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ON THE OPTIMAL PRICING POLICY OF A MONOPOLIST

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Abstract

The paper presents a simple explanation of price dispersion by a monopolist assuming only that consumers arrive in a random order and are served on a first come, first serve basis. A firm can sometimes increase its profits by charging two different prices for the same good and rationing sales at the lower price. However, it is never necessary to charge more than two prices, and a single price is sufficient as long as either the marginal revenue curve is everywhere downward sloping or the marginal cost of production is constant.

1. Introduction

When will a monopolist charge the same price for every unit that it sells? It is well known that a monopolist may have an incentive to charge different prices to different consumers if it can identify the demand curve of the different consumers and prohibit arbitrage. Even if individual consumers cannot be identified, it is sometimes possible to price discriminate by offering quantity discounts. If the cost of search is explicitly modelled, Salop (1977) has shown that a monopolist can sometimes benefit from randomizing over its prices across outlets (or presumably across time) in order to exploit differences in the search costs of agents with different reservation values. In this note, I examine another, less sophisticated, explanation of nonuniform pricing by a monopolist.

Suppose a large but fixed number of consumers arrive in random order to buy a homogeneous good. Before any consumers arrive, the firm puts a price tag on each unit it wishes to sell. Upon arriving, each consumer then purchases at the lowest available price up to the point where the price of the next unit exceeds his reservation value. Unless the firm has a constant (or decreasing) marginal cost of production, it may have an incentive to charge different prices for different units of the good.

For simplicity, suppose the firm has 300 units to sell. There are 100 consumers. Each consumer is willing to purchase a single unit at a price of 2 and 4 additional units at a price of 1. Clearly, if the firm must charge the same price for all of the units, it will charge a price of 1, resulting in a total revenue of 300. However, if we permit the firm to charge a different price for different units, it could increase it revenue by selling the first 250 units at price 1 the last 50 units at price 2. The first 50 consumers will purchase the

250 units at price 1. The next 50 who arrive will purchase 1 unit each at price 2. The revenue of the firm is consequently increased from 300 to 350.

The next three sections contain a characterization of the optimal pricing policy for any left continuous, downward sloping demand curve generated by a large number of consumers. For a fixed level of output, I show that the optimal pricing policy can be found as the solution to a linear programming problem in the space of nonnegative measures on the space of prices. From this, a number of properties of the solution can be established. First, the firm need never charge more than two prices to maximize its revenue. Second, in contrast to the case where the monopolist is constrained to use a single price, the **marginal revenue** must be a nonincreasing function of the quantity. In fact, it is precisely in those cases where the single price marginal revenue curve is strictly increasing over some range that the firm will have an incentive to charge more than two prices. Finally, at any level of output at which the firm charges more than two prices, the marginal revenue is constant. This implies immediately that if the firm can produce output at a constant marginal cost, then its revenue can be maximized by choosing a level of output for which a single price is optimal.

In Section 7, I discuss the welfare implications of using a two price policy and in Section 8, I conclude with a brief discussion of some applications.

Although I will analyze the problem in the context of a static problem, similar results can be established in a dynamic model where the flow demand is stationary demand and the cost function refers to the flow of production. In this case, a firm with an upward sloping marginal cost curve might find it optimal to randomize from day to day (or week to week) over two different prices in order to achieve a flow demand equal (on average) to its flow supply.

In any case, the argument of this note depends critically on an implicit informal appeal to the law of large numbers to identify the realization of the effective demand curve with its average. If we explicitly take into account the discreteness of the product space and the consumers, we are confronted with a complicated integer programing problem. For this problem, Krishna and Perry (1985) have been able to show that the firm will charge a single price when marginal cost is zero; otherwise, very little seems to be known about the optimal pricing policy.

2. The Model

A large number of consumers visit a firm in random order to purchase a homogeneous good. Before any consumers arrive, the firm must choose the quantity to be offered for sale and the price of each individual unit. When it is his turn to purchase, a consumer purchases from the available units up to the point where the price of the next unit exceeds the price he is willing to pay for it. Assuming that goods are perfectly divisible, a **pricing policy** is then a measure Q on the set of prices $[0,\infty)$, where, for any subset of prices P, Q(P) denotes the number of units for sale at those prices. We suppose that the demand curve of any positive measure of consumers is proportional to the aggregate demand curve $D(\cdot)$.

To simplify the mathematics, we will initially suppose that $D(\cdot)$ is a nonincreasing step function. That is, there is a finite set of prices $\{p_0,\ldots,p_{n+1}\}$ such that $0 \equiv p_0 < p_1 < \ldots < p_{n-1} < p_n < p_{n+1} \equiv \infty$, $0 = D(\infty) < D(p_n) < \ldots < D(0) \leq \infty$, and $D(p) = D(p_i)$ for $p \in (p_{i-1},p_i]$. Since the firm has no incentive to charge any price outside of the set $\{p_0,\ldots,p_n\}$, a pricing policy Q can be represented as a vector $(q_0,\ldots,q_n)$, where $q_i = Q(p_i)$ is the number of units for sale at price p_i.

We proceed as follows. First, we determine the set of pricing policies which are consistent with the sale of q (or less) units of the good. Then we determine which pricing policies from this set maximize the profits of the firm. Finally, we examine how the pricing policy and revenue change with the number of units sold.

3. The Optimal Pricing Policy of a Firm Selling a Fixed Quantity

Since there is no aggregate uncertainty, we may suppose the firm plans to sell every good to which it attaches a (finite) price. Since consumers always purchase at the lowest available price, we may also assume that the goods are sold in order of their price. For notational ease, let $D_i = D(p_i)$ and let E_i be the measure of excess demand at price p_i **after** the goods offered at prices p_0 to p_i have been sold. Then, since the units priced at p_0 are sold first, it follows that

$$E_0 = D_0 - q_0 = D_0[1-(q_0/D_0)].$$

Suppose that $E_0 \geq 0$. Then the excess demand for goods priced at p_1 is equal to the excess demand at price p_0 times the fraction of those consumers who stay in the market when the price rises from p_0 to p_1 minus the supply of goods offered at price p_1. The excess demand at price p_0 is just E_0. The fraction of consumers who stay in the market is the ratio of the number of consumers with reservation value greater than or equal to p_1 to the number of consumers with reservation value greater than or equal to p_0, (D_1/D_0).[1] The supply of goods offered at price p_1 is just q_1. Consequently,

$$E_1 = E_0[D_1/D_0] - q_1 = D_1[1-[(q_0/D_0)+(q_1/D_1)]].$$

Proceeding by induction, we may conclude that as long as $E_{i-1} \geq 0$, then

$$E_i = E_{i-1}[D_i/D_{i-1}] - q_i = D_{i-1}[1-\sum_{j=0}^{i-1}(q_j/D_j)][D_i/D_{i-1}] - q_i$$
$$= D_i[1-\sum_{j=0}^{i}(q_j/D_j)].$$

If all of the units offered are to be sold then E_n must be nonnegative. Consequently, any pricing policy must satisfy:

(1) $\sum_{j=0}^{n}(1/D_j)q_j \leq 1.$

Suppose the firm must sell q units, where $q \in [0,D(0)]$.[2] Then in addition to constraint (1), its pricing policy must satisfy:

(2) $\sum_{i=0}^{n}q_i = q.$

The problem of the firm wishing to sell q (or less) units is therefore:

Problem I: Choose a pricing policy $Q = (q_0,\ldots,q_n) \geq 0$ to maximize $\sum_{i=0}^{n}p_iq_i$ subject to constraints (1) and (2).

Problem I is a linear programming problem in $(q_0,\ldots,q_n)$ with two constraints. To characterize the solution, we appeal to the Kuhn–Tucker conditions (see e.g. Dixit (1976), p. 63). A pricing policy $(q_0,\dots,q_n) \geq 0$ is a solution to Problem I if and only if relations (1) and (2) are satisfied and there is a $\lambda \in (-\infty,\infty)$ and a $\mu \geq 0$ such that

(3) $p_i - \lambda - (\mu/D_i) \leq 0$ for $i = 0,\dots,n$, and $\sum_{i=0}^{n}[(p_i-\lambda)-(\mu/D_i)]q_i = 0$;

(4) $\mu[\sum_{i=0}^{n}(q_i/D_i) - 1] = 0$.

These conditions imply

LEMMA 1: (a) Constraint (1) is satisfied with equality if and only if $q \geq D_n$. (b) If $q < D_n$, then $q_i = 0$ for $i < n$.

PROOF: If $q < D_n$, then, since D is nonincreasing, constraint (2) implies $\sum_{i=0}^{n}(q_i/D_i) \leq (1/D_n)\sum_{i=0}^{n}q_i \leq q/D_n < 1$. Conversely, if $\sum_{i=0}^{n}(q_i/D_i) < 1$, then equation (4) implies $\mu = 0$. It then follows from relation (3) that $\lambda \geq p_n > p_i$ for $i < n$ and hence, from relation (3), that $q_i = 0$ for $i < n$. Therefore, $q = q_n < D_n$. Q.E.D.

Using Lemma 1 we may establish

PROPOSITION 1: For a fixed level of sales, a monopolist can always achieve its maximum revenue by charging no more than two prices for different units.[3]

PROOF: Let λ and $\mu \geq 0$ satisfy relations (1) to (4) for some vector $(q_0,\dots,q_n) \geq 0$. Let $\ell = \min\{i: q_i > 0\}$ and let $h = \max\{i: q_i > 0\}$ and suppose $p_\ell < p_h$. Then it follows from Lemma 1 that constraint (1) holds with equality. Since D is nonincreasing, it then follows from equation (2) that $[1/D_h]q < 1 < [1/D_\ell]q$. Consequently, we may choose $\hat{q}_\ell \geq 0$ and $\hat{q}_h \geq 0$ such that $\hat{q}_\ell + \hat{q}_h = q$ and $\hat{q}_\ell/D_\ell + \hat{q}_h/D_h = 1$. Let $\hat{q}_i = 0$ for $i \neq \ell,h$. Then it is easy to check that λ and μ also satify relations (3) to (4) for $(\hat{q}_1,...,\hat{q}_n)$. Consequently, $(\hat{q}_1,...,\hat{q}_n)$ is a solution to Problem I. Q.E.D.

4. Properties of the Revenue Function

We examine next how the optimal policy and the revenue change with an increase in the number of units to be sold. The argument in the previous section depends in no essential way on the assumption that the demand curve was a step function (nor on the fact that it was nonincreasing).[4] For the remainder of the paper, therefore, we will take $D(\cdot)$ to be any nonnegative right continuous function defined for all positive prices with the property that $D(p) > 0$ for some $p > 0$ and $\lim_{p\uparrow\infty} pD(p) = 0$.[5] Let $\bar{p} = \sup\{p: D(p) > 0\}$ and $\bar{D} = D(\bar{p})$.[6] We continue to use the notation $D_i \equiv D(p_i)$ for any $p_i > 0$.

Given p_ℓ and p_h, (q_ℓ,q_h) will denote a price policy Q which offers quantities q_ℓ and q_h at prices p_ℓ and p_h respectively and zero quantity elsewhere. Let $D_h = D(p_h)$ and $D_\ell = D(p_\ell)$. We adopt the convention that, if $p_\ell \neq p_h$, then $p_\ell < p_h$. For any nonnegative value q, let R(q) denote the maximum revenue that can be obtained from selling q units or less. We will refer to $R(\cdot)$ as the revenue function.

We will use the following implication of Lemma 1 and Proposition 1.

LEMMA 2: Suppose $\hat{q} \geq \bar{D}$ and, for prices p_h and p_ℓ, $(\hat{q}_\ell, \hat{q}_h)$ is an optimal policy. Then, for any $q \in [D_h, D_\ell]$,[7]

(5) $R(q) = p_\ell D_\ell[(q-D_h)/(D_\ell-D_h)] + p_h D_h[(D_\ell-q)/(D_\ell-D_h)]$,

and $R(q)$ can be attained with a policy (q_ℓ, q_h) defined by:

(6) $q_\ell/D_\ell + q_h/D_h = 1$, and

(7) $q_\ell + q_h = q$.

PROOF: Suppose, for some $\hat{q} > \bar{D}$ and prices p_ℓ and p_h, policy $(\hat{q}_\ell, \hat{q}_h)$ attains $R(\hat{q})$. Then there is a $\lambda \in (-\infty, \infty)$ and a $\mu > 0$ which, together with $(\hat{q}_\ell, \hat{q}_h)$, satisfies relations (1) to (4). Now consider any $q \in [D_h, D_\ell]$ and let $q_\ell, q_h \geq 0$ be the unique solution to equations (6) and (7). Then, together with λ and μ, (q_ℓ, q_h) satisfies relations (1) to (4). Therefore, (q_ℓ, q_h) is optimal and hence $R(q) = p_\ell q_\ell + p_h q_h$. Solving equations (6) and (7) for q_ℓ and q_h as functions of q then yields equation (5). Q.E.D.

Using Lemma 2, we may establish

PROPOSITION 2: (a) The revenue function is concave. (b) If it is optimal for the firm to charge two prices to sell quantity q, then marginal revenue is constant over some interval around q.

PROOF: Part (a) follows from the fact that Problem I is a concave programming problem. (See e.g. Dixit (1976), chapter 3.) To establish part (b), suppose, for some $\hat{q} > 0$, policy $(\hat{q}_\ell, \hat{q}_h)$ attains $R(\hat{q})$, where $p_\ell < p_h$ and $\hat{q}_\ell, \hat{q}_h > 0$. Then relation (3) implies that there is a $\lambda \in (-\infty, \infty)$ and a $\mu > 0$ such that $D_\ell[p_\ell - \lambda] = D_h[p_h - \lambda] = \mu \geq 0$. This implies that $D_\ell > D_h$. It then follows from equations (5) and (6) that $\hat{q}/D_\ell = [\hat{q}_\ell + \hat{q}_h]/D_\ell < 1 < [\hat{q}_\ell + \hat{q}_h]/D_h = \hat{q}/D_h$ and hence that $\hat{q} \in (D_h, D_\ell)$.[8] But then Lemma 2 implies that, for any $q \in [D_h, D_\ell]$, $R(q)$ is defined by equation (5). Since this function is linear in q, part (b) is proved. Q.E.D.

The concavity of $R(\cdot)$ implies that marginal revenue is downward sloping.[9] The constancy of the marginal revenue over those ranges of q which support a two price optimum is a consequence of the fact that equations (6) and (7) are linear in q_h, q_ℓ, and q. Note that these two equations also imply that, when the firm charges more than one price, an increase in total sales requires that it increase its sales of the lower priced good and decrease its sales of the higher priced good.[10]

5. The Optimal Choice of Quantity and the Conditions for a Single Price

When the monopolist is constrained to charge a single price, the revenue function need not be concave. In this section, we demonstrate that is is precisely when the single price revenue function is not concave that the firm has an incentive to charge more than one price at some output levels.

Let $R_s(q)$ be the revenue to the firm when it is constrained to sell q units at the single price $D^{-1}(q)$[11].

LEMMA 3: At any quantity $q > 0$, there is a $p_{\ell}, p_h \in (0,\infty]$ and an $\alpha \in [0,1]$ such that

(8) $\quad R(q) = \alpha R_s(D_{\ell}) + (1-\alpha) R_s(D_h)$.

PROOF: If $q \leq \bar{D}$, let $p_{\ell} = \bar{p}$, $p_h = \infty$, and $\alpha = q/\bar{D}$ Then Lemma 1 implies that

$$R(q) = \bar{p}q = \alpha p\bar{D} = \alpha R_s(\bar{D}) = \alpha R_s(\bar{D}) + [1-\alpha] R_s(0).$$

If $q > D(\bar{p})$, then it follows from equation (5) of Lemma 2 that, for some pair of prices p_{ℓ}, p_h, there is an $\alpha \in [0,1]$ such that

$$R(q) = \alpha p_{\ell} D_{\ell} + (1-\alpha) p_h D_h = \alpha R_s(D_{\ell}) + (1-\alpha) R_s(D_h).$$

In either case we obtain equation (8). Q.E.D.

Using Lemma 3, we may establish

PROPOSITION 3: The firm can maximize its **revenue** at all output levels using a single price if and only if the single price revenue function is concave.

PROOF: Let $H = \{(r,q) \in [0,\infty)^2 \colon r \leq R(q)\}$ and $H_s = \{(r,q) \in [0,\infty)^2 \colon r \leq R_s(q)\}$. Since the firm always has the option of using a single price, it follows immediately that $R(q) \geq R_s(q)$ for all $q \geq 0$. Therefore, $coH_s \subset coH$.[12] Furthermore, the concavity of $R(\cdot)$ implies that $coH = H$, and hence that

$coH_s \subset H$. But Lemma 3 implies that $H \subset coH_s$. Therefore, $H = coH_s$.

To complete the proof, note that $H_s = coH_s$ and hence $R(q) = R_s(q)$ for all q if and only if $R_s(\cdot)$ is a concave function. Q.E.D.

The profit maximizing quantity depends upon both the revenue and the cost functions. Unless the single price marginal revenue function is concave, it is always possible to choose a cost function so that the marginal revenue curve cuts the marginal cost curve at a quantity where the profit maximizing firm must charge two distinct prices. However, the next proposition implies that such a cost function must display increasing marginal cost.

PROPOSITION 4: If the cost function is concave, then the firm can always maximize its **profit** by charging a single price.

PROOF: Let $C(\cdot)$ be the cost function and suppose profit is maximized at output level $\hat{q}$. Let $q^* = \sup\{q: R(q) = R(\hat{q})\}$. Then the concavity of $R(\cdot)$ implies $R(\cdot)$ is continuous and hence that revenue is maximized at q^*. Now suppose that the firm **must** charge two prices to maximize profits, then Proposition 2 implies that there is an $m \in (-\infty,\infty)$ and an $\epsilon > 0$ such that $R(q) = R(q^*) + m(q-q^*)$ for any $q \in [q^*-\epsilon, q^*+\epsilon]$. Since q^* maximizes profit, it follows that $R(q^*) - C(q^*) \geq R(q^*-\epsilon) - C(q^*-\epsilon) = R(q^*) - m\epsilon - C(q^*-\epsilon)$, and hence that $C(q^*) - C(q^*-\epsilon) \leq m\epsilon$. But then the concavity of $C(\cdot)$ implies that

$$\begin{aligned} R(q^*+\epsilon) - C(q^*+\epsilon) &= R(q^*) - C(q^*) + m\epsilon - [C(q^*+\epsilon)-C(q^*)] \\ &\geq R(q^*) - C(q^*) + m\epsilon - [C(q^*)-C(q^*-\epsilon)] \geq R(q^*) - C(q^*). \end{aligned}$$

Consequently, profits are also maximized at $q^* + \epsilon$, contradicting the definition of q^*. Q.E.D.

Proposition 4 implies in particular that, if marginal cost is constant, a monopolist never need charge more than one price.

6. An Example With Two Prices

For any demand function with points of discontinuity, there will be a range of output levels at which it is optimal to charge more than one price. I consider here such a demand curve and contrast the optimal pricing policy and the corresponding revenue function with the pricing policy and revenue function which result when the firm is constrained to charge a single price.

Consider the following demand function:

$$D(p) = 0 \quad \text{for} \quad p > 2;$$

$$D(p) = 1 \quad \text{for} \quad 1 < p \leq 2;$$

$$D(p) = 5 \quad \text{for} \quad 0 < p \leq 1;$$

If the firm is constrained to charge a single price, then it will earn a revenue of $2q$ for $q \leq 1$ and a revenue of q for $1 < q \leq 4$. The marginal revenue will be undefined at $q = 1$. Now suppose we permit the firm to charge more than one price. Then for $q \leq 1$, its optimal pricing strategy is to charge a price of 2 for all units. For $1 < q \leq 4$, its optimal policy is to charge price 1 for the first $5(q-1)/4$ units and price 2 the remaining $(5-q)/4$ units. The marginal revenue is 2 for the first unit and 3/4 for the next four units.

The example is illustrated in Figure 1. The demand curve is presented in the top half of the figure. In the bottom half of the figure are the two revenue functions. Both the optimal revenue function and the single price revenue function are identical for the first unit. At $q = 1$, the single price revenue function R_s has a downward discontinuity whereupon it increases with slope 1 up to $q = 5$. In contrast, the optimal revenue function is found by connecting the single price revenue at $q = 1$ with the single price revenue at $q = 5$ with a straight line. Over this range of outputs, the firm will charge a price of 1 for some units and a price of 2 for the other units.

7. The Welfare Implications

In this section, I briefly discuss the welfare implications of the two price policies. Since the single price policies are always available, the profits of the monopolist must be higher whenever it strictly prefers a two price policy. The implications for the welfare of the consumers, however, are more ambiguous.

Suppose it is optimal for the firm to use a two price policy and consider the corresponding single price policy which results in the same level of aggregate sales. Since some consumers are rationed at the lower price when the two price policy is used, the social net benefit must be higher under the single price policy.[13] If, in addition, the optimal single price policy results in a **higher** level of output, then social welfare (and the welfare of consumers) is increased even further. The only case in which the two price policy can **increase** the level of welfare for either the consumers or the society as a whole is when it results in a higher level of output.

In general, the level of output under the optimal two price policy may be either larger or smaller than the level of output under the optimal single price policy. Suppose the demand curve is same as in Section 6. Then, in either a single price or a two price system the allocation of goods is efficient since all of the demand at price 2 is satisfied if any units are sold at price 1.[14] Consequently, social benefits will be higher under the pricing regime which results in the highest level of output.

Suppose, for instance, that marginal cost is αq, where $0 < \alpha < 1$. Then a monopolist using a two price policy will equate the marginal revenue of 3/4 to marginal cost to obtain an output level of $(3/4)(1/\alpha)$. In contrast, a monopolist using only a single price will either produce 1 unit at price 2 for a profit of $(4-\alpha)/2$ or produce $1/\alpha$ units at price 1 for a profit of $1/(2\alpha)$. Then if $\alpha = 1/2$, the single price profit is maximized by charging price 2. There is no consumer surplus and the profit is 7/4. With a two price policy, the monopolist produces 3/2 units of output. It charges price 1 for 5/8 of the units and price 2 for the remaining 7/8. The consumers receive a surplus of 1/8 and the firm earns a profit of 29/16 for a total benefit of 31/16. If $\alpha = 1/4$ and the firm must charge a single price, it sells 4 units at price 1. Consumer surplus is 1 and profit is 2 for a total benefit of 3. With a two price policy, the firm produces 3 units. It sells the first 5/2 units at price 1 and the remaining 1/2 unit at price 2. The consumers receive a surplus of 1/2 and the firm earns a profit of 9/4 for a total benefit of 11/4.

One additional word of caution. This analysis supposes that the presence of rationing does not affect the consumers' decisions on **when** they visit the store. However, if consumers know that they will be served on a first come first serve basis, there is an obvious incentive to visit earlier than they would have otherwise. As long as there is no relation between the **shape** of their demand curves and the timing of their visits, the decision problem of the firm will be unaffected. However, to the extent that the decision to move earlier is costlier to consumers, it imposes an additional cost which should also be included in the welfare analysis.

8. Some Applications

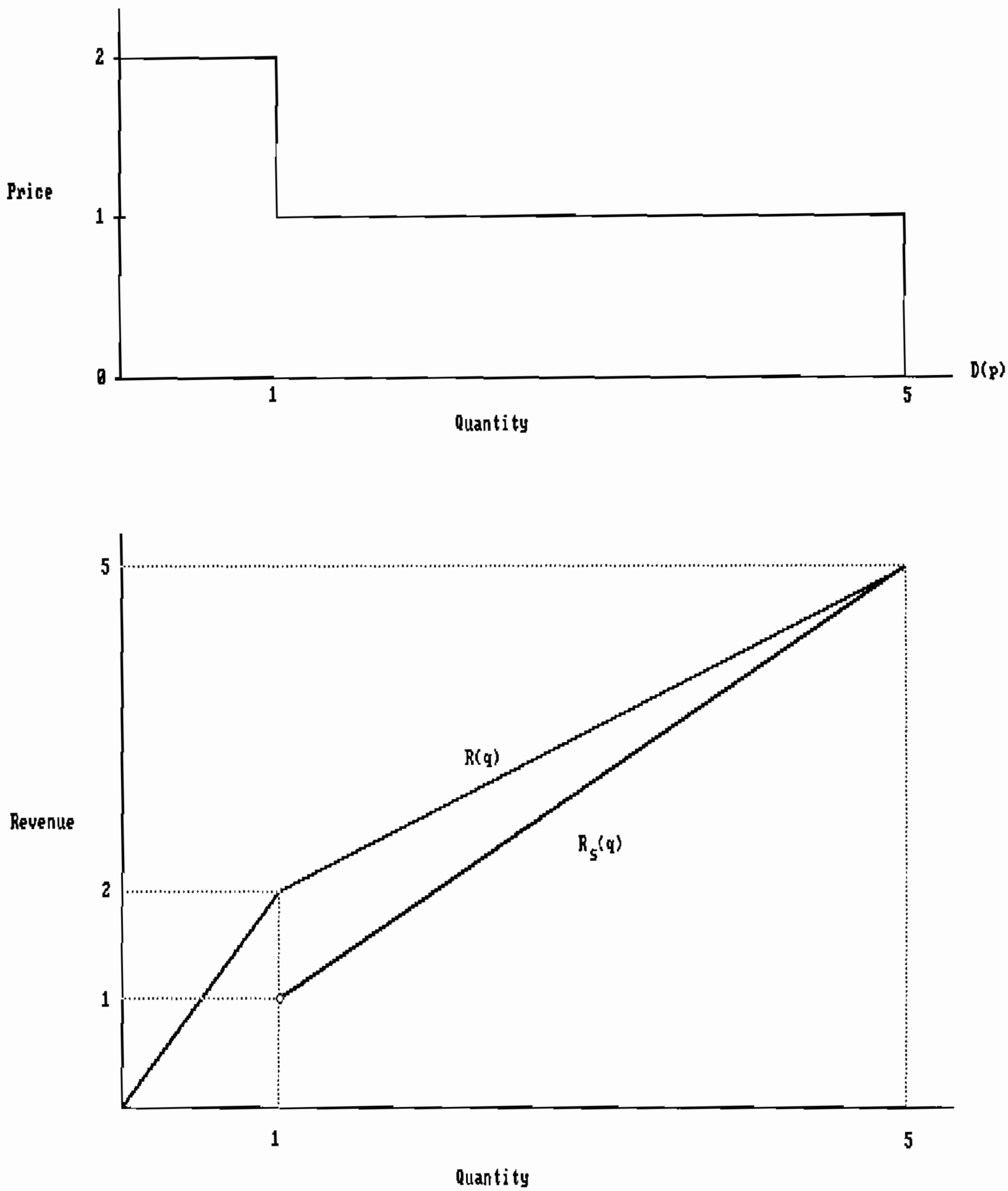
Does this model ever explain the pricing policy of a firm? It is not enough simply to find examples where the firm charges different prices for the same good, nor even where it rations the supply at random. Both the demand curve and the cost function must satisfy certain properties. First, the marginal function of the firm must increasing over some range of output, which means that the demand curve must look something like a step function. This condition is most likely to be satisfied if we view the population as containing two or more relatively distinct types so that the distribution of reservation values has more than one peak. There must also be some practical limits on capacity so that the marginal cost is increasing at the profit maximizing output level.

One plausible candidate for an application of this model is the pricing policy of airlines. Through various restrictions on the conditions of the sale, airlines are effectively able to segment their demand into groups with very different elasticities of demand. Round trip tickets in which the customer leaves and returns on the same weekday are frequently more expensive than round trip tickets with a layover of a week or more. The same day tickets are typically purchased by businessmen with a relatively low elasticity of demand, while the other tickets are more likely to be purchased by vacationers which considerably higher elasticities. Even within the latter group, however, there may still be some homogeneity that can only be exploited by **rationing** a limited supply of low price super–saver fares.

Except that the consumer must purchase the ticket a week or two in advance and cannot change his travel plans, the lower fares offer exactly the same service as the regular fare. Although these restrictions may explain why super saver fares must sold at a discount, they do not explain why the supply of super saver–fares is rationed. If the airline is not trying to exploit hetereogeneity of its consumers, a more sensible policy would be to raise the price of super–saver fares until the supply equals demand. What I suspect is happening is that the airlines realize that many purchasers of super–saver fares are willing to purchase tickets at a higher price, but it has not effective way of identifying this population. By rationing the supply of super–saver fares and selling the remaining tickets at a much higher price, it makes more money than it would it it raised the price of every ticket.

Another application of the model might be to the pricing policies of supermarkets. The FTC "super market unavailabilty rule" prohibits a firm from advertising a good unless its supply of the good is sufficient to meet a "reasonable" demand at the advertised price. Presumably, the intention is to eliminate "bait and switch" tactics in which the firm lures "naive" consumers into the store at the low price and then forces them to purchase the product at a higher price. This model suggests that such a policy might be optimal even with fully informed rational consumers if the firm is trying to exploit differences in the reservation values of its customers.[15] In this case, the welfare analysis above suggests that although there is some bias towards imposing a single price rule, there may be instances in which the net social gain and possibly the welfare of the average consumer is reduced by eliminating the pricing practices proscribed by the FTC regulations.

FIGURE 1. Optimal Two–Price Revenue Versus the Single Price Revenue.



Footnotes

[1] If $D_0 = \infty$, then the demand at any higher price is unaffected by the supply of goods at price $p_0 = 0$. In this case, we could simply start by considering the excess demand at p_1.

[2] For the remainder of the paper, we assume that $q \in [0,D(0)]$.

[3] In fact, this result is quite general. If it exists, the maximum of a linear programming problem can always be acheived at a vector for which the number of positive elements is less than or equal to the number constraints. (See e.g. Gale (1960), p.84). Even more generally, the maximum of a convex function over a convex set can always be acheived at an extreme point (Rockafellar (1970), p.348). It is easy to show that any extreme point of a set defined by two linear functions and nonnegativity constraints can be positive at at most two components. This fact was first pointed out to me by Stan Reiter. (See Chernoff and Reiter (1954)).

[4] If the price space is $[0,\infty)$, then relations (1) and (2) may be expressed as (1′) $\int Q(dp) = 1$ and (2′) $\int [1/D(p)]Q(dp) \leq 1$. The objective of the firm is then to choose a nonnegative measure Q to maximize $\int pQ(dp)$ subject to constraints (1′) and (2′). This is still an (infinite dimensional) linear programming problem. Since the constraints also satisfy the necessary constraint qualifications (see e.g. Dixit (1976), p.54), the infinite dimensional analogues to relations (1) to (5) are still necessary and sufficient conditions for a solution.

[5] Some such condition is needed to guarantee that a solution to Problem I

always exists.

[6] $\bar{p}$ plays the same role as p_n in Section 3. Lemma 1 is still valid if we replace p_n with $\bar{p}$.

[7] As we shall see in the proof of Proposition 2, relation (3) implies that $D_\ell < D_h$ whenever $p_\ell < p_h$.

[8] At this point we could use standard duality results to establish the Lemma. Relation (3) uniquely determines λ which is equal to the derivative of $R(\cdot)$. (See e.g. Dixit (1976), chapter 6).

[9] At almost all q, $R(\cdot)$ is differentiable, so that the marginal revenue is well defined. If $R(q)$ is not differentiable, identify the right–derivative of $R(q)$ as the **marginal revenue** of q.

[10] There is a close analogy here to the Rybcynski line in the standard two by two sector general equilibrium model (see e.g. Jones (1965)).

[11] $D^{-1}(q)$ is the **highest** price such that $D(p) = q$.

[12] coH denotes the convex hull of H.

[13] The fraction of consumers whose demand is satisfied at p_ℓ is $q_\ell/D_\ell = 1-(q_h/D_h)$. Suppose $D(p) = q$. Then if $q_\ell + q_h = q$, the difference between the total benefit under the single price and the optimal two price policy is

$$(q_h/D_h)\int_p^{p_h} D(x)dx - (q_\ell/D_\ell)\int_{D_\ell}^{p} D(x)dx > q_h(p_h-p) - q_\ell(p-p_\ell)$$
$$= R(q) - R_s(q) \geq 0.$$

[14] When the firm charges a single price of 1, it is appropriate to suppose that the high reservation demand is completely satisfied since it would be if the demand curve were strictly decreasing throughout.

[15] In fact, the model of this paper is actually embedded in the analysis of Katz and Nelson (1986) on the welfare of implications of the FTC regulations.

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