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AN EXAMINATION OF THE EQUILIBRIUM SPECIFICATION AND STRUCTURE OF PRODUCTION FOR CANADIAN TELECOMMUNICATIONS

by

Jeffrey I. Bernstein

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C. V. STARR CENTER FOR APPLIED ECONOMICS

NEW YORK UNIVERSITY
FACULTY OF ARTS AND SCIENCE
DEPARTMENT OF ECONOMICS
WASHINGTON SQUARE
NEW YORK, N.Y. 10003

An Examination of the Equilibrium Specification and Structure of Production for Canadian Telecommunications

Jeffrey I. Bernstein

Department of Economics

Carleton University

Ottawa, Ontario,

The National Bureau of Economic Research,

and

The C.V. Starr Center for Applied Economics

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Abstract

Multiple output models of Canadian telecommunications production are estimated under different equilibrium specifications. A specification test is conducted between the short and long-run equilibrium models and the long-run equilibrium is rejected. In order to capture the nature of the disequilibrium a dynamic cost of adjustment model is estimated for Bell Canada. There are significant adjustment costs and it is estimated that for $1.00 of capital expenditures it costs the carrier an additional $0.30 to install the new capital into the production process.

Returns to scale, productivity growth and price elasticities are estimated from the dynamic cost of adjustment model. In this context there are significant economies of scale, with returns to scale estimated to be 1.50. Scale economies appear to be robust across equilibrium specifications. The average annual productivity growth rate is estimated to be 1.32, which is greater than the estimates from long-run equilibrium models, and consistent with estimates for total Canadian manufacturing.

1. Introduction*

Econometric analysis of the multioutput technologies pertaining to telecommunications services has generally maintained the assumption that carriers are in long-run equilibrium. This assumption implies that all factors of production can be costlessly adjusted so the carrier instantaneously determines long-run factor demands. The major purpose of this paper is to test the assumption of long-run equilibrium for Canadian telecommunications,

Models incorporating various equilibrium specifications are estimated for Bell Canada. Three equilibrium specifications are investigated. The differences in equilibria centre on the role of investment in equipment and structures. It is assumed that a carrier must incur adjustment costs in order to change it's capital stock. In this way capital is distinguished from the other factors of production. First, a short-run model is estimated where the demands for the non-capital inputs are conditioned on the existing capital stock, which is fixed due to the existence of adjustment costs. Second, a long-run model is estimated with no adjustment costs so that capital appears as any other input. Third, a dynamic cost of adjustment model is estimated, which generalizes both the short and long-run models. In this model the demand for capital and non-capital inputs are determined and are influenced by adjustment costs.

The surveys by Fuss [1983] and Kiss and Lefebvre [1986] show that the vast amount of applied research relating to telecommunications production by Bell Canada and AT & T has been in the context of long-run equilibrium. Recently, for AT & T in a single product framework Christensen, Cummings and Schoech [1983] have estimated a short-run equilibrium model. In addition, Schankerman and Nadiri [1986] have estimated and tested a short-run equilibrium model and have accepted the latter over the long-run equilibrium specification. There have not been any multiple output models for AT & T which have relaxed the assumption of long-run equilibrium. For Bell Canada, in both single and multiple output contexts the long-run equilibrium assumption has been maintained. In this paper both short-run equilibrium and dynamic cost of adjustment models are estimated for Bell Canada.

In section 2 of the paper the theoretical development takes place. The dynamic cost of adjustment model is derived and the short and long-run equilibrium models are shown to be special cases of the former. The empirical implementation of the models occurs in section 3. Included in this section is the test pertaining to the nature of the equilibrium and the indicators showing the deviation between the short and long-run equilibria. Sections 4 and 5 describe the estimates of returns to scale, productivity growth and factor substitution. Lastly, there is a summary section.

2. Theoretical Development

The purpose of this section is to develop the theoretical model which forms the basis for the estimation of the models of production. The production technology of a telecommunications carrier can be represented as

$$(1) \qquad T(y(t), K(t), v(t), I(t), A(t)) = 0$$

where T is a transformation function, y is an ℓ-dimensional vector of outputs, K is an m-dimensional vector of quasi-fixed factors, v is an n-dimensional vector of variable factors, I is the vector of additions to the quasi-fixed factors, and A is an indicator of technological change.[1] The presence of the vector of quasi-fixed factor additions (or the gross investment vector) implies that there are adjustment costs associated with changing the quasi-fixed factors. These costs are internal to the production process and are manifested by the foregone output when resources are diverted from output production to quasi-fixed factor installation (see Treadway [1971], Mortenson [1973] and Epstein [1982]).

The accumulation of the stocks related to the quasi-fixed factors is governed by

$$(2) \qquad K_i(t+1) = I_i(t) + (1-\delta_i)K_i(t) \qquad i = 1,\ldots, m$$

where δ_i is the depreciation rate on the ith stock and $0 < \delta_i < 1$.[2]

In this dynamic cost of adjustment model, the objective of the carrier is to minimize the expected present value of the costs of hiring the variable factors, purchasing and installing the quasi-fixed inputs. These costs are denoted by,

$$(3) \qquad \Sigma_{s=t}^{\infty} E(t)\alpha(t,s)[w^T(s)v(s) + p_I^T(s)I(s)]$$

where $\alpha(t,s)$ is the discount factor between periods t and s, w(s) is the vector of variable factor prices, p_i is the vector of quasi-fixed factor acquisition prices and E(t) is the conditional expectations operator. The carrier minimizes (3) by selecting the variable and quasi-fixed factors subject to equations (1) and (2) given prices, outputs, technological change and discount rate. The expectations operator is conditional on current information concerning the future values of the exogenous variables. The superscript T relates to vector transposition.

The problem facing the carrier can be solved in two stages. The first stage relates to the determination of the variable factor demands conditional on the quasi-fixed factor demands (and thereby on gross investment levels). Variable factor demands are determined from

$$(4) \qquad c^V = w_1 G_1(y,K,v_2,\ldots,v_n,I,A) + \Sigma_{j=2}^{n} w_j v_j \ ,$$

where G_1 is the first variable factor requirement function and it is derived from (1) by solving for v_1, and c^V represents the variable cost which pertain to hiring the variable factors and installing the quasi-fixed factors.[3]

Minimizing variable cost by selecting the variable factor demands yields demand functions for these factors which depend on the variable factor prices, output quantities, quasi-fixed factors, gross investment flows and the rate of technological change. This, in turn, implies that the minimized variable cost function can be written as

$$(5) \qquad c^{v} = C^{v}(w,y,K,I,A) \ .$$

Moreover, by an application of Shepherd's Lemma to the variable cost function (as opposed to the total cost function) variable factor demand functions are,[4]

$$(6) \qquad v_j = \frac{\partial C^{v}}{\partial w_j}(w,y,K,I,A) \qquad j = 1,\ldots,n \ .$$

Equation set (5) and (6) constitute the solution to stage one of the carrier's production problem. This stage can be defined as the short-run equilibrium relating to the variable factors of production.

Stage two of the problem pertains to the determination of the quasi-fixed factor demands. This stage encompasses the dynamic features of the model because the stocks of the quasi-fixed factors are accumulated through time. It is important to notice that the variable factor demand functions (equation set (6)) are not independent of the intertemporal aspects of the model, because they depend on the quasi-fixed factors. Thus as the quasi-fixed factor demands change through time the variable factor demands are also affected.

The solution to the second stage is derived by substituting the variable cost function and the stock accumulation equations into the expected present value of the costs of production and adjustment (namely (3)), which yields

$$(7) \qquad \Sigma_{s=t}^{\infty} E(t)\alpha(t,s)[C^{V}(w(s),y(s),K(s),$$

$$K(s+1) - (I_m-\delta)K(s),A(s)) + p_I^T(s)(K(s+1) - (I_m-\delta)K(s))] ,$$

where I_m is the m dimensional identity matrix and δ is the diagonal matrix of depreciation rates for the m quasi-fixed factors. The solution to stage 2 involves the selection of the quasi-fixed factor demands through time by minimizing (7). The quasi-fixed factor for any period (s+1) is determined in period s and based on information in period s. The conditions characterizing this solution are

$$(8) \qquad \alpha(t,s)(\frac{\partial c^{V}(s)}{\partial I_j(s)} + p_{Ij}(s)) + E(s)\alpha(t,s+1) \frac{\partial c^{V}(s+1)}{\partial K_j(s+1)} -$$

$$\frac{\partial c^{V}(s+1)}{\partial I_j(s+1)} (1-\delta_j) - p_{Ij}(s+1)(1-\delta_j)) = 0 \qquad \begin{matrix} j = 1,\ldots,m, \\ s = t,\ldots,\infty. \end{matrix}$$

Equation (8) points out that a telecommunications carrier sets the discounted cost of increasing a quasi-fixed factor in period s (which is known in period s) to the expected discount cost-reduction from having the larger quasi-fixed factor in period s + 1. The marginal cost in period s consists of the marginal cost of adjustment and the marginal cost of acquisition, which is denoted by the purchase price. The expected marginal cost-reduction in period s + 1 consists of the decline in variable cost from entering the period with a larger quasi-fixed factor and also the consequent decline in adjustment and acquisition costs. From equation set (8) the demands for the quasi-fixed factors depend on expected future prices, outputs, discount rate and technological change. These demands are therefore forward looking.

The equilibrium conditions of the carrier are denoted by equations (5), (6) and (8). These equations define a short-run equilibrium because adjustment costs prevent the instantaneous attainment of the long-run demands of the quasi-fixed factors and thereby also the variable factors.

In most models used to estimate the production structure of a telecommunications carrier a long-run equilibrium is specified. These long-run equilibrium conditions (see Fuss [1983] and Schankerman and Nadiri [1986]) can be derived as a special case of the equilibrium conditions (5), (6) and (8). Indeed the specialization centres around equation set (8). Assume that the carrier is in long-run equilibrium such that $\Delta K(t) = 0$ for all time with the variable factor prices, and the capital acquisition prices expected to change by some constant rate. In addition, the discount rate is non-stochastic and changes by the same constant rate as the prices, while output and technological change are not expected to change, and marginal adjustment costs are zero in the long-run equilibrium. Thus equation set (8) can be rewritten as

$$(9) \qquad -\frac{\partial c^{v}(s+1)}{\partial K_j(s+1)} = p_{Ij}(s)(1+\theta)[r+\delta_j] \quad j = 1,\ldots,m,\ s = t,\ldots,\infty\ .$$

where r is the constant real interest rate and $\alpha(s,s+1) = [(1+r)(1+\theta)]^{-1}$ with θ defined as the rate at which prices and the discount rate change over time. The rental rates for the quasi-fixed factors are given by the right side of equation set (9). The rental rate on any one quasi-fixed factor is equal to the negative of the decline in variable cost from increasing the quantity of that quasi-fixed factor. Equation set (9) are the envelope conditions and implicitly define the long-run demands for the quasi-fixed factors.[5] Thus equations (5), (6) and (9) characterize a long-run equilibrium.

There are a number of ways to specify the equilibrium of a telecommunications carrier. First equations (5) and (6) define the short-run equilibrium of the variable factor demands which are conditioned on the quasi-fixed factors, as well as factor prices and output quantities. Second, equations (5), (6) and (8) define the short-run equilibrium relating to both the variable and quasi-fixed factors. Third, equations (5), (6) and (9) define a long-run equilibrium. In this paper all three types of equilibrium conditions are estimated as separate models.

3. Empirical Implementation

The estimation of each of the different types of equilibrium conditions necessitates that a functional form for the variable cost function must be specified. It is assumed that the variable cost function is a second order approximation to any general variable cost function. Thus

$$
\begin{aligned}
\ln(c^v/w_m) = {} & \beta_0 + \beta_L \ln w_L + \Sigma_{i=1}^{2} \beta_i \ln y_i + \beta_K \ln K \\
& + \beta_A \ln A + \beta_I \Delta K + .5[\beta_{LL}(\ln w_L)^2 + \\
& \Sigma_{i=1}^{2}\ \Sigma^{2}_{j=1} \beta_{ij} \ln y_i \ln y_j + \beta_{KK}(\ln K)^2 + \beta_{AA}(\ln A)^2 \\
& + \beta_{II}(\Delta K)^2] + \Sigma_{i=1}^{2} \beta_{Li} \ln w_L \ln y_i \\
& + \beta_{LK} \ln w_L \ln K + \beta_{LA} \ln w_L \ln A + \beta_{LI} \ln w_L \\
& \Delta K + \Sigma_{i=1}^{2} \beta_{iK} \ln y_i \ln K + \Sigma_{i=1}^{2} \beta_{iA} \ln y_i \\
& \ln A + \Sigma_{i=1}^{2} \beta_{iI} \ln y_i \Delta K + \beta_{KA} \ln K \ln A \\
& + \beta_{KI} \ln K \Delta K + \beta_{AI} \ln A \Delta K \ ,
\end{aligned}
$$

with $\beta_{ij} = \beta_{ji}$ i, j = 1, 2. There are now two variable factors, labor and materials (n = 2), two outputs, local and toll (ℓ = 2) and a single quasi-fixed factor, capital (m = 1). The wage rate is normalized by the material's factor price, $\omega_L = w_L/w_m$ and variable cost is also normalized by the price of materials. The normalization guarantees that the variable cost function is homogenous of degree one in the variable factor prices.[6] Adjustment costs are represented by net investment

($\Delta K(t) = K(t+1) - K(t)$) rather than by gross investment.[7] It is also assumed that marginal adjustment costs are zero in long-run equilibrium when $\Delta K = 0$. This assumption is made in many dynamic cost of adjustment models (see Denny, Fuss and Waverman [1981], Morrison and Berndt [1981], Pindyck and Rotemberg [1982] and Bernstein and Nadiri [1984]). The parameter restrictions implied by this assumption are $\beta_I = \beta_{LI} = \beta_{1I} = \beta_{2I} = \beta_{KI} = \beta_{AI} = 0$. These restrictions have the effect of creating a separation in the variable cost function between the variable cost of production and the cost of adjustment. Thus equation (10) can be decomposed into

$$(10.1)\qquad \ln(c_p^v/w_m) = \beta_0 + \beta_L \ln\omega_L + \Sigma_{i=1}^2 \beta_i \ln y_i + \beta_K \ln K + \beta_A \ln A + .5[\beta_{LL}(\ln\omega_L)^2 + \Sigma_{i=1}^2 \Sigma_{j=1}^2 \beta_{ij} \ln y_i \ln y_j + \beta_{KK}(\ln K)^2 + \beta_{AA}(\ln A)^2] + \Sigma_{i=1}^2 \beta_{Li} \ln\omega_L \ln y_i + \beta_{LK} \ln\omega_L \ln K + \beta_{LA} \ln\omega_L \ln A + \Sigma_{i=1}^2 \beta_{iK} \ln y_i \ln K + \Sigma_{i=1}^2 \beta_{iA} \ln y_i \ln A + \beta_{KA} \ln K \ln A$$

and

$$(10.2)\qquad c^a = .5\beta_{II}(\Delta K)^2$$

where $c^v = c_p^v + c^a$ with c_p^v variable production cost and c^a adjustment cost.

From the specification of the variable production cost function, the labor cost share of variable production cost is

$$(11) \qquad s_L = \beta_L + \Sigma_{i=1}^{2}\beta_{Li}\ln y_i + \beta_{LK}\ln K + \beta_{LA}\ln A$$

where $s_L = w_L v_L / c_p^v$. Notice that since the marginal cost of adjustment is zero in long-run equilibrium then the separability between variable production and adjustment costs implies that investment does not affect the labor share of variable production cost. The short-run equilibrium for the variable factors of production are denoted by equations (10.1) and (11), with the variable cost share of materials derived from $1 - s_L = s_m = w_m v_m / c_p^v$.

In order to characterize the complete short-run equilibrium it is necessary to derive equation set (8) based on the variable cost function given as (10) for the single quasi-fixed factor. In this case (8) becomes

$$(12) \qquad \beta_{II}\Delta K(t)(1+r(t)) + w_K(t) + E(t)[\beta_K + \beta_{KK}\ln K(t+1) + \beta_{LK}\ln\omega_L(t+1) + \Sigma_{i=1}^{2}\beta_{iK}\ln y_i(t+1) + \beta_{KA}\ln A(t+1)] (c^v{}_p(t+1)/K(t+1)) - \beta_{II}E(t)\Delta K(t+1) = 0 ,$$

where $w_K(t) = p_I(t)[(1+r)(1+\theta) - (1-\delta)(E(t)p_I(t+1)/p_I(t))]$ is the rental rate on capital at any point in time and not necessarily in long-run equilibrium.[8] Equation (12) implicitly defines the demand for capital. The adjustment cost affects the demand for capital. From (12), an increase in the contemporaneous demand for capital causes current adjustment to increase by $\beta_{II}\Delta K(t)(1+r)(1+\theta)$ but this larger capital demand lowers expected future adjustment cost by $\beta_{II}E(t)\Delta K(t+1)$. The short-run equilibrium conditions of the carrier consists of equations (10.1), (11) and (12).

The long-run equilibrium condition for capital, represented in general terms by (9), can now be written as a special case of (12),

$$(13) \qquad s_K(t) + \beta_K + \beta_{KK}\ln K(t) + \beta_{LK}\ln\omega_L(t) + \Sigma^2_{i=1}\beta_{iK}\ln y_i(t) + \beta_{KA}\ln A(t) = 0 ,$$

where $s_K(t) = p_I(t)(1+\theta)[r+\delta]K(t)/c_p^v(t)$ is the ratio of capital cost to variable production cost in the long-run equilibrium. The complete set of equations representing the long-run equilibrium are denoted by (10.1), (11) and (13).

There are three models to be estimated; equations (10.1) and (11), equations (10.1), (11) and (12), and equations (10.1), (11) and (13).[9] Estimating equation sets (10.1) and (11) or (10.1), (11) and (13) are straightforward . However, estimating equation set (10.1), (11) and (12) is more complicated because equation (12) contains expected future values of both exogenous and endogenous variables. To estimate equations of this sort, namely Euler equations, Hansen and Singleton [1982] developed a generalized methods of moments estimator. Moreover, as shown by

Pindyck and Rotemberg [1982] when the error terms in Euler equations are conditionally homoscedastic, the estimator is equivalent to the nonlinear three-stage least squares estimator developed by Jorgenson and Laffont [1974] and Amemiya [1977]. This estimation procedure involves instrumenting $\Delta K(t+1)$, $c_p^v(t+1)$, $\omega_L(t+1)$, $y(t+1)$ and $A(t+1)$ in equation (12). Although the selection of a set of instruments is somewhat arbitrary, Wickens [1982] has suggested the instruments can be obtained from a set of equations which determine the expected future values of the variables to be instrumented. The equations define the future value of the variables in terms of variables which are already known in the current period. The estimation of this system of equations yields the expected future values of the variables needed in equation (12).

Before the estimation of the models, the nature of the error terms in the equations must be discussed. First the error terms in equations (10.1) and (11) represent optimizing errors. Second, the errors in (12) (and(13)) represent unanticipated information or surprises which become available after the time the investment decision is made. Hence the conditional expected value of the error is zero at the time the investment decision is made.[10] It is also assumed that the errors are jointly normally distributed with zero mean and for each of the three models there is a positive definite symmetric covariance matrix.

Most of the data used in this paper were obtained from Kiss [1981]. The sample period was 1952-1979. The local output quantity was represented by a Tornqvist quantity index of gross production based on three output categories (local services, directory advertising and miscellaneous services). The toll output quantity was represented by a Tornqvist quantity index of gross production based on seven output categories (intra-Bell message toll services, Canada message toll services, U.S. and overseas message toll services, WATS, TWX, private line toll services, miscellaneous other toll services). The individual output categories were defined in terms of deflated revenues with various price indices developed for the ten individual categories. The labour input quantity was a Tornqvist quantity index in which hours worked within each labour category were weighted by their respective shares in total labour compensation. There were six occupational groups distinguished; telephone operators, plant craftsmen, clerical, other non-management, formen and supervisors and other management. The wage rate was defined as the total labour compensation divided by the Tornqvist labour input quantity index. The capital input was defined as a Tornqvist index for gross plant in service which consisted of six categories; land, buildings, central office equipment, outside plant, station equipment and general equipment. The Tornqvist capital input quantity index was formed by using category shares of capital costs. The rental rate or factor price of capital was obtained from Fuss and Waverman [1980]. The rental rate consisted of the telephone plant price index multiplied by the sum of the cost of financial capital (which was a weighted average of the costs of debt and equity capital) and the depreciation rate. From this product was netted the expected rate of inflation. This net magnitude was then modified to account for income taxes and capital cost allowances.[11] The materials input is a Tornqvist quantity index consisting of nine categories; maintenance (contract and non-labour expenses), vehicles and tools, rentals, house services (e.g. electricity, fuel oil, etc.), postage, printing and stationery, travel and transfer, research and development, and miscellaneous. The materials price was defined as the materials cost divided by the materials quantity index.

The estimation results for equation set (10.1) and (11) are presented in Table 1. The sample period is 1953-1979 and the maximum likelihood estimator was used. From Table 1 we can see that the fit of the equations was quite good and the majority of parameters were significant. In addition, the variable production cost function was increasing in the variable factor prices for each year in the sample; the function was non-decreasing in local output for each year and non-decreasing in toll output except for the first six years (1953-1959); the function was decreasing in the capital stock except for the first five years and 1963. The variable production cost function was concave in the variable factor prices for each year in the sample and was convex in the capital stock for each year. With respect to technological change, the variable selected was a binary variable equalling 1 from 1958-1971 and 0 everywhere else.[12] This period represented the era of significant technological change over the sample for Bell Canada. The model with the binary variable outperformed all the other models (relating to equations (10.1) and (11)) with or without technological change indicators. In addition, the preferred model had technological change entering linearly into the variable cost function.

Table 1: Variable Factor Demand Short-Run Equilibrium

Parameter	Estimate	Standard Error
β_0	-24.1211	15.9399
β_L	1.3886	0.5625 E-01
β_1	- 1.0135	12.0650
β_2	- 2.2375	1.6026
β_K	10.0694	13.2021
β_A	- 0.1888	0.4545 E-01
β_{LL}	0.1578	0.2423 E-01
β_{11}	4.3409	1.5628
β_{22}	0.2522 E-01	0.2003
β_{KK}	- 0.7174	3.9969
β_{L1}	0.4624 E-01	0.1863 E-01
β_{L2}	0.3918 E-02	0.1093 E-01
β_{LK}	- 0.1567	0.1642 E-01
β_{12}	- 0.8201	0.6605
β_{1K}	- 2.0029	2.6509
β_{2K}	0.8290	0.6743
Log of Likelihood Function	140.758	
Equation	R^2	DW
Variable Production Cost	0.9938	1.9907
Labor Share	0.8708	1.3525

The estimation results for the long-run equilibrium model, which pertains to (10.1), (11) and (13), are presented in Table 2. In the long-run, the variable cost function was increasing in the variable factor prices, increasing in local and toll output quantities and decreasing in the capital stock for each year in the sample. In addition the variable cost function was concave in the variable factor prices and convex in the capital stock.

It is possible to test whether Bell Canada actually minimizes total costs and is thereby in long-run equilibrium. Schankerman and Nadiri [1986] develop a test which is a special case of the Hausman [1978] specification test. Let β_C be parameter vector in (10.1), let β_L be the parameter vector in (11) and let β_K be the parameter vector in (13). Now short-run equilibrium of the variable factors is a maintained hypothesis so that $\beta_L \subset \beta_C$. In addition partition the parameter vector $\beta_C = (\beta_C^1, \beta_C^2)$ where β_C^1 appears in (10.1) but not in (13) and β_C^2 appears in (10.1) and (13) so that $\beta_C^1 = \beta_K$. Thus estimating equation set (10.1), (11) and (13) imposes the restriction that $\beta_C^1 = \beta_K$, while estimating (10.1) and (11) does not impose any restrictions on β_C^1. The test statistic is $M = N(\tilde{\beta}-\hat{\beta})^T \hat{V}^{-1}(\tilde{\beta}-\hat{\beta}) \approx X^2(n)$ where N is the number of observation, $\tilde{\beta}$ is the consistent estimator from (10.1) and (11), $\hat{\beta}$ is the consistent estimator from (10.1), (11) and (13) (i.e. with the restrictions $\beta_C^1 = \beta_K$), $\hat{V}$ is the consistent estimator of V, with $V = V_S - V_L$ and V_S is the asymptotic variance,covariance matrix from $N^{.5}(\tilde{\beta}-\beta)$, while V_L is the asymptotic variance - covariance matrix from $N^{.5}(\hat{\beta}-\beta)$. The M statistic is asymptotically distributed as a chi-squared distribution with q degrees

Table 2: Long-Run Equilibrium

Parameter	Estimate	Standard Error
β_0	- 7.6070	2.8923
β_L	1.4467	0.5731 E-01
β_1	2.9319	2.0241
β_2	- 0.7150	0.7760
β_K	1.7770	0.5300
β_A	- 0.1231	0.3039 E-01
β_{LL}	0.1541	0.2241 E-01
β_{11}	- 1.5952	0.8493
β_{22}	- 0.8522	0.1538
β_{KK}	- 0.9819	0.1952
β_{L1}	0.4485 E-01	0.2407 E-01
β_{L2}	0.5307 E-02	0.7731 E-02
β_{LK}	- 0.1625	0.2096 E-01
β_{12}	0.5341 E-01	0.3256
β_{1K}	0.7946	0.2258
β_{2K}	0.1452	0.5213 E-01
Log of Likelihood Function	169.054	
Equation	R^2	DW
Variable Production Cost	0.9874	0.9497
Labor Share	0.8783	1.2928
Capital Stock	0.6363	0.9388

of freedom where q is the number of parameters in equation (13) (i.e. β_K) which represents the number of parameter restrictions. The value of M is 67.328 and the critical value of $X^2_{0.005,\ 5} = 16.750$. Hence the assumption of long-run equilibrium is rejected for Bell Canada.

Since the long-run equilibrium model does not capture the way capital decisions are made, we now turn to the estimation of the dynamic cost of adjustment model represented by equations (10.1), (11) and (12). The nonlinear three stage least squares estimator was used. The instruments for the lead values of the exogenous variables in equation (12) were obtained from first order autoregressive processes. The lead values of the exogenous variables in (12) were the relative factor price and the local and toll output quantities. The instruments for the lead values of the endogenous variables were obtained from first order logarithmic equations with the regressors defined by the set of exogenous variables. The lead values of the endogenous variables in (12) were the capital stock and variable production cost. The estimation results for equation set (10.1), (11) and (12) are presented in Table 3. From Table 3 we can observe that the fit of the model was very good, the majority of parameters were significant. At all points in the sample, which was 1954-1978, the variable production cost function was increasing in the variable factor prices, decreasing in the capital stock and increasing in local output. The cost function was also increasing in toll output for all years except 1955 and 1963. The function was also concave in the variable factor prices and convex in the capital stock for all of the years.

Table 3: Dynamic Cost of Adjustment Model

Parameter	Estimate	Standard Error
β_0	-11.9152	3.9744
β_L	1.3742	0.6251 E-01
β_1	6.3863	2.5780
β_2	- 0.9480	0.8632
β_K	0.7237	0.4803
β_A	- 0.1875	0.3270 E-01
β_{LL}	0.1670	0.2409 E-01
β_{11}	- 1.7421	0.9432
β_{22}	- 0.1083	0.1668
β_{KK}	- 0.4741	0.1837
β_{L1}	0.2775 E-01	0.2936 E-01
β_{L2}	0.4158 E-03	0.7945 E-02
β_{LK}	- 0.1406	0.2485 E-01
β_{12}	0.2083	0.3580
β_{1K}	0.3241	0.2124
β_{2K}	0.8576 E-01	0.4807 E-01
β_{II}	0.2479 E-03	0.1726 E-03
Equation	R^2	DW
Variable Production Cost	0.9997	1.3724
Labor Share	0.9625	1.2896
Capital Stock	0.6936	1.0720

The adjustment cost parameter β_{II} was significant at the 0.10 significance level. This means that marginal adjustment cost is not zero when net investment is not zero. Thus, the firm is not in long-run equilibrium. From equation (12) the present discounted value of future net variable cost reductions due to an increase in capital equals the rental rate plus the current marginal cost of adjustment. The latter would be zero in long-run equilibrium. Hence a measure of the disequilibrium between the short and long-runs is the ratio of the marginal adjustment cost to the rental rate. Table 4 shows this ratio. On average over the sample a $1.00 expenditure on capital costs Bell Canada an additional $0.30 of adjustment costs. In addition, marginal adjustment cost relative to the rental rate has generally increased over the period 1955-1978.

An alternative indicator of the disequilibrium between the short and long-runs is the percentage difference between the capital stocks in the two runs. This difference is given in Table 4. The short-run capital was derived from the solution to equations (10.1) and (12), while the long-run capital was derived from the same two equations except with $\Delta K = 0$ and the other variables assumed to be changing at some constant rate. The long-run capital stock has been diverging from the short-run stock at a somewhat greater rate as the time period progressed. On average over the sample the long-run stock was about 10 percent greater than the short-run stock for Bell Canada.

Table 4: Measures of the Difference Between Short and Long-Run Equilibria

Year	$\beta_{II}\Delta K/w_K$	$(K^L-K^S)/K^S$
1955	0.1637	0.0696
1962	0.2438	0.0893
1970	0.4128	0.1363
1978	0.3665	0.1339
mean	0.2967	0.1073

4. Returns to Scale and Productivity Growth

Returns to scale and productivity growth are important indicators of the nature of the production technology for telecommunications services. Returns to scale is related to the change in costs as output quantities expand by some common rate. Productivity growth is related to the change in costs as technological change occurs. The empirical research on these technology indicators has been conducted in long-run static or short-run static equilibrium contexts (see Fuss [1983] and Kiss and Lefebvre [1986] and the references cited there-in), but not in a dynamic cost of adjustment, multiple output framework. The existence of adjustment costs affects the degree to which outputs and technological change alter costs and thereby affects returns to scale and productivity growth.

Returns to scale or the scale elasticity has been shown to be related to variable production cost (see Caves, Christensen and Swansen [1981] and Bernstein and Nadiri [1984] by

$$SE = (1 - \Sigma_{i=1}^{m} \partial \ln c_p^v / \partial \ln K_i)/\Sigma_{j=1}^{\ell} \partial \ln c_p^v / \partial \ln y_i \ .$$

The scale elasticity equals the inverse of the sum of the output elasticities of variable production cost after netting out the sum of the quasi-fixed factor elasticities.[13] In the present context, with a single capital stock and two outputs, the scale elasticity and its components are presented in Table 5.

Table 5: Cost and Scale Elasticities

Year	Local	Toll	Capital	Scale
1955	1.8288	0.0000*	-0.6717	0.9148
1962	1.3819	0.0259	-0.7447	1.2393
1970	0.8277	0.1267	-0.8429	1.9311
1978	0.5484	0.0738	-0.8054	2.9018
mean	1.2304	0.0599	-0.7927	1.5043

*The actual elasticity was -0.0015 which was not statistically different from 0. All other toll elasticities were positive except for 1963 which was -0.0217.

The first feature from Table 5 is that increases in toll output generated very small increases in variable production cost relative to the effect resulting from local output changes. On average a 1 per cent increase in toll output caused variable production cost to increase by only 0.05 per cent. When local output grew, cost increased by 1.23 per cent. The estimate of the output decomposition of variable production cost changes seems to be new to the empirical literature on telecommunications production structure. These results provide some evidence in favor of the conventional wisdom that producing additional local services is relatively more costly than producing additional toll services. What is somewhat surprising is the magnitude of the percentage cost differentials at the margin between the two outputs. The second result from Table 5 relates to the effect of capital on variable production cost. As the capital stock increases, variable production cost declines but less than in an equiproportional manner. A 1 per cent increase in capital causes cost to decline on average by 0.80 per cent.

The first three columns of Table 5 are the components of the scale elasticity. The latter is found in the last column. The scale elasticity generally increased over the sample such that on average there was increasing returns to scale with an elasticity of 1.50. However, there was not a monotonic trend in the scale elasticity. In this two output dynamic cost of adjustment model Bell Canada exhibited increasing returns to scale. The estimate of scale economies obtained in this study is in line, with those estimated from static, multiple output models (see Kiss and Lefebvre [1986]). Thus an important conclusion is that scale economies estimated from multiple output static models appear to be robust with respect to the equilibrium specification of the model.

The evaluation of productivity growth can be conducted in terms of the common rate of increase in the outputs as technological change occurs, given the inputs. Alternatively, productivity growth can be defined as the common rate of decrease in the inputs as technological change occurs, given the outputs. In terms of the variable production cost function,

$$PG_y = (-\partial \ln c_p^v/\partial t)/\Sigma_{j=1}^{\ell} \partial \ln c_p^v/\partial \ln y_i$$

$$PG_v = (-\partial \ln c_p^v/\partial t)/(1 - \Sigma_{i=1}^{m} \partial \ln c_p^v/\partial \ln K_i)$$

where PG_y represents the effect of technological change on the outputs and PG_v represents the effect on the inputs.[14] Technological change is not represented by time, in this model, but rather by a dummy variable which is 0 from 1954 to 1957, 1 from 1958 to 1971 and 0 from 1972 to 1978. Thus the shift in the variable production cost function (i.e. the numerator in PG) between the years t and t + s is

$$- \partial \ln c_p^v/\partial t = -(\ln c_p^v(t+s) - \ln c_p^v(t))/s .$$

In evaluating $\ln c_p^v(t+s)$ and $\ln c_p^v(t)$ all variables other than the binary variable are held fixed.[15] This means that by interpretting period t + s as the period with the binary variable equal to 1 and period t with the binary variable equal to 0 then the difference in variable production cost equals the estimate of the coefficient on the binary variable. This estimate is -.1875. Next interpretting t + s - t as the number of periods for which the dummy variable is 0, which is 11, yields the decline in variable production cost to be 1.7046 per cent (or (.1875/11)100). Thus technological change occurred for Bell Canada such that variable production cost decreased by an annual average of 1.71 per cent over the period 1954 to 1978.

The figure of 1.7046 per cent is the numerator (in percentage terms) in the formulas for productivity growth. The denominator in PG_y is given by the mean of the sum of the local and toll output elasticities of variable production cost. This figure is 1.2903. Thus PG_y for Bell Canada was approximately 1.32 per cent per year over the period 1954 to 1978. The denominator in PG_v is given by the mean of 1 minus the capital elasticity of variable production cost. This figure is 1.7927. Thus PG_v was 0.95 per cent per year over the period 1954 to 1978.

Models under the maintained hypothesis of long-run equilibrium have estimated the decline in production costs arising from technological change given the output levels (i.e. PG_v) to be between 0.55 and 0.80 per cent see (Denny et al [1981]). In addition, the effect of technological change on output growth given all the inputs (PG_y) was estimated to be between 0.85 and 1.15 per cent. These magnitudes have been considered too low (see Fuss [1983]) in light of the results obtained for Canadian manufacturing as a whole (see May and Denny [1979]). They estimated that PG_y was 0.95 per cent for Canadian manufacturing between 1946-76. Yet from casual evidence, technological change occurred more rapidly in telecommunications compared to total manufacturing. In this dynamic cost of adjustment model both estimates of productivity growth are significantly greater than in the static long-run models, and are consistent with the results obtained for Canadian manufacturing. It appears that the assumed flexibility of capital has caused an underestimate of productivity growth rates in Canadian telecommunications. A similar result was also obtained by Schankerman and Nadiri [1986] for AT & T in a short-run equilibrium model for the variable factors of production which were conditioned on the quasi-fixed factors.

5. Factor Substitution in the Short and Long-Runs

Adjustment costs cause quasi-fixed factors to be relatively more price inelastic in the short compared to the long-run. In addition, these costs generate variations in the degree of factor substitution or complementarity for both variable and quasi-fixed inputs between the two production runs. In fact it is possible for the complementarity or substitutability between factors to change between the short and long runs as the quasi-fixed factors become relatively more flexible.

The short-run factor price elasticities are derived from equations (10.1), (11) and (12) by differentiating with respect to ω_L and w_K. The results are presented in Table 6. The own price elasticities are all negative and quite inelastic. The variable factors had a relatively constant elasticity throughout the sample. The own price elasticity of the quasi-fixed factor became more inelastic through the sample. The cross price elasticities between the variable factors is just the negative of their respective own price elasticities, since the factor demand functions are homogenous of degree 0 in the variable factor prices. Since there are two variable factors then, of course, they are short-run substitutes. In addition, increases in the prices of labor and materials increased the demand for capital, with the latter exerting a stronger effect. In the short-run the rental rate on capital does not affect the variable factor demands. It is the stock of capital which affects variable factor demands. Thus there is an asymmetry in the cross price effects pertaining to capital. In this context, however, capital was a short substitute for labor and materials.

The long-run factor price elasticities are derived in the same

Table 6: Short-Run Factor Price Elasticities

Factor Price	Factor Demand	Year	Elasticity
Labor	Labor	1955	-0.1243
		1962	-0.1518
		1970	-0.1622
		1978	-0.1596
Materials	Materials	1955	-0.1809
		1962	-0.1742
		1970	-0.1668
		1978	-0.1690
Capital	Capital	1955	-0.3525
		1962	-0.1859
		1970	-0.0917
		1978	-0.0705
Labor	Capital	1955	0.0462
		1962	0.0214
		1970	0.0164
		1978	0.0148
Materials	Capital	1955	0.3063
		1962	0.1392
		1970	0.0753
		1978	0.0558

manner as the short-run except with $\Delta K(t) = 0$ and the other variables in equations (10.1), (11) and (12) assumed to be changing at a constant rate. The long-run price elasticities are presented in Table 7. In the long-run the own price elasticities were still inelastic but with the disappearance of adjustment costs the factors became relatively more elastic since it was less costly for Bell Canada to adapt to changing prices. This result was especially true for labor and capital. Materials was the most price inelastic factor of production in the long-run. This finding is different than that obtained in the short-run where, on average, the own price elasticities were quite similar. The own price elasticities estimated in the long-run equilibrium models surveyed by Fuss [1983] and Kiss and Lefebvre [1986] show that the capital elasticity is at the high end, the materials elasticity at the low end and labor elasticity in the middle of the long-run findings.

The cross price elasticities are also presented in Table 7. In the long-run capital and materials and capital and labor were substitutes. This finding is generally consistent with the short-run results. However, in the long-run labor and materials were complements, which was the converse to the short-run case. Capital adjustment costs affected the interrelationships between the variable factors of production. As capital become relatively more flexible due to the adjustment costs becoming zero, the variable factors changed from substitutes to complements. Generally, the long-run equilibrium models find that capital is a substitute for the other inputs, and that labor is a substitute for materials. This latter finding is consistent with the short-run result estimated in this paper but not in the long-run where labor and materials became complements.

Table 7: Long-Run Factor Price Elasticities

Factor Price	Factor Demand	Year	Elasticity
Labor	Labor	1955	-0.8309
		1962	-0.7875
		1970	-0.7409
		1978	-0.7631
Labor	Materials	1955	-0.1032
		1962	-0.1202
		1970	-0.1300
		1978	-0.1256
Labor	Capital	1955	0.7211
		1962	0.6033
		1970	0.5016
		1978	0.5493
Materials	Labor	1955	-0.0757
		1962	-0.1091
		1970	-0.1302
		1978	-0.1208
Materials	Materials	1955	-0.2650
		1962	-0.2942
		1970	-0.3152
		1978	-0.3051
Materials	Capital	1955	0.2148
		1962	0.2531
		1970	0.2567
		1978	0.2574
Capital	labor	1955	0.9065
		1962	0.8966
		1970	0.8711
		1978	0.8839
Capital	Materials	1955	0.3682
		1962	0.4144
		1970	0.4452
		1978	0.4307
Capital	Capital	1955	-0.9358
		1962	-0.8564
		1970	-0.7583
		1970	-0.8067

6. Conclusion

Short and long-run equilibrium models of multioutput production were estimated for Bell Canada. A specification test was conducted and it was found that Bell Canada was not in long-run equilibrium. In particular, the hypothesis that capital could be treated as a variable factor of production was rejected. In order to model capital decisions a dynamic cost of adjustment model was estimated. The cost of adjustment parameter was significant. Bell Canada had not adjusted to its long-run equilibrium level of capital. Indeed, it was estimated that for every $1 of capital expenditures it costs Bell Canada an additional $.30 of adjustment cost in order to install the new capital into the production process. The capital stock for Bell Canada was approximately 10 per cent below its long-run level due to the existence of adjustment cost.

Returns to scale and productivity growth were estimated within the context of the dynamic cost of adjustment model. Productivity growth estimates were grater than those obtained from the static long-run equilibrium models and appear to be more in line with estimates for Canadian manufacturing as a whole. The common average annual rate of output growth due to technological change for Bell Canada was 1.32 per cent, while for manufacturing the estimate of the growth rate was 0.95 per cent. The scale elasticity obtained from the multiple output dynamic cost of adjustment model was consistent with those estimated from the multiple output static models. The average scale elasticity was 1.50. Increasing returns to scale for Bell Canada appears to be robust across equilibrium specifications of factor demands within multiple output frameworks. Furthermore the results clearly showed that increasing returns to scale arose because of the samll effect that toll output exerts on variable production cost. A 1 per cent increase in toll output generates only a 0.05 percentage increase in variable production cost while a 1 per cent increase in local output causes cost to increase by more than 1.20 per cent.

Adjustment costs also affected the substitution possibilities between the factors of production. In the short-run labor and materials were substitutes, but in the long-run when adjustment costs are zero labor and materials became complementary factors. However, in both the short and long-runs capital was a substitute for both labor and materials. Factor demands did respond to changes in their respective factor prices. The price effects were highly inelastic. In fact with nonzero costs of adjustment the price elasticities for each factor averaged approximately -0.20 per cent, which were significantly more inelastic than those obtained from models maintaining the assumption of long-run equilibrium.

Notes

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1. The transformation function is increasing in outputs and the additions to the quasi-fixed factors, and decreasing in all the factors. In addition the transformation function is concave in the outputs and inputs.

2. The stocks are in fixed proportion to the services for the quasi-fixed factors. The problem of quasi-fixed factor utilization is not addressed in this paper as usual in models used to estimate the production structure of a telecommunications carrier.

3. The right side of equation (4) pertains to variable production costs (which are the costs of hiring the variable factors) and the costs of adjustment (or installation). However, because both K and I are given in the first stage of the problem then adjustment costs are also exogenous in this stage. The notation (t) is also suppressed unless ambiguity arises.

4. The variable cost function is increasing in the variable factor prices, output quantities and gross investment flows, and decreasing in the quasi-fixed factors. The function is also concave and homogenous of degree one in the variable factor prices, concave in the gross investment levels and convex in the quasi-fixed factors.

5. The general model is not dependent on the assumption of a non-stochastic discount rate. However, to characterize the envelope condition in the usual manner it is necessary to divide each of the Euler equations by the discount rate. This operation can only be carried out if the discount rate is non-stochastic. It is assumed from now on that the discount rate is non-stochastic.

6. The other conditions (see Footnote 4) on the variable cost function are not imposed.

7. Since depreciation is constant the gross and net investment are in a fixed proportion to each other. The empirical results were not affected by the use of net rather than gross investment.

8. The rental rate defined in the long-run equilibrium by the right side of (9) is a special case of $w_k(t)$. In the long-run equilibrium $E(t)p_I(t+1)/p_I(t) = 1+\theta$. Thus in the long-run $w_k(t)=p_I(t)[(1+r)(1+\theta)-(1-\delta)(1+\theta)]=p_I(t)(1+\theta)(r+\delta)$ which is precisely the right side of (9).

9. In equation set (10), (11) and (12) there is no need to estimate equation (10.2) since the only parameter in this equation is also in (12). Hence no new information is added by including (10.2). Thus the system to be estimated is (10.1), (11) and (12).

10. The error term in the variable cost function can also represent technology shocks. However, if the errors in the labour share or capital demand equations then the error in the variable cost function must be correlated with the variable factor prices and capital. In this case consistent estimation of the equations becomes much more difficult.

11. The expected rate of inflation was calculated by Fuss and Waverman [1980] as the difference between the Government of Canada savings bond rate and the real rate of return of 3 per cent. Strictly speaking this may not be consistent with the present model. In this context the expected rate of inflation is forward looking. However, the components of the rental rate were not available to us to permit a modification of the formula. Although we did re-estimate the model using the rental rate in Kiss [1981] which was defined in a manner similar to that in Fuss and Waverman except that the expected rate of inflation was assumed to be zero. In fact, the estimation results were hardly affected by the difference. The rental rate from Fuss and Waverman only extended to 1978. In 1979 we used .2374 which was based on a moving average.

12. The binary variable entered as lnA=B where B=1 from 1958-1971 and 0 everywhere else. The model with this variable outperformed models with a time trend, the percentage of telephones with access to direct distance dialing, the percentage of telephones with access to modern switching facilities or with an index of the last two variables. Performance of the models was based on the satisfaction of the regularity conditions relating to the variable production cost function.

13. The definition of the scale elasticity does not imply that the quasi-fixed factors are at their long-run equilibrium magnitudes. The quasi-fixed factors are evaluated at their short-run equilibrium levels. In addition the definition of scale elasticity is conditioned on holding adjustment costs fixed.

14. The definition of PG_v is conditioned on holding adjustment costs fixed. In addition, $PG_y = PG_v \cdot SE$. These definitions of productivity growth are consistent with any scale elasticity and not just constant returns to scale (see Caves, Christensen and Swanson [1981]).

15. This method is consistent with Diewert's [1976] quadratic approximation lemma.

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