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COLLAPSING EXCHANGE RATE REGIMES: AN EXTENDED FRAMEWORK

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Abstract

This paper extends the Flood-Garber linear discrete time model of a collapsing exchange rate regime. Uncertainty is introduced through random monetary policy, uncertain access to external credit, and susceptibility to random price shocks. The extended model does not assume purchasing-power-parity or interest rate parity, and the demand for real balances is made sensitive to real income and currency substitution motives. An endogenous conditional collapse probability is derived, as is a shadow exchange rate that serves to provide a lower bound for the post-collapse exchange rate.

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1. INTRODUCTION[1]

Models of exchange rate crises and collapsing exchange rate regimes depict deficits in the balance of payments leading to a gradual erosion and then sharp speculative attack on central bank foreign exchange reserves. This depletes reserves to the lowest level acceptable to the central bank and serves to eliminate the means by which the government can intervene in the foreign exchange market. When this occurs, the overvalued exchange rate can no longer be sustained and the exchange regime collapses. The collapse can take the form of either a discrete devaluation of the fixed exchange rate or a switch to a floating exchange rate generally accompanied by a sharp currency depreciation. This paper presents a linear discrete time model of a collapsing exchange rate regime. It is a direct extension of Flood and Garber (1984) but departs from that model by relaxing some of its strong assumptions. I relax the assumptions of purchasing power parity and interest rate parity, and allow real income and currency substitution motives to influence the demand for real balances. Further, whereas Flood and Garber [F-G] introduce uncertainty only through randomized domestic credit creation, the extended model of this paper also introduced random price shocks and shocks in the market for external credit.

[1]This paper has benefited from comments and suggestions by Peter Kenen, Bill Branson, Bob Cumby and Sue Collins. Princeton University and the C.V.Starr Center of New York University are gratefully acknowledged for their support. Any errors or inconsistencies are, of course, my own.

Following Salant and Henderson's (1978) application of a Hotelling model of natural resource depletion to a speculative attack on government gold stocks, Krugman (1979) applied the speculative attack model to the foreign exchange market. Krugman showed that when future government policy is known with certainty, a balance of payments problem passes through three stages: a period of gradually declining reserves, a sudden speculative attack and a post-crisis period during which the currency is allowed to depreciate. When monetary policy and the quantity of reserves that the government is willing to commit to maintain the existing exchange rate are not known, the timing and magnitude of the speculative attack are dependent upon the estimates of individuals who have incomplete knowledge.

In the linear discrete-time model of a collapsing fixed exchange rate developed by Flood and Garber (1984), the expected timing of the collapse is linked to an uncertain rate of domestic credit creation. The intuition behind the basic collapse model is that the probability of an exchange regime collapse equals the probability that an attack on central bank reserves will prove rewarding to speculating agents. Defining the shadow exchange rate, $\tilde{S}(t+1)$, as the exchange rate that would prevail in a post-collapse equilibrium, the standard argument is that a collapse will occur as soon as this shadow exchange rate is expected to exceed the prevailing controlled rate, $\bar{S}(t+1)$. This will lead to a speculative attack on the currency that depletes reserves down to a critically low level. If the exchange regime is abandonned, the speculating agents will profit by a minimum of $[\tilde{S}(t+1) - \bar{S}(t+1)]$ per unit of foreign currency held.

This representative agent model generates a forward discount on a weak fixed rate currency and succeeds at partially explaining the 'peso problem'.

The model has been used for generating time series of the estimated probability that the exchange regime will collapse one period in the future and for deriving a lower bound for the post-collapse sustainable exchange rate.

As a well formulated discrete time framework, the F-G model has demonstrated shown much promise in empirical observation. Indeed, the potential for applied research on collapsing exchange rate regimes has been demonstrated by recent empirical work.[2] Van Wijnbergen (1985) considered the sustainability of the move from a flexible to a fixed exchange rate as part of efforts to control inflation in Brazil, Argentina and Israel. By applying the basic collapse model, he was able to illustrate the linkage between unsustainable policy mixtures and their post-collapse inflationary consequences. Blanco and Garber (1986) applied the basic model to the Mexican experience from 1973 to 1982 and estimated one-period ahead probabilities that the Mexican peso would collapse. Collins (1986) studied the expected timing of realignments within the European Monetary System. Cumby and van Wijnbergen (1987) applied a similar model to the crawling peg experience of Argentina between 1979 and 1981. Despite some troubling assumptions embedded in the basic model, these applications performed well qualitatively.

The problematic assumptions of the basic model include: purchasing-power-parity, interest-rate-parity, and demand for real money balances insensitive to real income and currency substitution motives. In this paper I extend the basic model to eliminate these assumptions and also introduce

[2]See Blanco and Garber (1986), van Wijnbergen (1985), Collins (1986), Cumby and van Wijnbergen (1987), and Goldberg (1987b).

an additional level of uncertainty through random price shocks and shocks to external credit supplies. It is shown that empirical results based on the basic model will be biased if any of these extensions are statistically significant.

2. THE EXTENDED MODEL

This section presents the extended model of speculative attacks on a currency and discusses the conditions under which they will lead to a collapsed exchange rate regime. The starting point of this work is the linear discrete time model of Flood and Garber (1984). The points of departure are the relaxed assumptions of purchasing power parity and interest rate parity, as well as the treatment of demand for real balances as sensitive to real income levels and currency substitution motives. I make small country assumptions by keeping the foreign price level and associated price shocks as exogenous, but unlike the basic model I allow for a nontraded good. [3] Further departures from the Flood and Garber (F-G) model are in the sources of uncertainty affecting this country. F-G allowed for uncertain domestic credit creation, a condition that translates into fiscal policy but not uncertain inflation since purchasing power parity was assumed. I maintain uncertain domestic credit creation, but add random price shocks and uncertain access to supplies of external capital. This is important because the government will be shown to lose some of its control over the creation of domestic credit.

[3] Connolly and Taylor (1984) show how expectations of a devaluation, or change in exchange rate regime, can be reflected in relative prices of domestic and foreign goods. These dynamics are not the focus of this paper and are not modelled herein.

Under a fixed exchange rate, any excess supply of domestic credit is met with an offsetting decline in central bank foreign exchange reserves so that the supply of money is fairly stable. When reserves are no longer available for equilibrating the money market, the exchange rate will have to adjust. The timing, magnitude and probability of this adjustment and the speculative attacks on central bank reserves that may occur are the central focus of this paper. Throughout, the transition or collapse studied is one in which a fixed exchange rate gives way to a flexible rate or a steeply devalued fixed rate. [4]

The basic equations of the model are given below.

1) $M^d(t)/Q(t)= a_0- a_1i(t)+ a_2y(t)- a_3E[S(t+1)-S(t): \xi(t)]/S(t)$

2) $i(t)= i^*(t) + E[S(t+1)-S(t): \xi(t)]/S(t) + r(t)$

3) $Q(t)= \alpha P(t) + (1-\alpha)P^*(t)S(t)$

4) $P(t)/S(t)= P*(t) + \Omega'(t)$

5) $\Omega'(t)= \rho(t) + \Omega(t)$

6) $M^s(t)= R(t) + D(t)$

7) $D(t+1)= D(t) + \mu(t) + \epsilon(t)$

8) $\epsilon(t)= \gamma(t) - \phi(t)$

9) $M^s(t)= M^d(t)$

Equation 1 represents the demand for real balances, nominal money M(t) divided by the domestic price index Q(t). Demand is sensitive to real income, y(t), portfolio motive through the nominal interest rate, i(t), and currency substitution motives through the expected rate of depreciation of the exchange rate, S(t). The exchange rate is defined as domestic currency

[4]The model needs only minor modification to be applied to the case of unsustainable crawling exchange rates.

per unit of foreign exchange. Expectations are taken using the information set $\xi(t)$.

Equation 2 represents the linkage between the domestic interest rate and the foreign interest rate, $i^*(t)$. The spread is comprised of the expected change in the exchange rate plus some term that I will broadly interpret as a risk premium. The broad interpretation is because domestic interest rates are often distorted in pursuit of policy objectives so that $r(t)$ may diverge from the asset market interpretation of a risk premium. Further, if the domestic economy is rationed in its access to external credit supplies, the direct linkage between capital markets may be hindered.

Equation 3 defines the domestic price index as a weighted sum of the price of domestic nontraded goods, $P(t)$, and the domestic price of traded goods, $P^*(t)S(t)$. The share of nontraded goods in domestic expenditures, α, is taken as exogenously given when determining the dynamics of the exchange rate. This marginal propensity to consume need not be fixed, however, and comparative statics exercises may be applied so that we are able to distinguish between expenditure switching and expenditure reducing complexities in demand patterns.

Equation 4 breaks the assumption of short run purchasing-power-parities (PPP) by permitting deviations $\Omega'(t)$. In Equation 5 the deviations are divided into a systematic component, $\rho(t)$, and a short run or random component, $\Omega(t)$. The validity of various forms of PPP assumptions has been debated on both theoretical and empirical grounds. Flood and Garber modelled the strict form of PPP by ruling out both short and long run deviations from parities. Herein, however, the true form of the relation between domestic and foreign prices is left open-ended to be determined in the specific applications of the model.

Equation 6 defines the money supply as the sum of domestic credit in circulation, D(t), and the stock of central bank foreign exchange reserves, R(t). Foreign exchange reserves are valued in domestic currency.

Two sources of variability directly influence domestic credit creation. The first source, $\gamma(t)$, is a control variable that can be manipulated by monetary authorities. For example, domestic credit creation above trend may be attributed to surprise shortages in government revenues or unanticipated expenditures for which no revenues are available. It may also be that the government had expected to borrow in world capital markets to meet these shortages rather than to print money. Consequently, it is important to consider the uncertain availability of external credit flows into the troubled economy. This uncertainty is captured by $\phi(t)$. Positive shocks indicate unanticipated availability of external capital that would decrease the amount of domestic credit creation needed for meeting internal deficits. Negative shocks indicate unanticipated restricted access to external capital that lead to further increases in domestic credit creation above trend rate $\mu(t)$. Shocks in external credit availability may be country specific or global.

By solving for money market equilibrium we can illustrate the movement of central bank reserve stocks under a fixed rate system or of the exchange rate under a flexible exchange rate system. Because the money supply is largely fixed when the exchange rate is fixed, an increase in domestic credit, without an adequate increase in demand for money, leads to a loss of central bank reserves equal to the excess supply of domestic credit. If, for some reason, reserves were not permitted to decline, the exchange rate would have to adjust. Thus, with this information, we can derive the exchange rate that would clear the money market if the currency were instantaneously allowed to float. This rate is called the shadow exchange rate, $\tilde{S}(t)$. The shadow rate provides a miniumum bound for the rate that would be prevail if speculators attacked central bank reserves, depleted reserves to a critically low level, and forced the collapse of the exchange regime.

Following the approach used by Flood and Garber (1984), the method of undetermined coefficients is applied to generate a solution for the path of the shadow exchange rate. Positing $\tilde{S}(t) = \lambda_0 + \lambda_1 M(t)$, and combining equations (1) through (9), the solution for the shadow rate in period t becomes:

10) $\tilde{S}(t) = \left\{1/\ a\{P^*(t) + \alpha\ \Omega'(t)\}\right\}\left\{\ \{(a_1 + a_3)/a\}\mu + M(t)\ \right\}$

where $a = a_0 + a_2 y(t)\ -\ a_1[\ i^*(t) + r(t)\ ]$

As in F-G, the probability of an exchange regime collapse in this representative agent model is equal to the probability that the one period ahead shadow exchange rate will exceeds the one period ahead controlled or announced rate. The rationale for that formulation is straight-forward.

Since the government's commitment to a controlled rate gives speculators unrestricted access to central bank foreign exchange reserves, speculators operating in period (t), who expect the shadow rate to exceed the controlled rate in period (t+1), will purchase foreign exchange reserves at the end of period t. Since all agents are assumed identical, they will all attack at the same moment, thereby driving reserves down to the critical floor. The collapse will occur instantly since there are no longer any resources to be applied in defense of the weak currency.

As assessed in period t, the probability that the exchange regime will collapse, $\pi(t)$, is the probability that the shadow rate will exceed the controlled rate in period t+1.

$$11)\ \pi(t) = \Pr\left\{ \tilde{S}(t+1) - S(t) \geq 0 \right\}$$

$$= \Pr\left\{ \{1/a[P^*(t+1) + \alpha\Omega'(t+1)]\}\{((a_1+a_3)/a)\mu + M(t+1)\} - S(t) \geq 0 \right\}$$

Recall that $M(t+1) = D(t) + \mu(t) + \gamma(t+1) - \phi(t+1) + R(t+1)$, and that PPP deviations $\Omega'(t+1)$ are comprised of systematic and random components. Substituting and rearranging terms yields:

$$12)\ \pi(t) = \Pr\left\{ S(t)a\alpha\Omega(t+1) - \gamma(t+1) + \phi(t+1) \leq K(t+1) \right\}$$

where $K(t+1) = ((a + a_1 + a_3)/a)\mu + D(t) + R(t+1) - S(t)a[P^*(t+1) + \alpha\rho(t+1)]$

If a speculative attack were to deplete reserves to the minimum permissible level, then R(t+1) would be replaced by R_c in equation 12. This is the procedure that has been followed in the literature, leaving a probability expression that would need to be evaluated based on the probability distribution of the summed right hand side random variables. In the F-G basic model the solution was a straight-forward evaluation of an integral since their was only one source of randomness affecting the demand for real money balances. The solution to equation 12 is complicated by the presence of three sources of randomness, each of which may be characterized by a distinct statistical distribution. To solve the model we must apply a convolution integral and find the joint distribution of the sum of the random variables.

The expected probability that the shadow rate will exceed the fixed rate next period has three random influences and numerous components that are known with 'reasonable' certainty. The most relevant sources of randomness enter through domestic monetary policy, price shocks, and external liquidity shocks. To solve for a closed form solution for $\pi(t)$, the general and specific solutions are derived by first setting up the system and then by assuming hypothetical distributions for the random components. Rewriting the problem as

13) $\pi(t) = \Pr\left\{ Z(t+1) \le K(t+1) \right\}$,

the random variables are redefined and characterized by probability density functions (pdfs):

$$Y_1(t+1) = \gamma(t+1) \quad \text{with pdf} \quad f_{y1}(Y_1)$$

$$Y_2(t+1) = \phi(t+1) \quad \text{with pdf} \quad f_{y2}(Y_2)$$

$$X\ (t+1) = S(t)a\alpha\Omega(t+1) \text{ with pdf } f_x(X)$$

$$Z\ (t+1) = S(t)a\alpha\Omega(t+1) - \gamma(t+1) + \phi(t+1) \text{ with pdf } f_z(Z)$$

$$= X(t+1) - Y_1(t+1) + Y_2(t+1)$$

The probability density of Z(t+1) will be of the form:

14) $f_z(Z) = \int \int f_x(X)[1/S(t)a\alpha]\ f_{y1}(Y_1)\ f_{y2}(Z+Y_1-X)\ dY_1 dX$

so that

15) $\pi^s(t) = \int_{-\infty}^{K(t)} f_z(Z)\ dz$.

is the probability that the shadow rate will exceed the fixed rate next period. To provide a direct parallel to F-G and to give a more specific form to this function, hypothetical pdfs are introduced.

Suppose that large positive shocks to domestic credit creation are more likely than large negative shocks. Similarly, suppose a domestic inflationary shock is more likely than a deflationary shock. Then, it is reasonable to represent $\Omega(t+1)$ and $\gamma(t+1)$ as exponentially and independently distributed with zero means and variances of $(1/w)^2$ and $(1/\lambda_1)^2$ respectively.

16) $\Omega(t+1) = -1/w + \omega(t+1)$, with $f_x = f_\omega$ and

$$f_x(X(t+1)) = \begin{cases} w \exp[-w\,\omega(t+1)] & \text{if } \omega(t+1) \geq 0 \\ 0 & \text{if } \omega(t+1) < 0 \end{cases}$$

17) $\gamma(t+1) = -1/\lambda_1 + \nu_1(t+1)$, with $f_{y1} = f_\gamma$ and

$$f_{y1}(Y_1(t+1)) = \begin{cases} \lambda_1 \exp[-\lambda_1\,\gamma(t+1)] & \text{if } \gamma(t+1) \geq 0 \\ 0 & \text{if } \gamma(t+1) < 0 \end{cases}$$

It is difficult to posit the shape of shocks to the supply of external credit. For parallelism, they are assumed to be exponentially distributed about zero with a variance of $(1/\lambda_2)^2$.

18) $\phi(t+1) = -1/\lambda_2 + \nu_2(t+1)$, with $f_{y2} = f_\phi$ and

$$f_{y2}(Y_2(t+1)) = \begin{cases} \lambda_2 \exp[-\lambda_2\,\phi(t+1)] & \text{if } \phi(t+1) \geq 0 \\ 0 & \text{if } \phi(t+1) < 0 \end{cases}$$

The shift variables, $1/w$, $1/\delta_1$ and $1/\delta_2$ are included so that the random variables are distributed mean zero. Redefining $K(t+1)$ as $K'(t+1)$ to capture these shift variables, the pdf of Z is derived by integrating over equation 19.

19) $f_z(Z) = \int_0^\infty \int_{x-z}^\infty (w\lambda_1\lambda_2/Sa\alpha)\exp(-wX)\exp(-\lambda_1 Y_1)\exp[-\lambda_2(Z+Y_1-X)]\,dY_1\,dX$

The solution of this integral gives the pdf for the summed random variables,

20) $$= \left\{\omega\lambda_1\lambda_2/\ S(t)a\alpha(\lambda_1+\lambda_2)(w+\lambda_1)\right\}\left\{\ \exp(-\lambda_1 Z)\ \right\}$$

where Z is bounded by $[-\infty, K'(t+1)]$. Integrating over the range of Z yields $\pi(t)$, interpreted as the probability of collapse.

21) $\pi(t) = \left\{\omega\lambda_2/[S(t)a\alpha(\lambda_1+\lambda_2)(\omega+\lambda_1)]\right\} \exp[\lambda_1\ K'(t+1)]$

where

$$K'(t+1) = ((a+a_1+a_3)/a)\mu + D(t) + R_c - 1/\lambda_1 + 1/\lambda_2 - S(t)a[P^*(t+1) + \alpha\rho(t+1) - \alpha/\omega]$$

3. COMPARATIVE STATICS

Using comparative statics, we can sign the sensitivity of $\pi(t)$ to various state variables and control variables. The relationship between domestic credit creation and the probability of collapse is illustrated in Figure 1.

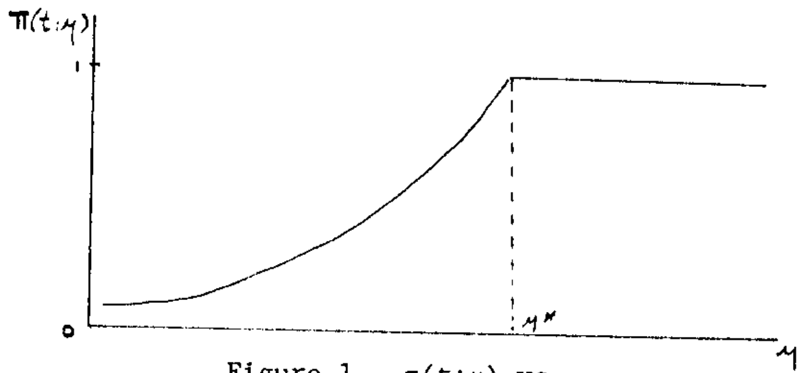

Figure 1. $\pi(t:\mu)$ vs. μ

It can be shown that there exists some $\mu=\mu^*$ such that $\pi(t:\mu^*)=1$. For all $\mu < \mu^*$, $d\pi/d\mu > 0$ and $d^2\pi/d\mu^2 > 0$, implying a concave mapping. There is not, however, a finite $\mu > 0$ that satisfies $\pi(t:\mu)=0$. To completely eliminate the probability of collapse $K'(t+1)$ would have to equal $-\infty$, a condition unlikely to be achieved by contracting domestic credit. Though $\pi(t)$ can not be eliminated, it can certainly be reduced by lowering μ.

The probability of collapse:

is positively correlated with the variability of domestic credit creation,

22) $\delta\pi(t)/\delta[1/\lambda_1] > 0$ if $1/\lambda_1 - 2\lambda_1 < w + \lambda_2$,

is positively correlated with the variability of supplies of external credit, as long as external credit markets are characterized by more uncertainty than internal money markets,

23) $\delta\pi(t)/\delta[1/\lambda_2] > 0$ if $1/\lambda_2 \geq 1/\lambda_1$ (sufficient condition),

is positively correlated with the degree of randomness attributed to price shocks,

24) $\delta\pi(t)/\delta(1/w) > 0$ if $w < (w + \lambda_1)a\alpha S(t)$,

is negatively correlated with the share of the consumption basket held by nontraded goods,

25) $\delta\pi(t)/\delta\alpha < 0$,

is negatively correlated with the 'risk premium', $r(t)$, on domestic investments. This results is questionable, however, since the meaning of $r(t)$ remains vague and its possible dependence on $\pi(t)$ remains unspecified,

26) $\delta\pi(t)/\delta r(t) < 0$.

The probability of collapse is negatively correlated with the level of real income. This reflects the increased transactions demand for real balances that reduces some of the excess supply of domestic credit. The exception, intuitively, is when increased $y(t)$ brings about a more than proportionate worsening of the current account due to increased import demand,

27) $\delta\pi(t)/\delta y(t) < 0$,

decreases as systematic deviations from PPP rise. The price wedge between traded and nontraded goods serves to remove some of the direct pressure from the exchange rate by making nontraded domestic goods relatively less expensive,

28) $\delta\pi(t)/\delta\rho(t) < 0$,

and finally, increases as the leakage through currency substitution increases, though it should be noted that a nonlinear rather than a linear currency substitution term would probably better capture the real world behavior of the variables,

29) $\delta\pi(t)/\delta a_3 < 0$.

The statistical significance of these terms is important when we interpret the results of the empirical studies of exchange rate crises.

4. IMPLICATIONS FOR EMPIRICAL STUDIES

Previous empirical studies of exchange rate crises include: Blanco and Garber (1986), Collins (1987), van Wijnbergen (1985), and Cumby and van Wijnbergen (1987). For the most part, the assumptions of the basic Flood and Garber (1984) collapse model were retained in these applications. Consequently, any collapse probability (and shadow exchange rate) estimates generated were understated to the extent that prices and external credit supplies exhibit short term randomness. The probability estimates were overstated to the extent that there are nontraded goods in the domestic consumption basket, to the extent that there are positive and significant risk premia and systematic deviations from purchasing-power-parity, and to the extent that real income and currency substitution motives influence the demand for real balances. Goldberg (1987) finds these extensions statistically significant in applications to the experiences of Mexico and Argentina.

Despite the extensions to the basic collapse model that were made in this paper, much additional work on modelling collapsing exchange rates needs to be undertaken. The models still assume a neat representative agent world where the internal and external environment is characterized by basically passive agents. The introduction of dynamic and strategic decision-making into models of exchange rate crises should be a strong priority in future research in this area.

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