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Joint Exploitation of a Productive Asset: A Game-Theoretic Approach

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ABSTRACT

It is generally believed that when two or more economic agents jointly exploit a common productive asset, there will be a tendency towards overuse or overconsumption, if there is no possibility of making binding commitments regarding the rates of use or consumption. Lancaster(1973) and Levhari and Mirman (1980) have studied specific examples of this phenomenon from a game-theoretic point of view, and in each case demonstrated the existence of a Pareto-inefficient Nash equilibrium of the corresponding dynamic game.

However, dynamic games often have multiple equilibria, and the question remains whether the games studied by these authors - and generalizations of those games - have efficient as well as inefficient equilibria. Indeed, the theory of repeated games suggests this possibility, although it must be emphasized that these games are not strictly repeated, since there is a state variable, namely the current stock of the productive asset, that changes through time in response to the players' actions.

In the present paper we explore the set of equilibria of a game-theoretic model of this type. We do find, in fact, that under certain circumstances there may be efficient as well as inefficient equilibria. Moreover, some of these equilibria have interesting features that are not present in repeated games.

In a repeated game, an important role is played by equilibria in which an inefficient equilibrium of the corresponding static game is repeated indefinitely. Under certain circumstances, especially if the players' discount rates are sufficiently low, an efficient outcome of the repeated game can be sustained as an equilibrium outcome by the players' threats to revert to the inefficient equilibrium in case of a deviation from the efficient path.

The equilibria of the dynamic games we study here that correspond to the repeated static equilibria are those in which each player uses a strategy in which his action at any date is independent of the current state of the state variable (the stock of the asset); we might call these "extreme equilibria." In these equilibria, the players run down the stock of the asset as fast as possible. By analogy with the terminology of repeated-game theory, we define a *trigger strategy equilibrium* to be a Nash equilibrium in which the players threaten to revert to an extreme equilibrium whenever a player is caught deviating from the target efficient path. The effectiveness of such threats depends, of course, on the "detection technology," i.e., on how much extra utility the deviating player can gain before his deviation is detected by the other players. In the model we study, efficient trigger-strategy equilibria may exist from some starting states but not others. More precisely, there is a stock level, say y', such that a trigger-strategy equilibrium exists from starting stocks greater than or equal to y', but not from those strictly less than y'. (This statement is meant to include the cases in which y' is zero or infinite.)

Under some circumstances, there may exist a new kind of equilibrium, which we call a *switching equilibrium*. We show that, in our model, whenever y' is positive (and finite), there is an open

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interval I with upper endpoint y' such that, from any starting stock in I there is an equilibrium of the dynamic game with the following structure: the players follow an inefficient but growing path until the stock reaches the level y', and then follow a trigger strategy (efficient) after that.

The use of a continuous-time model enables us to conveniently decouple the delay of information from the time interval between decisions. Although this leads to some conceptual and mathematical difficulties, we believe that it is an important contribution of our analysis.

In the continuous-time model that we study here, the stock at date t, $Y(t)$, evolves according to the differential equation,

$$Y'(t) = \eta[Y(t)] - c_1(t) - c_2(t) .$$

where (for the case of two players), $c_1(t)$ and $c_2(t)$ are the rates of consumption of the asset by players 1 and 2, respectively. The "production function," η, is assumed to be concave, and to take the value zero at both zero and some positive stock level. The strategy of each player determines his consumption rate at each time as a function of the previous history of the process, possibly with some delay. We assume that each player's utility for the game is equal to his total discounted consumption over the (infinite) duration of the game. This linearity of a player's utility in his consumption is the main special assumption of the model; however, we believe that many of the qualitative features of our results do not depend essentially on this linearity.