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ABSTRACT

Coalition voting (CV) is a voting procedure for electing a parliament under a party-list system of proportional representation. As under approval voting (AV), voters can vote for as many parties as they like, but unlike AV, each party does not receive one vote. Instead, each voter has a "party vote," which is divided evenly among all parties of which the voter approves.

These fractional approval votes determine the seat shares of each party in the parliament. In addition, each voter has a "coalition vote," which counts for the <u>set</u> of all parties (and proper subsets) of which the voter approves. Whereas party votes determine which sets of parties have a majority of seats, coalition votes determine which of these majority sets has the greatest approval <u>as a coalition</u>. The majority coalition that has all its parties approved of by the most voters becomes the governing coalition.

In order to have their coalition votes count, voters must vote for enough parties to constitute a majority coalition. By placing a premium on precisely the most approved of majority coalitions, CV also encourages parties, before an election, to reconcile their differences and form coalitions that are likely to have broad appeal. Such coalitions, insofar as they formulate coherent policies, facilitate voter choices, producing a convergence of voter and party/coalition interests.

Theoretical properties of CV are analyzed, and optimal strategies of voters and parties are investigated. CV's most likely empirical effects in faction-ridden multiparty systems, like those of Israel and Italy, are also considered. Although CV is radically different from AV in the way votes are allocated and aggregated, the physical act of voting--for as many alternatives as one likes--is the same, commending CV as a relatively simple and practicable reform to promote consensus in party-list systems.

COALITION VOTING

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1. Introduction

Coalition voting (CV) is a voting procedure for electing a parliament under a party-list system of proportional representation (PR). It is designed to induce voters to vote for a set of one or more parties that could win a majority of seats in parliament and thereby form a government.

By giving voters an inducement to vote for majority coalitions, CV also encourages parties to coalesce, even before an election. Indeed, because CV places a premium on precisely the majority coalitions that are most compatible--as defined by voter choices--it makes it advantageous for parties in such a coalition to urge their supporters not just to vote for them alone but also to vote for other parties in the coalition. In this manner, the bane of multiparty systems under PR--disincentives to coalition formation--is attenuated.

To be more specific, under CV voters vote for parties, which receive seats in parliament proportional to the number of votes they receive, as is the case in most party-list systems. Under CV, however, the usual restriction on voting for exactly one party is lifted so that voters can vote for as many parties as they like.

This feature of CV smacks of approval voting (AV), whereby voters can vote for as many candidates as they like in multicandidate elections without PR (Brams and Fishburn, 1978, 1983). But unlike AV, if a voter votes for more than one party, each party does not receive one vote--it depends on how many parties the voter voted for under CV, as will be illustrated shortly.

Since AV was first proposed more than ten years ago, it has generated much interest as well as a good deal of controversy (see, e.g., the recent exchange between Saari and Van Newenhizen, 1988, and Brams, Fishburn, and Merrill, 1988); we shall draw comparisons between it and CV later. We simply note here that AV and CV resemble each other only in the physical act of voting: the voter indicates on a ballot all the alternatives (candidates under AV, parties under CV) that are considered acceptable.

It is the manner in which votes are allocated and then aggregated, and how alternatives are selected, that makes CV radically different from AV as a voting system. Although a voter does not vote twice under CV, two different kinds of votes are distinguished. Specifically, a voter possesses

1. One party vote, which is divided equally among all parties that the voter votes for or approves of (we shall use the language of "approval" because of this shared feature with AV);
2. One coalition vote, which counts for the set of all parties--and every nonempty proper subset of this set--of which the voter approves.

For example, assume that there is a set of five parties, {1,2,3,4,5}, and a voter approves of all members of the subset {1,2,3}. Then parties 1, 2, and 3 each receives 1/3 of a (party) vote from this voter; in addition,

{1,2,3}, {1,2}, {1,3}, {2,3}, {1}, {2}, and {3}

each receives one (coalition) vote because, except for the empty set ∅, these are all the subsets of {1,2,3}.

We assume that any subset of parties, or coalition, can form; the complement of this coalition defines its countercoalition. (The latter may or may not represent a united opposition--we use the term here only to indicate the subset of parties not in the coalition.) Thus, {1,2,3} in the previous example is a coalition, and {4,5} is its countercoalition.

In section 2, we shall define what is meant by a "majority" and a "minimal majority" coalition; the latter is assumed to be the only kind that can form a government. Next, "governing" coalitions, which are a subset of minimal majority coalitions, will be defined and illustrated. In section 3 we shall derive some properties of CV, focusing particularly on its "monotonicity." How seats are apportioned to parties, and how a governing coalition is selected, are discussed in section 4.

In section 5 we analyze spatial competition in a simple three-party system with voters who have single-peaked preferences. Conditions under which the left, right, and center parties--and coalitions of these--can become governing are derived. This example serves as a springboard for a more general consideration of optimal strategies of voters and parties under CV, especially as they might be influenced by polls, in section 6.

In section 7 we briefly consider possible uses of CV in party-list systems like those of Israel and Italy, both of which currently have more than a dozen parties represented in their parliaments and in which consensus on major policy issues has been difficult to achieve. In the case of Israel, we suggest that the seven weeks of divisive maneuvering to form a coalition government after the 1988 parliamentary election might have been obviated by CV, even if the resulting coalition would not have been different (which is by no means clear). In section 8 we conclude with an assessment of CV as a practicable reform, comparing it with AV.

CV is obviously more complex than AV, representing a delicate balance between the desire of voters

- on the one hand, to cast all their party vote for their favorite party to maximize its proportion of seats in a parliament;

- on the other hand, to vote for enough parties to form a majority coalition, ensuring that their coalition vote counts (votes for some but not all members of a majority coalition receive no "partial credit") by contributing to its becoming governing.

Plausibly, this mix of incentives will push voters toward constructing winning coalitions that best mirror their preferences.

Parties, in turn, will be motivated to mirror these preferences, indicating to voters before the election agreeable coalition partners and joint policy positions that will mitigate the voters' choice problem. Indeed, precisely because voters can indicate acceptable (and hence nonacceptable) coalitions directly on their ballots--and these help determine the election outcome--CV forces the parties to pay greater heed to their preferences. In particular, parties will have a strong incentive to iron out differences with potential coalition partners in order to pose reasonable coalition voting strategies to the voters during the campaign.

2. Majority, Minimal Majority, and Governing Coalitions

Denote the m individual parties by the numbers 1, 2, 3, . . ., m, and coalitions (subsets) of parties by $C_1, C_2, \ldots, C_n$. Call the coalition of all parties, or grand coalition, $C = \{1,2,\ldots,m\}$. For every coalition C_j, define the countercoalition $\overline{C}_j$ to be $\overline{C}_j = C \backslash C_j$, or the difference of C and C_j--that is, the set of all parties that are in C and not in C_j. The partition of C into a coalition C_j and a countercoalition $\overline{C}_j$, where $C_j \cup \overline{C}_j = C$, will be referred to as partition j.

The number of possible coalitions and countercoalitions in a system is equal to the number of possible two-way divisions of the parties. Since there are m parties, and each may be a member of a coalition or its countercoalition, there are 2^m ways of dividing the parties into coalitions and countercoalitions, which includes the empty set Ø and the grand coalition C. However, half of these divisions will be the same, except for labeling one subset the coalition and its complement the countercoalition. Because these divisions are indistinguishable except for labeling, the number of <u>distinct</u> coalitions and partitions is $n = 2^{m-1}$.

Other notation that will be helpful is the following:

V = set of voters;

v = |V| = number of voters;

$v(C_j)$ = number of voters who vote for precisely the members of coalition C_j (j = 1, n).

Call voters who do not abstain <u>concerned</u> and those who do <u>unconcerned</u>. Because unconcerned voters have no effect on the vote totals of either parties or coalitions, we exclude them from the subsequent analysis, making $v(\emptyset) = 0$.

To define different kinds of coalitions, it is necessary to take account of both the party and the coalition votes of each coalition C_j. Recall that voters who vote for party i give varying numbers of (fractional) votes to that party, depending on how many <u>other</u> parties they approve of on their ballots. Specifically, the number of votes of party i is

$$p(i) = \sum_{k \in V} f_{ki},$$

where

$$f_{ki} = \begin{cases} 0 \text{ if voter k does not vote for party i} \\ 1/(1+j) \text{ if voter k votes for party i and j other parties} \end{cases}$$

Thus, only if a voter votes exclusively for party i does i receive one full vote. Otherwise, this voter's vote is divided evenly among the (1+j) parties approved of, so each receives 1/(1+j) votes.

The rationale for evenly dividing one party vote is two-fold:

1. Equality of voters. Each voter counts equally in the apportionment of parliamentary seats (more details on seat apportionment will be given in section 4). If there are 100,000 voters and 100 seats to be filled, for example, each voter accounts for 1/1000 seats, whether or not voters choose to concentrate their representation on one party or spread it across several.

2. No-breakup incentive for parties. Assume that the division of votes is even but that a voter who votes for more parties casts more votes *in toto*. For example, if a voter votes for two parties, assume that each party receives 2/3 of a vote instead of 1/2. Then it might be profitable for a party to split, assuming its supporters continue to vote for both parts, to maximize its vote total and therefore seats in parliament.

Allowing voters to distribute one vote unevenly across parties--or several votes, as under cumulative voting (presently used in a few local jurisdictions in the United States) and *panachage* (used in Luxembourg, Norway, Sweden, and Switzerland)--would obviously give voters more freedom to express themselves. But it would introduce complexities that would not only make voting impracticable in many systems but also alter the strategic incentives that we shall analyze in detail later.

To return to the definition of terms we will need, the number of *party votes* of C_j is

$$p(C_j) = \sum_{i \in C_j} p(i),$$

or the sum of the party votes of all parties in C_j. [We write p(i) for p({i}) when C_j = {i}.] By contrast, the number of "coalition votes" of C_j (to be defined below) does not reflect the contributions of each voter who votes for one or more parties in C_j. Rather, for a voter's coalition vote (one for each voter) to count for a coalition C_j, a voter must vote for <u>all</u> parties in C_j.

The number who do so will include at least the $v(C_j)$ voters who vote for precisely the parties in C_j. But this number also includes the voters who vote for <u>supersets</u> of C_j (which include C_j as a subset). Define the number of <u>coalition votes</u> of C_j to be

$c(C_j)$ = number of voters who vote for all members of C_j.

This is the number of voters who approve of all parties in C_j--and perhaps others as well.

So far we have distinguished the party votes of C_j, which total $p(C_j)$, from the coalition votes of C_j, which total $c(C_j)$. To be a <u>majority coalition,</u> C_j must have at least as many party votes as $\bar{C}_j$, or $p(C_j) \geq p(\bar{C}_j)$. If this inequality is reversed, then $\bar{C}_j$ is the majority coalition for partition j of the parties. (If the inequality is an equality, then C_j and $\bar{C}_j$ are both majority coalitions.) Thus, there is at least one majority coalition for each partition j of the parties.

Let $M = \{M_1, M_2, \ldots\}$ be the set of majority coalitions, which will be a subset of the set of coalitions $\{C_j\}$ defined by all partitions j. By eliminating from M all elements that are proper supersets of other elements--that is, majority coalitions that have superfluous members--one obtains M*, the subset of M that includes only its "minimal" majority elements, or the set of <u>minimal majority coalitions.</u>

Presumably, if an $M_i \in M$ is a majority coalition that does not need the votes of one or more of its parties to maintain its winning edge against a countercoalition, the superfluous members will not be adequately compensated and consequently defect. (They may not be superfluous if there is uncertainty about who exactly is a loyal coalition member; we shall consider the possible effects of incomplete information on voting and coalition strategies in section 6.) The defection of some superfluous members will not necessarily render a coalition minimal winning according to the "size principle" (Riker, 1962), because one coalition's minimal majority may be larger than another coalition's, as will be illustrated in section 4.

Finally, define the set of <u>governing coalitions</u> G to be the subset of M^* whose elements have the greatest coalition vote. Except in the event of ties, there will be one minimal majority coalition $M_i \in M^*$ that has more coalition votes than any other--that is, has more voters approving of all its parties--and this coalition will be designated as governing.

This means, operationally, that this coalition will be assigned a majority of seats in parliament. How the size of this majority is determined, and how seats are allocated to the individual parties that compose it, will be discussed further in section 4. In the remainder of this section, we illustrate the forgoing concepts with a simple example.

<u>Example 1</u>. $C = \{1,2,3\}$ and $v = 3$, so there are exactly three parties and three voters. Assume that the first voter votes for $\{1,2,3\}$, the second voter for $\{1,2\}$, and the third voter for $\{3\}$, so the party votes for each party are:

$p(1) = p(2) = 1/3 + 1/2 = 5/6;$

$p(3) = 1/3 + 1 = 4/3.$

Similarly, the party votes for all two-party coalitions are:

$p(\{1,2\}) = 5/6 + 5/6 = 5/3;$

$p(\{1,3\}) = p(\{2,3\}) = 5/6 + 4/3 = 13/6.$

[Henceforth we shall simplify $p(\{1,2\})$ to $p(12)$, and likewise for other subsets of C.] Because the three two-party coalitions all have more party votes than their one-party countercoalitions, $M = \{12,13,23,123\}$, where the elements of M are the coalitions of parties indicated. Since C is a superset of the other coalitions in M, $M^* = \{12,13,23\}$. Finally, $G = \{12\}$ because $c(12) = 2$ (from voters 1 and 2), whereas $c(13) = c(23) = 1$ (from voter 3). Observe that the two-party coalition with the fewest party votes, 12, is the governing coalition because it is supported by more voters as a coalition than any other coalition in M^*.

3. Properties of Coalition Voting

In this section we shall illustrate CV with more examples and prove several theorems that highlight certain properties of the system, especially its monotonicity. To begin with, it is easy to show that if more than half the voters vote for a single party, it is the governing coalition. More precisely,

Theorem 1. If $v(i) > v/2$, then party i is the sole member of G.

Proof. $v(i) > v/2 \Rightarrow \{i\} \in M \Rightarrow \{i\} \in M^*$. Moreover, $c(i) > v/2$, $\emptyset \notin M$, and $c(S) < v/2$ for every other nonempty coalition S. Hence, $G = \{i\}$. □

Theorem 1 does not extend to coalitions of two or more parties, where $\{i\}$ is replaced by C_j and $|C_j| \geq 2$. The reason is that a nonempty proper subset of C_j may also be in M; if so, it would eliminate C_j from M^*, so C_j cannot be in G.

Whereas Theorem 1 gives a sufficient condition for a single party to be in G, it is certainly not a necessary condition, as illustrated by

Example 2. C = {1,2,3}, v = 100, and v(12) = v(13) = 50. Thus, no voter votes for just one party, but 50 voters vote for the coalition 12 and 50 voters vote for the coalition 13.

The number of party votes, $p(C_j)$, of all singletons versus complementary pairs are as follows (where distinguishable, the fractional party votes derived from 12 are shown first, those derived from 13 second, in the sums below):

$p(1) = (1/2)(50) + (1/2)(50) = 50$ vs. $p(23) = (1/2)(50) + (1/2)(50) = 50$;

$p(2) = (1/2)(50) = 25$ vs. $p(13) = (1/2)(50) + (1)(50) = 75$;

$p(3) = (1/2)(50) = 25$ vs. $p(12) = (1)(50) + (1/2)(50) = 75$.

Thus, the majority coalitions, which receive at least as many party votes as their countercoalitions, are M = {1,12,13,23,123}.

Because all coalitions in M are supersets of {1} or {23}, M* = {1,23}. G = {1} because c(1) = 100 and c(23) = 0, which seems sensible since party 1 is the only coalition approved of by all voters.

In Example 2, note that {1} maximizes the coalition vote among all members of M as well as being a member of M*. This illustrates

Theorem 2. Some coalition that maximizes the coalition vote among all members of M is a member of M*.

Proof. Suppose $M_j \in M$ has at least as many coalition votes as all other members of M. If $M_j \notin M^*$, then it must include a smallest proper subset, say M_i, that is in M and has the same coalition vote as M_j. It follows that $M_i \in M^*$. □

Theorem 2 assures us that G (which maximizes the coalition vote among all members of M*) maximizes the coalition vote among all members of M as well. True, there may be another coalition in M (M_j in the proof of Theorem 2) with the same maximum coalition vote that is not a member of M*; but because it is not minimal, it cannot be in G.

Party 1's unique presence in G in Example 2 is undermined in

Example 2'. Same as Example 2, except one new voter, v(23) = 1, is added, making v = 101. Then M = {12,13,23,123}, M* = {12,13,23}, and G = {12,13(Mj i the proof of Theorem 2) with the same maximum coalition vote that is not a member of M*; but because it is not minimal, it cannot be in G.}.

Party 1's importance, nevertheless, is still felt as the one common member of both coalitions in G, which would presumably give it great bargaining power over parties 2 and 3 if the latter both desire to be in the governing coalition that actually forms. We shall return to such issues in section 4.

It may seem paradoxical, nonetheless, that by adding one voter, party 1 (with 100 coalition votes) is forced to share power with party 2 or party 3. True, coalitions 12 and 13 each have a majority of party votes (75.5 out of 101)--unlike party 1 with 50 party votes--but each garners only 50 coalition votes, making each half as approved of, as a unit, as party 1 by itself. However, admitting a coalition to membership in M that has fewer party votes than its complement can lead to an anomalous situation-- wherein members of M are approved of by only a minority of voters--as illustrated by

<u>Example 3</u>. $C = \{1,2,3,4\}$, $v = 81$, and $v(1) = v(2) = 30$; $v(4) = 11$; $v(123) = 10$. To begin with, consider the party votes received by one of the two big parties (e.g., 1) versus its complement:

$$p(1) = 30 + (1/3)(10) = 100/3 \text{ \underline{vs}. } p(234) = (1)(30) + (1/3)(10 + 10) + (1)(11) = 143/3.$$

Not only does party 1 lose to coalition 234 (and party 2 to coalition 134), but party 1 is approved of by only 40 of the 81 voters, less than a majority. As will be proved shortly, requiring that a coalition, in order to be a member of M, receive more party votes than its complement precludes members of M from being approved of by only a minority of voters.

Next, consider the party votes of the two-member and three-member coalitions that include parties 1 and 2:

$$p(12) = (1)(30 + 30) + (1/3)(10 + 10) = 200/3 \text{ \underline{vs}. } p(34) = (1/3)(10) + (1)(11) = 43/3;$$

$$p(123) = (1)(30 + 30) + (1)(10) = 70 \text{ \underline{vs}. } p(4) = 11;$$

$$p(124) = (1)(30 + 30) + (1/3)(10 + 10) + (1)(11) = 243/3 \text{ \underline{vs}. } p(3) = (1/3)(10) = 10/3.$$

Because all the coalitions that include parties 1 and 2 surpass their countercoalitions in party votes, M includes them, the other 3-party coalitions, and C: $M = \{12,123,124,134,234,1234\}$; eliminating supersets, $M^* = \{12,134,234\}$, and $G = \{12\}$.

Observe that 70 of the 81 voters approve of one or more members of coalition 12; other majorities approve of one or more members of other coalitions in M. That members of M receive approval from at least half of all voters is no accident, as shown by

<u>Theorem 3</u>. For every majority coalition in M, at least half the voters vote for one or more parties in the coalition.

Proof. If less than half of the voters vote for one or more parties in C_j, then $p(C_j) < v/2$ and, therefore, $C_j \notin M$. Hence, if $C_j \in M$, then at least half of the voters must have voted for parties in C_j. □

Even if members of a coalition are approved of by at least half the voters, this coalition will not be a member of M unless it receives more party votes than its complement. As a case in point, party 1 in Example 2' is approved of by 100 of the 101 voters, but its complement, coalition 23, receives more party votes, though only 51 voters approve of at least one of its members. On the other hand, because coalition 23 receives fewer coalition votes than 12 and 13 (in fact, only one), G = {12,13}.

The monotonicity of voting systems has been extensively studied (Fishburn, 1982; Bolger, 1985), but the meaning of monotonicity in the case of CV is not apparent. The interpretation we shall use is the following (with refinements to be illustrated later):

> Assume party i is in coalition S, $|S| \geq 2$, and $\{i\} \in G$. Suppose $n > 0$ voters vote for S. If the n voters who vote for S switch their support to a proper subset, S', that contains i, all else unchanged, then {i} is the sole member of G for the changed case.

Before proving the monotonicity of CV, we prove two helpful lemmas.

Lemma 1. Assume $|C\backslash\{i\}| \geq 1$. If $p(i) \geq p(C\backslash\{i\})$, then $c(i) \geq c(C\backslash\{i\})$.

Proof. Assume $p(i) \geq p(C\backslash\{i\})$. Contrary to the conclusion of the lemma, suppose that $c(i) < c(C\backslash\{i\})$. Then more voters vote for precisely $C\backslash\{i\}$ or a proper superset of $C\backslash\{i\}$ than {i}. But the only proper superset of $C\backslash\{i\}$ is C; because these voters contribute the same number of coalition votes to both {i} and $C\backslash\{i\}$, they can be ignored. For the remaining voters, $c(C\backslash\{i\}) = v(C\backslash\{i\})$, because voters voting for a proper subset of $C\backslash\{i\}$ would not contribute coalition votes to $C\backslash\{i\}$. Thus, $c(C\backslash\{i\}) > c(i)$ implies more voters vote for precisely $C\backslash\{i\}$ than for precisely $\{i\}$ or a proper superset of $\{i\}$ (excluding C). Now the $v(i)$ voters who vote for precisely $\{i\}$ contribute one party vote to $\{i\}$, and the voters who vote for a proper superset of $\{i\}$ contribute at least as many party votes to $C\backslash\{i\}$. Hence, if $c(C\backslash\{i\}) > c(i)$, $v(C\backslash\{i\}) > v(i)$, which implies $p(C\backslash\{i\}) > p(i)$, a contradiction. □

Lemma 2. If $\{i\} \in G$, then

(1) if $p(i) > p(C\backslash\{i\})$, $G = \{i\}$;

(2) if $p(i) = p(C\backslash\{i\})$, either $G = \{i\}$ or $G = \{i, C'\}$, where C' consists of the members of $C\backslash\{i\}$ that receive votes.

Proof. Assume $\{i\} \in G$. Then $p(i) \geq p(C\backslash\{i\})$. If this inequality is strict, then $\{C\backslash\{i\}\} \notin M$ and so cannot be in M* or G. Consequently, $C_j \in M$ only if $\{i\} \in C_j$. But these C_j's $\in M$ get deleted in forming M* since they are supersets of $\{i\}$. Thus, if the inequality is strict, $G = \{i\}$. Next, assume $p(i) = p(C\backslash\{i\})$ so that $p(i) = p(C\backslash\{i\}) = v/2$. Then both $\{i\}$ and $C\backslash\{i\}$ are in M. Clearly, $\{i\} \in M^*$, so no superset of $\{i\}$ is in M*. Let D (possibly empty) be the set of parties with no party votes, and let $C' = (C\backslash\{i\})\backslash D$. Then $p(C') = v/2$, so $C' \in M^*$. Moreover, if $C'' \neq C'$ has $v/2$ party votes and does not contain i, then C'' is a proper superset of C' and so is not in M*. □

What Lemma 2 establishes is that in order for both $\{i\}$ and one $C' \subseteq C\backslash\{i\}$ to be in G, they must tie not only in party votes but also in coalition votes. This can occur only if $v(i) = v(C')$ and, in addition, no voters vote for any C_j other than $\{i\}$ and C'. In other words, the voters are polarized--half vote for precisely $\{i\}$ and the other half for precisely C'.

Theorem 4. CV is monotonic.

Proof. Initially, $p(i) \geq p(C\backslash\{i\}) = p(C')$, where C' is as defined in Lemma 2. After the voters who vote for S switch their support to a proper subset, S', that contains i, $p(i) > p(C')$. By Lemma 2, G = {i}. □

Also by Lemma 2, it is possible that G = {i,C'} before the switch, but because {i} is assured of a strict majority of party votes after the switch, {i} will be the only member of G, thereby precluding the polarization of voters that might have existed before the switch.

To illustrate Theorem 4, consider

Example 4 (Monotonicity). C = {1,2,3,4}, v = 14, and v(1) = 6, v(23) = 4, and v(1234) = 4. It is easily seen that M = {1,12,13,14,123,124,134,234, 1234}, M* = {1,234}, and G = {1}. Now suppose that the four voters who vote for S = {1,2,3,4} switch to S' = {1,2,3}. This increases p(1) from 7 to 7 1/3, which eliminates {2,3,4} from M and therefore from M* and G. With this exclusion, M* = G = {1} after the switch.

In general, a switch increases {i}'s party vote, which either breaks a tie with C\{i} (as in Example 4) or increases {i}'s victory margin. In either case, {i} is ensured of sole membership in G by Lemma 2.

If we replace {i} in Theorem 4 by a larger set T, $|T| \geq 2$, Theorem 4 no longer holds. Instead, "subset monotonicity" may obtain, as illustrated by

Example 5 (Subset Monotonicity). C = {1,2,3,4}, v = 8, and v(1) = 3, v(2) = 2; v(3) = 1; v(134) = 2. Then M = {12,13,14,123,124,134,1234}, M* = {12,13,14}, and G = {13,14}. Let T = {13}, and note that $T \in G$. Now suppose that the two voters who vote for S = {1,3,4} switch to S' = {1,3}. Then M = {1,12,13,23,123,124,134,234,1234}, M* = {1,23}, and G = {1}. In this case, T is no longer in G after the change, but {1}, a subset of T, is in G.

The reason for party 1's elevation to unique governing status after the switch is that the extra votes it receives from the switchers make it a member of M, which displaces {1,3} from M* and therefore from G.

It is worth noting that if the two voters who shrink their approval set from {1,3,4} to {1,3} prefer party 3 to party 1, it is reasonable to suppose that they would prefer coalition 13 to party 1. Even if they are indifferent between or prefer party 1 to party 3, they still might prefer 13 to either party by itself because of the "balance" that the two parties in G provide.

In analyzing the monotonicity of CV, we shall not consider whether a proper subset of an approved set is better or worse than the set. Rather, we shall focus on the issue of whether voters, when they reduce the number of parties they approve of, hurt those they eliminate and help those that remain. In the case of Example 5, the two voters who eliminate party 4 from their approved set preclude it from being in G--either by itself or in a coalition with other parties--which is presumably responsive to their changed interests.

CV may also be said to be responsive to voters' changed interests in the following unusual situation:

- G = A before the switch, and G = B after the switch;
- $A \cap B = \emptyset$;
- All the parties in both A and B are members of S;
- All the parties in B, but not all the parties in A, are members of S'.

We shall refer to this situation as "null-set monotonicity" because the intersection of G before and after the switch is empty; it is illustrated by

<u>Example 6 (Null-Set Monotonicity)</u>. C = {1,2,3,4}, v = 9, and v(12) = 3; v(34) = 2; v(1234) = 4. Then M = {12,123,134,234,1234}, M* = {12,134,234},

and G = A = {12}. Now suppose that the four voters who vote for S = {1,2,3,4} switch to S' = {1,3,4}. Then M = {34,123, 134, 234,1234}, M* = {34,123}, and G = B = {34}.

Thus, G = B after the change does not overlap with G = A before the change. Moreover, party 1, a member of coalition G = A before the change, is excluded from membership in G = B after the change even though the change does not deprive it of approval. In fact, {1} receives more party votes after the change, but B = {34}, with two members, receives still more as a coalition.

CV, nevertheless, is responsive to the change in Example 6: G = B = {34} after the change precisely because all parties in B, but not all parties in A, are in S'. The reason A = {12} loses its governing status is that this coalition is not in M after the change and so cannot be in M* and G. On the other hand, B = {34} is not only a member of M and M* after the change but, with {12} eliminated from membership in M*, receives a larger coalition vote than {123}, the only other coalition in M* after the change.

We conclude that CV possesses the desirable property of monotonicity if one of the governing coalitions before the switch is a singleton (Theorem 4). Indeed, this coalition always becomes the unique governing coalition if it was not before. But if a former governing coalition is not a singleton but a larger coalition, then there is no guarantee that a switch that favors all its members will be sufficient to ensure that that coalition stays governing. As Example 5 demonstrated, one of its members may jump to governing status by itself.

Finally, a switch that favors only some but not all parties in a former governing coalition may lead to an entirely new governing coalition--all of whose parties are favored by the switch--as Example 6 demonstrated. Yet even in this case, it seems fair to say that the system is responsive to changes in approval patterns and so is basically monotonic. Furthermore, CV is rooted in majority rule: a majority of voters, by voting for a single party, can ensure that it is governing (Theorem 1); and every majority coalition, which potentially can become governing, has at least some of its parties supported by a majority of voters (Theorem 3). Thus, CV would appear to be fundamentally democratic in character.

4. Apportioning Seats to Parties and Choosing a Governing Coalition

If G has only one member, seats in parliament are apportioned to G and its complement according to the number of party votes that they receive. The apportionment of seats to the governing coalitions and their complements in our first four examples are shown below:

Example 1: p(12) = 5/3 (55.6%) vs. p(3) = 4/3 (44.4%)

Example 2: p(1) = 50 (50.0%) vs. p(23) = 50 (50.0%)

Example 2': p(12) = 75.5 (74.8%) vs. p(3) = 25.5 (26.2%)

p(13) = 75.5 (74.8%) vs. p(2) = 25.5 (26.2%)

Example 3: p(12) = 230/3 (82.3%) vs. p(34) = 43/3 (17.7%)

Since there is more than one governing coalition in Example 2', the choice between coalitions 12 and 13 might be made randomly.

Once seat assignments are made to the (governing) coalition and the (nongoverning) countercoalition, seats are assigned to parties within the governing/nongoverning coalitions, again according to the numbers of party votes. Thus, all parties, and the coalitions that subsume them, receive seats in accordance with the numbers of their party votes, which for the individual parties are as follows:

Example 1: Parties 1 (27.8%), 2 (27.8%), 3 (44.4%)

Example 2: Parties 1 (50.0%), 2 (25.0%), 3 (25.0%)

Example 2': Parties 1 (49.5%), 2 (25.2%), 3 (25.2%)

Example 3: Parties 1 (41.2%), 2 (41.2%), 3 (4.1%), 4 (13.6%).

The fact that governing coalitions will have at least as many party votes as their countercoalitions ensures that they will have at least as many seats in parliament. Thereby they can never be minorities, and, as the examples illustrate, will generally be strict majorities. In the case of a tie in party votes, as in Example 2, the governing coalition (party 1) might be given a tie-breaking vote to ensure that it can defeat coalition 23, which otherwise would be blocking.

Methods for apportioning seats in parliament, based on party vote totals, have been analyzed by Balinski and Young (1978, 1982), who recommend the Jefferson method. Their arguments for this method over others are persuasive, but this is a separate issue from that discussed here, which is to change the basis on which integer assignments of parliamentary seats are made from plurality voting (whereby voters are restricted to voting for one party) to CV.

Although CV is meant to give a boost to the most approved of coalitions, it is the parties themselves--or, more properly, their leaders--that must decide whether a governing coalition will actually form. By definition, a governing coalition will have a parliamentary majority, but so may other coalitions of parties. In Example 3, G = {12} with 82.3% of the vote, but a coalition of either party 1 or party 2 with party 4 also provides a parliamentary majority, with a much trimmer 54.8% of the seats.

In fact, coalition 12's overwhelming majority might be its Achilles' heel, at least insofar as the size principle is applicable (Riker, 1962). A coalition of either 14 or 24 is much closer to the minimum winning coalition than the size principle prescribes as optimal (under certain conditions).

Although coalition 12 is approved of by more voters than coalitions 14 or 24--whose members are jointly approved of by no voters--it is certainly possible that coalitions other than those that are governing may, in the end, strike a deal, despite the wishes of the voters. With a majority of seats, these coalitions could presumably assume the reins of power, even if the vast majority of voters did not approve of them.

There are at least two ways of countering deal-making that blatantly defies voter approval. One is to permit only governing coalitions to try to form a government; if they do not succeed, new elections would be held. A less drastic procedure would be to give governing coalitions priority in trying to form a government. Only if they fail would other coalition possibilities be entertained, perhaps in descending order of the size of the coalition vote of the different minimal majorities.

In the latter case, the parties to the negotiations would know what alternatives, if any, there were to fall back on if a deal fell through. Because the numbers approving of the various minimal majority coalitions would be known, the repercussions from flouting voter desires would be starkly evident.

In Example 2', there are two governing coalitions, 12 and 13. Both include party 1, so presumably it would be 1's choice as to which partner to accept. Although coalition 23 also controls a majority of seats, collectively its members are supported by only 1 (out of 101) voters.

This severe lack of approval of coalition 23 would seem to render it a dubious choice. But perhaps the fact that it holds a (bare) majority of seats and theoretically could form a government if party 1 fails to reach an agreement either with party 2 or party 3 would prevent party 1 from exploiting its presence in both governing coalitions and forcing party 2 or party 3 to "give away the store" to be selected as a coalition partner.

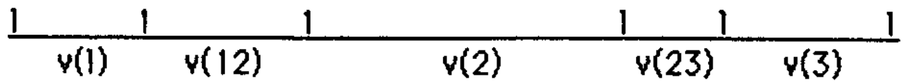
5. Spatial Competition: A Simple Example

Under CV, a voter faces the choice of voting for a favorite party, in which case it receives one full party vote, or dividing this vote among two or more parties. If one's favorite party by itself is unlikely to receive more party votes than a countercoalition of all the other parties, then it will not be a member of M (or M*) and therefore cannot be in G, except in concert with other parties.

Even in the latter case, one's vote for a favorite party would contribute nothing to the coalition vote of the (multimember) coalition, which is what determines whether it becomes governing. To be sure, one's vote for a single favorite party will give it maximum help in the apportionment of seats--whether it is in G or its countercoalition--but it will not help in its selection over other members of M*.

This choice can be abetted only if one votes for all parties of a coalition in M*. Thereby one contributes not only party votes to the members of this coalition but also one coalition vote, which will enhance its chances of selection over other members of M*.

To illustrate the calculation of optimal strategies of a voter, we first consider a situation in which C = {1,2,3}. Assume that the three parties can be arrayed on a left-right continuum such that party 1 is on the left, party 2 is in the center, and party 3 is on the right. Furthermore, assume that voters have single-peaked preferences and vote either for the single party closest to them or for the two parties closest to them, with the numbers of voters choosing single parties and pairs of parties as follows:



The lengths of the segments indicate the numbers of voters that choose the different voting strategies and are meant only to be illustrative. We assume that each party receives a vote from some voter and, in addition, that at least one voter votes for only one party [so $v(1) + v(2) + v(3) > 0$].

Observe that no voters choose parties 1 and 3 [i.e., $v(13) = 0$] because these parties--near different ends of the continuum--can never be the two parties closest to any voter. Wherever the positions of voters on the continuum are, they will generally be closer to one party than either of the others. Yet voters might approve of a second choice as well as a first because they think their favorite party cannot be a member of M by itself.

To determine M, consider the party vote totals of every singleton versus every pair:

$p(1) = v(1) + v(12)/2$ vs. $p(23) = v(2) + v(3) + v(23) + v(12)/2$;

$p(2) = v(2) + v(12)/2 + v(23)/2$ vs. $p(13) = v(1) + v(3) + v(12)/2 + v(23)/2$;

$p(3) = v(3) + v(23)/2$ vs. $p(12) = v(1) + v(2) + v(12) + v(23)/2$.

Party 2, in the center, will have at least as many votes as a counter-coalition of 13 if $p(2) \geq p(13)$, or

$$v(2) \geq v(1) + v(3). \qquad (1)$$

Thus, party 2 must have at least as many stalwarts (i.e., voters who vote only for it) as parties 1 and 3 combined. The nonstalwarts have no effect on party 2's victory because they cancel each other out--party 2 and countercoalition 13 garner exactly the same support [$v(12)/2 + v(23)/2$] from these voters.

If inequality (1) holds, then not only is party 2 in M but so also are coalitions 12 and 23. These coalitions are in M if $p(12) \geq p(3)$ and $p(23) \geq p(1)$. Given these nonstrict inequalities, (1) and the assumption that someone votes for each party imply the even stronger strict inequalities:

$$v(12) + v(1) + v(2) > v(3);$$

$$v(23) + v(2) + v(3) > v(1).$$

These eliminate parties 1 and 3 from M. Moreover, because the coalitions 12 and 23 are supersets of party 2, they will have fewer coalition votes than party 2 unless $v(2) = 0$, which by (1) implies $v(1) + v(2) + v(3) = 0$, contrary to an earlier assumption. Thus, if inequality (1) is satisfied, $G = M^* = \{2\}$. Furthermore, since party 2 cannot be in G if (1) fails, we have

<u>Proposition 1</u>. The center party is in G, and in fact is the sole member of G, if and only if it has at least as many stalwarts as the left and right parties combined.

When will a party on the left or right, perhaps combined with the center party, be the governing coalition? We will use the party-vote comparisons given earlier and one coalition-vote comparison to find conditions for party 1 on the left, and then coalition 12, to be in G. Comparable conditions can be given for party 3 on the right and coalition 23. Note that coalition 13, because it receives no coalition votes, can never be a governing coalition.

Party 1 will be in M (and necessarily M*, because it has no nonempty proper subsets) if $p(1) \geq p(23)$, or

$$v(1) \geq v(2) + v(3) + v(23). \qquad (2)$$

Similarly, party 3 will be in M* if

$$v(3) \geq v(2) + v(1) + v(12). \quad (3)$$

Adding inequalities (2) and (3) yields

$$2v(2) + v(12) + v(23) \leq 0,$$

which is possible only if

$$v(2) = v(12) = v(23) = 0,$$

which contradicts our assumption that party 2 receives a vote from some voter. Thus, parties 1 and 3 cannot both be in M* and therefore in G, so we have

<u>Proposition 2</u>. If one noncenter party is in G, then the other noncenter party is not in G.

Assume party 1 is in M* so that inequality (2) obtains. Party 2 will be in M*, as shown earlier, if inequality (1) obtains. Adding these two inequalities yields

$$2v(3) + v(23) \leq 0,$$

which is possible only if

$$2v(3) + v(23) = 0,$$

which contradicts our assumption that party 3 receives a vote from some voter. Thus, party 1 (or party 3) and party 2 cannot both be in M* and therefore in G, so we have

<u>Proposition 3</u>. If one noncenter party is in G, then the center party is not in G.

It is possible, however, for a noncenter party and its complement both to be in G, as when $v(1) = v(23) = v/2$.

Whenever none of the individual parties is in G, coalition 12, coalition 23, or both are in M^*. To determine which of these coalitions is governing, we check which has more coalition votes than the other. If $c(12) > c(23)$, then 12 is in G; if $c(23) < c(12)$, then 23 is in G; and if $c(12) = c(23)$, then both 12 and 23 are in G. Since we assume that no voters vote for the grand coalition C, $c(12) = v(12)$ and $c(23) = v(23)$. When the v expressions are substituted for the c expressions, we have

<u>Proposition 4</u>. If none of the individual parties is in G, then G contains either the left-center or the right-center coalition of two parties, depending on which coalition has more nonstalwart supporters. [It contains both if $v(12) = v(23)$.]

To summarize, Proposition 1 establishes how much (stalwart) support the center party must have in order to be the governing coalition. If this condition is not met and one noncenter party is governing, Propositions 2 and 3 establish that neither of the other parties can be governing by itself. Generally speaking, then, only one individual party can be governing at a time.

Inequalities (2) and (3) say how much (stalwart and nonstalwart) support each noncenter party must receive to be governing. Given that no individual party is governing, Proposition 4 demonstrates that the left-center or right-center coalition will be governing if it has more (nonstalwart) support than its counterpart on the other side of the continuum. Comparing different possibilities,

- the choice of the center party depends only on how much stalwart support it has versus the other two;

- the choice of a noncenter party depends on how much of both stalwart and nonstalwart support it has;
- the choice of a coalition of two parties--assuming that no individual party has sufficient support to be governing--depends only on how much nonstalwart support it has.

6. Optimal Strategies of Voters and Parties

If polls can help voters determine which party, or coalition of parties, is likely to become governing, then the voters can better choose strategies that (1) enhance the chances that their favorite parties will be governing, either by themselves or in a coalition, and (2) increase the representation of their parties in the parliament, whether they are in the governing coalition or its countercoalition. For example, if a voter's preference for parties is 123 (in that order), and the poll indicates the possibility that either coalition 12 or coalition 23 will becoming governing, then the voter will be well advised to vote for the coalition 12 to boost its chances.

On the other hand, if coalition 12 is definitely favored, the voter probably should vote for party 1 to increase its representation in the likely governing coalition. However, if coalition 23 is definitely favored, the voter's best strategy would seem to be to vote (insincerely) for party 2 to diminish party 3's representation in the governing coalition. But in the latter situation, the voter might arguably vote (sincerely) for party 1 in order to magnify its size as the countercoalition.

Now party leaders, with access to the same information--and perhaps other information from private polls--can draw similar inferences. Their focus, however, will be less on voting strategies and more on coalition strategies to help guide the voter to what they consider their optimal choices. Thus, if the polls indicate a toss-up between coalitions 12 and 23, it would behoove parties 1 and 3 to urge their supporters also to vote for party 2 to help ensure that the coalition of which they are members becomes governing.

Acceptance of this advice would tend to equalize the sizes of the parties in each coalition and therefore in the parliament. This could mean a diminution in the strength of the leading parties if they are not dominant enough to win on their own but must, in addition, seek (shared) votes from supporters of other parties. By suggesting coalition voting strategies to such voters, it would be hard for the leading parties to refrain from urging their own supporters to cast their nets wider by voting for the coalition.

Yet if either coalition 12 or 23 is favored in our earlier example (say, 12), party 1 leaders would presumaby urge some of their supporters <u>not</u> to choose a coalition strategy but instead to remain a stalwart for the party. This would increase 1's seat total and thereby strengthen its case for being awarded more ministerial posts in the government. Party 2, because it is in both potential governing coalitions, can afford to ignore the polls and simply exhort its followers to vote only for it so as to enlarge its role in whichever governing coalition forms. On the other hand, if coalition 13 were a possibility, party 2 could not afford to be so cavalier and might suggest a coalition voting strategy to at least some of its supporters.

Through such tactics, the parties can aid the voters, before the election, in choosing voting strategies that advance the voters' interests. Even if a party's participation in a governing coalition seems not in the cards, it still can exploit poll information to try to enhance either the opposition as a whole by encouraging support of a countercoalition of which it is a member, or its position in a countercoalition by encouraging an exclusive vote for itself.

7. Possible Uses

CV seems an attractive procedure for coordinating if not matching voters' and parties' interests in a party-list system. Because the most compatible (i.e., approved of) majority coalitions get priority in the formation of a government, both voters and parties are encouraged to weigh coalition possibilities. Even before the election, it seems, parties might negotiate alliances and offer joint policy positions, better enabling voters to know what a coalition stands for if it should form the next government.

Under AV, by contrast, candidates do not form alliances but instead enjoy a premium in votes by taking positions that are acceptable to as many voters as possible--without, at the same time, appearing bland or pusillanimous, which might cost them votes. The fractional approval votes of CV give it some flavor of AV, but, just as significant, the coalition votes help to ensure that both voters and parties think beyond single best choices to coherent coalitions that can govern. This is not a factor in single-winner elections, to which AV is best suited.

But, as noted earlier, AV and CV are equivalent in allowing the voter to make multiple choices. Thus, CV does not impose greater burdens than AV on the voter, although the allocation and aggregation of votes, and the determination of a governing coalition, makes calculations under CV somewhat more complex.

Nevertheless, its basic thrust is clear--to give voters an incentive to build majority coalitions, especially if they do not view any single party as far and away the best or invincible. If this is the case and they are intensely committed to a party, voters will maximize a party's representation by voting for it alone but, in the process, forfeit any role in the selection of a preferred coalition that includes it.

The Israeli parliamentary election in November 1988 might have benefitted from CV. Neither Labor on the left nor Likud on the right won a majority of seats; an agreement with different combinations of the religious parties could have given either major party the necessary seats to form a government. Consequently, both parties flirted with the religious parties, who won collectively 15 percent of the seats in the Knesset, over a period of several weeks in an effort to forge an alliance, but both failed in the end to reach an agreement. (Actually, there were several tentative agreements--even some that seemed binding--and bitter recriminations when these collapsed.)

After seven weeks of incessant bickering and tortuous negotiations, Labor and Likud decided to extend their tenuous relationship, which had been first established after the 1984 election. For the majority of (secular) Israelis, this outcome probably was preferable to an alliance of one of the major parties with the religious parties--and certainly to most American Jews, who were strongly opposed to the orthodox stand of these parties on the question of "who is a Jew."

Under CV, Israeli voters, anticipating that no single party would win a majority of seats, would have been able directly to express themselves on what coalitions of parties they found they could best live with, if not like. With 27 parties running, and 15 that actually gained one or more seats in the Knesset, the opportunities for building coalitions would have been manifold.

As in the election itself, it appears likely that about two-thirds of the voters would have voted for Labor or Likud. A substantial number of these voters probably would have voted for both major parties, at least as compared to voting for one of the major parties and the religious parties. But more likely still, broad coalitions on the left and right probably would have emerged, reflecting the main secular dimenstions of Israeli politics (Bara, 1987). On the other hand, if the religious parties had significantly moderated their demands-- anticipating that otherwise they would be left out of a governing coalition--then they might have made themselves acceptable to greater numbers of voters, probably on the right.

Whatever the alignments, there almost certainly would have been considerable coalition voting, so the governing coalition probably would not have been decided by a miniscule proportion of the electorate voting for majority coalitions. But what sentiment the coalitions would have reflected is difficult to say a priori. What seems clear is that small and relatively extremist parties, while keeping their most fervent supporters and their seats proportionate to their numbers, would have hurt their chances of being in a governing coalition unless they broadened their appeal and allied themselves with other parties that could boost their chances.

The alliance of Labor and Likud that was finally consummated might not have been different under CV if the religious parties had persisted in their demands and the smaller parties on the left and right had not tried to ally themselves with the major parties. But surely the path to achieving this result would have been smoother.

This possible consequence of using CV is by no means slight, because the weeks of infighting among politicians, and large-scale demonstrations by citizens, not only wracked the society during the protracted negotiatitions but engendered deep cleavages that remain today. However, it is also possible that a more compatible grouping of parties would have emerged and won under CV.

Israel may be a special case--one observer has characterized her coalitions as "fragile" (Seliktar, 1982)--but she is not alone in experiencing difficulties in establishing durable coalitions under a party-list system. As another example, Italy has had 48 governments since World War II, an average of more than one a year.

Lijphart (1984) classifies Italy as a "consensus" democracy precisely because cleavages in the society have necessitated restraints on majority rule. To be sure, the frequent shifts in Italian governments probably have not had a great impact on the country's economy, national defense, or foreign policy, in part because the same people--mostly Christain Democrats--have governed for over 40 years, shuffling ministry portfolios among one another (Haberman, 1989).

In fact, there have been only 18 persons since World War II to serve as prime minister. But, as Haberman (1989) reports, with 13 parties and a style of government that is more patronage-driven than policy-driven,

> many Italian political leaders sense that the national mood is changing, that people have grown tired of the old politicking and want governments that last longer than the average life of a flashlight battery. There is a desire for predictable, enforceable and sustainable policies, they say, and it has grown keener with the approaching economic integration of Europe in 1992.

Indeed, we believe CV could help to promote what Haberman (1989) reports is

> a widely shared desire: a restructuring of political forces to give Italian voters a choice between a few dominant parties or

coalitions, offering clear programs and capable of taking turns at the helm.

In the concluding section we shall briefly recapitulate our findings and make our normative stance more explicit.

8. Conclusions

CV diminishes the problem of politicians' ignoring voters' interests. Indeed, it induces party leaders to try to reconcile their differences, insofar as possible, before an election and might even encourage mergers of parties. Since voters benefit from coalitions' surfacing before, not after, an election, their task of choosing a coalition, as allowed under CV, is facilitated.

Thus, CV seems well equipped to produce a convergence of voter and party/coalition interests, which Downs (1957) suggested, to the contrary, would be opposed in multiparty systems: parties would find it in their interest to be as ambiguous as possible about compromises they would make to enter a coalition, thwarting voters who seek clarity in order to make more informed choices in the election. If vagueness is dispelled at all, it is only after the election, when forced upon the parties by the necessity of forming a government.

The coalition-inducing and voter-responsive properties of CV should commend it to politicians in a party system like Israel's, which has been repeatedly torn by corrosive conflict that reflects in part the divided nature of that society, or Italy's, whose ephemeral governments seem ill-suited to making the tough choices required for the future. Similarly, in other countries (e.g., Belgium) that may be divided along ethnic, linguistic, racial, religious, or other lines--or simply have a tradition of factional conflict--

CV would seem to offer hope for promoting greater reconciliation and compromise, not because it is in the public interest (however defined) but rather because it is in the parties' self-interest if they desire to participate in a governing coalition.

These advantages of CV, in our opinion, justify a somewhat complex vote-aggregation and coalition-selection process. The intricacies of the Hare system of single transferable vote (STV), which has been widely used in both public and private elections, are no less easy to understand--even by mathematicians (Brams, 1982)--but these do not seem to burden the average voter.

In fact, because the voters under CV have only to indicate approval of parties--but do not have to rank them (as under STV)--CV facilitates the task of voting, particularly if there is a large number of parties. In addition, the parties, fueled by their desire to be in a governing coalition, will, in all likelihood, present the voters with reasonable coalition strategies, which should further ease the voters' burdens. All in all, we see no unusual practical difficulties in implementing CV in party-list systems.

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