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ENDOGENOUS CURRENCY SUBSTITUTION, INFLATIONARY FINANCE, AND WELFARE

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Roberto Chang

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NEW YORK UNIVERSITY
FACULTY OF ARTS AND SCIENCE
DEPARTMENT OF ECONOMICS
WASHINGTON SQUARE
NEW YORK, N.Y. 10003

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Roberto Chang
Department of Economics
New York University
269 Mercer Street - Third Floor
New York, NY 10003

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Abstract

In some inflationary countries, a foreign currency has replaced the domestic currency as a store of value, unit of account, and even means of payment. This paper studies the relationship between this "currency substitution" (CS) phenomenon and the theory of inflationary finance. CS poses several problems to our understanding of inflationary taxation because it implies that some agents choose to evade the inflation tax. The following questions arise: How does CS limit the government power to impose the inflation tax? If some agents can evade the tax, who pays the tax? What are the welfare effects of policies that deal directly with CS?

I answer these questions in the context of a model in which both CS and inflation arise as endogenous equilibrium outcomes. Because of transactions costs, private agents substitute foreign for domestic currency only if the domestic rate of inflation is sufficiently high. Because of heterogeneity, however, the relative cost of substitution differs across agents.

I characterize competitive equilibria given a constant value of the fiscal deficit to be financed by inflation. For small deficits, no CS arises in equilibrium. In another, higher range of deficits, equilibria exist in which some agents engage in CS. For some values of the deficit, both CS and non CS equilibria exist.

The maximal revenue from inflation depends on the willingness of the private sector to engage in CS, which in turn depends on transactions costs. I characterize exactly the distributional effects of inflation. Finally, I analyze the welfare consequences of taxes on holding of foreign exchange and exchange controls. CS is shown to be inefficient for the country as a whole, because seigniorage accrues to foreigners.

I. Introduction

Many countries use inflation as a tax on domestic currency balances to finance government expenditure. In some inflationary episodes, a foreign currency (such as the US dollar) has replaced domestic currency as a store of value, unit of account, and even means of payment. This phenomenon has been called "currency substitution" (CS) [1]. CS poses several questions to our understanding of inflationary finance because it implies that at least some agents in the economy can evade the inflation tax. How does CS limit the government's ability to impose the inflation tax? If some agents can evade the tax, who pays the tax? What are the welfare effects of policies that deal directly with CS, such as a prohibition on the use of foreign currency?

Earlier models of inflationary finance and currency substitution (Bailey (1956), Girton and Roper (1981)) followed the aggregative approach of postulating demand functions for national and foreign currencies instead of deriving them from the optimizing behavior of private agents. Although aggregative models provided many insights, they could not address the questions we have mentioned. Because they did not specify the fundamentals of the economy, they could not explain CS and inflation finance simultaneously. Their aggregative nature prevented them from characterizing the distributional effects of inflation. They cannot be amended to study the effects of the changes in the financial structure that would be brought about by foreign exchange controls or taxes on the holdings of foreign currency.

In this paper I study inflationary finance in a simple overlapping generations model in which private agents have the option of holding foreign currency or domestic fiat money and where I explicitly include the transactions costs associated with CS. Since this is a model of optimizing agents, it does not suffer from the same shortcomings as aggregative models. The model explains both CS and inflation as endogenous equilibrium outcomes which depend on the parameters of the model - government policies, preferences, technology, and the relative costs of using domestic and foreign currency.

I follow Bailey (1956) in arguing that transactions costs are crucial for understanding the welfare costs of inflation. The novelty of this paper is to show that transactions costs also determine the nature of CS. I model transactions costs in a very simple way. Private agents can use either domestic or foreign currency as assets. There is a given amount of government deficit to be financed by inflation. Under a flexible exchange rate regime, the real return on home currency is smaller than the return on foreign currency. However, individuals must pay a fixed cost in order to hold foreign currency. This cost represents the deadweight losses involved in substituting currencies. The need to verify the authenticity of foreign bills, the time spent in dealing with foreign exchange traders, or the cost of obtaining information about the relevant exchange rate are examples of such a cost. This formulation, admittedly simple, allows us to answer the questions raised above.

I present results on the existence of monetary equilibria with CS, the relationship between the maximal revenue from inflation and CS, and the welfare aspects of inflationary finance. With respect to the existence of equilibria, the paper shows that for small values of the government deficit inflation is low and no CS arises in equilibrium. In another, higher range of deficits, equilibria exist in which some agents hold the domestic currency and some the foreign currency. For some values of the deficit, both CS and non CS equilibria exist. This implies that expectations of inflation may become self fulfilling. High expected inflation leads some agents to substitute foreign for domestic currency. This increases the inflation rate needed to finance the deficit, justifying the inflationary expectations. If people expected low inflation instead, they would hold only domestic currency. The inflation rate needed to finance the fiscal deficit would be then lower, confirming the expectations of low inflation.

I discuss how CS is related to the government's power to obtain inflationary revenue. In particular, the maximal revenue that the government can obtain from inflation depends on the private sector's willingness to engage in CS, which in turn depends on preferences and the transactions cost. The revenue maximizing inflation rate is a decreasing function of the transactions cost, and goes to zero as transaction costs go to zero. This contrasts with the earlier work by Friedman (1971), Auernheimer (1974), and Calvo (1978). In their papers, the revenue maximizing inflation rate depends on the rate of growth of the economy and the interest elasticity of the aggregate demand for money. My research directs attention towards the financial technology as the main constraint on the government's seigniorage power.

In contrast with earlier analyses, the distributional aspects of inflation finance are completely described. A main finding is that if both equilibria with and without CS exist, the equilibrium without CS is Pareto superior to the CS equilibrium. Another consequence of the analysis is that any CS equilibrium is inefficient for the country as a whole since CS involves paying seigniorage to the issuer of foreign currency [2].

The effects of taxes on foreign currency holdings or exchange controls on equilibria and private welfare can be precisely described. I show in particular that the imposition of exchange controls may improve matters if the economy settles down to a low inflation equilibrium. However, other highly inflationary equilibria appear and therefore exchange controls may make everybody worse off.

The paper proceeds as follows: Section II describes the model. Individual behavior is analyzed in Section III. Section IV defines competitive equilibria and characterizes competitive equilibria for a given value of the government deficit. Welfare issues and policy implications are analyzed in Section V. Section VI concludes.

II. The Model

We will study a simple overlapping generations model of a small open economy [3]. Time is discrete. At the beginning of each period $t = 1,2,...$ a new generation of N "rich" and n "poor" individuals is born. There is no population growth and generations are identical. Each agent in generation $t \geq 1$ lives for two periods, youth and old age. In addition, there is an agent ("generation zero") who lives only during period one. Therefore, in each period there are two generations alive and any two generations can coexist for at most one period.

There is a single, nonstorable consumption good which is freely traded. Each agent born at $t \geq 1$ is endowed with a positive amount of the consumption good when young. The size of these endowments is the only difference between rich and poor agents. Let e_r and e_p denote the endowments of young rich and poor agents, respectively ($e_r > e_p > 0$). I will often denote the "type" of a given agent by a subscript $h = r,p$, where r (resp. p) corresponds to a rich (resp. poor) agent.

Agents receive no endowments of goods when old. To finance old age consumption, a young agent must exchange part of his endowment for national currency ("pesos") or foreign currency ("dollars"). Holding pesos is costless. In contrast, in order to hold dollars a young domestic resident must bear a positive fixed cost B.

B is a measure of transactions costs, and is supposed to capture the inconvenience of engaging in currency substitution. Examples of such a cost are the need to verify the authenticity of foreign bills, the time and effort involved in dealing with foreign exchange traders, and the cost of obtaining information about the relevant exchange rate. In this paper all these costs are summarized in B. In a more complete model, B would be itself a function of other variables such as the extent of CS itself. As a first approximation to the problem, however, I make the assumption without apology.

Generation zero has no endowment of the consumption good but possesses, at the beginning of $t = 1$, some given amounts of pesos and dollars M_0 and F_0.

To make the argument as simple as possible, we will assume that pesos and dollars are the only assets available. It will become clear that other assets could be included by specifying the transactions costs associated with them.

The dollar price of consumption is determined in the world market and equals one. Let p_t denote the peso price of consumption and x_t the spot exchange rate (pesos per dollar). Free trade implies that in equilibrium $p_t = x_t$.

This country has a government that must use inflation to finance an exogenously given sequence of expenditures on the consumption good. For clarity, I assume that the government can levy no other taxes and therefore government expenditure is equal to the <u>primary fiscal deficit</u>[4]. Each period the government issues as many pesos as needed to finance its expenditure:

(1) $$M_t = M_{t-1} + p_t g_t \qquad t = 1,2,3,\ldots$$

where M_t is the supply of pesos and g_t denotes government expenditure or the fiscal deficit at t.

Foreigners do not hold pesos, and international borrowing and lending is ruled out by assumption. As a consequence, the change in the quantity of dollars held by residents of this country equals its trade surplus:

$$(2)\quad F_t - F_{t-1} = N(e_r - k(f_r^t) - c_{r1}^t - c_{r2}^{t-1}) + n\,(e_p - k(f_p^t) - c_{p1}^t - c_{p2}^{t-1}) - g_t \qquad t = 2,3,\ldots.$$

$$F_1 - F_0 = N(e_r - k(f_r^1) - c_{r1}^1) + n\,(e_p - k(f_p^1) - c_{p1}^1) - c^0 - g_1$$

where F_t = quantity of dollars in the home country at the end of period t.

f_p^t (f_r^t) = dollar holdings of poor (rich) agents born at t

c_{hj}^t = consumption, in j^{th} period of life, of an agent of type h = r,p born at t

and k(.) is a function such that k(0) = 0 and k(f) = B if f > 0.

Notice that transactions costs are included in this accounting by the presence of the function k(.).

III. Individual Behavior

Each agent has perfect foresight and takes prices as given. Agents in their old age have a trivial decision problem: since no utility is derived from unspent cash balances, they inelastically supply their currency holdings in exchange for the consumption good. This section analyzes the decision problem of a young agent h born at t. The main result of this section is that agent h will decide to hold either pesos or dollars according to whether the inflation rate is smaller or larger than some threshold rate, which in turn depends on his wealth and preferences. For most of this section, the superscript h will be used only when necessary.

Agent h must choose consumption in youth and old age, and currency holdings in youth, so as to maximize lifetime utility subject to his lifetime budget constraint. Let $u(c_1,c_2)$ be the utility function (which is common for all agents except generation zero) defined on consumption in youth (c_1) and old age (c_2). Agent h chooses $(c_1, c_2, M, f) \geq 0$ to maximize $u(c_1,c_2)$ subject to:

$$(3) \qquad p_t c_1 + M + x_t f \leq p_t(e_h - k(f))$$
$$p_{t+1} c_2 \leq M + x_{t+1} f$$

where e_h equals e_p (resp. e_r) if the agent is poor (resp. rich) [5]. The effect of the transaction cost B on agent h's problem is captured by the presence of k(.) in his budget constraint.

We will assume that u(.,.) is strictly increasing in both arguments and strictly concave. I also assume that $u_1(c_1,c_2) \to \infty$ as $c_1 \to 0$ and $u_2(c_1,c_2) \to \infty$ as $c_2 \to 0$, where $u_i(.,.)$ denotes the i^{th} partial derivative of u.

Only equilibria in which domestic currency has value are considered. Using the fact that $p_t = x_t$ in equilibrium, the budget constraint (3) can be rewritten as:

$$(4a) \qquad c_1 + m + f \leq e_h - k(f)$$
$$(4b) \qquad c_2 \leq (1+\pi_t)^{-1} m + f$$

where $m = M/p_t$ and $1+\pi_t = p_t/p_{t+1}$. The consumer chooses c_1, c_2, m and f to maximize utility subject to (4). His optimal choice depends on prices only through π_t, the inflation rate between t and (t+1). The budget constraint (4) shows that the consumer pays the inflation tax if he chooses to hold domestic currency. By holding dollars, he evades the tax, but bears the transactions cost B.

This problem differs from the standard consumer problem because the budget constraint (4) is not convex due to transactions costs. However, the problem can be simplified much further because transactions costs are fixed. We observe that an optimizing agent will never hold both pesos and dollars. If inflation is nonpositive ($\pi_t \leq 0$) he will only hold pesos. Suppose that $\pi_t > 0$ and agent h chooses positive values for both m and f. Then, a marginal substitution of f for m would leave (4a) unchanged, but would increase the RHS of (4b). Since u is increasing in c_2, this cannot be optimal.

As a consequence, agent h's problem can be decomposed in two parts. First he decides which currency to hold. Second, given that decision, he maximizes utility by choice of consumption in youth and old age. He will hold pesos if the maximum utility attainable by holding pesos (and paying the inflation tax) is higher than the maximum utility attainable by holding dollars and paying the fixed cost B.

Suppose that agent h decides to hold pesos. Then f = 0 and k(f) = 0 in (4), and his market opportunities are summarized by:

$$(5) \qquad c_1 + (1+\pi_t)\, c_2 = e_h$$

If agent h decides not to hold dollars, he maximizes $u(c_1,c_2)$ subject to (5). This has the form of the standard consumer problem. If the <u>indirect utility function</u> is defined by:

$$(6) \qquad v(\pi,y) = \max\ u(c_1,c_2) \quad \text{s.t. } c_1 + (1+\pi)c_2 = y$$

then, when the inflation rate is π_t, the maximum utility that agent h can obtain from holding pesos is $v(\pi_t, e_h)$.

On the other hand, if agent h decides to hold dollars, $m = 0$ and $k(f) = B$ in (4), and his problem reduces to choose c_1 and c_2 to maximize utility subject to:

$$(7) \qquad c_1 + c_2 = e_h - B$$

Thus, the maximum utility agent h can attain from engaging in currency substitution is $v(0, e_h - B)$.

Define π_h^* by:

$$(8) \qquad v(0, e_h - B) = v(\pi_h^*, e_h)$$

π_h^* is the rate of inflation at which agent h is exactly indifferent between holding pesos and dollars. It is the smallest inflation rate at which agent h decides to evade the inflation tax by holding foreign currency. If inflation is smaller than this threshold rate, the existence of transactions costs make it optimal to hold pesos and pay the inflation tax. If the inflation rate is higher than π_t^*, agent h bears the cost B to evade the inflation tax by holding dollars.

Since $v(.,.)$ is increasing in its first argument, π_h^*, if it exists, is unique, strictly positive if B is positive, and is increasing in B [6]. From (8), π_h^* depends on e_h and the utility function. I will assume that the utility function is such that π_h^* exists for all $0 < B < e_h$, and that it is decreasing in e_h for given B [7].

Figure 1 illustrates the solution to agent h's problem. The length of OD is $e_h - B$ and the slope of DD' is -1. Thus, DD' is agent h's budget line if he holds dollars. The utility associated with the indifference curve U_oU_o is $v(0, e_h-B)$. On the other hand, the length of OA is e_h. AA' is drawn such that it is tangent to U_oU_o. The slope of AA' is, therefore, $-(1+\pi_h^*)$. When the inflation rate is below π_h^*, agent h can choose a point in a higher indifference curve than U_oU_o. In Figure 1, AG is the budget line associated with holding pesos for an inflation rate smaller than π_h^*. In contrast, AH is the budget line associated with using pesos if inflation is larger than π_h^*. In this case, agent h will hold dollars and attain U_oU_o.

[INSERT FIGURE 1 HERE]

In what follows, π_r^* denotes the threshold rate for rich agents, and π_p^* the threshold rate for the poor. Under our assumptions, $\pi_r^* < \pi_p^*$: the inflation rate at which the rich engage in currency substitution is smaller than the threshold rate for the poor. This captures the observation that it is relatively cheaper for wealthier people to use foreign currency.

For further reference, we define the <u>savings function</u> s by:

$$(9) \qquad s(\pi,y) = \arg\max_{0\leq s\leq y} \; u(y-s, \; (1+\pi)^{-1}s)$$

Under our assumptions on the utility function, $s(\pi,y)$ is single valued and continuous. I will assume that s is strictly decreasing on π [8]. For given $y > 0$, s is strictly positive if π is finite and approaches a nonnegative limit as π increases to infinity.

The savings function represents the demand for assets of agent h. Our discussion shows that agent h saves only in pesos if $\pi_t < \pi_h^*$, and only in dollars if $\pi_t > \pi_h^*$. Summarizing the results in this section, the demand for pesos of an agent of type h born at t is given by:

$$(10) \qquad m_h^t = m(\pi_t, e_h) = s(\pi_t, e_h) \qquad \text{if } \pi_t < \pi_h^*$$
$$= 0 \qquad \text{if } \pi_t > \pi_h^*$$
$$= \text{either } 0 \text{ or } s(\pi_t, e_h) \qquad \text{if } \pi_t = \pi_h^*$$

and his demand for dollars is:

$$(11) \qquad f_h^t = f(\pi_t, e_h) = 0 \qquad \text{if } \pi_t < \pi_h^*$$
$$= s(0, e_h - B) \qquad \text{if } \pi_t > \pi_h^*$$
$$= \text{either } 0 \text{ or } s(0, e_h - B) \qquad \text{if } \pi_t = \pi_h^*$$

IV. Competitive Equilibrium and Endogenous Currency Substitution

Given a sequence of government deficits $\{g_t\}$ and initial quantities of pesos and dollars M_0 and F_0, a (perfect foresight) competitive equilibrium is a sequence of price levels $\{p_t\}$, a sequence $\{M_t\}$ of peso supplies, and a nonnegative pair (m_h^t, f_h^t) of asset demands for each agent $h = r,p$ and $t = 1,2,3,\ldots$ such that:

(i) Given π_t, m_h^t and f_h^t are given by (10) and (11)

(ii) For all t, $M_t/p_t = m_t = Nm_r^t + nm_p^t$

(iii) Given $\{g_t\}$ and $\{p_t\}$, the sequence $\{M_t\}$ satisfies (1) for all t.

Condition (i) requires that each agent maximize utility given current and anticipated prices. Condition (ii) requires that the real supply of pesos be equal to the demand for pesos in every period. It is straightforward to show that (ii) ensures that the market for foreign exchange clears as well. Condition (iii) requires that the government be able to finance its deficit in each period.

Competitive equilibrium in this economy depends on the exogenous sequence $\{g_t\}$ of government deficits. For the rest of the paper, we shall restrict attention to permanent deficits, i.e., we will assume $g_t = g \geq 0$ for all t. In this section I will discuss the existence and uniqueness of competitive equilibria associated with a given g. I show that, under some conditions on π_h^*, a given value of g can give rise to at most two kinds of competitive equilibria: non CS regimes in which only pesos are held in equilibrium, and CS regimes in which both pesos and dollars are held along the equilibrium path. These regimes are stationary in the sense that the equilibrium inflation rate and all real quantities constant. Furthermore, no jumps across regimes are possible if deficits are permanent. I finally show that for g in a nontrivial range both CS and non CS regimes exist.

IV.a. Non CS Regimes

In a non CS regime $f_h^t = 0$ for all t and h, and therefore $\pi_t \leq \pi_r^*$ for all t. From (1), $1+\pi_t = m_t/(m_{t+1}-g)$. Using (10) and the equilibrium condition (ii), it can be seen that a non CS equilibrium regime can be characterized by a sequence $\{m_t\}$, that for all $t \geq 1$ satisfies $\dfrac{m_t}{m_{t+1}-g} \leq 1 + \pi_r^*$ and

$$(12) \qquad Ns\left(\frac{m_t}{m_{t+1} - g} - 1,\ e_r\right) + ns\left(\frac{m_t}{m_{t+1} - g} - 1,\ e_p\right) = m_t$$

Equation (12) is just the reflected offer curve commonly found in the overlapping generations literature [9]. It is an equilibrium requirement on m_t and $m_{t+1}-g$. In Figure 2, the locus OC is the graph of (12). It is easy to show that OC cuts the 45^o line from below [10]. I also assume that the utility function is such that OC is convex.

Since $1+\pi_t = m_t/(m_{t+1}-g)$, each point in OC defines an inflation factor equal to the inverse of the slope of the ray connecting it to the origin O. Since savings functions are single valued, each ray emanating from O can cross OC only once. The ray OA has slope $(1+\pi_r^*)^{-1}$. I shall assume:

[A1] The slope of OA is larger than the slope of OB, where B is the point at which the derivative of OC is one.

[A1] means that the inflation rate at which rich people start holding dollars is smaller than the inflation rate which would maximize steady state seigniorage if the economy was closed. This is the case if the transaction cost B is sufficiently small relative to e_r [11].

The value of government deficits g is measured on the southbound axis. Given g, equal to OO′ for instance, the equality $m_{t+1} = m_t$ is represented by 45^o lines emanating from the g-axis, such as O′Z′. Of course, OZ represents $m_{t+1} = m_t$ if $g = 0$.

INSERT FIGURE 2 HERE

The study of non CS competitive equilibria is easy from Figure 2. The equilibrium equation (12) is the offer curve OC, and the condition $m_t/(m_{t+1}-g) \leq \pi_r^*$ implies that equilibrium paths must remain above the ray OA.

Consider <u>stationary</u> equilibria in which $m_t = m$ for all t. From (1) $\pi_t = \pi = g(m-g)^{-1}$. Thus, given g, a non CS stationary equilibrium is a value of m ≥ 0 such that $g(m-g)^{-1} \leq \pi_r^*$ and

(13) $$Ns(g(m-g)^{-1},e_r) + ns(g(m-g)^{-1},e_p) = m.$$

In Figure 2, the arc AS completely describes the set of non CS stationary equilibria. For each point in AS, one can find $g \geq 0$ such that there is a steady state equilibrium. Consider S', for instance. The corresponding value of g can be found by drawing a 45^{o} line through S' and finding its intersection with the g-axis. Thus, the value of g for which S' represents a stationary equilibrium is OO'. The gross inflation factor associated with this equilibrium is the inverse of the slope of OS' and, by construction, it is lower than $1+\pi_r^*$. On the other hand, points on OS cannot be stationary equilibria because they are associated with inflation rates higher than π_r^*. Thus, S" cannot be a stationary equilibrium because the slope of OS" is smaller than the slope of OA. Notice that this is true even when there is a stationary no CS equilibrium associated to OO'. By inspection, it is clear that there is a unique stationary equilibrium for any g in OO". Therefore we have:

<u>Proposition 1</u>: Assume [A1]. Then there is a unique non CS stationary equilibrium for each $g \in [0, g^u]$, where $g^u = \pi_r^*(1+\pi_r^*)^{-1} \{Ns(\pi_r^*, e_r) + ns(\pi_r^*, e_p)\}$ [12].

The limit g^u is found by inserting $\pi_r^* = g(m-g)^{-1}$ in (13).

Thus, for each value of g in $[0,g^u]$ there is a stationary non CS equilibrium $m^*(g)$, say. From inspection of Figure 2 it can also be seen that any solution of (12) for which m_1 is different from $m^*(g)$ will violate one of the equilibrium conditions. Suppose, for example, that $g = OO'$ and $m_1 > m^*(g)$. Then m_t increases without bound. If $m_1 < m^*(g)$, then m_t would evolve along the arc OBA in finite time, and the inflation rate would increase above π_r^*. Hence we can state

Proposition 2. Assume [A1]. Then the only non CS equilibrium regimes are the stationary ones described in Proposition 1.

An implication of Proposition 2 is that, regardless of the values of M_0 and F_0, given $g \in [0, g^u]$ the economy converges in the first period to a non CS stationary equilibrium. In the first period, generation zero supplies his currency holdings inelastically, and spends F_0 in the world market. The price level jumps so as to make the real value of pesos equal to the steady state value: $M_0/p_1 = m^*(g) - g$. There is a trade deficit exactly equal to F_0 in period one, and balanced trade thereafter [13].

V.b. CS Regimes

CS regimes are equilibria along which both pesos and dollars are held. It then must be the case that $f_r^t > 0$ and $m_p^t > 0$ for all t, which requires that $\pi_r^* \le \pi_t \le \pi_p^*$ for all t. Since $1+\pi_t = m_t/(m_{t+1}-g)$, a CS equilibrium regime is characterized by a sequence $\{m_t\}$ that for all $t \ge 1$ satisfies $1+\pi_r^* \le m_t/(m_{t+1}-g) \le 1+\pi_p^*$ and

$$(14) \qquad ns\left(\frac{m_t}{m_{t+1} - g} - 1 \,,\, e_p \right) = m_t$$

Equation (14) has the same form as (12). In fact, it is the reflected offer curve of an economy inhabited only by poor agents. In a CS regime, rich agents pay the transaction cost B to "leave" the market for domestic currency and avoid inflation.

The analysis of (14) is analogous to the analysis of (12). The graph of (14) is the locus OC" in Figure 3. The slopes of OA" and OD are $(1+\pi_r^*)^{-1}$ and

$(1+\pi_p^*)^{-1}$ respectively. In drawing Figure 3, I have assumed that transaction costs are small in the following sense:

[A2] The slope of OD is greater than the slope of OB", where B" is the point at which the slope of OC" is equal to one.

[A2] means that the rate of inflation at which poor people start to hold dollars is smaller than the revenue maximizing inflation rate in a closed economy inhabited by poor agents [14].

[INSERT FIGURE 3 HERE]

By construction, each point in OC" satisfies (14). Also, each point in OC" defines an inflation factor equal to the inverse of the slope connecting that point with the origin. Therefore, CS equilibrium regimes must stay within the cone bounded by OA" and OD. Since (14) must be satisfied, CS equilibrium paths must lie on the arc A"D.

Given g, a stationary CS equilibrium is a value $m > 0$ such that $m_t = m$ satisfies (14) and $\pi_r^* \leq g(m-g)^{-1} \leq \pi_p^*$. Each point in A"D, such as H, is a stationary CS equilibrium for some value of g. This value is given by the intercept on the g-axis of the 45^o line that goes through that point. Thus, the value of g for which H is a steady state is the distance OO". Using the same arguments as in Propositions 1 and 2, it is straightforward to prove:

<u>Proposition</u> <u>3</u> Under [A2] there is a unique stationary CS equilibrium for each g in the interval $[g_1, g_2]$, where $g_1 = \pi_r^* (1+\pi_r^*)^{-1} ns(\pi_r^*, e_p)$ and $g_2 = \pi_p^* (1+\pi_p^*)^{-1} ns(\pi_p^*, e_p)$. Moreover, this is the only CS equilibrium.

Proposition 3 implies that given the policy $g \in [g_1, g_2]$, the economy may converge immediately to an stationary CS regime in which the real value of domestic currency is constant at, say, $m^{**}(g)$. The price level jumps as to equate the real value of pesos to the steady state value: $M_0/p_1 = m^{**}(g) - g$. The economy adjusts the quantity of dollars to the desired steady state quantity through a trade surplus or deficit: $F_1 - F_0 = Ns(1, e_r - B) - F_0$. The balance of trade is zero thereafter, because the aggregate demand for dollars is constant.

V.c. Mixed regimes

So far we have shown that for $g \in [0, g^u]$ there is a unique equilibrium without CS, and that for $g \in [g_1, g_2]$ there is a unique equilibrium with CS. One could conjecture the existence of mixed equilibria in which agents hold dollars in some periods but hold only pesos in some other periods. However, it turns out that, given a feasible value of g, shifts across CS and non CS regimes are impossible in equilibrium.

Proposition 4 Under [A1] and [A2] there cannot be equilibria with $F_t = 0$ and $F_{t+1} > 0$ for any $t \geq 1$.

The proof is by contradiction. Suppose there is an equilibrium in which for some $T \geq 2$ $F_{T-1} = 0$ and $F_T > 0$. Then, it must be the case that $\pi_T \geq \pi_r^* \geq \pi_{T-1}$. This implies that $m_{T-1} < m_T < m_{T+1}$ [15]. But then the economy settles on a path leading to autarky on the CS offer curve (14) and in finite time π_t would be greater than π_p^*. In terms of Figure 3, if the economy switches from the non CS offer curve to the CS offer curve OC" at T, m_T is to the left of

$m^{**}(g)$. This cannot be an equilibrium.

The same kind of argument implies that there cannot be an equilibrium in which $F_t > 0$ and $F_{t+1} = 0$ for any t. Thus mixed regimes cannot occur for the class of permanent deficits that we are considering.

V.d. Multiplicity of Equilibria

Although for given g there can be at most one equilibrium with CS and at most one equilibrium without CS, there is a range of values of the fiscal deficit g for which both equilibria exist. From the definitions of g_1 and g^u in Propositions 1 and 3, $g_1 < g^u$. Hence:

Corollary For each g in $[g_1, \max\{g_2, g^u\}]$ there are two equilibria, one with CS and another without CS.

This Corollary defines precisely, in terms of preferences and technology, the range of fiscal deficits for which different expectations about inflation may become self validating. For any g in this range, expectations of high inflation may lead the rich to hold foreign currency. In this case, the demand for national currency comes only from the poor, real domestic balances are small, and the inflation rate needed to finance g is high, confirming expectations. If, on the contrary, rich agents expect lower inflation, they will hold domestic currency. Because this increases the base of the inflation tax, the inflation rate required to finance g is low and expectations become self fulfilling.

Figure 4 summarizes the results of this section. The vertical axis measures the level of the government deficit g. The equilibrium rate of inflation is measured on the horizontal axis. Non CS equilibria must be on the curve OH', which graphs:

$$(15) \qquad \Gamma(\pi) \equiv \pi(1+\pi)^{-1} \{Ns(\pi,e_r) + ns(\pi,e_p)\} = g$$

which is derived from (13) by using the steady state budget constraint $g = \pi(1+\pi)^{-1}m$. If currency substitution were not possible, $\Gamma(\pi)$ would be the level of revenue that the government could obtain from applying the inflation tax π [16]. With currency substitution, not all these levels of inflationary revenue are feasible: non CS equilibrium regimes must satisfy (15) and also the requirement $\pi \le \pi_r^*$. Thus, the set of non CS equilibrium regimes are given by the bold arc OA. Figure 1 shows that there is a unique non CS equilibrium for each g in $[0,g^u]$ [17]

CS equilibria must lie on the curve OH, which graphs:

$$(16) \qquad \gamma(\pi) \equiv \pi(1+\pi)^{-1} ns(\pi,e_h) = g$$

Equation (16) is derived from (14) by assuming stationarity and using $g = \pi(1+\pi)^{-1}m$. Its meaning is similar to (15). $\gamma(\pi)$ is the level of revenue that the government would be able to collect in the absence of CS by applying the inflation tax <u>only to poor agents</u>. This is because in CS equilibria the rich pay the transaction cost B and evade the inflation tax. In addition to (15), CS equilibria must satisfy $\pi_r^* \le \pi \le \pi_p^*$. Thus, the set of CS equilibria is the bold arc BC. Under [A2], there is a unique CS equilibrium for each $g \in [g_1,g_2]$ [18].

Two equilibria exist for g in $[g_1,g^u]$. In contrast, equilibrium is globally unique for g in the intervals $[0,g_1)$ and $(g^u,g_2]$. Notice that inflation is always locally increasing in g, even when we focus on the CS equilibria described by BC. In contrast, earlier papers (Bailey (1956), Phelps and Shell (1969)) associated multiplicity of equilibria with inflation rates decreasing in the government deficit [19].

[INSERT FIGURE 4 HERE]

V. Policy Issues and Private Welfare

Now we can use our results in the preceding section to answer the questions of the Introduction: How does CS affect the government's ability to use the inflation tax? What are the distributional effects of the inflation tax? What are the welfare effects of policies that deal directly with CS?

V.a. Inflationary Finance and Currency Substitution

In this model, currency substitution arises as the optimal response of private agents to the government's attempt to impose the inflation tax. The cost of evading inflationary taxation is summarized by the transactions cost B. Thus, the government's power to obtain inflationary revenue depends on B, which in turn summarizes many institutional factors. A deeper analysis of inflationary taxation requires the explicit modelling of these institutional factors. However, we can still make some observations based on the above analysis.

A measure of the government's power to impose the inflation tax is the maximal revenue from inflation [20], which in this model is Max $\{g_2, g^u\}$. Now, Propositions 1 and 3 imply that g_1, g_2, and g^u all depend on π_r^* and π_p^* and hence on B. As B becomes smaller, these bounds become smaller, and they go to zero as B goes to zero. This only means that the maximal revenue that the government can extract from inflation depends directly on the difficulty with which people can substitute domestic currency with other assets. When that substitution is costless, the government cannot obtain any revenue from inflation [21]. Notice that the revenue maximizing inflation rate is equal to π_p^*. Given preferences, π_p^* depends on B only, and thus on the available financial technology.

Neglecting the possibility of currency substitution may lead to incorrect policy analysis. In particular, models that take the aggregate demand for (domestic) money as structural tend to overestimate the potential for inflationary revenue. Consider, for example, a country with a history of small government deficits and low inflation. Our model implies that no currency substitution will be observed. If a researcher estimated the demand for money based on past observations, he will probably obtain a good fit. But if policy makers tried to use his study to increase inflationary revenue, currency substitution would emerge and a "structural break" in the demand for money equation would be observed [22].

It is likely that transactions costs decrease with time, due to technological progress and improvements in communications. Thus, the power to use inflation as a tax tends to disappear. It is also plausible that B depends on past inflation. This would be the case if agents pay a once-and-for-all costs for new institutional arrangements in order to deal with inflation. In this case, a government that uses inflationary finance will find it more difficult to use the inflation tax in the future. Moreover, once inflation reaches a certain point, reducing the inflation rate may be very difficult even if the fiscal deficit decreases . Thus, high inflation may be a "hysteresis" phenomenon.

V.b. Who Pays the Inflation Tax?

Our model allows for an analysis of the distributional effects of inflation. This is a topic that has not been formally analyzed in the literature, although there is the widespread feeling that inflation is a "regressive" tax.

If equilibrium is unique, then the distributional effects of inflation are clear. If $g \in [0, g_1)$, a no CS equilibrium obtains and both rich and poor agents pay the inflation tax. Higher inflation makes both types of agents worse off. If $g \in (\mathrm{Max}\ \{g_2, g^u\},\ g_2]$, a CS equilibrium obtains and only the poor pay the inflation tax. Higher inflation make the poor worse off but does not affect the rich. The common notion of inflation as a regressive tax applies here in a precise sense.

The most interesting case is $g \in [g_1,\ \mathrm{Max}\ \{g_2, g^u\}]$. In this case both CS and non CS equilibria exist. The following Proposition shows that these two equilibria can be Pareto ranked.

Proposition 5 If $g \in [g_1,\ \mathrm{Max}\ \{g_2, g^u\}]$, all agents are better off in the non CS equilibrium than in the CS equilibrium.

Proof: Fix $g \in [g_1,\ \mathrm{Max}\ \{g_2,\ g^u\}]$. Three kinds of agents must be considered: generation zero, poor agents born at $t \geq 1$ and rich agents born at $t \geq 1$. The welfare of generation zero depends only on the real value of his money balances at $t = 1$. Since the real value of dollars is fixed, his utility depends only on M_0/p_1. In the non CS equilibrium, $M_0/p_1 = m^*(g) - g$. In the Cs equilibrium, $M_0/p_1 = m^{**}(g) - g$. Since $m^*(g) > m^{**}(g)$, generation zero prefers the non CS equilibrium.

In any equilibrium, the utility of poor agents born at $t \geq 1$ is $v(\pi,\ e_p)$.

Since π is higher in the CS equilibrium than in the non CS equilibrium, and v is decreasing in π, the poor prefer the non CS equilibrium.

Finally, let π_a be the inflation rate in the non CS equilibrium. The utility of the poor in a non CS equilibrium is thus $v(\pi_a, e_r)$. In a CS equilibrium, it is $v(0, e_r - B) = v(\pi_r^*, e_r)$. But $\pi_a \leq \pi_r^*$. Therefore $v(\pi_a, e_r) \geq v(\pi_r^*, e_r) = v(0, e_r - B)$. Rich agents are at least as well off in the non CS equilibrium as in the CS equilibrium, and strictly better off if $\pi_a < \pi_r^*$, i.e., if $g < g^u$. This completes the proof.

Notice that when there are both CS and non CS equilibria, holding dollars is individually but not collectively rational for the rich. In the CS equilibrium, the rich hold dollars because the inflation rate is high. If they did not hold dollars, the inflation rate would be low enough to justify holding only pesos. Currency substitution appears to be a "coordination failure".

V.c. Foreign Exchange Taxes and Exchange Controls

Suppose that the government can impose a foreign exchange tax, a fixed real tax T contingent on the holding of foreign currency. Assume that the proceedings of the foreign exchange tax are used to finance part of the deficit. From the viewpoint of private agents, this tax is equivalent to an increase of the cost B, and therefore of π_r^* and π_p^*. In terms of Figure 4, the vertical lines $\pi = \pi_r^*$ and $\pi = \pi_p^*$ shift to the right, and the value of g associated with CS equilibria would decrease by NT. Given g, an appropriate choice of T may succeed in eliminating currency substitution. However, other equilibria with high inflation may appear. Thus, we reach the startling conclusion that a foreign exchange tax may eliminate currency substitution and make everybody worse off!

Aside from these engineering problems, the distributional effects of the foreign exchange tax depend on the initial value of g. If g is in the interval $[g_1, \text{Max}\{g_2, g^u\}]$, the tax will benefit all agents. If $g \in (g^u, g_2]$, the rich will be worse off and the poor better off with the tax. Notice that if the foreign exchange tax effectively eliminated currency substitution, nobody would pay it.

The above analysis applies directly to the imposition of exchange controls, i.e., direct prohibitions on the use of foreign currency. A prohibition on the use of dollars is equivalent to an infinite foreign exchange tax. Our analysis implies that a prohibition of this sort may be Pareto improving if $g \in [g_1, \text{Max}\ \{g^u, g_2\}]$, but have distributional consequences if $g > \text{Max}\ \{g_2, g^u\}$. Moreover, other high inflation equilibria appear.

The effectiveness of foreign exchange taxes or prohibitions depends on the government's ability to enforce them. One would expect the government's abilities of a government to be rather limited if it has to rely on inflation finance in the first place. Thus one should ask if there are other policies which may be easier to implement.

In this economy, a Pareto improving policy is the issue of alternative assets to foreign bills. Suppose that the government can costlessly issue a one period indexed bond, which is sold for p_t at t and pays p_{t+1} at t+1. Assume that individuals have to pay the transactions cost B if they hold this bond. Then, the set of equilibria in which these bonds are held coincides exactly with the set of CS equilibria. However, in the first period the government would make a surplus of $Ns(0, e_r - B)$, which could be redistributed to the private sector in a Pareto improving fashion.

This surplus is possible because in a CS equilibrium the economy acquires the steady state amount of dollars by running a trade surplus in the first period. These dollars are never spent in consumption. Therefore, the home country gives up seigniorage, which accrues to foreigners. This is exactly the argument discussed by Fischer (1982). By issuing its own debt, the government could recover this seigniorage. Of course, the feasibility of such a policy depends on the relative costs of issuing such a bond and the cost, for the private sector, of holding it [23].

VI. Final Remarks

One contribution of this paper is to show explicitly how transactions costs determine the interaction between currency substitution and inflation. The model presents foreign exchange as an asset whose return dominates domestic currency but with associated transactions costs. It should be now clear that other assets can be included in the model as long as we specify the transactions costs associated with them. My emphasis on foreign exchange comes from my belief that the transactions costs associated with currency substitution are low.

The attractiveness of foreign exchange as a store of value depends of course on the exchange rate regime, which I assumed to be flexible. With a fixed or predetermined exchange rate, currency substitution becomes less profitable and the arguments in this paper must be modified. Needless to say, if the government is pursuing an inflationary policy, fixing the exchange rate brings about a different set of problems. Also, we must remember that the arguments in this paper follow whenever there is an alternative asset to domestic currency.

The fact that in equilibrium individual portfolios are degenerate are not essential for the results. What our model is missing is a story about the "transactions role" of money, as opposed to the "store of value" function. Still, the point is that in an inflationary environment agents will reduce their holdings of national currency to the strict minimum to carry out purchases. If inflation is high, there is currency substitution and domestic currency is not held as a store of value. We can obtain nondegenerate portfolios by adding a cash in advance constraint or including domestic currency in the utility function, but these devices do not add anything interesting to the results obtained here.

I assumed that the cost of holding dollars was fixed at B. This need not be the case, and more complex forms for the cost function k(.) may be investigated. Also, k(.) may depend on the extent of currency substitution in the economy. At the end, the shape of k(.) may be an empirical question. A message of this paper is that the answer to that question has important consequence for economic policy.

Appendix

Existing analyses of the revenue maximizing inflation rate assume that the government chooses the rate of growth of the money supply as opposed to the level of government deficit. This Appendix shows that the results of the main text are not altered if we assume that the government fixes money supply growth. In particular, it is still the case that the revenue maximizing inflation rate is π_p^*.

Let the money supply evolve according to the fixed rule:

$$[X1] \qquad M_t = qM_{t-1} \qquad t = 1,2,\ldots$$

where $q \geq 1$ is the choice variable of the government. By definition:

$$[X2] \qquad 1+\pi_t = \frac{p_{t+1}}{p_t} = \frac{qm_t}{m_{t+1}}$$

It follows that a perfect foresight equilibrium with no CS is a sequence (m_t, π_t) such that :

$$[X3] \qquad Ns\left(\frac{qm_t}{m_{t+1}} - 1, e_r\right) + ns\left(\frac{qm_t}{m_{t+1}} - 1,\ e_p\right) = m_t$$

and:

$$[X4] \qquad \frac{qm_t}{m_{t+1}} \leq 1+\pi_r^* \qquad \text{all } t$$

[X3] has the same form as equation (12), the only difference being that [X3] defines a relationship between (m_{t+1}/q) and m_t. In Figure 5, OC is the graph of [X3]. The $m_{t+1}=m_t$ line is a ray from the origin with slope 1/q. The condition [X4] states that admissible equilibrium paths stay above the point F in the curve OC. If the slope of OB is 1/q, $m^*(q)$ is the only possible equilibrium with no CS. Thus, there is a unique non CS equilibrium for q in $[1,\ 1+\pi_r^*]$.

An equilibrium with CS is a sequence $\{m_t\}$ that satisfies:

[X5] $$ns\left(\frac{qm_t}{m_{t+1}} - 1,\ e\right) = m_t$$

[X6] $$1+\pi_r^* \leq \frac{qm_t}{m_{t+1}} \leq 1+\pi_p^*$$

In Figure 5, OC' is the graph of [X5], and [X6] implies that equilibrium paths must stay on the arc PQ. Therefore, by the same arguments of the text, it can be seen that there is a unique CS equilibrium for each q in $[1+\pi_r^*, 1+\pi_p^*]$, and that these are the only possible equilibria (no shifts across regimes are possible). In addition, if B = 0, the only policy consistent with equilibrium is q = 1, i.e., a constant money supply. Notice that given q, equilibrium is always unique, as opposed to the case in the text.

[INSERT FIGURE 5 HERE]

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Footnotes

[1]The standard reference on CS is Girton and Roper (1981).

[2]This gives precise content to the arguments in Fischer (1982).

[3]See Azariadis (1987) or Sargent (1987) for an introduction to overlapping generation models with money.

[4]Alternatively, let g_t be the difference between government expenditures and lump sum taxes on youthful income, and e, E, denote endowments net of tax.

[5]Notice that short sales of currency are not allowed.

[6]These properties follow from the fact that $v(\pi,y)$ is decreasing in π and increasing in y.

[7]For π_h^* to exist for all $0 < B < e_h$ it is sufficient that $u(c_1,0) = 0$. A sufficient (but not necessary) condition for π^*_h to be decreasing on e_h is that $u(.,.)$ be homogenous of degree one. In this case, $v(\pi,y) = yv(\pi,1)$, and therefore π_h^* is defined by $v(\pi_h^*,1) = (e_h-B)v(0,1)/e_h$.

[8]Equivalently, I assume that first and second period consumption are gross substitutes.

[9]See Cass et al. (1980) or Azariadis and Guesnerie (1983)

[10]In Figure 2, we have used the fact that $s(.,.)$ is decreasing in its first argument, which implies that $m_{t+1}-g$ is an increasing function of m_t.

[11]The proof of this assertion is trivial and follows from (12) and (15) below.

[12] This is a sketch of a formal proof of Proposition 1 : a stationary equilibrium without CS can be defined as a pair (π, g) s.t. $\pi \leq \pi_r^*$ and $g = \pi(1+\pi)^{-1}[Ns(\pi, e_r) + ns(\pi, e_p)]$. [A1] ensures that the RHS of this equation is strictly increasing in π for $0 \leq \pi \leq \pi_r^*$. The Proposition follows.

[13] The model rules out multiplicity of non CS equilibria because the availability of assets with a fixed nonnegative rate of return imposes an upper bound on the rate of inflation that is consistent with equilibrium> The inflation rates associated with nonstationary paths converging to autarky violate that upper bound. The argument is similar to those of Brock and Scheinkman (1980) and Obstfeld and Rogoff (1983), but unlike them we do not rely on the possibility of government backing of fiat money.

[14] Again, the proof of this assertion follows from (14) and (16) below.

[15] Because $F_{T-1} = 0$, $m_{T-1} = Ns(\pi_{T-1}, e_r) + ns(\pi_{T-1}, e_p)$. $F_T = 0$ implies $m_T = ns(\pi_T, e_p) \leq ns(\pi_{T-1}, e_p) < m_{T-1}$. In turn, $1+\pi_T = m_T\,(m_{T+1} - g)^{-1} \geq 1+\pi_r^* \geq 1+\pi_{T-1} = m_{T-1}(m_T - g)^{-1}$, which implies $m_T < m_{T+1}$

[16] The relationship $\Gamma(\pi) = g$ is therefore a monetary Laffer curve

[17] In Figure 1, α_1 is the value of π that maximizes $\Gamma(\pi)$ in (15). By [A1], $\pi_r^* < \alpha_1$. Thus, the role of [A1] is to ensure that the CS equilibrium is unique.

[18] Assumption [A2] implies that $\pi_p^* < \alpha_2$, where α_2 is the maximizer of $\gamma(\pi)$. Thus, [A2] ensures the CS equilibrium is unique.

[19]The consequences of relaxing [A1]-[A2] are easily derived from the reflected offer curves. If $(1+\pi_r^*)$ is smaller than the slope of OB in Figure 2, then for some range of feasible government deficits two non CS equilibria would appear. The equilibrium with lower inflation would be determinate and locally unstable, and the other one locally stable and indeterminate. In other words, this would be similar for the results for a closed economy. However, a unique non CS equilibrium would still obtain for g close to zero. The consequences of relaxing [A2] are analogous: uniqueness of CS equilibria is not guaranteed. Finally, the argument leading to Proposition 5 is not valid, and shifts between regimes may occur. In particular, this may open the possibility of "cycles". In short, a vast multiplicity of equilibria obtains when [A1]-[A2] are relaxed. This is not a shortcoming of my model, but of the overlapping generations structure (Woodford (1984)). Indeterminacy of equilibria also arises if gross substitutability is not assumed. In particular, if the slope of the offer curve is less than minus one at the point where it cuts the 45^o line, then the non CS equilibria are indeterminate. This also opens the way to cyclical equilibria (Azariadis and Guesnerie (1983)).

[20]The question of the revenue maximizing inflation rate has been studied by Friedman (1971), Auerheimer (1974) and Calvo (1978). They assume that the government fixes the rate of growth of the money supply and ask which rate maximizes inflationary revenue. This is a slightly different question than the one I consider in the text. However, the Appendix shows that the main results of the text remain unchanged.

[21]Assuming that the inflation rate in the rest of the world is zero.

[22]This is just a consequence of the Lucas Critique (Lucas (1976)).

[23] It may also be noticed that there may be an incentive for the government to default on the indexed bonds, which may make the bond issues unfeasible.

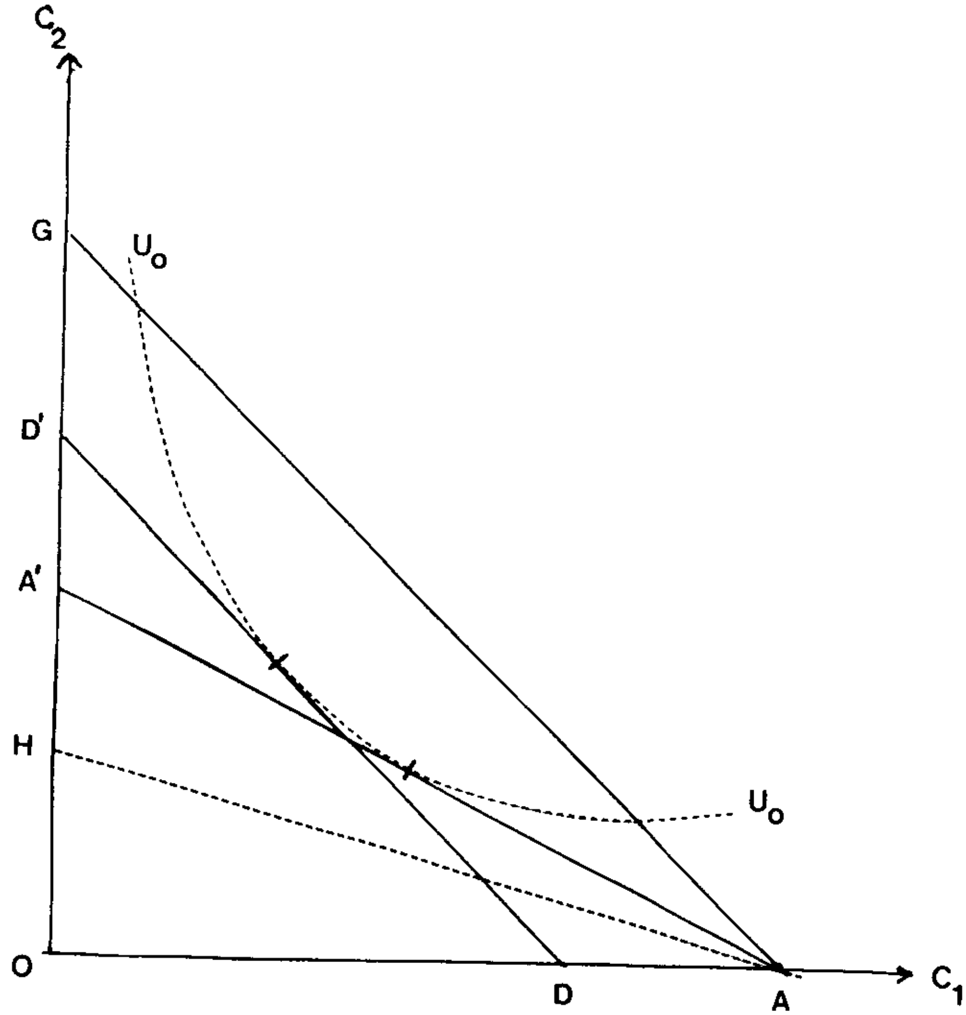

Figure 1

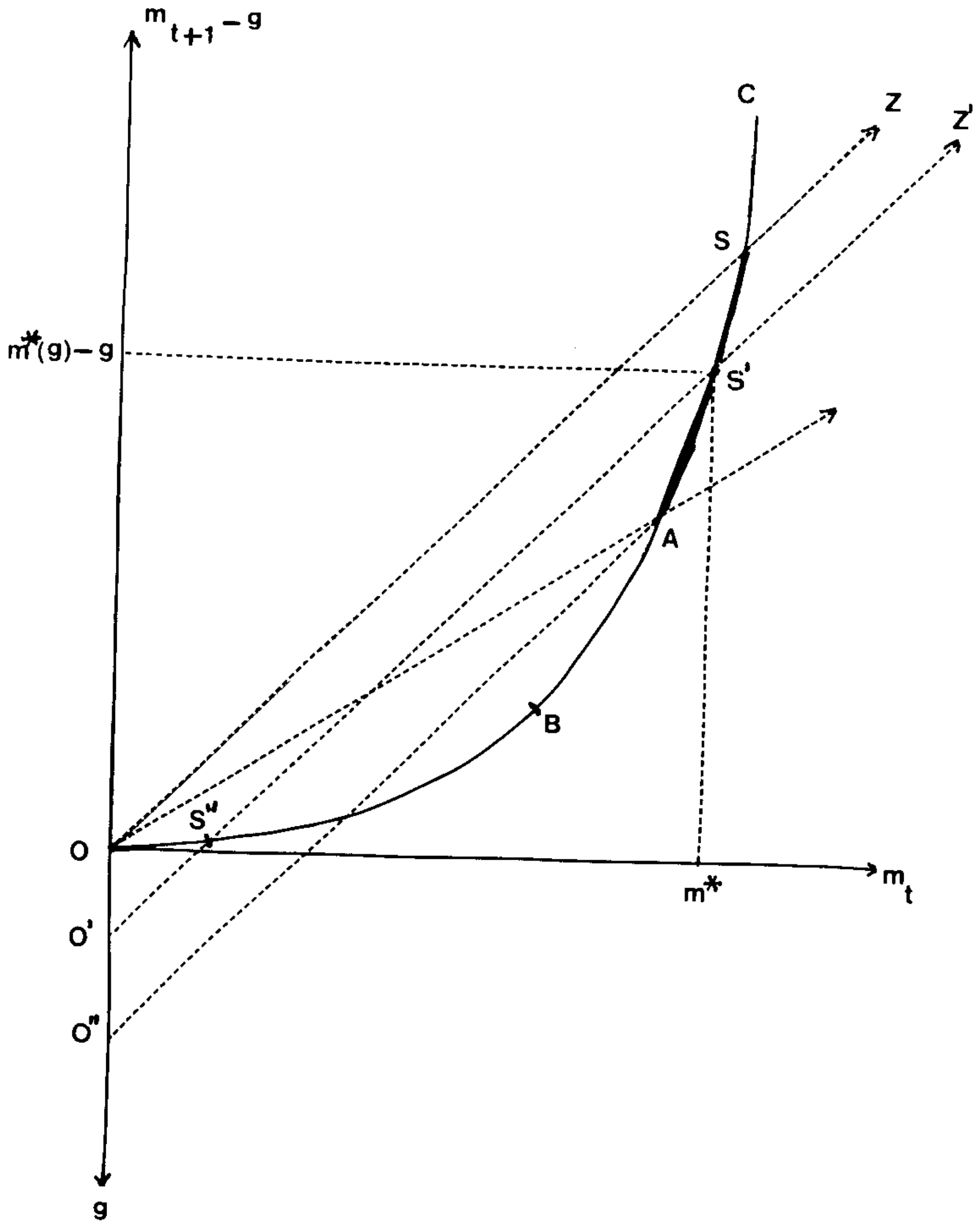

Figure 2

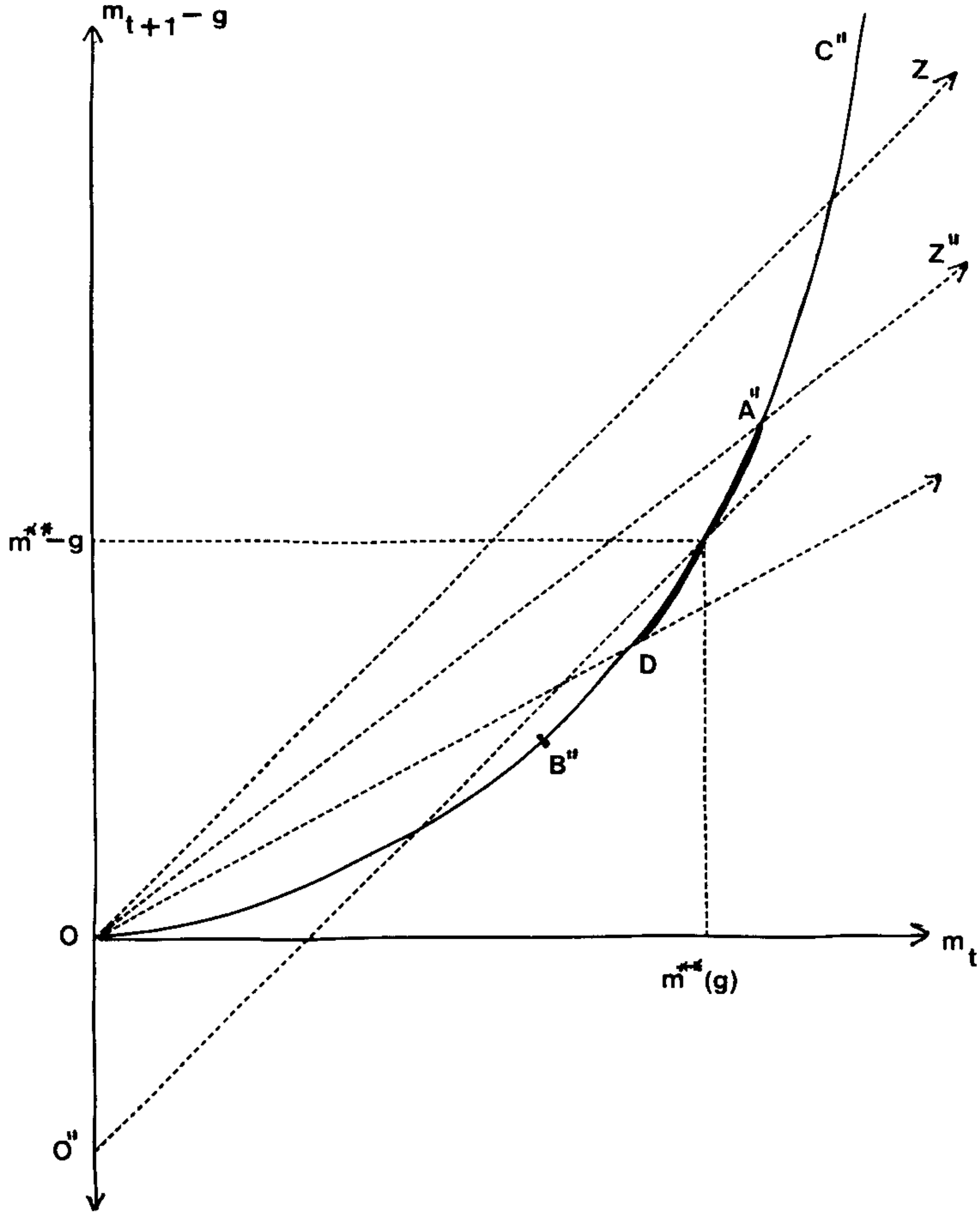

Figure 3

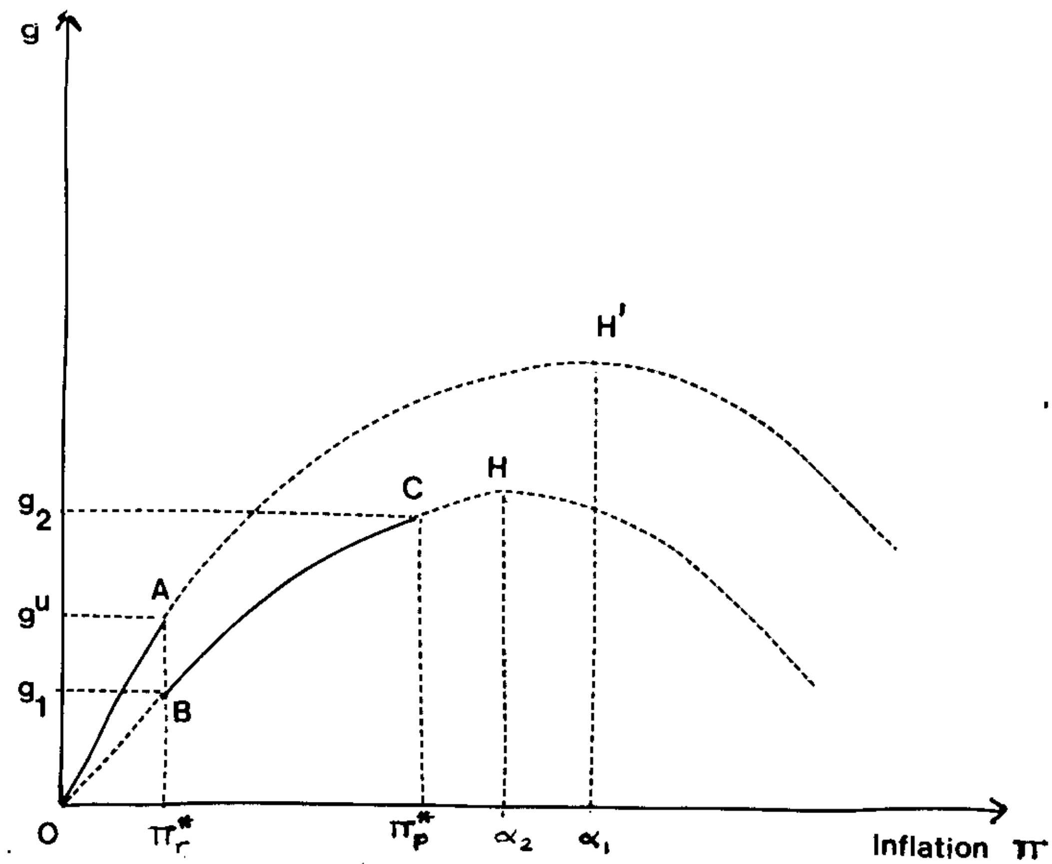

Figure 4

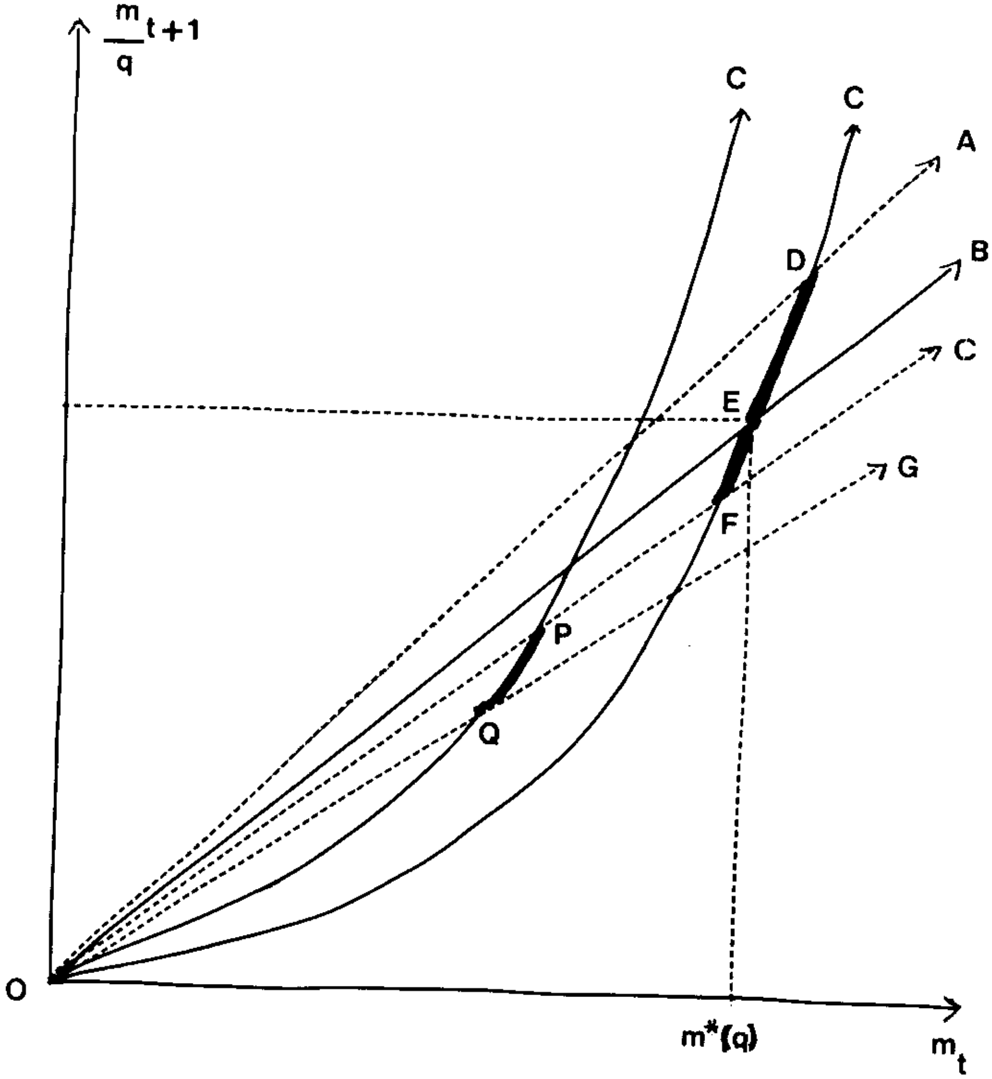

Figure 5