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Abstract: An example of a dynamic game between a union and a firm is constructed where an equilibrium path starts out with non-cooperative strategies and switches to cooperative strategies as the level of employment increases.

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It is well-known that in repeated games it may be possible to sustain a cooperative equilibrium with the discount rate sufficiently small, if players adopt strategies that threaten to revert to non-cooperative actions when their opponents deviate from cooperative behavior. A similar situation arises in dynamic games where a cooperative solution can be maintained as a subgame perfect equilibrium under threats to revert to other credible strategies, like, for example, stationary Markov (feedback) strategies. However, in dynamic games, whether a cooperative solution can or cannot be sustained depends not only on the discount rate, but also on the value of the state variable at the particular point in the game. Since the state variable evolves with each play, it is possible to have an equilibrium in which agents play their stationary (feedback) strategies below a critical level of the state variable where a cooperative solution cannot be sustained. Then, after crossing the critical level, they can switch to cooperative strategies which are supported by threats of reverting to stationary (feedback) strategies. It is also possible to have situations where cooperative equilibria can be sustained only for initial conditions above a critical value of the state variable, and where initial conditions below that critical value result in the use of non-cooperative stationary strategies that lead to a non-cooperative steady state.

Recently some authors explored the enforcability of cooperative solutions in dynamic games. Cave (1984) has developed and extended the early work of Levhari and Mirman (1980) on the great fish war. Oudiz and Sachs (1986) have discussed related issues with respect to macroeconomic and international policy coordination. Nevertheless, the relation between the state of the system and the enforcability of a cooperative solution or the possibility of the eventual (together with the impossibility of the immediate) emergence of cooperation along an equilibrium path are issues that have not yet been explored in detail.[1] Below we give examples of dynamic games where the situations described above will arise.

We consider a simple bargaining game between a labor union and a firm. The firm is subject to adjustment costs in changing its labor force and has a linear production technology. Its profits are given by

(1) $$\sum_{t=0}^{\infty} (a\ell_t - w_t\ell_t - b(\ell_t - \ell_{t-1})^2)\beta^t$$

for ℓ_{-1} given, where $a\ell_t$ is the output at time t, ℓ_t is employment, w_t is the wage, $b(\ell_t - \ell_{t-1})^2$ is the adjustment cost and β is the discount factor. The employment level is confined to the interval $[0,\bar{\ell}]$, where $\bar{\ell}$ is the maximum labor available. The union would like to choose wages to maximize its discounted stream of wage bills, given by

(2) $$\sum_{0}^{\infty} \beta^t w_t \ell_t$$

The structure of bargaining is such that every period the union first announces a wage and then the firm announces an employment level.

We first consider an equilibrium with stationary strategies. The union chooses a policy function $W: [0,\bar{\ell}] \to [0,\infty)$ which selects a wage t as a function of the amount of employment at t-1: $w_t = W(\ell_{t-1})$. The firm chooses a policy function $\ell_t = L(w_t, \ell_{t-1})$ where $L: [0,\infty) \times [0,\bar{\ell}] \to [0,\bar{\ell}]$. A pair of functions (W, L) is an equilibrium if W maximizes (2) given L and L maximizes (1) given W. Given values of specific parameters, a subgame perfect equilibrium can be obtained using dynamic programming methods. The first order conditions associated with the firm's and union's optimization problems (after eliminating the value functions for the firm and the union) are,

$$(3) \qquad a - w_t - 2b(\ell_t - 1_{t-1}) = \beta[2b(\ell_{t+1} - \ell_t) - \ell_{t+1}\cdot dW(\ell_t)/d\ell_t]$$

$$(4) \quad \ell_t + w_t\cdot dL(w_t,\ell_{t-1})/dw_t = \beta(dL(w_t,\ell_{t-1})/dw_t)\left[(dL(w_{t+1},\ell_t)/d\ell_t)/(dL(w_{t+1},\ell_t)/dw_{t+1})\right]\ell_{t+1}$$

Given a, b, β and $\bar{\ell}$, the parameters of the optimal linear policy function of the form $\ell_{t+1} = k_0 + k_1\ell_t \;\; k_2 w_{t+1}$ and $w_{t+1} = c_1 + c_2\ell_t$ can be computed using the undetermined coefficients method. (Of course, concavity restrictions and transversality conditions can also be shown to hold for the union's and the firm's optimization problem.) We will denote the value functions of the firm and the union by $F^s(\ell_0)$ and $U^s(\ell_0)$. Both $U^s(\ell_0)$ and $F^s(\ell_0)$ are quadratic functions, satisfying the dynamic programming equations below:

$$(5) \quad F^s(\ell_0) = b_0 + b_1\ell_0 + b_2\ell_0^2 = \max_{\ell_1\in[0,\bar{\ell})} w(\ell_0)\cdot\ell_1 - b(\ell_1 {}^-\ell_0)^2 + \beta F^s(\ell_1)$$

$$(6)\quad U^s(\ell_0) = a_0 + a_1\ell_0 + a_2\ell_0^2 = \max_{w_1\in[0,\infty)} w_1 L(w_1,\ell_0) + \beta U^s(L(w_1,\ell_0))$$

Substituting the policy functions into (5) and (6) and equating coefficients, the parameters of the value functions can then be computed.

Note that the dynamics of employment for this equilibrium with stationary strategies will be given by

$$(7)\quad \ell_{t+1} = (k_0 + k_2c_1) + (k_1 + k_2c_2)\ell_2$$

with the steady state $\tilde{\ell} = (k_0+k_2c_1)/(1-(k_1+k_2c_2))$.

A cooperative solution is obtained by maximizing a weighted sum of (1) and (2) with respect to sequences of w_t and ℓ_t. Assigning unequal weights, given the linearity in the problem, leads to a solution which allocates all of the surplus to the agent which is more heavily weighted. For convenience we restrict the wage w_t to $[0,\bar{w}]$ for all t, where $\bar{w}$ can be arbitrary or can be the wage that attains the maximum surplus so that

$$(8)\quad \bar{w}\hat{\ell}_1 = S(\ell_0) = \max_{\ell_1\in[0,\bar{\ell}]} a\ell_1 - b(\ell_1 - \ell_0)^2 + \beta S(\ell_1)$$

where $\hat{\ell}_1$ is the value of ℓ_1 that maximizes the right side of (8) and $S(\ell_0)$ is the maximum surplus.

For simplicity we avoid unequal weights that generate corner solutions and which allocate the entire surplus to the firm or union. With equal weights it is easy to check from first order conditions that the wage rate is indeterminate at all times and the optimal sequence for employment is the one which solves (8). A particular wage trajectory then results in a particular allocation of the surplus between the firm and the union. We restrict our attention to cooperative equilibria with constant wage trajectories since any prespecified allocation of the surplus $S(\ell_0)$ can be attained with a constant wage trajectory. The optimal employment trajectory that maximizes the right-hand side of (8) is given by

$$(9) \qquad \begin{cases} \ell_t = \ell_{t-1} + z & \text{if } \ell_{t-1} + z \leq \bar{\ell} \\ \ell_t = \bar{\ell} & \text{otherwise} \end{cases}$$

where $z = a/2b(1-\beta)$. This solution is obtained from the first order conditions corresponding to the solution of (8) by imposing conditions to assure that transversality conditions hold.[2] Note that for any cooperative solution the employment trajectory will always hit the boundary $\bar{\ell}$. A choice of a constant wage $\hat{w} \in [0,\bar{w}]$ then results in a cooperative solution which allocates the total surplus between the union and the firm. We will denote the values of a cooperative solution associated with $\hat{w}$ that accrue to the firm and the union by $F^c(\ell_0;\hat{w})$ and $U^c(\ell_0,\hat{w})$.

We can now study whether a cooperative solution can be sustained if the strategies of the agents involve threats to revert to an equilibrium with stationary strategies. Let

$$H_t = \{(\ell_0, \ell_1 \ldots \ell_{t-1}, w_1, w_2 \ldots w_{t-1}) \mid \ell_i \in [0,\bar{\ell}],\ w_j \in [0,\infty) \text{ for } i = 0,1 \ldots t-1 \text{ and } j = 1,2 \ldots t-1\}$$

and let h_t be an element of H_t. Let the strategy of the firm at time t be given by the function L_t: $H_t \times [0,\infty) \rightarrow [0,\bar{\ell}]$ such that

$$\ell_t = L_t(h_t, w_t) = \begin{cases} \ell_{t-1} + z & \text{if } w_i = w^c \text{ for all } i \le t \\ k_0 + k_1\ell_{t-1} + k_2\ell_{t-1}^2 & \text{otherwise.} \end{cases}$$

Let the strategy of the union at time t be given by the function W_t: $H_t \rightarrow [0,\infty)$ such that

$$w_t = W_t(h_t) = \begin{cases} \hat{w} & \text{if } \ell_i = \ell_{i-1} + z \quad \text{for all } i \le t-1 \\ c_1 + c_2\ell_{t-1} & \text{otherwise.} \end{cases}$$

Under these strategies, which punish the opponent for deviating from cooperative behavior by reverting forever to an equilibrium with stationary (feedback) policies, the firm and the union must compute whether it pays to deviate. The value to the firm from deviating is given by

$$F^D(\ell_{t-1}) = \max_{\ell_t \in [0,\bar{\ell}]} (a - w_t)\ell_{t+1} - b(\ell_t - \ell_{t-1})^2 + \beta F^s(\ell_t)$$

while the value to the union from deviating, since it moves first, is simply the value of the equilibrium associated with the stationary strategies, that is $U^D(\ell_{t-1}) = U^s(\ell_{t-1})$. Therefore, to be enforced, a cooperative equilibrium starting from ℓ_0 and described by (9) must have $U^c(\ell;\hat{w}) \geq U^D(\ell)$ and $F^c(\ell;\hat{w}) \geq F^D(\ell)$ for all ℓ along its path. However, the possibility that a cooperative equilibrium is only enforceable for some interval $(\ell^*,\bar{\ell})$ which contains $\tilde{\ell}$, the steady state of the equilibrium associated with stationary strategies, raises the possibility of an equilibrium sequence where agents use stationary (feedback) strategies up to ℓ^* and then switch to cooperative strategies. For such a possibility, $\tilde{\ell}$ must be stable under stationary strategies. If a critical ℓ^* exists, that is if $U^D(\ell) \leq U^c(\ell;\hat{w})$ <u>and</u> $F^D(\ell) \leq F^c(\ell;\hat{w})$ for some $\hat{w}$ and any $\ell \in [\ell^*,\bar{\ell}]$, but not for $\ell \in [0,\ell^*)$ and any $\hat{w} \in [0,\bar{w}]$, we can consider the following equilibrium pairs of strategies for the firm and the union. Let the strategy of the firm at t be given by the function $\tilde{L}_t\colon H_t \times [0,\infty) \to [0,\bar{\ell}]$ such that

$$\ell_t = \tilde{L}_t(h_t,w_t) = \begin{cases} \ell_{t-1} + z & \text{if } \ell_{t-1} \in [\ell^*,\bar{\ell}] \text{ and } w_i = \hat{w} \\ & \text{whenever } \ell_{i-1} \in [\ell^*,\bar{\ell}] \text{ for } i \leq t \\ k_0 + k_1\ell_{t-1} + k_2\ell^2_{t-2} & \text{otherwise} \end{cases}$$

Let the strategy of the union at t be given by the function $\tilde{W}_t\colon H_t \to [0,\infty)$ such that

$$w_t = \tilde{W}_t(h_t) = \begin{cases} \hat{w} & \text{if } \ell_{t-1} \in [\ell^*,\bar{\ell}] \text{ and } \ell_i = \ell_{i-1} + z \\ & \text{whenever } \ell_{i-1} \in [\ell^*,\bar{\ell}_0] \text{ for } i < t-1 \\ c_1 + c_2\ell_{t-1} & \text{otherwise.} \end{cases}$$

We now give parameter values a, b, β and $\bar{\ell}$ which generate a subgame perfect equilibrium for the sequence of strategy pairs $(\tilde{W}_t, \tilde{L}_t)$, t=0,.... with the property that stationary strategies are used until employment reaches ℓ^* and cooperative strategies are played from then on.

For a = 1.5, b = 1, β = 0.2 and $\bar{\ell}$ = 0.75, the equilibrium stationary strategies are given by

$$(9) \qquad \ell_t = 0.393231 + 0.45130\ell_{t-1}$$

$$(10) \qquad w_t = 0.67695 + 0.95926\ell_{t-1}.$$

The non-cooperative steady state is $\tilde{\ell}$ = 0.7150. The value functions corresponding to stationary strategies are:

$$F^s(\ell_{t-1}) = 0.22183 + 0.408296\ell_{t-1} - 0.765116\ell^2_{t-1}$$

$$U^s(\ell_{t-1}) = 0.426296 \quad 0.784630\ell_{t-1} - 0.451304\ell^2_{t-1}$$

For the cooperative solution, the path of employment is given by

$$(11) \qquad \ell_t = \ell_{t-1} + 0.9375$$

If we set the constant cooperative wage at $\hat{w}$ = 1.362332, the critical value of ℓ is given by ℓ^* = 0.67.

For $\ell < \ell^*$, $F^c - F^D < 0$ and the cooperative equilibrium cannot be supported. For $\ell > \ell^*$, $F^c - F^D > 0$ and $U^c - U^D > 0$ so that a cooperative equilibrium can be supported. For ℓ_0 = 0.64, the firm and the union will at first play their stationary strategies given by (9) and

(10). In the second period, $\ell_1 = 0.681 > \ell^*$ so that the firm and union will switch forever to cooperative strategies, given by (11).

Of course, as the discount rate is increased (discount factor decreased), the range of cooperative wages $\hat{w}$ that are sustainable shrinks. For sufficiently high discounting there does not exist a $\hat{w}$ and a corresponding cooperative equilibrium that can be sustained from any value of ℓ.

It is also possible to construct examples where the steady state under stationary strategies, $\tilde{\ell}$, is stable but is below ℓ^*. Then for $\ell_0 \in [0,\ell^*)$, the equilibrium strategies lead to convergence to $\tilde{\ell}$ but for $\ell_0 \in [\ell^*,\bar{\ell}]$, a cooperative solution can be implemented as an equilibrium for the industry. Therefore, whether efficient allocation and production can or cannot be sustained depends critically on initial conditions. If we slightly alter the previous example so that $\bar{\ell} = 1$ and $\hat{w} = 1.45$, we obtain $\tilde{\ell} = 0.715 < 0.87377 = \ell^*$. For $\ell_0 < \ell^*$, $F^c < F^D$ and a cooperative equilibrium cannot be sustained. Under stationary strategies employment converges to $\tilde{\ell}$. For $\ell_0 \geq \ell^*$, the cooperative equilibrium is implemented and employment converges to $\bar{\ell}$.

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FOOTNOTES

[1] Note that switching from cooperative to non-cooperative strategies without ever returning to cooperative ones cannot occur along an equilibrium trajectory because players will have the incentive to deviate earlier and therefore immediately.

[2] Solutions to the first order conditions (given by a second order difference equation) which grow at the rate of $1/\beta$ do not satisfy transversality conditions and are eliminated by appropriate choice of ℓ_1.