2+n\rho^2)^{1/2} - (1-\rho^2)^{1/2}\right] \tag{10}$$

and

$$\gamma = \frac{\sigma}{\sigma_\varepsilon}(1-\rho^2)^{1/2} . \tag{11}$$

Thus

$$h_i(\varepsilon) = \bar{\theta} + \beta\sum_{j=1}^{n}\varepsilon_j + \gamma\varepsilon_i \qquad i = 1,\ldots,n, \tag{12}$$

and since β and γ obey (10) and (11), $h \sim N(\mu,\Sigma)$, as required.

Having solved for the reduced form parameters, we now turn to the parameters of primary interest, namely, the structural form parameters.

We try the following linear reaction functions:

$$\phi_i(x_{-i},\varepsilon_i) = a + b\sum_{j\neq i}x_j + c\varepsilon_i \qquad i = 1,\ldots,n. \tag{13}$$

Note that ϕ_i is symmetric--it does not explicitly depend on i. Since the ϕ_i are functions, not correspondences, eq. (4) must hold as an equality for each i:

$$h_i(\varepsilon) = \phi_i[h_{-i}(\varepsilon),\varepsilon_i] \text{ for all } \varepsilon \in R^n. \tag{4)'$$

That is, for all ε,

$$\bar{\theta} + \beta\sum_j\varepsilon_j + \gamma\varepsilon_i = a + b\sum_{j\neq i}(\bar{\theta} + \beta\sum_k\varepsilon_k + \gamma\varepsilon_j) + c\varepsilon_i \tag{14}$$

$$= a + (n-1)b\bar{\theta} + b[(n-1)\beta + \gamma]\sum_j\varepsilon_j + (c - b\gamma)\varepsilon_i .$$

Equating coefficients yields three restrictions:

$$\bar{\theta} = a + (n - 1)\bar{\theta}b,$$

$$\beta = b\gamma + (n - 1)b\beta$$

and

$$\gamma = c - b\gamma,$$

from which we obtain the solution for the structural parameters in terms of the reduced form parameters:

$$a = \frac{\bar{\theta}\gamma}{(n - 1)\beta + \gamma}, \quad b = \frac{\beta}{(n - 1)\beta + \gamma}, \quad \text{and}$$

$$c = \frac{\gamma[n\beta + \gamma]}{(n - 1)\beta + \gamma}$$

To check the plausibility of these solutions, consider for instance the special case of $\rho = 0$ (independence). Then $\beta = 0$ so that B is diagonal, and $b = 0$ so that there is no strategic interaction, and $\gamma = c = \sigma/\sigma_\varepsilon$, which is easily seen to yield $F(x)$.

Because $b < (n - 1)^{-1}$, each game has a unique equilibrium. To see this, let $\|x\| = \max_i |x_i|$. Then the operator $T_\varepsilon : X^n \to X^n$ defined by $T_\varepsilon x = \{\phi_i(x_{-i},\varepsilon_i)$ for $i = 1,\ldots,n\}$ is, for each $\varepsilon \in R^n$, a contraction with modulus $(n-1)b$; that is, $\|T_\varepsilon x - T_\varepsilon x'\| \leq (n-1)b\|x - x'\|$ for any x and x' in R^n. Since a solution to (4)' is, for each fixed ε, the fixed point of T_ε, it must be unique.

It is of interest to see what happens to the structural coefficients as the number of players gets large. If ρ and σ^2 are fixed as $n \to \infty$, this experiment enlarges the size of the game while keeping fixed the correlation coefficient between any pair of the x_i's. The interesting parameter is b, which measures the degree of interaction among the players. Let $\bar{b} = (n - 1)b$ so that if $\bar{x}_{-i} \equiv (n - 1)^{-1} \sum_{j \neq i} x_j$,

$$\phi(x_{-i}, \varepsilon_i) = a + \bar{b}\bar{x}_{-i} + c\varepsilon_i .$$

Thus $\bar{b}$ is the effect that a unit increase in the average decision of others has on x_i. We have $\bar{b} < 1$, but $\bar{b}$ is increasing in n, and

$$\lim_{n\to\infty} \bar{b} = 1,$$

a strong form of strategic complementarity, a situation similar to the one in example 1.

The multiplier can also be calculated explicitly. This is the effect of a unit increase in ε_i on the sum of all the players' decisions:

$$\frac{\partial}{\partial \varepsilon_i} \sum_j h_j(\varepsilon) = n\beta + \gamma = \frac{\sigma}{\sigma_\varepsilon} (1 + n\rho^2 - \rho^2)^{1/2} = O(n^{1/2}).$$

On the other hand, the effect of ε_i on $n^{-1}\Sigma h_j$, the average decision, is $O(n^{-1/2})$.

5. Factor Analysis and the Number of Apparent Aggregate Shocks

In this section we briefly show that even when the true model contains no aggregate shocks, there is in general no upper bound on the number of "aggregate shocks" that one might extract via factor analysis.

Let $H(x;\theta)$ be a family of distributions on X indexed by $\theta \in R^k$. Let $\mu(\theta)$ be a distribution on R^k. If the θ's were actually a k-vector of aggregate shocks distributed according to μ, the distribution of the observables would be

$$\hat{F}(x) = \int_{R^k} \prod_{i=1}^{n} H(x_i;\theta)d\mu(\theta). \tag{15}$$

Assume for simplicity that the individual components of θ are independent under μ. If a sufficiently large sample of the x's is available, one can, in principle at least, discover that k factors were responsible for the covariance among the x's, and it is then tempting to conclude that there exist k independent sources of comovement among the x's.

As the previous two sections show, however, this conclusion is unwarranted. By Theorem 1 and Lemma 1, any $\hat{F}$ can be generated from games containing no aggregate shocks; the θ's could be just agglomerations of individual shocks, caused by strategic interaction. Both examples have a structure that is representable as a one-factor model, but as is clear from (15), there need be no upper bound on k, the dimensionality of θ, since the number of parameters that it generally takes to describe a distribution (in this case H) can be infinite.

6. Conclusion

This paper has argued in favor of a micro shock explanation of aggregate risk. I conclude with what I regard to be a strong intuitive argument in favor of it. If the economy were really driven by a few aggregate shocks or sunspot variables, we would have been able to identify them long ago. Or, if the aggregate shocks are of a different special nature or identity in each instance, then at least we should be able to identify them after they have "done their work." Our continuing inability to identify these shocks is the main argument in favor of the micro shock approach. The latter says, in effect, that we shall never be able to predict aggregates--it is simply impossible to account for all or even most of the sources of aggregate risk.

APPENDIX

The purpose of the appendix is to show that (9)' and (9)" imply (10) and (11). We begin by noting that

$$2\beta(\beta + \gamma) = (\beta + \gamma)^2 - (\gamma^2 - \beta^2).$$

Substituting into (9)", this yields

$$(\beta + \gamma)^2 + (n - 2)\beta^2 + \beta^2 - \gamma^2 = \rho\sigma^2/\sigma_\varepsilon^2,$$

that is,

$$(\beta + \gamma)^2 + (n - 1)\beta^2 - \gamma^2 = \rho\sigma^2/\sigma_\varepsilon^2 \ . \qquad \text{(A.1)}$$

Subtracting (A.1) from (9)' yields

$$\gamma^2 = (1 - \rho^2)\sigma^2/\sigma_\varepsilon^2 \qquad \text{(A.2)}$$

which implies (11) of the text. Now substitute for γ into (9)" to obtain an expression in β alone:

$$2\beta(\beta + \frac{\sigma}{\sigma_\varepsilon}\sqrt{1 - \rho^2}) + (n - 2)\beta^2 = \rho^2\sigma^2/\sigma_\varepsilon^2,$$

that is,

$$n\beta^2 + \frac{2\sigma}{\sigma_\varepsilon}\sqrt{1 - \rho^2} - \frac{\rho^2\sigma^2}{\sigma_\varepsilon^2} = 0,$$

a quadratic in β, with the solutions

$$\beta = \frac{1}{2n}\{-\frac{2\sigma}{\sigma_\varepsilon}\sqrt{1 - \rho^2} \pm \sqrt{\frac{4\sigma^2}{\sigma_\varepsilon^2}(1 - \rho^2) + 4n\frac{\rho^2\sigma^2}{\sigma_\varepsilon^2}}\}.$$

Choosing the larger solution and simplifying leads to eq. (10).

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