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ABSTRACT

Several models are discussed in which aggregate fluctuations result from self-fulfilling revisions of agents' expectations ("sunspot equilibria"). It is shown that this is possible in models with infinite lived agents if financial markets are imperfect; both cash-in-advance constraints and constraints upon lending between agents of different types are discussed. The paper concludes with a general discussion of the extent to which business cycle theories of this kind yield testable predictions.

Keynes argued in the General Theory that aggregate instability was largely caused by fluctuations in investment expenditures, and that the volatility of investment was due, among other things, to the dependence of investment decisions upon entrepreneurs' beliefs about an uncertain future, which beliefs were themselves subject to a great deal of variation that might be unrelated to any changes in anything other than market psychology itself. (See especially chapters 5 and 12.) While Keynes described the volatility of expectations in more colorful language than most of his contemporaries, the theme is not an uncommon one in business cycle theories of the quarter century prior to the publication of the General Theory. Lavington (1921, pp. 171-2), for example, attributes business cycles to "the inherent instability of business confidence"; he argues that an initial increase in the confidence of some producers, "whether or no this confidence is justified", leads to actions which are themselves "a real cause of increased confidence on the part of many other producers", so that a boom results. Hawtrey (1950, pp. 341-347) attributes the trade cycle to "the inherent instability of credit", which in turn is propelled by self-validating expectations of two sorts -- fluctuations in the expectations of businessmen about the state of demand, which can be self-fulfilling because of relatively passive accommodation by banks of changes in the demand for credit (p. 344), and fluctuations in the beliefs of bankers about the extent to which they can extend credit without causing a dangerous deterioration of their reserves, which depends upon the rate at which other bankers are expanding their own supplies of credit (pp. 342-343).

Self-fulfilling revisions of expectations have played very little role, on the other hand, in the business cycle models of recent decades. This has probably occurred for two reasons. First, to the extent that expectations constitute a causal factor capable of variation independent of movements in other, observed, state variables, one would seem to be unable to predict economic outcomes. Hence, such effects would simply contribute to the stochastic shock terms tacked on to the equations of a formal econometric model of the business cycle, and need not be modeled explicitly. Second, recent approaches to business cycle theory have stressed the importance of grounding all the posited structural relations upon rational individual choice, and accordingly -- especially in the case of business cycles that are understood to represent a recurrent state of affairs -- it is argued that individuals should be assumed to have "rational expectations" as well. As accounts of spontaneous movements in the state of business confidence ("animal spirits") are usually assumed to imply that businessmen's beliefs are irrational[1], such accounts of the nature of fluctuations are judged inconsistent with the general goal of explaining aggregate outcomes in terms of rational individual choice.

We would like to argue, to the contrary, that such an account of the nature of aggregate fluctuations is entirely consistent with the aims and methods of modern equilibrium business cycle theory, as represented, in particular, by the work of Lucas and of Kydland and Prescott. For explicit equilibrium models may have stationary rational expectations equilibria in which expectations (and hence equilibrium prices and allocation of resources) vary in response to state varibles that in fact convey no information about changes in preferences, resources or technological possibilities. These variables are often called "sunspot variables" and the equilibria "sunspot equilibria" (after Cass and Shell (1982)).[2] We propose that these equilibria be interpreted as representations of repetitive fluctuations in which spontaneous revisions of agents' expectations are self-fulfilling, i.e., they produce a changed outcome such that the changed expectations are validated. They thus indicate a role for volatile expectations as an independent causal factor in aggregate fluctuations, and -- insofar as fluctuating equilibria are possible even when preferences and technology are completely constant -- an "inherent instability" of certain competitive economies of the sort asserted by the prewar authors.[3]

Early examples of stationary sunspot equilibria may not have appeared to hold much promise as empirically relevant models of aggregate fluctuations. On the one hand, solutions of this kind were found to exist in the case of some ad hoc linear macroeconomic models (Taylor, 1977; Shiller, 1978). But the stationary sunspot equilibria occurred only for parameter values outside what was considered the range of possible empirical relevance. And it was thought possible that the sunspot equilibria existed as solutions only because the models in question were not explicitly derived from the constrained optimization problems of rational agents; a complete list of the conditions needed for a certain pattern of events to represent a true equilibrium might rule out some of the apparently valid solutions.[4] One the other hand, solutions of this kind were found to exist in certain quite stylized examples of general equilibrium models (Shell, 1977; Azariadis, 1981; Cass and Shell, 1982). These examples made it clear that sunspot equilibria could exist in rigorously specified models, but their empirical relevance for the explanation of aggregate fluctuations remains unclear.

The present paper exhibits a new class of models that may possess stationary sunspot equilibria -- stationary infinite-horizon competitive economies with a small number of infinite lived agents, and financial constraints of various types. We believe that this class of examples is of particular interest in demonstrating that models of aggregate fluctuations that result from self-fulfilling expectations represent a promising approach to equilibrium business cycle theory -- an alternative to the more familiar approaches that assume an intrinsically stable economy which is however subject to repeated exogenous shocks to tastes, technology, or the like. First of all, the structure of the models is similar to that found in other recent exercises in equilibrium business cycle theory (such as those of Kydland and Prescott), except for the introduction of certain financial constraints. One consequence is that, as in the work of Kydland and Prescott, here the parameters that determine the quantitative predictions of the model have reasonably clear empirical correlates, so that one can say a good deal about what are plausible magnitudes quite apart from considerations of what values are needed in order for the model to predict aggregate fluctuations of the sort that occur. Second, the method of analysis used here (unlike the prior general equilibrium literature) characterizes equilibrium fluctuations in terms of linear stochastic difference equations of the kind used in time series econometrics.

Finally, the connection argued for here, between financial constraints and the possibility of self-fulfilling revisions of expectations, echoes a theme from earlier, less formal discussions of aggregate instability. All of the prewar authors mentioned above lay great emphasis upon dysfunctional behavior of the institutions controlling the availability of finance in their accounts of the business cycle, and proposals for the regulation of financial markets figure prominently among their suggested reforms.[5] Among contemporary macroeconomic theorists, Axel Leijonhufvud has on several occasions articulated a view similar to our own. He argues (1981, pp. 73-78, and pp. 120-123) that serious macroeconomic instability occurs only when the exhaustion of liquid "buffer stocks" causes liquidity constraints to bind, and that in such a case instability results because revisions of expectations about future incomes become self-fulfilling.[6] The examples presented here (in which specifications are chosen such that certain liquidity constraints always bind) attempt to model formally this conception of an economy "outside the corridor", although, for reasons of analytical tractability, they describe economies in which the condition is permanent rather than merely episodic.

Section 1 presents some general observations regarding the need for market imperfections if expectations are to be self-fulfilling. Section 2 presents examples of financially constrained economies in which expectations are self-fulfilling, stressing the formal similarity between these examples and examples from the overlapping generations literature. Section 3 shows that expectations may continue to be self-fulfilling even when the financial constraints described in Section 2 are weakened in various respects. Section 4 then addresses the sense in which models with multiple equilibria have testable consequences or can generate useful predictions, in the light of these examples.

I. INCOMPLETE MARKETS AND SELF-FULFILLING EXPECTATIONS

Before presenting specific examples of economies with imperfect financial intermediation in which "sunspot" fluctuations can occur in a stationary rational expectations equilibrium, a few general remarks are appropriate on the role of financial constraints in making possible equilibrium phenomena of this kind.

Remark 1. In the case of a competitive economy with a finite number of agents (the kind we consider here) and a convex technology (i.e., no increasing returns to scale), if a complete set of Arrow-Debreu contingent claims markets exists (as in, say, the sort of stochastic growth models treated by Kydland and Prescott (1980, 1982)), then there are no equilibria in which "sunspot" variables affect the allocation of resources.

The reason is simple. First of all, an allocation in which "sunspot" variables matter is Pareto inefficient (in terms of the ex ante expected utility of agents). For any such allocation x, consider the allocation $\hat{x}$ which in each state assigns to each agent a consumption vector equal to the expected value of the vector he receives in x. This is feasible because of the convexity of production sets; but it also increases the ex ante expected utility of each agent who receives a stochastic allocation in x, because of the concavity of the utility functions.[7] Then, since Arrow-Debreu equilibria are Pareto efficient (under standard assumptions -- no externalities, etc.), allocation x cannot be an equilibrium allocation.

This means that in any model of a competitive economy with no increasing returns to scale in which fluctuations are to be explained as "sunspot" equilibria, one must depart from the Arrow-Debreu structure in some way sufficient to prevent the usual proof of the First Welfare Theorem from holding. Of course there are many ways in which this might be done. An example constructed by Dave Cass, for example, shows that it is possible to have sunspot equilibria in a model with complete contingent claims markets; the example is an infinite horizon model (hence with an infinite sequence of goods) with overlapping generations of finite lived agents (hence an infinite sequence of agent types). As is well known, violations of the First Welfare Theorem are possible with complete markets in the case of infinite sequences of agents and goods.[8] This might suggest that overlapping generations (o.l.g.), rather than incomplete financial markets, are the crucial element in allowing for self-fulfilling equilibrium fluctuations; and indeed, most of the existing examples[9] of sunspot equilibria involve overlapping generations.

We wish, however, to direct attention elsewhere for several reasons. First of all, Barro (1974) has argued that while economic agents are finite lived, they often leave bequests to their descendants, and that an infinite sequence of successive generations of finite lived agents linked by bequests (chosen to maximize the utility of the givers, who have an altruistic concern for their descendants) will choose an infinite horizon consumption and saving program like that of a single infinite lived agent. While the Barro reduction is not without analytical difficulties,[10] it is not clear that these provide any reason to expect that stationary "sunspot" equilibria of the kind found in certain overlapping generations models can continue to exist when a large number of agents are linked by bequests. The most plausible argument for the robustness of the conclusions obtained for o.l.g. models is that many people do not leave bequests,[11] not because of the absence of intergenerational ties of sympathy, but because the descendants are expected to be so much better off that the desired bequest is negative; constraints upon the transfer of debts to one's descendants, and upon the degree to which the descendants can borrow early in life against their projected future earnings, are therefore crucial to the o.l.g. story.

Thus one must posit financial constraints after all; and the relevant constraints may not be correctly summarized by supposing that finite lived agents leave no bequest but can freely transfer income between different periods of their lives. If children cannot assume the debts of their parents or grandparents because creditors will not lend to the children, then a consistent model must not assume that the same individuals are free to borrow against their future income in order to finance consumption early in life. But once the borrowing constraints are introduced, it is not clear that one's qualitative conclusions are much changed by assuming infinite lived representative agents rather than o.l.g. In particular, stationary sunspot equilibria are possible with infinite lived agents subject to financial constraints. And in the cases considered in section 2, the financial constraints result in consumption demands and labor supply on the part of infinite lived agents that mimic decisions that would be made by a sequence of overlapping generations of finite lived agents -- a sort of converse to the Barro reduction.

Furthermore, in o.l.g. models with perfect financial markets, money is valued in equilibrium only as a store of wealth, and hence earns a rate of return similar to that obtained upon all assets. This is not observed in actual economies, and hence the way money is treated in such models is plainly inadequate.[12] If one supposes that monetary phenomena play an important role in aggregate fluctuations, then one will wish to introduce a transactions motive or precautionary motive for money holding into any proposed business cycle model. But money can supply "transactions" or "liquidity" services only in an economy in which financial intermediation is imperfect. Thus one is led to consider economies with financial constraints; and indeed, the type of constraints considered below are the sort that are introduced into intertemporal equilibrium models by authors such as Robert Lucas in order to model the demand for money. Again, once the constraints are introduced for this reason, it is not clear that anything important for business cycle modeling would be gained by continuing to consider overlapping generations.

Another problem with the existing literature on self-fulfilling expectations in o.l.g. models is that the examples of stationary sunspot equilibria presented thus far all involve two period lived agents. It is difficult to know how to interpret the "periods" in such models, and hence how to interpret the preference and technology parameters, so as to be able to ask whether stationary sunspot equilibria exist for empirically plausible parameter values or not. One possible inference from these examples,[13] for instance, is that the repetitive equilibrium fluctuations modeled occur at very low frequencies, since the "period" of the "cycles", however one chooses to measure it, is always greater than two periods and hence longer than the lifetime of the agents in the model. Yet "business cycles" last for only a fraction of a typical human lifespan, even of a typical person's period of economic productivity. The models presented below are not subject to this difficulty. Indeed it is shown in section 4 below that one of these models yields a quantitative prediction regarding the "period" of equilibrium aggregate fluctuations in terms of technological parameters and a labor supply parameter that have clear empirical significance; and that the "period" is of the correct order of magnitude when these parameters are assigned empirically plausible magnitudes.

Remark 2. In the example presented below, stationary sunspot equilibria exist because of the existence of a continuum of non-stationary perfect foresight (i.e., deterministic) equilibria, converging asymptotically to the same deterministic steady state (d.s.s.), and this sort of indeterminacy of perfect foresight equilibrium can be shown to be necessary and sufficient for the existence of stationary sunspot equilibria close to a d.s.s., in a broad class of stationary economies to which the models considered below belong.[14] Stationary sunspot equilibria that represent small fluctuations about a deterministic steady state are a particularly interesting case, as in this case a linear approximation to the equilibrium conditions is valid and the equilibrium fluctuations can be characterized using linear stochastic models. Such a linear representation is extremely useful in that it allows one to derive quantitative predictions regarding aggregate fluctuations that can be tested using the standard techniques of time series econometrics. Even if such formal testing is not contemplated, a linear stochastic model makes it easy for one to generate predictions regarding the relative variances of and covariances between different aggregate variables, which sorts of comparison make up most of the stylized facts about business fluctuations that one wishes to explain; hence the use of linear approximations in the work of Kydland and Prescott. Hence one's attention is directed to the question of when perfect foresight equilibrium is indeterminate in stationary deterministic economies.

One can show that in a competitive model with a finite number of infinite lived agents, perfect financial intermediation precludes indeterminacy.[15] The reason is as follows.[16] In the absence of financial constraints, competitive equilibrium must be Pareto optimal. All possible equilibrium allocations then must belong to the set F of Pareto optimal allocations. If there are H agents, points in F can be parameterized by H-1 coordinates, representing the relative weights on the utilities of the different agents. For each point in F, there is a unique set of relative prices for all goods representing the marginal rates of substitution and transformation between goods, and one can price each agent's consumption stream. Equilibria are then those points in F at which each agent's consumption stream has exactly the same value as his endowment stream plus initial assets. These amount to H-1 independent conditions (the last budget equation holds by identity if the others hold), so that, generically, solutions in F are locally isolated. Hence there cannot be a continuum of equilibria all converging to the same long run stationary allocation.

The existence of stationary sunspot equilibria that represent small fluctuations around a d.s.s., then, requires conditions under which such an argument is invalid. In the models we discuss in section 2, it is imperfect financial intermediation that prevents even perfect foresight equilibrium from being Pareto optimal, and invalidates the proof described. The proof also fails for other types of economies; in particular, it fails if one has an infinite sequence of agents (F is no longer a finite dimensional manifold), as in o.l.g. models.[17] But, for the reasons just discussed, the case of economies with imperfect financial intermediation seems particularly likely to be relevant to business cycle theory.

Cass and Shell (1982) suggest that "incomplete market participation" is crucial to the existence of sunspot equilibria, giving several examples of how this can result in the existence of sunspot equilibria in finite horizon competitive economies. This might seem to be a unifying principle that would subsume both the finance constrained economies of section 2 and the o.l.g. examples, as o.l.g. economies are often described as economies in which a "market imperfection" exists in that agents whose lives do not overlap are unable to trade. This would not, in our view, be correct. The o.l.g.

examples with sunspot equilibria are economies in which, as far as perfect foresight equilibrium is concerned, there is complete market participation -- all agents can and do trade in all markets that they have any desire to trade in (see Shell (1971)) -- and it is the indeterminacy of perfect foresight equilibrium that is responsible for the existence of stationary sunspot equilibria in such cases. And, as noted above, sunspot equilibria can exist in o.l.g. models even when all agents trade in a complete set of contingent claims markets. Hence incomplete markets and an infinite sequence of agents must be judged two distinct types of economic structure that can allow self-fulfilling expectations in equilibrium. Yet, as the examples below show, there is a certain formal similarity between the equilibrium conditions in models of the two kinds, and from this purely formal viewpoint it is not surprising that both kinds of economies allow for stationary sunspot equilibria.

Financial constraints, however, are not the only kinds of market imperfections (in economies with infinite lived agents) that can allow for equilibria in which fluctuations result from self-fulfilling revisions of expectations. In the example of Diamond and Fudenberg (1984), externalities associated with the matching technology through which potential traders meet, and in the example of Shlaifer (1985), externalities associated with the implementation of new technologies that other firms then copy, and non-price-taking behavior on the part of firms in determining the profits they can expect from implementation of such technologies, similarly allow for endogenous equilibrium fluctuations. Either of these mechanisms might turn out to be important for understanding aggregate instability, although thus far they have been illustrated only in quite special models that make it difficult to judge whether the sort of fluctuations exhibited can occur for empirically plausible parameter values.

II. STATIONARY SUNSPOT EQUILIBRIA IN ECONOMIES WITH FINANCIAL CONSTRAINTS

In this section, two relatively simple examples are presented in which the analogy with the equilibrium dynamics found in o.l.g. models is particularly evident. More complex (and perhaps more realistic) specifications of the financial constraints are discussed in section 3.

A. A Cash-in-Advance Monetary Economy

Consider a cash-in-advance economy of the kind treated by Wilson (1979) and Lucas and Stokey (1984). A single agent type exists, who is endowed with labor power each period and consumes the single perishable consumption good, produced instantaneously using labor (one unit of the good per unit of labor employed). Each agent wishes to maximize the expected value of

(2.1) $$\Sigma_{t=1}^{\infty} \gamma^{t-1} [u(c_t) - v(n_t)]$$

where $0 < \gamma < 1$ is a discount factor, c_t is consumption in period t, n_t is labor supplied in period t, and u and v are C^2 functions satisfying

(i) $u' > 0,\ u'' < 0,\ v' > 0,\ v'' > 0$

(ii) $v'(0) = 0,\ v'(n) \to \infty$ as $n \to \infty$

Assumptions (i) are the standard monotonicity and concavity assumptions, while assumptions (ii) guarantee a finite positive desired labor supply for any positive real wage.

Each agent must sell his entire output of the good (or, equivalently, his labor) each period on a competitive spot market, and purchase the good he consumes on this market as well. Furthermore, it is assumed that purchases must be paid for out of cash balances held at the beginning of the period; wages earned for period t labor supply cannot be spent until period t+1.[18] Thus agents maximize the expected value of (2.1) subject to the constraints

(2.2a) $$p_t c_t \leq M_t$$

(2.2b) $$M_{t+1} = M_t - p_t c_t + p_t n_t$$

where p_t is the period t money price of the good (and hence also the period t money wage) and M_t is the quantity of money held at the beginning of period t. It is assumed that an initial per capita money supply $M > 0$ is uniformly distributed among the agents at the beginning of period t.

Stochastic processes for (c_t, n_t, M_t) solve this optimization problem, for a given stochastic process for (p_t), if for all t

(2.3a) $$v'(n_t) = \gamma E_t[u'(c_{t+1})p_t/p_{t+1}]$$

(2.3b) $$u'(c_t) \geq v'(n_t)$$

and (2.2a) are satisfied, (2.2a) holds with equality whenever (2.3b) is a strict inequality, and $(M_t/p_t)u'(c_t)$ is bounded above. These stochastic processes describe a rational expectations equilibrium (r.e.e.) if in addition $c_t = n_t$ and $M_t = M$ at all times.

Let us restrict our attention to equilibria in which (2.2a) holds with equality at all times. Then $p_t = M/n_t$ at all times, and an equilibrium is a stochastic process for (n_t) satisfying

(2.4a) $$V(n_t) = \gamma E_t\, U(n_{t+1})$$

(2.4b) $$n_t \le \bar{n}$$

at all times, where $V(n) \equiv nv'(n)$, $U(c) \equiv cu'(c)$, and $\bar{n}$ is the quantity defined by $u'(\bar{n}) = v'(\bar{n})$. Note that there are no stochastic shocks in equations (2.4), yet a stochastic process for (n_t) may be a solution. For the sake of specificity, suppose that (x_t) is an i.i.d. random variable (with c.d.f. G) that agents observe, although it conveys no information about preferences, endowments, or technology, and let us look for stationary stochastic processes in which output may depend upon the history of realizations of this "sunspot" variable. Then a stationary r.e.e. is a function $n_t = \phi(x_t, x_{t-1}, x_{t-2}, \ldots)$ such that $\phi \le \bar{n}$ and

$$V(\phi(x_t, x_{t-1}, \ldots)) = \gamma \int U(\phi(x, x_t, x_{t-1}, \ldots))\, dG(x)$$

for all histories $(x_t, x_{t-1}, \ldots)$. One such equilibrium is the d.s.s. $\phi = n^*$ for all histories of x, where n^* satisfies $V(n^*) = \gamma U(n^*)$, i.e., $v'(n^*) = \gamma u'(n^*)$. (Since $\gamma < 1$, $n^* < \bar{n}$.) Under what conditions do there also exist stochastic equilibria (i.e., equilibria in which the sunspot variable affects the level of output) near the d.s.s. (i.e., such that n_t remains within a neighborhood of n^* for all histories of x)?

Linearization of (2.4a) around n^* yields

$$[v' + n^* v''] (n_t - n^*) = \gamma [u' + n^* u''] E_t(n_{t+1} - n^*)$$

where the derivatives of u and v are evaluated at n^*. This linear stochastic difference equation has stationary solutions of the form

$$(2.5) \qquad (n_t - n^*) = k \sum_{j=0}^{\infty} \left(\frac{v' + n^* v''}{\gamma(u' + n^* u'')}\right)^j (x_{t-j} - x^*)$$

for arbitrary k, where x^* is the mean of the distribution G(x), in the case that

$$(2.6) \qquad \left| \frac{v' + n^* v''}{\gamma(u' + n^* u'')} \right| < 1$$

(Note that this is also exactly the case in which there exists a continuum of non-stationary perfect foresight equilibria converging asymptotically to the d.s.s.) Let us suppose that the distribution G(x) has a compact support. (This involves no loss of generality, as x can always be rescaled by an appropriate nonlinear transformation.) Then when (2.6) holds, for small enough nonzero k, the "sunspot" solution (2.5) satisfies (2.4b). Furthermore, for k small enough, the fluctuations remain sufficiently close to the d.s.s. for the linear approximation to be valid. Hence, when (2.6) holds, there exists an infinite number of stationary sunspot equilibria near the d.s.s., and those of them in which the fluctuations are of sufficiently small amplitude are well approximated by (2.5).[19]

Note that since $v' = \gamma u'$, $v'' > 0$, and $u'' < 0$, (2.6) cannot hold unless the denominator of the fraction is a large negative quantity. Defining the elasticity of labor supply with respect to the expected real value of wages (in terms of consumption the following period) as

$$e = \frac{\gamma[u'(c)c + u''(c)c^2]}{v''(n)n^2 - \gamma u''(c)c^2}$$

one finds that (2.6) is equivalent to $|1 + e^{-1}| < 1$, or $e < -1/2$. Thus, a sharply backward-bending labor supply curve is required in order for stationary fluctuations of this kind to be possible.

This is reminiscent of Azariadis' (1981) findings regarding the existence of stationary sunspot equilibria in an o.l.g. monetary exchange economy, and of Grandmont's (1985) regarding the existence of deterministic equilibrium cycles in the same kind of model. In fact, the equilibrium conditions derived above, for the case in which the cash-in-advance constraint always binds, are identical to those obtained by Azariadis. Equation (2.3a) is exactly the first-order condition for optimal labor supply by an infinite sequence of overlapping generations of the period lived agents, who supply labor in the first period of life and consume in the second, and wish to maximize the expected value of $\gamma u(c_{t+1}) - v(n_t)$. The infinite sequence of budget constraints (2.2a) breaks up the optimization problem of the infinite lived representative agent into an infinite sequence of independent finite horizon optimization problems, resulting in the possibility of dynamics formally analogous to those of an o.l.g. model. But here the "periods" have nothing to do with human biology; they represent a time lag associated with the payments mechanism, and must thus be interpreted as quite short.[20] Hence equilibrium fluctuations in a model of this type occur on time scales much shorter than a human lifetime.

Note also that our results here do not depend upon the assumption that loan markets do not exist, only upon the assumption that shopkeepers require payment in cash at the time of a purchase, so that a payment lag exists. One may introduce a competitive loan market as follows (as in both Wilson (1979) and Lucas and Stokey (1984)). Suppose that each period is divided into two subperiods. A competitive loan market exists in the first subperiod; agents may trade money for promises to pay money at the beginning of the following period. In the second subperiod, the goods market is open, and agents make purchases using money that they hold at the end of the first subperiod. Suppliers receive payment for their goods during the second subperiod, which funds may be used to repay loans, or may be spent or lent in the following period. Then, since all agents are identical, the loan markets clear each period with no borrowing or lending. Hence, the equilibrium conditions remain those written above, plus an additional equation to determine the interest rate on money loans. ((2.3b) is in this case the condition that the nominal interest rate on money loans be non-negative.) Condition (2.6) continues to be a sufficient condition for the existence of stationary sunspot equilibria near the d.s.s.

Condition (2.6), however, requires preferences of a relatively implausible sort. It is often regarded as desirable to impose the condition of "gross substitutability" -- i.e., that excess demands for all goods be decreasing functions of the own price and increasing functions of the prices of all other goods. In the present context, this requires

$$\text{(iii)} \qquad u'(c) + cu''(c) > 0 \quad \text{for all} \quad c \geq 0$$

which in turn implies that $e > 0$, i.e., labor supply is an increasing function of the expected real value of the wage. It is known from the literature on indeterminacy of perfect foresight equilibrium in o.l.g. models that indeterminacy is not possible near a monetary steady state of an o.l.g. exchange economy if "gross substitutability" holds, regardless of the number of goods per period, the number of periods each agent lives, or the number of distinct agent types per generation.[21] On the other hand, indeterminacy is possible even with "gross substitutability" in the case of o.l.g. production economies.[22] This suggests that the introduction of production and capital may be required in the case of finance constrained infinite lived agents as well, in order for stationary sunspot equilibria to be possible without extreme and implausible assumptions on preferences.

B. A Production Economy with Concentrated Ownership of Capital

It is possible to introduce production using capital into the economy of the previous section in such a way that the resulting equilibrium dynamics are formally close to those found in the case of an o.l.g. production economy (specifically, the example of Reichlin (1985)). In order for the labor supply and consumption decisions of the representative worker not to be changed, it is necessary to consider an economy in which the capital stock is owned entirely by a second group of infinite lived agents.[23] (Reasons for the wage-earning households to never hold capital are discussed below.) It is also convenient (although not essential for anything done below) to assume that the agents who own the capital stock supply no labor; then aggregate capital accumulation and aggregate labor supply are each determined each period by the decision of one representative agent.

We assume furthermore that the owners of capital can spend the returns to capital in the same period in which the capital is used in production -- distributions of profits are not subject to the same time lag that wage distributions were assumed above to be subject to. This means that producers are extended credit, either directly by the other sellers of produced goods (we might suppose that the producers settle accounts with each other at the end of each period, after the goods market closes), or by a banking system[24] that creates whatever quantity of inside money producers demand in the first subperiod as long as the borrowers have capital to pledge as collateral for the loan.[25] (Which assumption is made does not matter for the business cycle dynamics described here, although it does matter when monetary policy is considered -- see footnote 41.) An assumption of this kind -- that investment expenditures are not cash-constrained in the same way that consumption expenditures are -- plays a prominent role in both the Keynesian and pre-Keynesian literature on business cycle dynamics.[26] It allows for variation in the velocity of money over the cycle, as the proportion of total expenditures that consist of investment expenditures changes, and is at the heart of Keynes' argument that investment varies independently of the level of savings.

We consider here only equilibria in which

$$r_{t+1} > p_t/p_{t+1} \tag{2.7}$$

in all cases, where r_{t+1} is the ex post gross real return to each unit of capital purchased in period t, so that the return on capital dominates that on money. In such a case, owners of capital hold no money. Each period, they choose a capital stock k_t (to be used in production in period t+1) and consumption q_t, subject only to the budget constraint

$$k_t + q_t \leq r_t k_{t-1} \tag{2.8}$$

In equilibrium, (2.8) holds with equality. The budget constraints of wage earners, on the other hand, continue to be (2.2a-b), with now $p_t n_t$ replaced by $w_t n_t$ in (2.2b), as the money wage w_t no longer equals the goods price p_t.

As in section A, the fact that these budget constraints hold need not mean that loan markets do not exist; it only requires that the two types of agent are unable to borrow from or lend to to one another. The absence of lending between the two types is subject to two interpretations, which rely upon quite different sorts of credit market imperfections, but lead to the same sort of business cycle dynamics.

Interpretation A. Under this interpretation, owners of capital are unwilling to lend to wage-earners, because of problems in enforcing repayment -- debtor's prisons no longer exist, human wealth is more easily moved by a fleeing debtor than are capital goods or inventories, and legal arrangements do not allow borrowers to sell themselves into servitude. It may be supposed that owners of capital have access to a perfect set of competitive financial markets; but since they are assumed to be all identical, in equilibrium they neither borrow nor lend among themselves and (2.8) holds.

For this interpretation to be consistent, we must show that in the equilibria under consideration,

$$u'(c_t) \geq \gamma E_t[u'(c_{t+1})r_{t+1}] \tag{2.9}$$

holds at all times, where c_t is the consumption of wage-earners in period t, so that wage-earners would like to borrow (if they could at the rate of return earned upon capital) rather than wishing to save (i.e., purchase capital themselves) at any point during the business cycle. Let us suppose that the owners of capital have stationary additively separable preferences and a discount factor $0 < \beta < 1$. Then in a d.s.s., $r_t = \beta^{-1}$ each period, and $c_t = c$ (a constant) each period. Condition (2.9) necessarily holds with strict inequality in the d.s.s., then, if $\gamma < \beta$ (wage-earners discount the future more). But in this case, condition (2.9) continues to hold (by a continuity argument) for any stochastic equilibrium involving sufficiently small fluctuations around the d.s.s.

This interpretation is a natural one in that it amounts simply to an extension of the sort of information and enforcement problems that presumably lie behind the cash-in-advance constraint. It is also consistent with the traditional emphasis in the Keynesian literature upon the sensitivity of consumption expenditures to current (as opposed to expected lifetime) expenditure as an important source of instability[27] and the existence of some degree of constraint of this sort is suggested by recent econometric studies.[28] Still, it may be doubted that wage-earners discount the future to such a degree, given that they do accumulate assets of various sorts, expecially houses and durable goods, and the assumption that they are unable to borrow against future wages may seem implausible given the quantity of consumer credit that now exists.[29] An alternative interpretation would be perfectly consistent with these observations.

Interpretation B. Under this interpretation, the owners of capital are distinguished not as the only agents who discount the future sufficiently little to be willing to save, but as the only agents who have knowledge of the production opportunities. Each agent of this type is an entrepreneur, who can operate a certain production technology at an arbitrary scale with constant returns. But the scale at which each entrepreneur produces is limited by the amount of savings he himself has to invest. Wage earners (or the intermediaries who invest their savings) are unwilling to invest in the projects of the entrepreneurs, because they themselves are unable to evaluate the profitability of the projects proposed by the entrepreneurs and they have difficulty preventing misuse of the delegated funds.[30] Under this interpretation, it may be supposed that non-entrepreneurial households have access to a perfect set of competitive financial markets (except for the payment lag associated with monetary exchange), but that as they are all identical, in equilibrium they neither borrow nor lend.

For this interpretation to be consistent, we must show that in equilibrium,

$$(2.10) \qquad u'(c_t) \leq \gamma E_t[u'(c_{t+1})r_{t+1}]$$

holds at all times. This means that wage earners are not willing to issue debt that promises to pay the same rate that entrepreneurs receive from their production projects; at that rate of return, they would like to lend, if only there were not the problem of monitoring the entrepreneurs. By the same argument as above, if $\gamma > \beta$ (wage earners discount the future less than entrepreneurs), (2.10) necessarily holds with strict inequality in any d.s.s., and likewise for any equilibrium involving small fluctuations around the d.s.s.

This intrepretation emphasizes the dependence of the level of production upon the flow of profits to firms out of which production must largely be financed, a linkage that is also emphasized in the business cycle theory of Kalecki.[31] Greenwald and Stiglitz (1986) suggest that such constraints play an important role in the propagation of aggregate fluctuations, although they do not consider the possibility of endogenous fluctuations that is of primary interest here. In section 3.B below, it is shown that the structure of the model is not greatly changed if it is assumed that wage earners do invest in production projects (through intermediaries), but that they do so through <u>debt</u> contracts rather than by contributing equity. With this extension, the postulated failure of financial intermediation becomes a particularly plausible one. For the moment, we continue to assume that no lending occurs.

Again we restrict our attention to equilibria in which the cash-in-advance constraint is always binding for wage earners, i.e., equilibria in which

$$u'(c_t)/p_t > v'(n_t)/w_t \tag{2.11}$$

at all times. (This is the analog of (2.3b)) when $w_t \neq p_t$). Then $p_{t+1}c_{t+1} = w_t n_t$ in every period, as in the model without capital, and again the first order condition for optimal labor supply is

$$V(n_t) = \gamma E_t U(c_{t+1}) \tag{2.12}$$

It is especially easy to see the possibility of stationary sunspot equilibria when we make certain simplifying assumptions regarding technology and the preferences of owners of capital. Let us suppose that owners of capital seek to maximize the expected value of[32]

$$\Sigma_{t=1}^{\infty} \beta^{t-1} \log q_t$$

In this case (given that the agent's entire wealth consists of the capital he owns at any point in time), such an agent consumes each period a constant fraction of his wealth, i.e., he chooses

$$q_t = (1-\beta) r_t k_{t-1} \tag{2.13a}$$

$$k_t = \beta r_t k_{t-1} \tag{2.13b}$$

In such a case, $k_t + q_t = \beta^{-1} k_t$. Let us assume furthermore a single sector production technology with fixed coefficients: m units of period t labor are employed with each unit of capital (period t produced good) to produce a units of output (inclusive of any undepreciated capital) that may either be consumed or used in the following period's production. Let us assume that $\beta a > 1$. Then equilibrium labor demand is always $n_t = mk_{t-1}$, and the condition for goods market clearing in period t is $c_t = ak_{t-1} - \beta^{-1}k_t$. Substituting these equations into (2.12) yields a stochastic difference equation for the capital stock

$$V(mk_{t-1}) = \gamma E_t U(ak_t - \beta^{-1}k_{t+1}) \tag{2.14}$$

Given any stochastic process for the capital stock that satisfies (2.14), it is possible to uniquely determine the evolution of r_t/p_t using (2.13), and of n_t and c_t using the expressions given above. The real wage is then given by

$$m(w_t/p_t) = a - r_t$$

Then, since at the end of each period the constant outside money stock M must be held by wage earners (as wages just received that they have not had an opportunity to spend), and since each period wage earners spend their entire money holdings, in equilibrium $M = w_t n_t = p_t c_t$ each period. This then determines the level of money prices each period. It remains only to check that the price and quantity movements thus derived satisfy (2.2b) (with w_t replacing p_t as noted above) and (2.7) all times, and either (2.9) or (2.10), depending upon the interpretation chosen. (If one supposes that <u>both</u> kinds of borrowing constraints apply, then neither (2.9) nor (2.10) need hold at all times).

A unique d.s.s. exists, corresponding to the stationary capital stock k^* that solves

$$V(mk^*) = \gamma U((a - \beta^{-1})k^*)$$

(Conditions (i)-(ii) and $\beta a > 1$ suffice for a unique solution $k^* > 0$ to exist.) Since a d.s.s. implies constant prices and $r = \beta^{-1} > 1$, (2.7) holds, and since the stationary labor supply and consumption of wage earners are such that $V(n) = \gamma U(c) < U(c)$, (2.2b) holds. Finally, if one assumes $\gamma < \beta$ (in accordance with interpretation A), (2.9) holds, and if one assumes $\gamma > \beta$ (in accordance with interpretation B), (2.10) holds.

We now wish to consider whether non-constant functions $k_t = \phi(x_t, x_{t-1}, \ldots)$ exist that satisfy (2.14), where again x_t is an i.i.d. sunspot variable with compact support. Again we can answer this, in the case of functions such that k_t remains close to k^* for all sunspot histories, by linearizing (2.14) around k^*. One obtains

$$(2.15)\quad m[v'+n^*v''](k_{t-1}-k^*) = \gamma[u'+c^*u''][a(k_t-k^*) - \beta^{-1}E_t(k_{t+1} - k^*)]$$

From the result of Blanchard and Kahn (1980), we know that (2.15) has a continuum of linear solutions in which the sunspot variable affects the evolution of the capital stock if the polynomial

$$\gamma[u' + c^*u''][\beta^{-1}\lambda^2 - a\lambda] + m[v' + n^*v''] = 0$$

has two roots λ with modulus less than one. Given assumption (iii), this occurs if and only if

$$(2.16)\qquad e > \frac{\beta a - 1}{2 - \beta a}$$

Again, one can then show that (2.16) is a sufficient condition for the existence of small-amplitude fluctuating solutions to (2.14).[33] And, in the case of solutions in which the amplitude of the fluctuations about the d.s.s. is sufficiently small, the inequalities that hold strictly in the d.s.s.

continue to hold. Hence all conditions for a stationary r.e.e. are satisfied.

Thus in the present case we find that stationary sunspot equilibria can exist near a d.s.s., even when labor supply responds positively to changes in the real wage. One might still wonder if (2.16) does not represent an extreme assumption about preferences. This seems not to be the case. Given that the capital/output ratio for the U.S. economy is about 3 years, and assuming a depreciation rate for the capital stock of about 10 percent per year (as in Kydland and Prescott (1982)), a plausible value for a is approximately $1 + .23\Delta$, where Δ is the length of a "period" in years. If (again following Kydland and Prescott) we suppose that the rate of time discount of owners of capital is 4 percent per year, a plausible value for β is approximately $1-.04\ \Delta$. In this case, (2.16) becomes approximately $e > .2\ \Delta$. Since Δ is surely much less than a year, the elasticity of labor supply required by (2.16) is quite small.

Note that in the fluctuating equilibria of this economy, it is possible to interpret the sunspot realization x_t as an unforecastable change in the expectations of producers regarding the income they will receive from production in period t, in response to which they change their desired level of investment expenditure. Since p_t and r_t are given by the equations

(2.17a) $$p_t = M[ak_{t-1} - \beta^{-1}k_t]^{-1}$$

(2.17b) $$r_t = k_t/\beta k_{t-1}$$

the budget of each producer in period t depends upon the level of expenditure of producers as a group. Suppose that a sunspot realization leads producers to expect a higher return r_t. Then each will wish to spend more. An increase in desired aggregate expenditure by producers as a group means an increased volume of credit extended to producers, which drives up the price level to the point where the consumption expenditures of wage earners (whose money expenditures are limited by the cash in-advance constraint (2.2a)) plus the desired purchases by producers do not exceed total output.[34] Given that the labor market must clear at a wage of $w_t = M/n_t = M/mk_{t-1}$, the price increase results entirely in increased returns to capital, in the amount given by (2.17b). Thus the income from capital that each producer receives is exactly the average expenditure of producers as a group -- in Kalecki's phrase, "capitalists get what they spend". Hence each producer, in deciding how much to spend on credit in a given period, must seek to determine how much others are spending, and so on. Under such circumstances, rumors of a change in the state of "business confidence" become self-fulfilling. Thus this model captures the Keynesian notion that the volatility of investment expenditure results in aggregate fluctuations, and that an important reason for the volatility of investment expenditures is their dependence upon the state of entrepreneurs' expectations.[35]

Yet for a repetitive pattern of fluctuations in expenditure to occur, the pattern of response to the rumors (that we may suppose are generated by a process of noisy communication among producers according to some regular stochastic law) must be such that labor markets in fact clear despite wage earners' understanding of the pattern of stochastic variations in the value of money. Hence, rather than arbitrary fluctuations in investment expenditure being possible, one must have a pattern satisfying (2.14). Thus the rational expectations hypothesis, rather than ruling out a role for fluctuating "animal spirits" in the causation of aggregate fluctuations, imposes restrictions upon the kind of pattern of fluctuations that can persist over time -- which is what allows a business cycle theory of this kind to have predictive content.

III. WEAKER FORMS OF FINANCIAL CONSTRAINT

The sort of financial constraint assumed in section 2.B is rather special -- no borrowing or lending between the two types of agents -- and reality is certainly much more complex. It is accordingly of interest to consider whether the sort of stationary equilibrium fluctuations shown to be possible in that case continue to be possible in economies in which the financial constraints are less severe.

A. Restricted Borrowing by Wage Earners

Let us first consider the restriction upon borrowing by wage earners assumed in interpretation A above. A more realistic model would allow some borrowing by such households. What degree of restriction upon their borrowing is necessary in order for the stationary sunspot equilibrium to exist?

Let us suppose that lenders allow such households to borrow any amount up to a real credit limit d; and for simplicity let us suppose that the interest that borrowers must pay upon such loans is variable and equal to the returns yielded by capital. (This simplifies our calculations, because the debt of wage-earning households is then a perfect substitute for capital in the portfolios of savers).

Let us restrict our attention to equilibria in which the credit limit is always binding. (Note that if $\gamma < \beta$, then in any d.s.s. the limit must bind, and likewise in the case of fluctuating equilibria near any d.s.s.) Then in each period $p_t c_t = w_{t-1} n_{t-1} + d p_t (1 - r_t)$. The first order condition for optimal labor supply is again

$$(3.1) \qquad V'(n_t) = \gamma E_t[u'(c_{t+1})w_t/p_{t+1}]$$

although this is no longer equivalent to (2.12) when $p_{t+1}\, c_{t+1} \neq w_t n_t$. Since the total wealth of savers at the beginning of period t must be $k_{t-1} + d$, (2.13b) becomes $k_t+d = \beta r_t(k_{t-1}+d)$, so that $k_t + q_t = \beta^{-1}k_t + (\beta^{-1}-1)d$. This implies that in any equilibrium

$$c_t = ak_{t-1} - \beta^{-1}k_t - (\beta^{-1} - 1)d$$
$$w_{t-1}n_{t-1}/p_t = c_t - d(1-r_t) = [a - \beta^{-1}(k_t+d/k_{t-1}+d)]k_{t-1}$$

Substituting thse expressions (plus the labor demand equation $n_t = mk_{t-1}$) into (3.1) yields

$$(3.2) \qquad V(mk_{t-1}) = \gamma E_t[u'(ak_t-\beta^{-1}k_{t+1}-(\beta-1)d)(a-\beta^{-1}\left(\frac{k_{t+1} + d}{k_t + d}\right))k_t]$$

A stationary solution $k^* > 0$ exists if and only if

$$v'(m(1-\beta)d/(\beta a-1)) < \gamma(a-\beta^{-1})u'(0)/m$$

If $u'(0)$ is finite, this places an upper bound on d beyond which there does not exist a d.s.s. of this form. But if $u'(c) \to \infty$ as $c \to 0$, a d.s.s. exists no matter how large d is, and the debt limit is always binding.

Equation (3.2) generalizes (2.14). Again we have a second-order stochastic difference equation for the capital stock, and again the existence of stationary fluctuating equilibria near the d.s.s. depends upon whether linearization of (3.2) around the d.s.s. has a characteristic polynomial with both roots of modulus less than one. It is evident that for small $d > 0$, the roots continue to be close to those found in the case $d = 0$, so that fluctuating equilibria continue to exist for certain parameter values.

Even if the debt limit varies over the business cycle, the conditions for existence of fluctuating equilibria are not greatly changed, as first order condition (3.1) continues to apply, and only the relations linking c_t and $w_{t-1}n_{t-1}/p_t$ with k_t are slightly changed. The equilibrium dynamics are much more affected if an individual wage earner's debt limit is tied to the hours that he chooses to work in the coming period. Then an increased labor supply in period t allows more consumption in period t by relaxing the borrowing constraint; as a result, n_t is not chosen solely with a view to the value in period t+1 of the wages earned. Preliminary investigations suggest that too great an effect of this sort rules out existence of the fluctuating equilibria near the d.s.s. Hence the plausibility of interpretation A depends crucially, not upon an assumption that wage earners cannot borrow at all, but that they cannot increase the amount they are allowed to borrow by committing themselves to longer hours of work.

B. Entrepreneurs Only Able to Obtain Debt Finance

The financial constraint assumed in Section 2.B is also unrealistically severe under interpretation B, for only the riskiest sorts of ventures are completely dependent upon the private funds of the entrepreneur for finance. A more plausible financial constraint is one under which the only sort of outside finance available is debt finance -- outside investors are unwilling to share the firm-specific risks with the entrepreneur, because of the difficulty of monitoring the reasons for unexpectedly low profits.[36]

We can introduce firm-specific risk by supposing that different entrepreneurs sell different products in a large number of distinct markets (though the different goods are produced using the same technology), and that in addition to the random fluctuations in aggregate spending modeled above, there are random shifts in consumer tastes that do not affect aggregate spending but affect the division of it between markets. In addition, let us suppose that there is a substantial penalty imposed upon entrepreneurs in the event of a default upon a loan repayment. Then the consequence of the riskiness of profits is that entrepreneurs will not be willing to leverage themselves too far, even if lenders offer an unlimited quantity of loans at a fixed interest rate.[37] Hence the level of aggregate investment will continue to depend upon entrepreneurs' expectations of profits out of which to finance that investment. Note that, under the preferences assumed, entrepreneurs continue to invest the same constant fraction of their wealth each period, regardless of the riskiness of the distribution of returns to such investment. If we suppose that the taste shocks are revealed (along with the current "sunspot" realization) before firms make their investment decisions, then (in a rational expectations equilibrium) each entrepreneur can predict his current period profits with certainty and so chooses to invest exactly fraction β of his wealth after repayment of debts (plus the amount he invests of newly borrowed funds). Because of the firm-specific demand shocks, the levels of investment chosen by different entrepreneurs will differ; but aggregate investment will be a fraction β of the aggregate wealth (after repayment of debts) of entrepreneurs -- which depends upon aggregate profits which depend upon aggregate expenditures -- which we may suppose are not affected by the taste shocks at all.

In what follows, then, we assume that the aggregate dynamics (of the general level of prices, of the aggregate capital stock, etc.) occur as if there were no firm-specific demand shocks at all; but we assume that there is a limit to the amount of leverage that can be added to a given quantity of entrepreneurial capital. (Whether the limit is imposed by lenders or self-imposed by entrepreneurs who fear the bankruptcy penalty is irrelevant.) Let us suppose that an entrepreneur's debt cannot exceed fraction θ (where $0 \leq \theta \leq 1$) of the total value of the capital goods he purchases for use in production. Let us, furthermore, consider only equilibria in which

$$r_{t+1} > i_t(p_t/p_{t+1}) \tag{3.3}$$

at all times, where i_t is one plus the nominal interest rate on funds borrowed in period t (before the goods market opens) and repaid in period t+1 (before the goods market opens). Then the debt limit always binds. It follows that the real wealth of the representative entrepreneur in period t (after repayment of debts and anticipating period t profits, as before) is $k_{t-1}(r_t - \theta i_{t-1}(p_{t-1}/p_t))$, and equations (2.13) become

$$q_t = (1-\beta)(r_t-\theta i_{t-1}(p_{t-1}/p_t))k_{t-1} \tag{3.4a}$$

$$k_t = (\beta/(1-\theta))(r_t-\theta i_{t-1}(p_{t-1}/p_t))k_{t-1} \tag{3.4b}$$

Wage earners save by lending to entrepreneurs, at a fixed nominal interest rate set in advance. In writing (3.4), we have assumed that the debt limit θ is set at such a level that no entrepreneurs fail to pay their debts in full; hence lenders regard the loans to be completely free of default risk. It follows that the interest rate at which wage earners are willing to lend is given by

$$i_t = \frac{w_t}{p_t}\frac{u'(c_t)}{v'(n_t)} \tag{3.5}$$

Again, in equilibrium, $M = w_t n_t$ each period; substituting this, the profit equation $r_t = a - m(w_t/p_t)$, and (3.5) into (3.4b) yields

$$(3.6) \qquad ak_{t-1} - (1-\theta)\beta^{-1}k_t = \frac{M}{p_t}\left(1 + \theta \frac{u'(c_{t-1})}{V(n_{t-1})} k_{t-1}\right)$$

Also, in equilibrium, $n_t = mk_{t-1}$ and $c_t = ak_{t-1} - (\theta+(1-\theta)\beta^{-1})k_t$. Also, substituting these into (3.6) allows us to derive a smooth positive-valued function

$$M/p_t = \psi(k_{t-2}, k_{t-1}, k_t)$$

defined on the domain $k_{t-2} > 0$, $ak_{t-1} > (\theta+(1-\theta)\beta^{-1})k_t > 0$. Multiplying (3.1) by n_t and substituting the above then yields

$$(3.7) \qquad V(mk_{t-1}) = \gamma E_t[u'(ak_t - (\theta+(1-\theta)\beta^{-1})k_{t+1})\psi(k_{t-1}, k_t, k_{t+1})]$$

Again we have a second-order stochastic difference equation for the capital stock. A d.s.s. is a $k^* > 0$ such that $k_t = k^*$ for all t solves (3.7). It follows that k^* must satisfy

$$mv'(mk^*) = \gamma u'((a-\theta-\beta^{-1}(1-\theta))k^*)(a-\gamma^{-1}\theta-\beta^{-1}(1-\theta))$$

Given assumptions (i)-(ii), a unique d.s.s. exists. It follows from (3.5) that in the d.s.s. $i_t = \gamma^{-1}$ each period, while it follows from (3.4b) that $r_t = \gamma^{-1}\theta + \beta^{-1}(1-\theta)$ each period, so that (3.3) holds with strict inequality as long as $\beta < \gamma$. It then follows that (3.3) also holds at all times in the case of small fluctuations about the d.s.s.

As before, the existence of fluctuating equilibria near the d.s.s. depends upon the roots of the characteristic polynomial of the linearization of (3.7) about the d.s.s. Since (3.7) reduces to (2.14) when $\theta = 0$, it is clear that stationary sunspot equilibria continue to exist, for a certain range of parameter values, for small positive θ.

On the other hand, one also finds that stationary sunspot equilibria near the d.s.s. are impossible in the case of sufficiently large θ. One of the necessary conditions for a polynomial $a_2\lambda^2 + a_1\lambda + a_0 = 0$ to have both roots of modulus less than one is that $a_0 + a_1 + a_2$ have the same sign as a_2. But in the linearization of (3.7), we find

$$a_0 + a_1 + a_2 = \gamma(a-\beta^{-1}(1-\theta)-\gamma^{-1}\theta)(\theta+\beta^{-1}(1-\theta)-a)k^*u'' + m^2k^*v''$$
$$a_2 = \gamma\beta^{-1}(1-\theta)u' + \gamma(a-\beta^{-1}(1-\theta))(\theta+\beta^{-1}(1-\theta))k^*u''$$

At any d.s.s., $a_2 + a_1 + a_2 > 0$ follows from assumption (i) on preferences. Therefore one must have $a_2 > 0$ for the stationary sunspot equilibria to exist, but it follows from (i) that $a_2 < 0$ for θ close to 1. Thus there is an upper bound on the debt-equity ratio that is consistent with the existence of fluctuating equilibria of this type, the exact location of which depends upon the technology and preference parameters.[38] This indicates the importance of the existence of distinct types of agents between whom financial intermediation is incomplete, in the generation of these equilibrium fluctuations, for in the limit $\theta \to 1$, the equilibrium conditions approach those of a model in which there is a single type of infinite lived agent who both supplies labor and owns the capital stock.

C. Borrowing Constraints in the Absence of a Payments Lag

In all the examples presented thus far, the postulated financial constraints include a cash-in-advance constraint that prevents wage earners from spending wages until the period after they are earned, and in all these examples this constraint is necessary in order for outside money to have a positive value in the d.s.s. and the fluctuating equilibria near it. Yet it is possible for outside money to be valued in a competitive equilibrium even when no such payment lags exist, i.e., even when it is assumed that all income can be spent in the same period in which it is earned. This is possible, for example, if agents are unable to borrow against future income, and their income stream is sufficiently variable that in a stationary equilibrium the borrowing constraint binds at certain times.[39] This kind of financial constraint also can allow stationary sunspot equilibria to exist.

In order to employ the methods of analysis used above, one needs a model with a d.s.s.; yet, as just noted, one also needs exogenous fluctuations in agents' incomes in order for the borrowing constraints to bind in such a d.s.s. One simple case in which this is possible is treated by Bewley (1984). Let there be two types of infinite lived agents -- type 1 with a large endowment in odd periods and a small endowment in even periods, and type 2 with endowments of exactly the opposite pattern. Let the endowments consist of perishable goods, so that fiat money is the only store of value. If the variation in endowment between periods is sufficiently great, and the agents' rate of time preference is sufficiently small, there exists a d.s.s. in which money in valued. In the d.s.s., type 1 agents hold money in odd periods, but would like to borrow (though they cannot) in even periods, while type 2 agents hold money in even periods and are borrowing constrained in odd periods. Bewley shows that this pattern of borrowing constraint every second period -- which continues to hold for any equilibria involving sufficiently small fluctuations about the d.s.s. -- causes the infinite lived agents to have demands like those of a sequence of two period lived agents, so that the conditions for a perfect foresight equilibrium in his model are identical to those of an o.l.g. model.[40]

It follows that stationary sunspot equilibria may exist near the d.s.s. of a Bewley economy; this occurs in exactly the cases that it would for an o.l.g. economy. Thus, for example, in the pure exchange example considered by Bewley, the existence of stationary sunspot equilibria requires preferences that fail to satisfy the "gross substitutability" condition. But it is possible to construct examples with production (analogs of the o.l.g. examples of Calvo (1978) and Reichlin (1985), for instance) in which stationary sunspot equilibria exist even when preferences satisfy that condition.

4. Multiple Equilibria and the Predictive Content of Business Cycle Theory

As noted in the introduction, one very general reason for resistance to theories of the business cycle that allow a role for spontaneous revisions of expectations is the view that a theory of this sort amounts to abandoning the attempt to explain or predict market phenomena. We have sought to show, by contrast, that if one postulates that the pattern of autonomous fluctuations in expectations -- at least in the case of the repetitive movement associated with business cycles -- must satisfy the conditions for "rationality" of expectations introduced into business cycle theory by Lucas, the kinds of self-fulfilling fluctuations in expectations that can occur are quite restricted. Nonetheless, all the models above are characterized by a large multiplicity of stationary equilibria in those cases in which stationary sunspot equilibria are shown to exist. It might seem that models with so many equilibria make very few testable preditions (so that, on one popular view, they have little content), and cannot be used (as other sorts of business cycle models aspire to be) in predicting the consequences of alternative policies.

We would like to suggest otherwise. First of all, it should be noted that in the above examples, while a large number of stationary rational expectations equilibria exist, in <u>all</u> of the fluctuating equilibria, the fluctuations are of the same type. This is most particularly true of the set of fluctuating equilibria in which the fluctuations are small enough that the linear approximation to the equilibrium conditions is adequate. Then, for example, in the case of the model of section 2.B, in all fluctuating equilibria the fluctuations in the capital stock take the form

$$\tilde{k}_t = a\beta\tilde{k}_{t-1} - (\beta a-1)(1+e^{-1})\tilde{k}_{t-2} + \epsilon_t \quad (4.1)$$

where $\tilde{k} = \log k_t - \log k^*$, and ϵ_t is the innovation in the stochastic process for $\tilde{k}_t$. (This follows from (2.15).) But then, since variations in output and hours also move proportionally with movements in the capital stock (but lagged one period), equation (4.1) also indicates the pattern of movements of these variables. Linearizing (2.17a) around the steady state and substituting (4.1) yields

$$\tilde{p}_t = -(1+e^{-1})\tilde{k}_{t-2} + (\beta a-1)^{-1}\epsilon_t$$

where $\tilde{p}_t = \log p_t - \log p^*$. Similarly one obtains equations expressing other period t state variables as linear combinations of $\tilde{k}_{t-1}$, $\tilde{k}_{t-2}$, and ϵ_t. Thus one obtains a single-factor linear stochastic model of the co-movements among aggregate variables. Even without specification of numerical values for the parameters, the model implies a large number of cross-equation restrictions upon the VARs that one should obtain. And none of the parameters are "free", in the sense that their values can only be inferred from the pattern of aggregate fluctuations observed; one has a reasonably definite idea of the value of each of them on independent grounds.

As an illustration, we present impulse response functions for the logarithms of output, investment, consumption, and the price level, in response to a one-time innovation ϵ_t that results in a 1% increase in the capital stock. (See Figure 1.) Here we have adopted the parameter specifications suggested in section 2.B, along with e = .25 and Δ = 3 months. Note that the response of output has the familiar "hump shape" predicted by a multiplier-accelerator model and characteristic of autoregressions of detrended U.S. GNP data. (The degree of "momentum" predicted by our model, however, is considerably less than that observed, unless a is set unrealistically large.) The responses of investment and consumption are both similar to those characteristic of a multiplier-accelerator model -- investment rises initially, then drops below its steady state level as output passes the "hump", finally approaching the steady state level from below, while consumption rises and falls back to its steady state level from above, tracking the output response with a lag. Also note that the model predicts that investment expenditures are several times as variable as consumption expenditures (although not so much more variable as is actually observed).

The model makes two predictions about variations in the price level over the business cycle. On the one hand, unexpected price increases coincide with unexpected increases in investment, which lead by one period positive innovations in output. This relationship is similar to that described by the

"Lucas supply curve", but the causality is entirely reversed -- here, unexpected increases in entrepreneurs' production plans cause an unexpected expansion of the volume of credit which causes the unexpected increase in the general price level. (It is the effect of this price increase on the real value of wages that causes consumption to decline in the period that the unexpected increase in investment occurs -- an example of what the Austrians called "forced saving".) On the other hand, <u>anticipated</u> price level movements are negatively correlated with anticipated output movements -- which fits with Prescott's (1983) observation that the (detrended) price level is a countercyclical variable.

Finally, the time scale over which aggregate fluctuations are predicted to persist is roughly that which is observed; for example, the model predicts that the deviation of output from its steady-state value will have fallen to about one-third of its maximum value six quarters after the initial revision of expectations. (As noted previously, the ability to predict fluctuations at "business cycle frequencies" is an important advantage of the present class of models over previous examples of the o.l.g. type.)

The model thus predicts a very specific pattern for aggregate fluctuations; only the <u>amplitude</u> of the fluctuations is not predicted, since different stationary rational expectations equilibria involve fluctuations of different amplitudes about the d.s.s. The fact that the amplitude of the fluctuations is theoretically indeterminate may be considered undesirable. But as far as the number of testable implications of the theory is concerned, the situation is no different than in the case of the model of Kydland and Prescott (1982). There the variance of output fluctuations is predicted, for any given variance of the productivity shock process; but in practice, the variance of the productivity shock process is a free parameter, inferred from the observed variance of output fluctuations.

The linear approximations used above are, of course, valid only in the case of "small" fluctuations; hence there may exist other stationary fluctuating equilibria that are not simply scaled-up versions of the ones described above. But even so, there are testable quantitative predictions of the model that apply to all stationary equilibria -- the stochastic process followed by the capital stock must satisfy (2.14), and so on.

Even when the linear approximation is valid, the fact that all fluctuating equilibria are simply scalar multiples of a single equilibrium depends upon the fact that in all the examples presented above, the number of stable roots of the characteristic polynomial exceeds the number of predetermined state variables by exactly one. In more complex models, one might obtain an even larger number of excess stable roots. In such a case, there would exist more than one possible pattern for stationary equilibrium fluctuations; but still, all the possible patterns would be linear combinations of a finite number of fluctuating equilibria.

The sort of theory proposed here also offers predictions about which competitive economies are unstable in the sense described and which should not be. Similarly, comparisons are possible between alternative policy regimes as to whether they allow stationary r.e.e. in which self-fulfilling revisions of expectations occur.[41] It is our belief that stabilization policy should be conceived as an attempt to design a stationary policy regime in which such fluctuations cannot occur in equilibrium. (Recall that, as explained in Section 1, fluctuations in the allocation of resources in response to sunspot variables cannot be efficient. Of course, in the examples presented here, the d.s.s. is not Pareto optimal either, because of the financial constraints, but still it seems likely that agents are generally better off in the d.s.s.) In this way, business cycle theory can yield useful policy prescriptions despite an inability to predict (because of the existence of multiple equilibria) exactly what outcome must result from some possible policy interventions.

FOOTNOTES

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1. It seems to have been Keynes' view that sudden changes in expectations need not coincide with a change in what is in fact likely to occur -- he emphasizes the lack of foundation for any particular belief. But Hawtrey (1950, p. 346) writes: "It is taken for granted that the optimism and the pessimism are <u>mistaken</u>. But in reality this is not so...Each state of expectation tends to bring about its own fulfillment." This comes close to anticipating the view taken here -- that spontaneous fluctuations in expectations can occur in a rational expectations equilibrium.

2. Sargent and Wallace (1984) propose the nomenclature "spurious indicators". They propose that solutions to a macroeconomic model involving such variables may provide an explanation of the instability of the value money observed during hyperinflationary episodes. By contrast, our interest

here is in stationary fluctuations.

3. One might alternatively model "inherent instability" by the existence of deterministic equilibrium cycles, as in Grandmont (1985a,b) or Benhabib and Nishimura (1984). A problem with this line of research -- as far as practical business cycle modelling is concerned -- is that the existence of such deterministic equilibrium cycles always depends crucially upon nonlinear aspects of the equilibrium conditions, whereas most of the techniques available for summarizing the observed properties of business cycles, and estimating and testing formal dynamic models, are concerned with linear models. Stationary sunspot equilibria (at any rate, those which involve small fluctuations about a d.s.s., as illustrated below), by contrast, may be adequately represented by linear time series econometric models. (It is worth noting, however, that deterministic equilibrium cycles also exist, for certain parameter values, in the case of all the financially constrained economies discussed below. On the relationship between deterministic cycles and stationary sunspot equilibria, see Woodford (1984, secs. 3-4).

4. See, e.g., Taylor (1972, pp. 1378, 1383), Gourieroux, Laffont and Monfort (1982, pp. 424-5).

5. Kohn (1984, 1985) discusses this analytical tradition and proposes a reconstruction in more modern terms.

6. Scheinkman and Weiss (1986) also show, in an explicit general equilibrium model, that borrowing constraints can result in equilibrium fluctuations in output that would not occur if a complete set of Arrow-Debreu markets existed, although self-fulfilling expectations play no role in their example.

7. The argument is due to Balasko (1983).

8. The first example was provided by Samuelson (1958); the crucial role of

the infinite sequences stressed by Shell (1971).

9. See, e.g., Shell (1977), Azariadis (1981), Azariadis and Guesnerie (1982, 1983), Woodford (1984), Peck (1984), Guesnerie (1985), and Grandmont (1985b).

10. E.g., Gale (1983, sec. 1.8; 1985) shows that bequest economies may have an infinite number of equilibria, only one of which corresponds to the one assumed by Barro, while Bernheim (1985) shows that application of the Barro argument to economies in which "dynasties" intermarry leads to a number of paradoxical conclusions.

11. See, e.g., Tobin (1980a). Note that sunspot equilibrium fluctuations are still possible in the case of some infinite lived "dynasties" of the Barro sort, as long as a sufficient number of familes are not linked by bequests; it is even possible when the Barro families own all of the non-human wealth in the economy. See Muller and Woodford (1985).

12. See, e.g., Tobin (1980b), McCallum (1983).

13. Sims (1984) raises this objection against the deterministic cycle model of Grandmont (1985a), but the point would apply equally to models of stationary sunspot equilibria like that of Azariadis (1981).

14. See Woodford (1985b).

15. This is not simply a trivial consequence of the proposition regarding Arrow-Debreu economies stated above. We are concerned here with a property of the perfect foresight equilibrium dynamics, for purposes of which "perfect intermediation" requires only that all agents can trade goods in different periods for each other at the same set of prices; there need not, for example, be any markets for claims contingent upon "sunspot" realizations.

16. The argument is due to Kehoe and Levine (1985, section 2). Their proof for exchange economies is easily extended to production economies of the sort

considered here.

17. This is true even under circumstances where perfect foresight equilibrium is Pareto optimal. For example, if non-depreciating land exists in an o.l.g. model, or one or more infinite lived agents (Barro "dynasties") coexist with the overlapping generations of finite lived agents, perfect foresight equilibrium must be Pareto optimal, but indeterminacy is still possible. See Muller and Woodford (1985).

18. It might seem that each agent should simply consume his own product; but we may suppose that there are actually a large number of differentiated goods, and that agents specialize in the production of a single good while they wish to consume all goods. If the model is completely symmetric across goods, symmetric equilibria can be characterized using the aggregate notation used here. The general argument is due to Lucas (1980).

19. Existence of these equilibria can be proved rigorously using a contraction mapping as in Woodford (1986). One can also prove the existence of stochastic solutions of the form $n_t = \phi(z_t)$, where the sunspot variable z_t is a finite-state Markov process, using the method of Azariadis (1981), or the existence of stochastic solutions of the form $n_t = \phi(n_{t-1}, x_t)$, where x_t is an i.i.d. sunspot variable with compact support, using the method of Farmer and Woodford (1984).

20. It is not clear exactly how long a "period" should be taken to be here, as in a simple model like this one it is at once the reciprocal of the transactions velocity and of the income velocity of money, whereas in complex economies like our own the former period is much shorter than the latter. In any event, a "period" should be only a fraction of a year.

21. See Kehoe, Levine, and Woodford (1986).

22. See the second example of Calvo (1978), or the example of Reichlin (1985).

23. Such a division of the population plays a particularly important role in the business cycle theory of Kalecki (1939, 1965). Our assumption here can also be viewed as a simple way, in an explicit general equilibrium model, of modeling the separation between the decisions of households and firms that plays a crucial role in models such as that of Greenwald and Stiglitz (1985).

24. See, e.g., Woodford (1985a).

25. Note that one need not assume that financial intermediaries treat different types of agents differently; one may suppose that all agents are required to finance expenditures out of either money balances held at the beginning of the period or loans that cannot exceed the income from capital expected to be received during the period. In equilibrium, wage earners choose not to hold capital, and so are cash-constrained each period.

26. For example, see Hawtrey (1950) and Robertson (1926) on money creation by banks in response to increased desire for investment by firms. The same mechanism is assumed by Kalecki (1939), who states that investment expenditures are "self-financing", while the consumption expenditures of workers are limited to their current wages. The assumption that certain categories of expenditures are "income-constrained" while others -- notably investment expenditures -- may be augmented by a release of cash from speculators' hoards, is quite explicit in Kaldor's (1939) account of the role of speculation in creating aggregate instability. (For a modern treatment of Kaldor's argument, see Kohn (1985).) A similar distinction between different categories of expenditures -- into "cash goods" versus "credit goods" -- is made by Lucas and Stokey (1984) as a way of allowing for velocity variations,

although there is no investment in their model.

27. See, e.g., Hall and Mishkin (1982), Flavin (1984). These studies do not, of course, indicate such a tight coupling between current (or recent past) income and current consumption as is asumed in the model presented here.

29. But the mere fact that wage earners can borrow, to a certain extent, may not rule out a modified version of interpretation A. See Section 3.A below.

30. Informational problems that may prevent entrepreneurs from having access to outside funds are discussed in Greenwald and Stiglitz (1985,1986).

31. See especially (1965, pp. 90-98). A similar finance constraint is shown to be a possible source of business cycles by Day (1967), who cites an earlier literature on such constraints. See also Steigum (1983) on the role of finance constraints in the theory of investment.

32. Such preferences are very special, but the preferences of these agents (apart from the value of β) have little effect upon business cycle dynamics since in any event (when "periods" are short) they consume only a small fraction of their wealth each period, so that k_t is chosen nearly as large as $r_t k_{t-1}$. See the discussion of "small" infinite lived agents in Muller and Woodford (1985, sec. 5.A).

33. See the proof in Woodford (1986).

34. This is just the "inflationary gap" mechanism of Keynes (1940), as well as the mechanism of "forced saving" whereby consumption is reduced in response to an increase in investment in the theories of Robertson (1926) and Hayek (1931).

35. See especially Keynes (1936, ch. 5).

36. The argument is developed in detail by Greewald and Stiglitz (1985).

37. This is just Kalecki's (1939) "principle of increasing risk".

38. It might seem that no such upper bound exists when u" is small; for when u" = 0, $a_2 > 0$ for all $\theta < 1$. But in the u" = 0 case, one can show from another of the Routh-Hurwicz conditions that one must have $\theta < (2-\alpha\beta)\gamma/(2\gamma-\beta) < 1$ in order for the stationary sunspot equilibria to exist.

39. Examples involving stochastic endowments are discussed by Scheinkman and Weiss (1986) and Levine (1985).

40. The structure of Bewley's example is also closely related to the "turnpike" economy of Townsend (1980), in which the borrowing constraints are derived from a particular postulated pattern of movement of infinite lived agents between different trading locations.

41. For example, in the case of the economy of section 2.B, one can show that, in the event that bank loans are the only source of credit for investment expenditures, control of the quantity of such loans by a monetary authority can make the d.s.s. the unique r.e.e. One such policy is to target total money expenditures in period t at ap^*k_{t-1}, where p^* is the constant price level in the d.s.s. If other sources of credit (e.g., trade credit) are sufficiently elastic, monetary policy is impotent, but in that case, variation of government deficit spending to offset variations in investment spending can stabilize. See Woodford (1986).

REFERENCES

Azariadis, C., "Self Fulfilling Prophecies," J. Econ. Theory 25 (1981), 380-396.

Azariadis, C., and R. Guesnerie, "Propheties Creatrices et Persistance des Theories," Revue Economique 33 (1982), 787-806.

__________, "Sunspots and Cycles", CARESS w.p. no. 83-22, revised March 1984.

Balasko, Y., "Extrinsic Uncertainty Revisited," J. Econ. Theory 31 (1983), 203-210.

Barro, R.J., "Are Government Bonds Net Wealth?" J. Political Econ. 82 (1974), 379-402.

Benhabib, J., and K. Nishimura, "Competitive Equilibrium Cycles", mimeo, N.Y.U., January 1984.

Bernheim, D., Paper presented at AEA meetings, New York, December 1985.

Bewley, T., "Dynamic Implications of the Form of the Budget Constraint," mimeo, Yale Univ., 1983.

Blanchard, O.J., and C. Kahn, "The Solution of Linear Difference Equation Models Under Rational Expectations," Econometrica 48 (1980), 1305-1311.

Calvo, G.A., "On the Indeterminacy of Interest Rates and Wages with Perfect Foresight," J. Econ. Theory 19 (1978), 321-337.

Cass, D., and K. Shell, "Do Sunspots Matter?" J. Political Econ. 91 (1983) 193-227.

Day, R.H., "A Microeconomic Model of Business Growth, Decay, and Cycles," Unternehmensforschung 11 (1967), 1-20.

Diamond, P., and D. Fudenberg, "An Example of Rational Expectations Business Cycles in Search Equilibrium," mimeo, M.I.T., August 1984.

Farmer, R.E.A., and M. Woodford, "Self Fulfilling Prophecies and the Business Cycle," CARESS w.p. no. 84-12, April 1984.

Flavin, M., "Excess Sensitivity of Consumption to Current Income: Liquidity Constraints or Myopia?" NBER w.p. no. (1341), May 1984.

Gale, D., "Money: In Disequilibrium", Cambridge Univ. Press, Cambridge, 1983.

————, CARESS working paper, 1985.

Gourieroux, C., J.-J. Laffont, and A. Monfort, "Rational Expectations in Dynamic Linear Models: Analysis of the Solutions," Econometrica 50 (1982), 409-426.

Grandmont, J.-M., "On Endogenous Competitive Business Cycles," Econometrica 53 (1985a), 995-1045.

————, "Stabilizing Competitive Business Cycles," CEPREMAP w.p. no. 8518, Paris, August 1985b.

Grandmont, J.-M. and G. Laroque, "Stability of Cycles and Expectations," CEPREMAP w.p. no. 8519, Paris, revised July 1985.

Greenwald, B., and J.E. Stiglitz, "Information, Finance Constraints and Business Fluctuations," mimeo, December 1985.

————, "Money, Imperfect Information, and Economic Fluctuations," mimeo, February 1986.

Guesnerie, R., "Stationary Sunspot Equilibria in an N Commodity World," mimeo, C.E.Q.C., Paris, October 1985.

Hall, R.E., and F.S. Mishkin, "The Sensitivity of Consumption to Transitory Income: Estimates from Panel Data on Households," Econometrica 50 (1982), 461-481.

Hawtrey, R.G., "The Trade Cycle," in American Economic Association, "Readings in Business Cycle Theory," Allen and Unwin, London, 1950. [Originally published in 1926.]

Hayek, F.A., "Prices and Production," Routledge and Kegan Paul, London, 1931.

Kaldor, N., "Speculation and Economic Stability," Review Econ. Stud., 1939.

Kalecki, M., "Essays in the Theory of Economic Fluctuations," Russell and Russell, New York, 1939.

————, "Theory of Economic Dynamics," Allen and Unwin, London, revised ed. 1965.

Kehoe, T.J., and D.K. Levine, "Comparative Statics and Perfect Foresight in Infinite Horizon Economies," Econometrica 53 (1985), 443-453.

Kehoe, T.J., D.K. Levine, A. Mas-Colell and M. Woodford, "Gross Substitutes in Large Square Economies," mimeo, U.C.L.A., April 1986.

Keynes, J.M., "The General Theory of Employment, Interest and Money," Macmillan, London, 1936.

————, "How to Pay for the War," Macmillan, London, 1940.

Kohn, M., "Monetary Analysis, the Equilibrium Method, and Keynes' General Theory," Dartmouth College w.p. no. 84-1, February, 1984.

————, "Policy Effectiveness and the Specification of Aggregate Demand: Keynes Saved by Robertson," mimeo, Dartmouth College, April 1985.

Kydland, F., and E.C. Prescott, "A Competitive Theory of Fluctuations and the Feasibility and Desirability of Stabilization Policy," in S. Fischer, ed., "Rational Expectations and Economic Policy," U. Chicago Press, Chicago, 1980.

————, "Time to Build and Aggregate Fluctuations," Econometrica 50 (1982), 1345-1370.

Lavington, F., "The English Capital Market," Methuen, London, 1921.

Leijonhufvud, A., "Information and Coordination," Oxford U. Press, Oxford, 1981.

Levine, D.K., "Liquidity with Random Market Closure," mimeo, U.C.L.A., August 1985.

Lucas, R.E., Jr., "Equilibrium in a Pure Currency Economy," in J.H. Kareken and N. Wallace, eds., "Models of Monetary Economies," F.R. Bank of Minn., Minneapolis, 1980.

————, "Studies in Business Cycle Theory," M.I.T. Press, Cambridge, 1981.

Lucas, R.E., Jr., and N.L. Stokey, "Money and Interest in a Cash-in-Advance Economy," CMSEMS w.p. no. 628, Northwestern Univ., October 1984.

McCallum, B.T., "The Role of Overlapping Generations Models in Monetary Economics," Carnegie-Rochester Conf. Series 18 (1983), 9-44.

Muller, W.J., and M. Woodford, "Determinacy of Equilibrium in Stationary Economics with Both Finite and Infinite Lived Consumers," mimeo, Carnegie-Mellon Univ., November 1985.

Peck, J., "On the Existence of Sunspot Equilibria in an Overlapping Generations Model," mimeo, Univ. Penn., April 1984.

Prescott, E.C., "Can the Cycle be Reconciled with a Consistent Theory of Expectations, or A Progress Report on Business Cycle Theory," mimeo, I.M.S.S.S., July 1983.

Reichlin, P., "Equilibrium Cycles and Stabilization Policies in an Overlapping Generations Economy with Production," mimeo, Columbia Univ., August 1985.

Robertson, D.H., "Banking Policy and the Price Level," King and Son, London, 1926.

Samuelson, P.A., "An Exact Consumption-Loan Model of Interest with or without the Social Contrivance of Money," J. Political Econ. 66 (1958), 467-482.

Sargent, T.J., and N. Wallace, "Exploding Inflation," mimeo, Univ. of Minn., January 1984.

Scheinkman, J.A., and L. Weiss, "Borrowing Constraints and Aggregate Economic Activity," Econometrica 54 (1986), 23-45.

Shell, "Notes on the Economics of Infinity," J. Political Econ. 79 (1971), 1002-1011.

————, "Monnaie et Allocation Intertemporelle," CNRS seminaire de E. Malinvaud, Nov. 21, 1977.

Shiller, R.J., "Rational Expectations and the Dynamic Structure of Macroeconomic Models," J. Monetary Econ. 4 (1978), 1-44.

Shlaifer, A., "Implementation Cycles," mimeo, M.I.T., August 1985.

Sims, C., Discussion, in "Abstracts for the Workshop on Price Adjustment, Quantity Adjustment, and Business Cycles," Preprint series no. 59, Institute for Math. and Apps., Univ. of Minnesota, February 1984.

Steigum, E., Jr., "A Financial Theory of Investment Behavior," Econometrica 51 (1983), 637-645.

Taylor, J.B., "Conditions for Unique Solutions to Stochastic Macroeconomic Models with Rational Expectations," Econometrica 45 (1977), 1377-1385.

Tobin, J., "Asset Accumulation and Economic Activity," Basil Blackwell, Oxford, 1980a.

————, "Discussion," in J.H. Kareken and N. Wallace, eds., "Models of Monetary Economies," F.R. Bank of Minn., Minneapolis, 1980b.

Townsend, R.M., "Models of Money with Spatially Separated Agents," in J.H. Kareken and N. Wallace, eds., "Models of Monetary Economies," F.R. Bank of Minn., Minneapolis, 1980.

Wilson, C., "An Infinite Horizon Model with Money," in J. Green and J.A. Scheinkman, eds., "General Equilibrium, Growth and Trade," Acad. Press, 1979.

Woodford, M., "Indeterminacy of Equilibrium in the Overlapping Generations Model: A Survey," mimeo, Columbia Univ., May 1984.

———, "Credit Policy and the Price Level," mimeo, Columbia Univ., August 1985a. To appear in W.A. Barnett and K.J. Singleton, eds., "New Approaches in Monetary Economics," Cambridge Univ. Press.

———, "Stationary Sunspot Equilibria," mimeo, Carnegie-Mellon Univ., December, 1985b.

———, "Stationary Sunspot Equilibria in a Finance Constrained Economy," mimeo, Columbia Univ., January, 1986. Forthcoming in J. Econ. Theory.