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THE EFFECT OF CHILD CUSTODY AND SUPPORT ARRANGEMENTS ON THE WELFARE OF CHILDREN AND PARENTS†

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ABSTRACT

Under functional form assumptions regarding the preferences of divorced parents, estimates of behavioral parameters are obtained from information on compliance with child support orders. The data utilized are from the Wisconsin Child Support experiment, and are collected from administrative records containing information on the characteristics of parents at the time of divorce, settlements, and subsequent compliance with orders. Our behavioral models perform relatively well in explaining observed compliance data.

In the second part of the analysis, we investigate the correspondence between observed child custody and support orders and those which are optimal under maintained assumptions regarding the objectives of institutional actors who play a role in determining settlements. We find that under some behavioral assumptions, the child support orders predicted from our models are not grossly out of line with those observed, though our models predict that many more cases should have produced joint custody awards.

1. Introduction

When a married couple with children obtains a divorce, at least four agents with differing objectives and resources are immediately involved. Children typically have little in the way of resources, and typically experience a great deal of psychological stress during the process of and following the divorce of their parents;[1] they are also likely to experience substantial decreases in their consumption levels (see, e.g., Beller and Graham (1985)). Mothers and fathers, considered as two separate groups of agents, must be viewed as having diverse objectives following a divorce (and even within intact marriages[2]). Fathers most often have significantly greater financial resources than mothers at the time of and following the divorce (Duncan and Hoffman (1985) and R. Weiss (1984)), though the mother may garner a greater amount of loyality from the children if she has served as their primary caretaker during the marriage.[3] Finally, the legal and social system which defines the rules under which the outcome is determined, and more importantly, has implicit or explicit independent valuations of those outcomes, must be viewed as a fourth agent.[4] What makes the issue of child custody and child

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[1]For a survey of some of the psychological effects of various custody arrangements on children, the reader is referred to the articles in Weithorn (1987).

[2]Current research in household consumption decisions has stressed the role of differences in preferences and resources between members of intact households in accounting for within household consumption allocations (see Manser and Brown (1980), McElroy and Horney (1981), and Chiappori (1988)). In order to compare behavior within and outside the marriage, Weiss and Willis (1985,1989) have posited invariant preferences for mothers and fathers, with pre- and post-divorce behavior differing due to changes in resource allocations (income and custody rights) and bargaining strategies. In this paper, we take the divorce as a given, and therefore assume nothing about pre- and post-divorce preferences of mothers and fathers. Whether or not preferences change following a divorce, the rules under which consumption allocations are determined (e.g., cooperative versus noncooperative behavior) are likely to change following the divorce. Rule changes, as well as resource shifts, account for differences in pre- and post-divorce welfare levels, aside from any redefinition of preferences.

[3]This argument is often advanced as a reason for awarding the mother physical custody, under the "best interest of the child" rationale (see Weitzman (1985), Chapter 8).

[4]Mnookin and Kornhauser (1979) analyze the role of legal institutions in determing final divorce orders through the differential bargaining power given

support determination such a controversial social issue and difficult analytical problem is the fact that the four agents involved are so diverse in terms of objectives and resources.

In this paper we examine the effect of custody and child support orders on the post-divorce welfare levels of these four groups of agents. We begin by specifying the preferences of the parents, which are defined over own consumption and the consumption of child services. Following a divorce, child "quality," as measured by total expenditures, continues to be a public good as it was during the marriage (see Weiss and Willis (1985,1989)). However, the valuation of child quality depends on the amount of time the divorced parent spends with the child; in this manner we can view the post-divorce preferences of the parents as being indexed by the proportion of time the child spends with them. We take it as given that the welfare level obtained with a given level of expenditures on own consumption and total parental expenditures on the child is a function of the amount of time spent with the child for each parent. Given the child support order, custody arrangement, and income levels, the expenditures on consumption by each parent and on the children is determined within a Nash equilibrium framework.

The institutional actor, referred to alternatively as the "judge" or the "court" in what follows, takes the equilibrium responses of the parents into account when deciding the custody and child support arrangements. The institutional agent is assumed to have preferences defined over custody (in some cases) and over total parental expenditures on the child. While the preferences of the child are not explicitly modelled, there is a presumption that the objective of the court reflects them to a substantial degree.[5]

Institutions also play a prominent role in the theoretical and empirical

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to the contestants. Elster (1989) examines the extent to which legal institutions can and should use rational decision rules in adjudicating custody cases. Cassetty (1978) and the papers in Cassetty (1983) look at the role of public policy in defining and enforcing custody and child support orders.

[5] Lazear and Michael (1988, Chapter 8) have set forth child support and alimony guidelines based on estimates from an intrahousehold consumption expenditure model, under several assumptions regarding institutional objectives. In contrast to the work performed here, their recommendations are based on an analysis of expenditure patterns rather than compliance data, and the issue of noncompliance is not explicitly addressed. They provide an excellent discussion of the problems inherent in selecting and appropriate institutional objective.

analysis of divorce settlements conducted by Weiss and Willis [(1985) and especially (1989)]. In their original paper on the subject (1985), the institutional agent's role was primarily to enforce divorce settlements. In their subsequent paper (1989), they focused attention on the role of the institutional agent in settling disputed cases when the mother and father could not come to an amicable agreement (they did not consider the problem of noncompliance explicitly in this analysis). The analysis we conduct here should be considered as complementary to theirs, in that our attention is focused primarily on parental choices regarding compliance with orders and the effectsof divorce settlements on expenditures on children. We make no distinction between settlements reached amicably or adjudicated in an adversarial procedure. Instead we view all settlements as being reached within an environment consisting of legal and political institutions and social norms. While we use the terms "court" and "judge" frequently below so as to avoid constant reference to the vaguely-specified institutional actor, the usage of these terms should not be taken to imply that the relevance of our analysis is restricted to cases in which settlements are adjudicated rather than merely "rubber-stamped" by the court.

In general, when setting the custody and support arrangements, the court must also take into account the contingency that they may not be honored. We assume throughout that there is perfect compliance with custody arrangements, while we allow for the possibility that ordered transfers from one parent to another may be ignored in the model which provides the framework for our analysis of the compliance decision. *Ex ante*, the judge cannot determine whether or not orders will be complied with, and so sets custody and child support optimally using an expected utility criterion. The divorce settlement affects both the expected value of the judge's objective in the compliance and noncompliance states and the probabilities of the two states.

The compliance decision is central to our analysis, since all behavioral parameters are estimated using only compliance information from a sample of individuals under court orders to make child support payments. While a numberof social scientists have investigated compliance empirically (including Chambers (1979), Beller and Graham (1985), Robins (1986), and Garfinkel and Oellerich (1989)), the focus of these studies is usually on the the effects of noncompliance on the post-divorce income allocation between fathers and mothers and the enforcement problem *per se*. We have introduced a number of assumptions regarding the preferences of fathers and mothers so as to better understand the behavioral motivation of noncompliance. With such an understanding, it may eventually be possible to consider how divorce arrangements should be structured so as to increase the welfare of all or a subset of the agents involved in divorce.

Using data from the Wisconsin Child Support experiments, we first report the results of our empirical analysis of the compliance decision. We are able to obtain estimates of the parameters characterizing the preferences of fathers and mothers, as well as of the distribution of a random variable which is instrumental in determining whether there is compliance with a given order. These behavioral estimates, while being of some independent interest, have been obtained primarily for use in determining the optimal custody arrangements and child support orders of institutional agents under various assumptions regarding their objective functions. We then proceed to compare the actual divorce settlements with those observed in the data. Under the assumption that the institutional agents act in an expected utility-maximizing manner, these comparisons shed some light on the nature of its objective function.

The plan of the paper is as follows. In Section 2 we provide an exposition of our modelling assumptions, characterize equilibrium expenditures of the parents on the child given the divorce settlement, and discuss some properties of the institutional agent's decision rules under a variety of assumptions concerning the compliance behavior of individuals ordered to pay child support (i.e., perfectly imperfect compliance, perfect compliance, and uncertain compliance). Section 3 contains a discussion of the properties of maximum likelihood estimators for the behavioral parameters of the model using only data on compliance decisions. In Section 4 we report estimates of probit models of the compliance decision, which include a number of characteristics of the parents not included in the behavioral model, as well as estimates from the structural model. Section 5 contains the results of some simulation exercises intended to assess the congruence between actual divorce settlements and those which would be observed under various objective functions for the institutional agent. Section 6 contains a brief conclusion.

2. Models of Child Support Orders and Welfare Determination

In this Section, we consider three models of the determination of child support orders and the behavioral reactions to those orders on the part of former husbands and wives. All the models discussed below share a number of features.

For analytic tractability, we make a number of simplifying functional form assumptions from the outset. Assumptions such as those imposed below are necessary if one is to produce testable implications regarding the behavior of parents and institutional agents in the aftermath of household dissolution.[6]

The utility function of each parent, which is defined over the consumption levels of a market good and child services, is of the Cobb-Douglas form. That is,

$$[1] \qquad u_p = \alpha_p \ell n(c_p) + (1-\alpha_p)\tau_p \ell n(k),$$

where c_p denotes the amount of consumption of the market good by p,

k denotes the level of child quality,

τ_p is the amount of time the child spends with parent p,

and α_p is a parent-specific preference parameter.

The price of the market good is normalized to unity. We adopt the normalization that the total amount of time available to be spent with the parents is one, so that $\tau_m = 1 - \tau_f$. The level of child services (or child quality) is given by

$$[2] \qquad k = e_f + e_m,$$

where e_p is the expenditure on the child made by parent p.

Each parent has an exogenously determined level of income, I_p. In particular, it is assumed that the level of income received by each parent is

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[6]The assumptions made here produce unique equilibrium levels of expenditures on the child by the parents. The uniqueness property is virtually indispensable if the ultimate object of the exercise is empirical analysis. Furthermore, the uniqueness property is necessary for purposes of analyzing the court's behavior, since it conditions its decisions on the optimal responses of the parents.

independent of the child support order, which is composed of the custody arrangement and, possibly, child support payments [mandated income transfers from one parent to the other].

A. Institutional Agent Sets Custody Arrangements Only

We begin by considering the case in which the institutional agent only has the power of determining the custody arrangement. In terms of our model, by the custody arrangement we mean the determination of the time that the father will spend with the child, τ_f $(=1-\tau_m)$. In what follows, we will define τ as being the proportion of time spent with the father (we will only explicitly subscript τ when it is necessary to avoid ambiguity). Since the institutional agent is assumed to only set τ, the expenditures on the child by the parents are totally "voluntary." We now consider the optimization problems faced by the parents and the institutional agent in such an environment.[7]

A1. The Parents' Child Expenditure Decisions

Each parent takes the time allocation parameter τ and the level of their own income, I_p, as given. In order to determine the equilibrium level of expenditures by both parents, we first specify the artificial problem of utility maximization *given the other parent's levels of expenditures on the child*, $e_{p'}$. Then to find the reaction function of individual p, define the problem

$$[3] \qquad V_p(\tau, I_p; e_{p'}) = \underset{e_p \in \mathbb{R}_+}{\text{maximum}}\ \alpha_p \ell n(I_p - e_p) + (1-\alpha_p)\tau_p \ell n(e_f + e_m); \qquad p=f,m.$$

Instead of working with [3], in which each parent is constrained to make nonnegative contributions to the child, we have chosen to define reaction functions in which parents can feasibly make negative contributions to child quality. Thus we define

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[7]This setting is not necessarily unrealistic even when child support orders are issued, if enforcement of such orders seldom occurs. In the data below, we will see that compliance with child support orders is far from perfect. The assumptions of this particular model may not be grossly out of line with what occurs *de facto*.

[3′] $\tilde{V}_p(\tau, I_p; e_{p'}) = \underset{e_p \in \mathbb{R}}{\text{maximum}}\ \alpha_p \ln(I_p - e_p) + (1-\alpha_p)\tau_p \ln(e_f + e_m)$; $p=f,m$.

Of course, $\tilde{V}_p(\tau, I_p; e_{p'}) \geq V_p(\tau, I_p; e_{p'})$ for all values of $(\tau, I_p, e_{p'})$. The nonnegativity contraints on parental contributions to the child are imposed when defining the Nash equilibrium. In Proposition 1, we demonstrate that the Nash equilibrium we define corresponds to that which would result from using the reaction functions associated with the constrained optimization problem given in [3].[8]

There exists a unique solution e_p^* associated with parent p's optimization problem [3′], which is of the form

[4] $e_p^* = a_p I_p - (1-a_p)e_{p'}$

where $a_p = (1-\alpha_p)\tau_p / [(1-\alpha_p)\tau_p + \alpha_p]$; $p=f,m$.

The functions given in [4], e_f^* and e_m^*, are the reaction functions of the parents.[9] Using these linear reaction functions, it is straightforward to solve for a [unique] fixed point $(\tilde{e}_f, \tilde{e}_m)$ defined as

[5]
$$\tilde{e}_f = e_f^*(\tilde{e}_m; I_f, \tau)$$
$$\tilde{e}_m = e_m^*(\tilde{e}_f; I_m, \tau).$$

Under our functional form assumptions, we find

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[8] The analysis of the effect of child support orders and the distribution of income across parents on total expenditures on the child is considerably simplified when the objective functions of the parents do not directly depend on the custody arrangement and when only interior solutions (where both parents spend money on the child) are considered, as in Weiss and Willis (1985). We have analyzed a more complex case because of our belief that incentives to comply with child support orders may be directly affected by custody arrangements, and due to the necessity of defining a model which was valid globally (i.e., for all values of parental income, custody arrangements, and child support orders) because of our primary goal of econometric implementation.

[9] As discussed above, for some values of τ, I_f, and I_m, either e_f^* or e_m^* can be negative. As long as $\tau \in (0,1)$, both cannot be negative, since total expenditures on the child would be negative, implying that the value of each parent's utility would be $-\infty$.

$$\tilde{e}_f = d(a_f I_f - (1-a_f)a_m I_m)$$

[6]

$$\tilde{e}_m = d(a_m I_m - (1-a_m)a_f I_f),$$

where $d = \{1-(1-a_f)(1-a_m)\}^{-1}$.

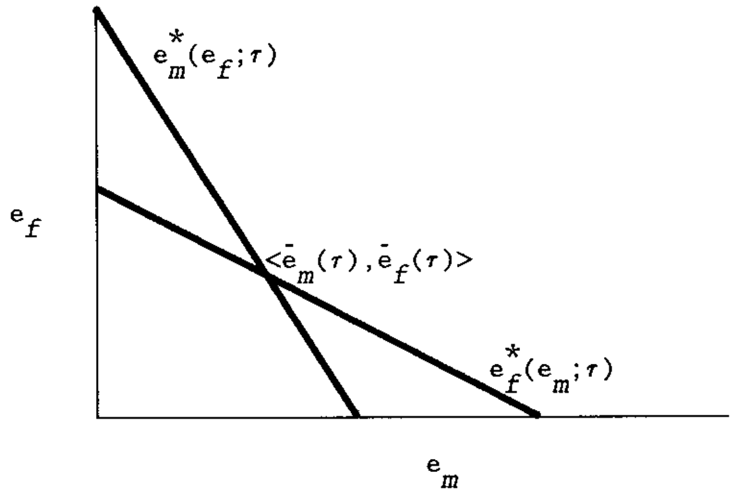
The pair $(\tilde{e}_f, \tilde{e}_m) \notin \mathbb{R}^2_+$ for some values of the parameters $\{\tau, I_f, I_m\}$. Since it is not feasible to spend negative amounts of income on a good, any equilibrium concept must insure that both the expenditures of the father and the mother on the child are nonnegative. The Nash equilibrium in child expenditures we now describe satisfies this feasibility condition.

Before stating the proposition formally, we illustrate the qualitative features of the Nash equilibrium produced by our modelling assumptions. Figure 1 contains a depiction of two equilibria, one in which both parents make positive expenditures on the child and one in which only one parent does. Figure 1a represents an example of a Nash equilibrium in which both parents make a positive expenditure on the child, since the intersection of the two linear reaction functions occurs in $\mathbb{R}_{++}$. In Figure 1b, the intersection of the reaction functions occurs at a point for which desired expenditures of the father are negative. Once we impose the nonnegativity constraint on the father's expenditure, we move along the mother's reaction function to the point it intersects the horizontal axis; this point becomes the stable Nash solution to the problem. An analagous argument holds for the case in which the mother's desired expenditures are negative. The Nash equilibrium in this case occurs at the intersection of the father's reaction function and the vertical axis. Also note that the "type" of equilibrium (one parent making expenditures versus both) depends on the income distribution, the preference parameters, and the custody arrangement.

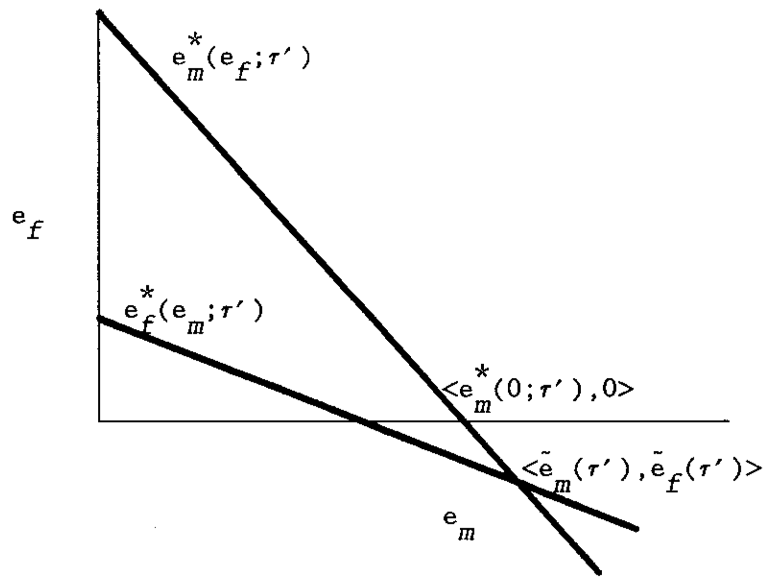
FIGURE 1

Determination of Nash Equilibrium Child Expenditure Levels

(1a) Illustration of an Interior Solution



(1b) Illustration of a Corner Solution



Proposition 1 *For all triplets* (τ, I_f, I_m) *there exists a unique level of expenditures on child quality* $(\bar{e}_f, \bar{e}_m)(I_f, I_m, \tau)$ *of the following form:*

$$
[7] \qquad
\begin{array}{ll}
(\bar{e}_f = 0\ ;\ \bar{e}_m = a_m I_m) & \textit{iff } \tau < b_f(I_f, I_m) \\
(\bar{e}_f = \tilde{e}_f\ ;\ \bar{e}_m = \tilde{e}_m) & \textit{iff } b_f(I_f, I_m) \le \tau \le b_m(I_f, I_m) \\
(\bar{e}_f = a_f I_f\ ;\ \bar{e}_m = 0) & \textit{iff } \tau > b_m(I_f, I_m)
\end{array}
$$

where b_f *is defined as* $0 = \tilde{e}_f(I_f, I_m, b_f)$,

and b_m *is defined as* $0 = \tilde{e}_m(I_f, I_m, b_m)$.

Proof: Since $\partial e_f^*/\partial e_m = -\ \alpha_f/[\alpha_f + (1-\alpha_f)\tau]$ and $\partial e_f/\partial e_m^* = -\ [\alpha_m + (1-\alpha_m)(1-\tau)]/\alpha_m$, $|\partial e_f^*/\partial e_m| < |\partial e_f/\partial e_m^*|$. Thus the reaction function of the mother cuts the reaction function of the father from above (see Figure 1). At the point of intersection of the two reaction functions, at least one parent must be making positive expenditures on the child. If the intersection occurs in $\mathbb{R}_{++}$, then the point of intersection is the stable Nash solution and both parents make positive expenditures on the child. If the intersection occurs at a point for which $\tilde{e}_f < 0$, then impose the nonnegativity constraint on the father's expenditures. If he spends zero, the mother's best response is given by her reaction function evaluated at zero expenditures by the husband. At this expenditure level, the father's desired expenditures remain negative, so that the stable Nash solution is at $\langle \bar{e}_f = 0, \bar{e}_m = e_m^*(0; I_m, \tau) \rangle$. If the intersection occurs at a point for which $\tilde{e}_m < 0$, a similiar argument establishes that the stable Nash solution is at $\langle \bar{e}_f = e_f^*(0; I_f, \tau), \bar{e}_m = 0 \rangle$.

The function $\tilde{e}_f$ $(\tilde{e}_m)$ is monotone increasing (decreasing) in τ. Since $\tilde{e}_f(I_f, I_m, 0) < 0$, and $\tilde{e}_f(I_f, I_m, 1) > 0$, there is a unique value b_f: $0 = \tilde{e}_f(I_f, I_m, b_f)$. Since $\tilde{e}_m(I_f, I_m, 0) > 0$ and $\tilde{e}_m(I_f, I_m, 1) < 0$, there is a unique value b_m: $0 = \tilde{e}_m(I_f, I_m, b_m)$. The ordering of τ with respect to b_m and b_f then establishes the qualitative nature of the equilibrium. ■

Given the decision rules [7], it is straightforward to conduct comparative statics exercises on equilibrium child expenditure levels. In particular, it is easy to show that that child expenditures by parent p are a nondecreasing function of his or her income and are a nonincreasing function of the income of parent p′. Also, child expenditures by parent p are a nonincreasing function of α_p and are a nondecreasing function of $\alpha_{p'}$.

Using these results establishing the existence and uniqueness of a Nash equilibrium in child expenditures by the parents, and the monotonicity of the equilibrium expenditure functions in income and preference weights, we are ready to consider the institutional agent's optimization problem.

A2. The Court's Custody Decision

As mentioned in the introduction, the institutional agent's objective function is extremely problematic to specify, primarily due to the difficulty in precisely identifying this entity. We will limit the class of objective functions considered to the following:

$$[8] \qquad u_j = u_j(\tau, e_f + e_m) = u_j^1(\tau) u_j^2(e_f + e_m),$$

where u_j^2 is quasi-concave in total child expenditures.

In the remainder of the analysis of the optimal decision of the institutional agent performed in this Section, we will set $u_j(\tau) = 1$, $\tau \in [0,1]$, so that the judge has no concern about the custody arrangement other than the effect of the custody arrangement on the sum of the expenditures on the child (this assumption is not required for the validity of any of the empirical analysis reported in Section 4, and is not imposed in performing many of the simulations reported in Section 5). The purpose of our analysis will be to show that *even if* the judge is concerned only with expenditures, optimal custody arrangements may result in joint custody or mother or father custody depending on the characteristics of the parents, which in this anal-ysis are limited to preference weights and income levels. In practice, insti-tutional actors seem to make custody decisions in large part based on their perceptions of social norms, their understanding of psychological theory, and historical precedent.[10] Any assumptions we could make regarding such notions would have

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[10]See Chapters 8 and 9 of Weitzman (1985) for descriptive evidence regarding the rules judges report they use in making custody and child support decisions. Even when cases are not disputed, family law attorneys play a large role in the settlements agreed to. In advising clients, these agents appear

an immediately apparent effect on our conclusions. We leave the investigation of such issues to another time, and focus here on the types of institutional decisions expected to be observed under a strict objective of maximizing expenditures on the child.

Since u_j^2 is quasiconcave, without loss of generality we can specify the objective of the judge to be the maximization of total expenditures on the child.[11] When the judge's choice is limited to the determination of the custody arrangement, the outcome is described as follows.

Proposition 2 *The judge will either assign full-time custody to the mother or the father, or will order joint custody, where the proportion of time with the father is given by* $\tau^* = \arg\max \{\tilde{e}_f(\tau)+\tilde{e}_m(\tau)\}$. *Define* $V_s = \tilde{e}_f(\tau^*)+\tilde{e}_m(\tau^*)$. *Then the optimal choice of the judge is*

$$[9] \qquad \tau_j^* = \begin{cases} 1 & \text{iff } I_f/I_m > (1-\alpha_m)/(1-\alpha_f) \text{ and } (1-\alpha_f)I_f > V_s \\ \tau^* & \text{iff } V_s > (1-\alpha_f)I_f \text{ and } V_s > (1-\alpha_m)I_m \\ 0 & \text{iff } I_f/I_m < (1-\alpha_m)/(1-\alpha_f) \text{ and } (1-\alpha_m)I_m > V_s \end{cases}$$

Proof: From Proposition 1, for any custody arrangement $\tau \in [0,b_f]$, only the mother spends positive amounts on the child, and her total expenditures are $a_m(\tau)I_m$. Since a_m is a decreasing function of τ, the maximum level of expenditures over this interval is at $\tau = 0$. By the same argument, for all $\tau \in [b_m,1]$, only the father spends positive amounts on the child, which are equal to $a_f(\tau)I_f$. Since a_f is an increasing function of τ, maximal expenditures over this interval occur at $\tau = 1$.

Consider the problem of maximizing the quantity

$$[10] \qquad \tilde{e}_f(\tau) + \tilde{e}_m(\tau) = \frac{a_m(\tau)a_f(\tau)}{1-(1-a_m(\tau))(1-a_f(\tau))} [I_m+I_f].$$

There is a unique solution to this problem,

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to be similiarly influenced by their perceptions of social norms and the rules judges use in adjudicating disputes.

[11] This is the case because the compliance is certain in this model. This statement is not true in the case of uncertain compliance.

$$[11] \qquad \hat{\tau} = \delta/(1+\delta), \quad \delta = \left[\frac{(1-\alpha_m)\alpha_f}{(1-\alpha_f)\alpha_m}\right]^{1/2}.$$

If $\hat{\tau} \notin [b_f, b_m]$, then the function $\tilde{e}_f(\tau)+\tilde{e}_m(\tau)$ is monotone on the interval $[b_f, b_m]$, and the maximum value of total expenditures on the interval is given by $\max[\tilde{e}_f(b_m), \tilde{e}_m(b_f)]$. If the maximum over the interval is $\tilde{e}_f(b_m)$, this quantity is exceeded by $(1-\alpha_f)I_f$. If the maximum over the interval is $\tilde{e}_m(b_f)$, this quantity is exceeded by $(1-\alpha_m)I_m$. Then if $\hat{\tau} \notin [b_f, b_m]$, the optimal arrangement cannot be joint custody. If $\hat{\tau} \in [b_f, b_m]$, then $V_s = \tilde{e}_f(\hat{\tau}) + \tilde{e}_m(\hat{\tau})$. If V_s exceeds both $(1-\alpha_m)I_m$ and $(1-\alpha_f)I_f$, joint custody is optimal, with the proportion of time assigned to the father equal to $\hat{\tau}$. ■

We depict two examples of the objective function of the institutional agent in Figure 2. In 2a, the court chooses sole custody of the mother, while in 2b, the optimal choice is joint custody. From Proposition 2, note that in a population in which all mothers share a common value of α_m and all fathers share a common value of α_f, joint custody awards will specify a common split of time between fathers and mothers (i.e., the division of time will be invariant with respect to the incomes of the parents). Of course, the parental distribution of income will determine whether or not joint custody is awarded.

B. Institutional Agent Sets Custody and Child Support Payments with Certain Compliance

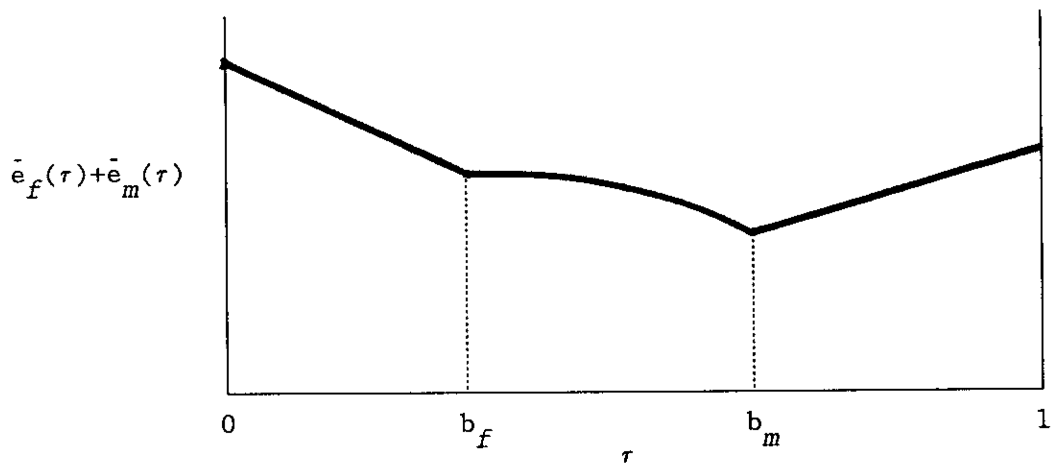
We now briefly consider the unrealistic case in which the court sets both the time spent with the father, τ, and the amount of money transfered from the father to the mother for the purpose of child support, T, and where the parents are assumed to comply with the order with probability one. Perfect compliance should not be taken too literally, however, since we will allow whichever parent receiving the subsidy to allocate it in a way consistent with the maximization of his or her own preferences, as we will continue to do throughout the paper. Note that the amount transfered from the father to the mother can be negative, indicating that the mother has been ordered to pay child support to the husband.

In this model, the judge is assumed to have discretion in ordering the pair (τ, T) subject only to the following constraints. As in model A, the

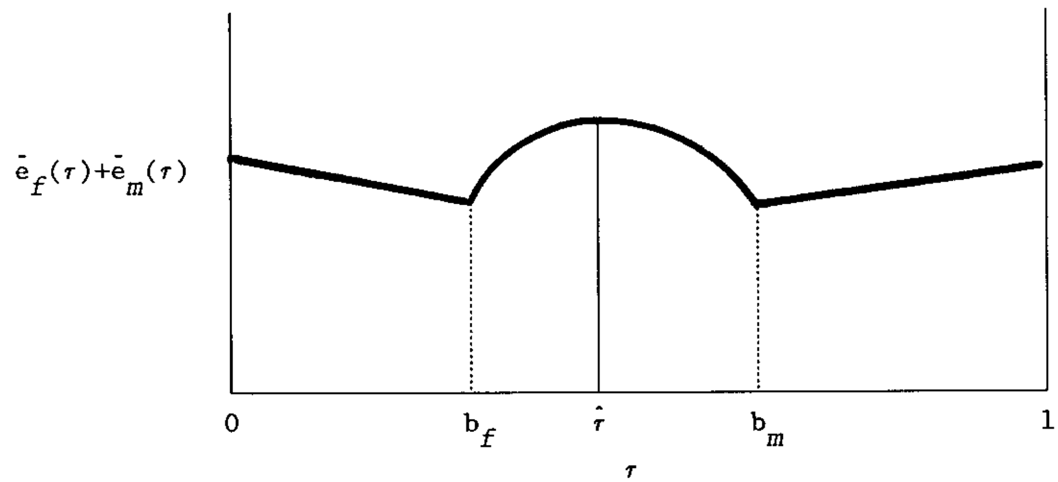
FIGURE 2

Total Expenditures on the Child as a Function of the Custody Arrangement

(2a) Maximum at Corner



(2b) Maximum at Interior



total time endowment is one, so $\tau \in [0,1]$; in addition, it must be the case that $T \leq I_f$ if $T > 0$ and $-T \leq I_m$ if $T < 0$. This ensures that the parent ordered to pay child support possesses the income to allow compliance with the order. Since the parent's behavior in this model includes no new considerations beyond those introduced in model A, we will turn immediately to the court's problem.

We will continue to assume that the court's only concern is in the maximization of dollar amounts spent on the child by both parents. If the court imposes *no child support orders*, then the court's problem reduces to that considered in Section A. Therefore, the court will only impose orders to increase the total amount of child expenditures *given the custody arrangement*, τ. Before turning to the details of the court's decision, we should bear the following fact in mind. If, in the voluntary contribution situation [Model A], the father spends nothing on the child, then ordering a transfer of income from the father to the mother will increase child expenditures. If the mother spends nothing on the child, then transfers from her to the father will increase child expenditures. If both spend positive amounts on the child voluntarily, transfers will reduce the expenditures by one parent and increase those of the other. These properties are reflected in the following.

Proposition 3 *Under the assumption of perfect compliance with child support orders, the optimal order is* $\{\tau=0, T=I_f\}$ *when* $\alpha_f > \alpha_m$ *, and is* $\{\tau=1, T=-I_m\}$ *when* $\alpha_f < \alpha_m$.

Proof: Under joint custody, when both parents make positive expenditures on the child, total expenditures on the child are only a function of the sum of the parents' income, so that the optimal transfer is indeterminant (see [10]). Under joint custody, when one parent makes no expenditure on the child, total expenditures on the child will be increased if that parent's income is transferred to the parent making positive expenditures; in this situation, total expenditures can be increased further by transferring all custody to the parent making positive expenditures. By this argument, the only possible arrangements to consider are sole custody of one parent who receives the other parent's total income in child support, or joint custody in which both parents make postive expenditures on the child so that the child support order is indeterminant.

The maximum amount of child expenditures under sole custody with a child support order as defined above is $\max\{(1-\alpha_f),(1-\alpha_m)\}[I_f+I_m]$. Under joint custody, when both parents voluntarily make expenditures on the child, the maximal level of child expenditures is given by V_s. Fix a value of α_f, and let $\alpha_m = \alpha_f = \alpha$. Then the maximal amount of child expenditures under sole custody is $(1-\alpha)[I_f+I_m]$. Under joint custody, $V_s = \{(1-\alpha)/(1+3\alpha)\}[I_f+I_m]$, so expenditures on the child are always greater under sole custody with orders than under joint custody when $\alpha > 0$. Now let $\alpha_f \neq \alpha_m$, and write $V_s(\alpha_f,\alpha_m)$. Note that V_s is symmetric, so that $V_s(x,y) = V_s(y,x)$, and that V_s is a decreasing function of both arguments. Then since $(1-\alpha)[I_f+I_m] > V_s(\alpha,\alpha)$, $\max\{(1-\alpha_f),(1-\alpha_m)\}[I_f+I_m] > V_s(\min(\alpha_f,\alpha_m),\min(\alpha_f,\alpha_m)) > V_s(\alpha_f,\alpha_m) = V_s(\alpha_m,\alpha_f)$. ∎

Of course, we do not observe such "extreme" child support orders in practice. There are several reasons why this may be the case, the first and most obvious being that we have misspecified the preferences of the institutional agent. Our assumption that the welfare of parents is of no direct concern to the institutional actor is no doubt inaccurate to some extent. However, in the remainder of the paper we will maintain this assumption, and examine another reason for the choice of lower levels of child support orders, the possibility of noncompliance.

C. Institutional Agent Sets Custody and Child Support Payments with Uncertain Compliance

In order to analyze the case in which we are ultimately interested, that of uncertain compliance, it is necessary to slightly modify the framework employed up to this point. The specification of the problem we have chosen has been primarily influenced by the data available to us. The reader should be aware that the approach taken here is only one of many which could profitably be considered.

C1. The Decision of Whether to Comply with a Child Support Order

The first question which must be addressed is, given an order $\{\tau,T\}$, what are the reasons for and results of noncompliance? We shall continue to assume that no violation of the custody arrangement, τ, is possible. The only issue regarding compliance is whether the parent ordered to transfer income actually satisfies this legal obligation.

For simplicity, assume that the father has been ordered to transfer income to the mother for purposes of child support [by far the most common type of order]. Given the custody arrangement, in the absence of a child support order, the optimal expenditures by the father will be $\tilde{e}_f(I_m,I_f,\tau)$, which is defined in Proposition 1. In this case, total expenditures on the child will be $\tilde{e}_m(I_m,I_f,\tau)+\tilde{e}_f(I_m,I_f,\tau)$, the maximal value of which is given in Proposition 2 for the case of no child support orders.

A judge will only issue orders when he or she desires an increase in the total expenditures on the child in excess of what would be received in Nash equilibrium with no stipulated transfers [determined in Proposition 1]. To accomplish this, the father must be ordered to pay in excess of $\tilde{e}_f(I_m,I_f,\tau)$, since any transfer to the wife will only be partially dedicated to the child's consumption. This problem of the wife's taxation of child support payments has been referred to by Weiss and Willis (1985) as the monitoring problem, though in our case, it seems equally apt to refer to it simply as the dead weight loss associated with child support transfers from the point of view of the institutional agent. The institutional agent must order enough of a transfer to increase total expenditures, which means that after making the transfer, the husband will not spend further income on child consumption. Then all expenditures on child consumption will be made by the mother, whose income after receiving the transfer is I_m+T. In this case, her expenditures on the child are determined from her reaction function, evaluated at income I_m+T and a father's child expenditure level of 0. By this argument, the value of compliance from the point of view of the father, is

$$[12] \qquad V_c(\tau,T) = \alpha_f \ell n(I_f-T) + (1-\alpha_f)\tau\ \ell n(e^*_m(I_m+T,0)).$$

While the judge's optimal child support order may leave the father making no voluntary expenditures on the child when he complies with the order under our maintained assumptions regarding the judge's behavior, we are of course uncertain regarding the actual decision rules used by the judge. Especially when it comes to specifying an econometric model of compliance decisions, *in which the custody arrangement and support order are taken as given*, it is necessary to allow for the possibility that the father would make voluntary contributions to the child even after honoring his obligations under the child support order. In this case, we have

$$[12'] \quad V_c(\tau,T) = \alpha_f \ell n(I_f - T - \bar{e}_f(I_f - T, I_m + T, \tau)) + (1-\alpha_f)\tau \ \ell n(\bar{e}_f(I_f - T, I_m + T, \tau) + \bar{e}_m(I_f - T, I_m + T, \tau))$$

We now must confront the difficult problem of precisely specifying what is meant by noncompliance. In the data analyzed below, all child support orders involve transfers from the father to the mother.[12] For *any* possible values of $\{\tau, T, \alpha_f, \alpha_m\}$, the father's welfare under noncompliance, V_n, will be at least as great as his welfare under compliance, V_c, when $T \geq 0$. Conversely, if the mother was ordered to pay child support, her value of noncompliance will always be at least as large as her value of compliance. Of course in either case, the parent order to receive the transfer is a passive agent, always strictly preferring receipt of the transfer to not receiving it. Clearly then, for the nonpassive agent to comply with a child support order, the utility function for the agent must be modified. We take a very straightforward approach in proposing a modification, so as to make the econometric implementation of the model as tractable as possible.[13]

First of all, we will take compliance to be an all-or-nothing phenomenon. An implication of this assumption is that all individuals under child support orders should fully pay them or not pay them at all. A significant proportion of individuals under orders only pay them in part, an observation which would appear to be inconsistent with our assumption. There may be some rationalization of this behavior by noting that compliance is defined over a period of time, which is typically greater than one year in the data analyzed below.

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[12] We have not included any cases in which the father was awarded custody of the child. Even in such cases, it is rare for a noncustodial mother to be asked to pay child support to the custodial father.

[13] Our usage of unspecified costs of noncompliance roughly parallels the manner in which Moffitt (1983) introduces fixed psychic costs of participation in welfare programs to study the problem of nonparticipation of eligible population members. Dubey *et al.* (1989) employ nonmonetary punishments for bankruptcy in their study of the general equilibrium effects of the phenomenon. As is the case here, they resort to this device for reasons of tractability.

Compliance may be a period by period phenomena, so that individuals either perfectly comply each period or transfer no money. Since we use time averages, partial compliance is consistent with this interpretation. Since our model is a static one, we will not be able to pursue this point further here.

If the individual does not comply with the order, we specify his net welfare to be

$$[13] \qquad V_n(\tau,T) = \alpha_f \ell n(I_f - \bar{e}_f(I_f,I_m,\tau)) + (1-\alpha_f)\tau \; \ell n(\bar{e}_f(I_f,I_m,\tau) + \bar{e}_m(I_f,I_m,\tau)) - \xi \; , = \tilde{V}_n(\tau,T) - \xi \; .$$

where ξ is a fixed (welfare) cost of deviating from the award. From our previous assumptions, and since ξ is assumed to be exogenous with respect to the father's allocation decision, it follows that [13] is the highest level of welfare the father can obtain if he chooses not to comply.

The father's choice of whether to comply is given by the following rule:

$$[14] \qquad \text{Comply with } (\tau,T) \Leftrightarrow V_c(\tau,T) \geq V_n(\tau) \; .$$

This rule will be used in defining estimators of the distributions of α_f, α_m, and ξ in the population.

C2. The Judge's Problem when Noncompliance is Possible

Given knowledge of the values of α_f , α_m , and ξ, as well as the values of I_f and I_m , the judge faces no uncertainty as to whether a particular child support order will be honored. Thus, if the judge had full information regarding the parameter values and values of the state variables for a particular case, he or she would presumably optimize their objective *subject to a known compliance "constraint."* If this were the case, no noncompliance would be observed.

Noncompliance with child support orders is common (approximately one-half of the individuals in the sample analyzed below do not comply). As is amply documented in numerous accounts of the actual process of custody and child support award determination, judges, attorneys, and other agents from legal or social welfare institutions face substantial uncertainty regarding the tastes and resources of fathers and mothers. In our simple model, uncertainty may enter only through the preference parameters of the parents, or the psychic cost to the father of not complying with a child support order.[14] We now explore the implications for the judge's decision of allowing uncertainty in each of the three parameters which characterize the compliance decision of the father.

It is necessary to introduce heterogeneity into the model if we are to obtain well-defined estimators for the behavioral parameters. It is also necessary if we are to conduct an analysis of the judge's decision problem when noncompliance can occur. It is interesting to note that the introduction of heterogeneity through the various parameters, while of little consequence econometrically,[15] is far from inconsequential behaviorally.

First, consider the case in which the judge knows the preference parameters, which are invariant across all cases he hears, but is uncertain as to the psychic cost of noncompliance for any given father. The judge is assumed to have perfect knowledge as to the distribution of ξ, G_ξ, in the population. We now generalize the judge's objective [8] to the case of uncertain compliance. Maintaining the assumption that the judge derives no direct satisfication from a particular custody arrangement [i.e., $u_j^1(\tau) = 1 \ \forall \ \tau$], the judge's expected utility from a given custody arrangement and child support order is

$$[15] \qquad E_\xi \, u_j = P(C|\tau,T) \; u_j(\bar{e}_f(C)+\bar{e}_m(C)|\tau,T) + (1-P(C|\tau,T)) \; u_j(\bar{e}_f(N)+\bar{e}_m(N)|\tau),$$

---

[14] We continue to assume throughout that the judge has complete information regarding each parent's income. In our static model, we do not address the incentives for noncompliance which occur when the custodial parent's income increases and/or the noncustodial parent's income decreases. While decreases in the income of the noncustodial parent after a settlement account for some failures to comply, a substantial number of fathers under child support orders fail to comply even when their income remains constant or increases after the settlement.

[15] In the sense that there is no *a priori* reason to prefer that one parameter be allowed to have a nondegenerate distribution in the population rather than any other. Of course, one particular parameterization may produce results more in line with our expectations than another, or may be preferable for reasons linked to tractability of the estimator. We will return to these issues in the following Section.

where C and N denote the events of compliance and noncompliance, respectively, and where we have dropped the superscript "2" on the judge's utility function for notational simplicity. The judge's decision affects both the probability of compliance, and the value of the judge's objective under the two possible outcomes. Note that under the event of noncompliance, when the child support order is ignored, only the custody arrangement determines the total amount of expenditures on the child. Under compliance, both τ and T affect the total expenditures, in general.

The judge's problem is

$$[16] \qquad V_j(I_f, I_m) = \underset{\{\tau, T\}}{\text{maximum}}\ E_\xi\ u_j\ .$$

Assuming differentiability of $E_\xi\ u_j$ for the moment, and fixing the value of τ at one of its two possible values, the first order condition for T given τ is

$$[17] \qquad 0 = P_T(C)\{u_j(C) - u_j(N)\} + P(C)[\partial u_j(C)/\partial T],$$

where $P_T(C)$ denotes the partial derivative of the compliance probability with respect to the child support order.

Proposition 4 *When compliance is probabilistic, the optimal child support order does not belong to the set* $\{-I_m, I_f\}$.

Proof: The sign of the child support order determines which party is the decision-maker and which is passive; if the sign if positive, the father is the decision-maker, and if negative, it is the mother. For any given τ, the probability of compliance with an order of either $-I_m$ or I_f is identically zero, since G_ξ is assumed to be absolutely continuous. Thus at either of these orders, the value of the judge's problem is the same as when $T = 0$.

For any optimal award, it must be the case that $u_j(C) > u_j(N)$. For $T > 0$, we have from [17] that the derivative of the court's objective function with respect to T is $P_T(C, T{=}I_f)[u_j(C, T{=}I_f) - u_j(N)]$, which is negative for all τ for which a transfer from father to mother is optimal. Since the judge's objective function is everywhere differentiable (with respect to T) on the interval $[-I_m, I_f]$, the maximum value of the objective cannot occur at I_f. For $T < 0$, the derivative of the court's objective function with respect to T is $P_T(C, T{=}-I_m)[u_j(C, T{=}-I_m) - u_j(N)]$, which is positive for all τ for which a transfer from mother to father is optimal. Then the optimal order must not stipulate a transfer $-I_m$ when the mother is ordered to pay the father. ■

Proposition 4 merely serves to demonstrate that direct concern for the welfare of parents on the part of the institutional agent is not necessary to explain why "extreme" orders such as those produced in Proposition 3 are not observed. When compliance is imperfect, the institutional agent must consider the effect of the order on the welfare of the agent ordered to pay when making an optimal choice of arrangements. Because there cannot be compliance when an agent is ordered to transfer all of his or her income to the other, such an arrangement can never (optimally) be ordered.

Because of the complicated form of the compliance probability, not much can be said concerning the nature of the solution(s) to the institutional agent's problem in the probabilistic compliance case. We do know that the agent's objective is everywhere differentiable in both τ and T, and that it is continuously differentiable almost everywhere (i.e., except at a countable number of points which correspond to values of τ and T at which the father or mother shift between regimes of positive and zero expenditures on the child). Therefore, any solution(s) to the institutional agent's problem are solutions to the first order conditions almost surely. There may exist a continuum of such solutions.

In the remainder of the paper, we restrict the custody choices of institutional agent to $\tau \in \{.1,.5\}$, which correspond to mother's custody and joint custody, respectively. For each value of τ, we find the optimal choice(s) of T, denoted $T^*(\tau)$, which are solutions to [17] with probability one. Let the value of the court's problem at any one of the elements of $T^*(\tau)$ be given by $V_j^*(\tau)$. Then the solutions to the optimal custody-support problem are

[18] $$\tau^* = \underset{\tau\in\{.1,.5\}}{\operatorname{argmax}} V_j^*(\tau)$$

$$T^* = T^*(\tau^*) \ .$$

Note that there may exist multiple solutions in terms of custody arrangements and transfers conditional on custody arrangements, that is, τ^* and T^* can be set-valued. The fact that there may exist multiple solutions to the judge's problem is not troublesome from the point of view of the econometric model formulated in Section 3, since all estimation is done conditional on the judge's decision. Behaviorally, it is important to recognize that in this model, the judge always prefers the event of compliance to noncompliance.

Consider a generalization of the above model of probabilistic compliance which allows for population heterogeneity in preference parameters. For econometric reasons, we will only allow heterogeneity in one of the parent's preference parameters, in this case, the mother's. Let the distribution of α_m be given by G_m, assumed to be absolutely continuous over some set $(a,b) \subseteq (0,1)$. The father is assumed to know the mother's value of α_m, while the judge does not.

In this problem, it is straightforward to generalize the judge's objective from [15] as

$$[19] \qquad E_m\, E_\xi\, u_j = \int_0^1 \{P(C;\alpha_m|\tau,T)\; u_j(\bar{e}_f(C;\alpha_m)+\bar{e}_m(C;\alpha_m)|\tau,T)$$

$$+ (1-P(C;\alpha_m|\tau,T))\; u_j(\bar{e}_f(N;\alpha_m)+\bar{e}_m(N;\alpha_m)|T)\}\; dG_m(\alpha_m).$$

By a slight extension of the arguments given in the proof of Proposition 4, this function is continuously differentiable almost surely. Therefore, all solutions to the optimal child support order problem (conditional on custody) are solutions to $\partial[E_m E_\xi u_j]/\partial T = 0$ with probability one. The first order conditions for this problem are linear combinations of the first order conditions given in [17] (for various values of α_m). Since each of these was associated with values of T^* in the open interval $(-I_m, I_f)$, the first order conditions associated with this problem produce solutions in $(-I_m, I_f)$. Given the values associated with the solutions to the first order condition [17] when custody is held constant, we can determine optimal custody arrangements as defined in [18].

Allowing heterogeneity in more than one parameter does not affect the qualitative characteristics of the judge's optimization problem. In all these cases, the judge's optimal child support order does not require the father to transfer all of his income to the mother, as was the case under the conditions of Proposition 3.[16] This does not result from any inherent concern the judge has for the father's welfare, but rather is a response to the threat of the father not to comply with a given order.

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[16] When the mother's preference parameter was smaller than the father's.

When the court has imperfect information regarding parameters influencing the expenditures on the child *given the observed state C or N*, the court may have ambivalent attitudes toward noncompliance behavior. When only the mother's preference parameter is heterogeneous, a father with a given custody arrangement and child support order may not comply for essentially two reasons. Consider the situation represented in Figure 3. In this illustration, there exists a nondegenerate distribution of maternal preferences (the particular density function displayed is generated from estimates discussed in Section 4). For a father with income I_f, direct costs of noncompliance ξ, and facing an order T and custody arrangement τ, compliance will be determined by his ex-wife's income I_m and the value of her preference parameter α_m. Conditioning on I_m, the relationship between compliance and α_m is nonmonotone, with noncompliance associated with high or low values of α_m and compliance with moderate values. When the mother is very "unselfish" (i.e., has a value of α_m close to zero), she is already providing a large level of expenditures on the child. The father then is "free-riding" on the mother's high level of child expenditures and does not comply with the child support order so as to spend more on his own consumption. The court would presumably wish to impose negative sanctions on fathers not complying for such reasons. On the other hand, fathers may not comply because the mother is "selfish" (i.e., has a value of α_m close to one). In the extreme case in which her value of α_m is 1, the mother would spend all child support transfers on herself, whereas the father would spend some portion of T on the child if he did not comply with the order. Then for some values of α_m, the *ex post* value of the court's problem would be larger under noncompliance than it would under compliance.[17] This problem clearly has a principal-agent structure to it, so the question naturally arises as to whether it is possible to design custody-child support arrangements so that the agents (the fathers in this case) take actions consistent with the maximization of the court's objective. We shall briefly return to this question in Section 6.

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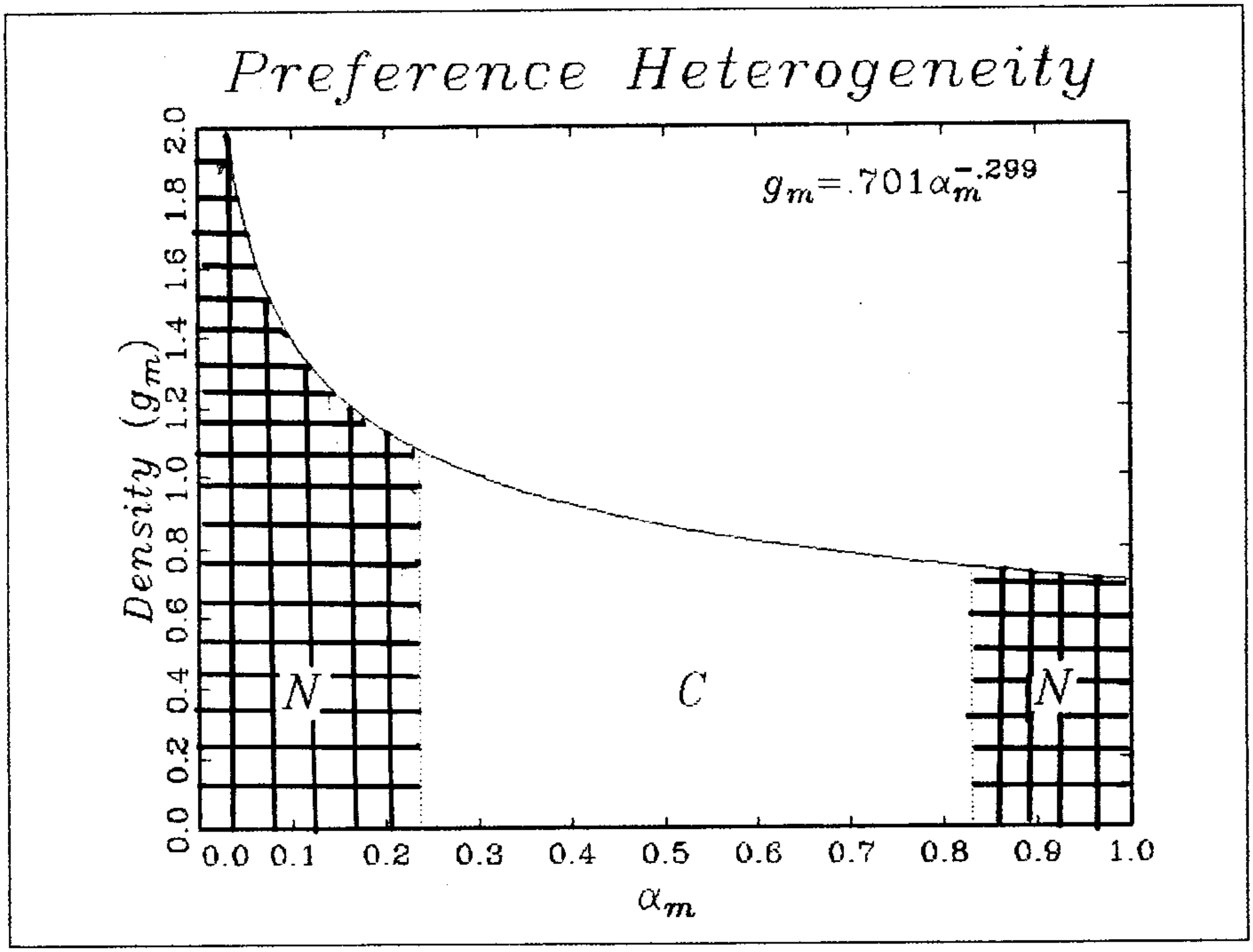
[17]That is, if it ultimately became aware of the true value of α_m in a particular case after the father had made his compliance decision.

FIGURE 3

The Compliance Decision and the Preferences of the Mother



D. *Summary of Section 2*

The results contained in this Section will be used throughout the remainder of the paper. The characterization of the Nash equilibrium of parent's expenditures on the child (Proposition 1) is crucial for the remaining propositions, and for the estimation of behavioral parameters, the topic of the next Section. Once the parents' behavior was described, our attention shifted to the behavior of the institutional agent under various constraints on his or her actions. We first showed that in the case in which the judge could *only* set custody arrangements, even if the objective of the court is not defined over the division of time between the two parents, optimal custody arrangements will in general be determined by the incomes of the two parents (Proposition 2). Furthermore, if the judge orders joint custody, the proportion of the time spent with the father is uniquely determined, and is a function only of preference parameters.

We showed that when the judge has no concern for the welfare of the parents, is free to set custody *and* child support levels, and is assured of compliance, it is optimal to order one parent to transfer all of his or her income to the other (Proposition 3). Which parent receives the income transfer and the custody of the child is independent of the distribution of income between the two parents, and is solely determined by whom has the lowest value of α_p (i.e., which is the least "selfish"). In Proposition 4, we showed that when compliance was not assured, it would never be optimal to order one parent to transfer all of his or her income to the other, not due to the court's concern for the welfare of either parent, but rather because such a transfer would guarantee noncompliance. Due to the complexity of the court's optimization problem, we have not attempted to derive further characterizations of the decision rules. We provide some numerical evidence on the rules below in the process of conducting an analysis of the conformity of actual settlements and those produced optimally under various assumptions regarding institutional objectives.

3. Estimation of Model Parameters

In this Section we discuss issues of identification and estimation of model parameters, where the dependent variable is the father's decision of whether to comply. In estimating these "conditional" compliance decisions, the child custody arrangment and support order are taken as given. We will estimate two versions of the model; in the first, we specify fixed values of the preference parameters α_f and α_m in the population, and a heterogeneous value of the welfare cost of noncompliance, ξ, the population distribution of which is denoted G_ξ. In the second model, we will continue to assume that all fathers share the same value of the preference parameter α_f, but we will allow population heterogeneity in both the direct welfare cost of noncompliance to the father and the mother's preference parameter α_m, which is distributed according to G_m. In both models, it will be assumed that the heterogeneous parameters are independently and identically distributed in the population; in particular, these random variables will be assumed independent of the incomes of the parents and the child support award and custody arrangement.

While we only use the compliance decision to estimate the behavioral parameters of the model, the second stage of our empirical work is an attempt to assess the adequacy of the models put forth in Section 2, both in terms of their ability to explain compliance behavior *given court orders*, and in terms of explaining the court orders themselves.

We now turn to the estimation of the model under the heterogeneous welfare cost of noncompliance specification. The welfare cost of noncompliance is assumed to be strictly positive; we take the distribution of ξ to be exponential, with associated density $g_\psi(\xi) = \psi exp(-\psi\xi)\chi[\xi \in \mathbb{R}_+]$.[18] Let the indicator variable d take the value 1 if the father complies with the child support order, and 0 otherwise. Then the log likelihood of the sample is

$$[20] \qquad \mathcal{L}(\alpha_m, \alpha_f, \psi) = \sum \{d \ln P(d=1|\alpha_m, \alpha_f, \psi) + (1-d) \ln (1-P(d=1|\alpha_m, \alpha_f, \psi))\}$$

From [12′], [13], and [14], the probability of compliance is

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[18]Since the only distributional information used in constructing the maximun likelihood estimates is the area in the right tail, we believe that the choice of distribution (in the class with support over the positive real line) is not likely to be of much consequence empirically.

$$[21] \quad P(d=1) = P\{ \alpha_f \ell n(I_f - T - \bar{e}_f(C)) + (1-\alpha_f)\tau \ \ell n(\bar{e}_f(C)+\bar{e}_m(C)) > \alpha_f \ell n(I_f - \bar{e}_f(N)) + (1-\alpha_f)\tau \ \ell n(\bar{e}_f(N)+\bar{e}_m(N)) - \xi \} ,$$

$$= \exp[-\psi \{ \alpha_f(\ell n(I_f - \bar{e}_f(N)) - \ell n(I_f - T - \bar{e}_f(C))) + (1-\alpha_f)\tau(\ell n(\bar{e}_f(N)+\bar{e}_m(N)) - \ell n(\bar{e}_f(C)+\bar{e}_m(C))) \}],$$

where the last line follows from the assumption that ξ is exponentially distributed with parameter ψ, and where C denotes the state variables of the equilibrium child expenditure levels of the parents in the case when the father complies with the child support order $[C=\{I_f-T, I_m+T, \tau\}]$ and N denotes the state variables when the father does not comply $[N=\{I_f, I_m, \tau\}]$.

We now turn to a discussion of the conditions required for the m.l. estimator of ϑ to be consistent and normally distributed asymptotically. Define the parameter vector $\vartheta = [\psi \ \alpha_f \ \alpha_m]'$ and the parameter space $\Omega = \mathbb{R}_+ \times (0,1)^2$. Let ϑ_0 be the population value of the parameters, with $\vartheta_0 \in \Omega$. From Proposition 1, we know that the log likelihood function is not continuously differentiable in the parameters. The problem occurs at the points b_f and b_m defined in Proposition 1. While $\partial\mathcal{L}/\partial\vartheta$ exists for all $\vartheta \in \Omega$, the first partials are not continuous at the points b_f and b_m.[19] This turns out not to be a serious problem, since the number of points at which the log likelihood function is not continuously differentiable with respect to ϑ is countable. Then, the gradient of the likelihood function is continuous almost surely in $\vartheta \in \Omega$, and the expected value of the gradient vector exists for all $\vartheta \in \Omega$. These, conditions, along with some other boundedness assumptions, are used by Huber (1967,1981) to demonstrate consistency of the m.l. estimator corresponding to a root of the equations $\partial\mathcal{L}/\partial\vartheta = \underset{\sim}{0}$. In terms of proving asymptotic normality, Huber requires additional conditions to hold which are analagous to Lipschitz conditions on the first partials of $\mathcal{L}$, all of which are met by our functional form specifications. In addition, it is necessary that $E(\partial\mathcal{L}/\partial\vartheta')$ have a non-singular derivative matrix at ϑ_0, given by $-\mathcal{I}(\vartheta_0)$, where $\mathcal{I}(x)$ denotes the

[19] Since the values b_f and b_m differ depending on whether the individual complies with the child support order or not, each individual contribution to the likelihood has four points in the parameter space at which the first partials are noncontinuous (those components of the gradient vector associated with α_f and α_m)

information matrix evaluated at x. Then Huber shows $\sqrt{I}(\hat{\vartheta}-\vartheta_0)$ is asymptotically normal with mean 0 and covariance matrix $\mathcal{I}(\vartheta_0)^{-1}$.

The preference parameters enter the log likelihood in an exceedingly complicated way, so as to insure that $\mathcal{L}$ is not globally concave over the parameter space. Thus, any root of the likelihood equations $\partial\mathcal{L}/\partial\vartheta = \underset{\sim}{0}$ may correspond to a local extremum of the function. We have searched for local maxima by starting the iterative estimation algorithm from a variety of points in the parameter space Ω. When the algorithms converged to different points, the point associated with the highest value of the log likelihood was chosen as the m.l. estimate. Of course, the point actually chosen is not guaranteed (even probablistically) to be the point in the parameter space associated with the global maximum of $\mathcal{L}$.[20]

A final issue involves the assumption that the parameter space is an open set. For example, consider the form of the likelihood function when $\alpha_f=\alpha_m=1$. In this case, the probability of compliance is $P(\ell n(I_f-T) > \ell n(I_f) - \xi) = exp\{-\psi[\ell n(I_f)-\ell n(I_f-T)]\} > 0$, thus $\mathcal{L}$ is well-defined [i.e., bounded] even for parameter values in the closure of the open set Ω. For some finite samples, it may happen that the maximum of the log likelihood occurs at $\hat{\alpha}_f=\hat{\alpha}_m=1$, which is not an element of Ω. In this case, the m.l. estimator would not be given by $\partial\mathcal{L}(\hat{\vartheta})/\partial\vartheta = 0$. However, the probability of such an event occurring when ϑ_0 is an element of Ω goes to zero as samples grow indefinitely large. In terms of the actual computation of the m.l. estimator, we found that when starting with large values of the preference parameters of the parents, there was a pronounced tendency for the function to move toward this point in the closure of Ω. Estimation of most models began from "small" values of the preference parameters.[21]

Now consider the estimation of behavioral parameters when both the welfare cost of noncompliance for the father and the preference parameter of the mother is random. We will continue to assume that the direct welfare cost of noncompliance is exponentially distributed in the population. We will also

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[20] Since initial consistent estimators of ϑ are not available to use for starting values, the algorithm can not be said to have converged to the root corresponding to the global maximum with probability one.

[21] Recall that low values of the preference parameters imply that parents are "unselfish" in regards to own versus child consumption.

assume that the direct welfare cost of noncompliance is independently distributed with respect to α_m. As before, we constrain α_m to lie in the unit interval, so that we have chosen a distribution function with the appropriate support. We posit

$$[22] \qquad G_m(\alpha_m) = \alpha_m^{\delta}, \qquad \delta > 0 , \alpha_m \in (0,1).$$

The power function distribution G_m is a special case of the Beta distribution.[22]

The parameter vector is now $\tilde{\vartheta} = [\psi \ \delta \ \alpha_f]'$, and the parameter space associated with this parameterization of the compliance decision is $\tilde{\Omega} = R_+^2 \times (0,1)$. The log likelihood function is

$$[23] \qquad \tilde{\mathcal{L}}(\psi,\delta,\alpha_f) = \sum \{d \ \ell n \ P(d=1|\psi,\delta,\alpha_f) + (1-d) \ \ell n \ (1-P(d=1|\psi,\delta,\alpha_f))\},$$

where the probability of compliance is derived through the following argument. Define the value of compliance for a father conditional on the mother's type, $V_c(\alpha_m) = \alpha_f \ \ell n(I_f - T - \bar{e}_f(C;\alpha_m)) + (1-\alpha_f)\tau \ \ell n(\bar{e}_f(C;\alpha_m)+\bar{e}_m(C;\alpha_m))$, where $\bar{e}_p(S;\alpha_m)$ is the equilibrium expenditure on the child by parent p when the father's compliance decision is S (equals C or N) given the mother's preference parameter α_m. Similarly, define the father's value of noncompliance [neglecting the direct welfare cost of noncompliance] conditional on the mother's type by $\tilde{V}_n(\alpha_m) = \alpha_f \ \ell n(I_f - \bar{e}_f(N;\alpha_m)) + (1-\alpha_f)\tau \ \ell n(\bar{e}_f(N;\alpha_m)+\bar{e}_m(N;\alpha_m))$. Conditional on the value of α_m, the probability of compliance is given by equation [21]. The unconditional probability of compliance is

$$[24] \qquad \begin{aligned} P(d=1) &= \int P(d=1|\alpha_m) \ dG_m(\alpha_m) \\ &= \int_0^1 \exp[-\psi \ \{\tilde{V}_n(\alpha_m) - V_c(\alpha_m)\} \ \delta\alpha_m^{\delta-1} d\alpha_m. \end{aligned}$$

Note that while this specification of the compliance decision seems slightly more general than the previous specification in which the distribution of the mother's preference parameter was assumed to be degenerate at some point on the open interval (0,1), statistically speaking, this is not the case. In particular, the second model does not "nest" the first, for there is

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[22] See Johnson and Kotz (1970, Chapter 24) for a discussion of the power function and Beta distributions. Note that when $\delta = 1$, the power function distribution simplifies to the uniform over the unit interval.

no way in which a random variable with a power function distribution can be degenerate at an interior point on the unit interval. While it is possible to conduct tests between the two specifications using procedures developed for non-nested models, the interpretation of such tests is much more problematic than it is when the models are nested.[23]

In terms of the properties of the log likelihood function $\tilde{\mathcal{L}}$, not much in addition to the discussion of the properties of $\mathcal{L}$ need be said. We assume that the true value of the parameters $\tilde{\vartheta}_0 \in \tilde{\Omega}$. By inspection of [24], we see that $\tilde{\mathcal{L}}$ is continuously differentiable in both ψ and δ; it is also almost surely differentiable in α_f by the following argument. For any given individual and for any fixed α_f there exist two values of α_m for which the partial derivative $\partial[V_n(\alpha_m)-V_c(\alpha_m)]/\partial\alpha_f$ is not continuous.[24] Since α_m is continuously distributed over the unit interval, the probability that α_m takes either of these two values is zero, and the first partial of the likelihood with respect to $\tilde{\vartheta}$ is continuous almost surely. All the other conditions required for Huber's proof of consistency and asymptotic normality are assumed to hold.

4. Empirical Results for the Compliance Decision

Our empirical analysis will span two Sections. In this Section, we discuss estimates of the compliance decision. In addition to presenting estimates of behavioral parameters from the two specifications of the "optimal" compliance decision developed above, we will present estimates from more conventional probit models. These estimates provide some benchmark with which to compare the estimates from our behavioral model; they also allow us to consider the effects of numerous other characteristics of fathers, mothers, and children, which for reasons of tractability could not be included in the behavioral analysis. In Section 5, we present an analysis of the determinants of the custody arrangement and child support order; for now, these are taken

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[23] For the development of non-nested hypothesis tests based on the likelihood function, see Cox (1961,1962).

[24] For given income levels and custody and support orders, and for fixed α_f, there exists one value of α_m such that α_f would lie at b_f for the compliance case, and another value of α_m such that α_f would lie at b_f for the noncompliance case.

as given.

The data used in this paper are gathered from court and payment records of divorce, separation, annulment, and paternity cases in 18 counties in Wisconsin. The population of cases from which the original sample was constructed was defined as all family court cases involving a child under 18 years of age. In each of the 18 counties, between 150 and 200 cases over the period 1980-1986 were randomly selected, with approximately equal numbers of cases being selected each year.

We work with a small subset of the original sample in this research. Naturally, we restrict our sample to divorce cases, since divorce is a basic premise of all the arguments developed in Section 2. Only cases from the years 1980-82 are utilized, and in none of the cases were mandatory guidelines regarding custody or child support operative.[25] Only couples with one child were chosen for inclusion in our sample. In part, this was done so that we would not have to consider the shared custody alternative,[26] or, more generally, the possibility of different custody arrangments for different children. In addition, only cases with joint custody or mother's custody were included.[27] All the cases included in the study had fixed custody arrangements and child support levels over the period of the sample. Finally, cases with missing data on the focal variables of the analysis were excluded.[28] We were left with

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[25]Beginning in 1984 child support orders were determined mechanically as a proportion of the non-custodial parents gross income (in cases of sole physical custody), where the proportion was an increasing function of the number of children involved. For purposes of analyzing custody and child support orders in the absence of external constraints on the court's choice, we wished to exclude cases adjudicated under these conditions.

[26]That is, designating the father as the custodial parent for some of the children and the mother as the custodial parent of the others.

[27]This essentially rules out cases in which the father is the sole custodial parent. Only two or three of otherwise eligible cases were excluded by this criterion.

[28]Missing information is a serious problem in this data set; well over half of all cases have missing data on some variable(s). This is particularly a problem with respect to information regarding income which was merged into this data set using state income tax returns. For example, in our final sample of 156 cases, only 106 have complete income information from state income tax returns [we have not used these data in this analysis since preliminary analysis suggested little difference in results whether income tax or court-provided income information was used]. In addition to data from state income tax returns, demographic information regarding the parents is often not

a total of 156 cases.

The variables used throughout the empirical analysis are standard demographic characteristics like the education of both parents and the age of both parents and the child. Information was also available on the length of the marriage. If the parents have joint custody, the joint custody variable takes the value 1, otherwise it is zero. The amount of the child support order and the incomes of the parents have been denominated in 1980 dollars. The dependent variable of the analysis, compliance, takes the value 1 if the father has paid *at least 90%* of his child support obligations over the sample period; otherwise it takes the value 0.

The data are described in Table 1. Approximately one-half of fathers in this sample did not comply with child support orders under our definition of compliance. Only 12% of fathers had joint custody arrangements. Average child support orders were about 15% of the average income levels of fathers in this sample. The mean income of mothers was slightly more than 50% of the mean income of fathers. Education levels of mothers and fathers are virtually identical, and both exhibit little variability in the sample. The average age of the child involved in the case was 6 years [recall that the child must be less than 18 for the case to be included in the original sample].

The behavioral models described in Section 2 yielded rules for the father to use in making a compliance decision; the arguments (state variables) appearing in these functions were the income of both parents, the custody arrangement, and the child support order. Under the functional form assumptions made in deriving these rules, the decision to comply would typically be a nonlinear function of these state variables. As a check on the importance of such nonlinearities, and on the (implicit) assumption that no other characteristics of the parents or child mattered in determining compliance, we estimated a series of probit models. The results are reported in Table 2.

In the first model estimated, we included the state variables of our analysis, though they were constrained to enter the probit index function in a linear manner. The signs of the coefficients are all consist with predictions from our model, with the exception of mother's income, which we would expect to have a negative sign. Only two of the coefficients are significantly dif-

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recorded.

TABLE 1

Descriptive Statistics for Sample of Divorced Parents with One Child (Sample Selection Discussed in Text)

Variable	*Mean*	*Std. Dev.*
Comply	.47	
Joint Custody	.12	
Amount of Order	159.77	96.35
Mother's Income	589.79	297.93
Father's Income	1073.20	617.77
Mother's Education	12.20	1.30
Father's Education	12.22	1.66
Mother's Age	28.55	7.64
Father's Age	30.43	7.62
Duration of Marriage	8.65	6.88
Age of Child	6.18	5.12

Sample Size = 156

TABLE 2

Probit Estimates of the Father's Compliance Decision

(Asymptotic Standard Errors in Parentheses)

Variable	*Specification* I	II	III[1]	IV
Constant	-1.021	-2.823	-2.463	-2.111
	(.323)	(1.172)	(.787)	(.603)
Joint Custody	.669	.640	-1.065	.745
	(.328)	(.336)	(1.803)	(.341)
Amount of Order ($\times 10^{-3}$)	-.172	-.679	10.129	9.049
	(1.330)	(1.394)	(4.972)	(4.354)
Mother's Income ($\times 10^{-3}$)	.295	.148	1.080	.333
	(.359)	(.377)	(1.503)	(1.298)
Father's Income ($\times 10^{-3}$)	.666	.630	1.155	1.046
	(.232)	(.240)	(.742)	(.593)
Mother's Education		.045		
		(.097)		
Father's Education		.112		
		(.075)		
Length of Marriage		.008		
		(.039)		
Child's Age		.005		
		(.050)		
Order^2 ($\times 10^{-6}$)			-23.074	-17.690
			(12.229)	(7.982)
M. Inc.^2 ($\times 10^{-6}$)			.125	-.054
			(.955)	(.945)
F. Inc.^2 ($\times 10^{-6}$)			-.397	-.134
			(.291)	(.166)

Table 2 continued on next page

TABLE 2
(continued)

	Specification			
	I	II	III	IV
Log Likelihood	-99.576	-97.571	-92.747	-95.266
Proportion of Correct Predictions	.551	.564	.590	.577

Tests of Specifications

Specifications	*Likelihood Ratio Statistic*	*df*	*Probability*[2]
I vs II	4.010	4	.405
I vs III	13.658	9	.135
I vs IV	8.620	3	.035
III vs IV	5.038	6	.539

Notes for Table 2:

1 Specification III includes all the variables in Specification I plus all possible squared terms and (first-order) interactions. The coefficients associated with the interaction terms have not been reported. None were individually statistically different from zero at conventional significance levels; neither were they jointly significant, as can be seen from the likelihood ratio test involving Specifications III and IV.

2 This is the probability that a chi-square distributed random variable with the relevant degrees of freedom exceeds the value of the likelihood ratio test statistic.

ferent from zero, the ones associated with father's income and the joint custody variable.

We next expanded the probit index to include parental education, the length of the marriage, and the child's age.[29] There was some reason to expect each of these coefficients to be non-zero. Our thoughts were that the parental eduction measures may be better measures of "permanent income" than the income measures collected at the time of the divorce, and so may be more accurate measures of the state variables of the decision rule. The child's age may affect the husband's evaluation of the reasonableness of a given order, since desired expenditures on the child may well be age-dependent. The duration of the marriage may proxy for differences in the information the father has about the mother's consumption patterns, the extent to which the father "objects" to the mother's taxation of child support payments, or a myriad of other factors.

Adding these variables (Specification II), we see that only the father's education has an effect on the probability of compliance which approaches being statistically significant. It works in the same direction as does the income of the father, so that the interpretation of it as a proxy for permanent income is not ruled out. Of course, it may also indicate that fathers with higher levels of education may have higher valuations of child versus own consumption; many interpretations are possible. The addition of all these other characteristics does not substantially change the estimates of the effects of joint custody and father's income on compliance.

In Specifications III and IV, we explore the possibility that the variables included in Specification I may enter the probit index nonlinearly. In Specification III, we have entered all the variables in Specification I, plus the squares of these variables [except for Joint Custody, which is a dummy variable], and all possible interactions [of which there are 6]. Conducting a likelihood ratio test of Specification III versus Specification I, we could not reject the hypothesis that the coefficients associated with all the nonlinear terms were jointly zero. However, Specification IV supplies support

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[29] We initially included the ages of the parents in the probit indexes in the compliance and child custody decisions, as well as the child support order regressions. We dropped these variables since they did not appear to be important in explaining sample variability in any of these characteristics.

for the presence of nonlinearities after the interaction terms have been dropped from the model. In particular, there is evidence that the probability of compliance is concave in the size of the order. There is also evidence, albeit weaker, that the compliance probability is a concave function of the father's income. Thus there is some evidence for nonlinear relationships between the state variables and the compliance probability.[30]

Table 3 contains m.l. estimates of the behavioral parameters in the model in which the preference parameters are assumed homogeneous within the populations of divorced mothers and fathers, and only the direct welfare cost of noncompliance is allowed to vary across fathers. The estimates of the preference parameters indicate that if either mothers or fathers had "full" custody [that is, the other parent was assigned no time with the child], the custodial parent would spend approximately half of his or her income on the child. Mothers appear to be slightly more inclined to spend their income on the child than do fathers. Given the m.l. estimates and their asymptotic sampling distribution, we are able to say that the probability that mothers actually are less "selfish" than fathers (i.e., $\alpha_m < \alpha_f$) is .76.

It is interesting to note that the behavioral model, which involves only three parameters, affords better predictions of compliance behavior [in a certain sense] than do the probit models presented in Table 2. From any well-defined discrete choice model, we can compute the probability that a given father will comply. If this probability is greater than .5, we will say that the model predicts that the father will comply. Comparing the predicted behavior, defined in this way, with the actual decision, we see that the behavioral model makes the correct prediction in 63% of the cases. No probit specification achieves a successful prediction rate of more than 59%, in spite of the fact that up to four times as many parameters are estimated. By this criterion, the behavioral model seems to perform adequately in actually fitting these data.[31]

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[30]Though it must be remembered that there is no explicit relationship between the estimates obtained from these probit models and the parameters of the behavioral model.

[31]As pointed out above, the parameter estimates from the behavioral and probit models are not directly comparable. Furthermore, neither estimator is defined with respect to the objective of maximizing the number of correct classifications using the criterion we use [estimators defined with respect

TABLE 3

Estimates of Preference Parameters from the Father's Compliance Decision with Fixed Value of the Mother's Preference Parameter

Parameter	*Point Est.*	*Asy. S.E.*
α_m	.470	.090
α_f	.497	.117
ψ	10.926	3.637

Log Likelihood = -106.030

Probability of Greater Altruism on the Part of Mothers

$\text{Prob}\{\alpha_m - \alpha_f < 0 \mid \hat{\alpha}_m - \hat{\alpha}_f = -.027 \;;\; \text{AVAR}(\hat{\alpha}_m - \hat{\alpha}_f) = .038\} = .761$

Comparison of Predicted and Actual Compliance Decisions

		Predicted		
		Comply	*Don't Comply*	
Actual	*Comply*	45	28	73
	Don't Comply	29	54	83
		74	82	

Proportion of Correct Classifications: .635

We next estimated the behavioral model in which both the direct cost of noncompliance and the mother's preference parameter were assumed to vary in the population [Table 4]. Estimates of the parameter of the distribution of welfare costs of noncompliance and the father's preference parameter are virtually identical across the two specications. The point estimate of the parameter of the distribution of the mother's preference parameter, .701, implies that the distribution is skewed to the right over the unit interval (see Figure 3). The mean of the distribution is .412, the standard deviation is .299, and the median value of α_m is .372. While the difference between the mean (or especially median) value of the preference parameter of the mother and the fixed preference parameter of the father is greater than was the case for the previous model, the estimates of α_f and δ indicate that approximately 38% of mothers are more "selfish" than the father.[32] This has some implications for the optimal decisions of the court, which we will explore in the next Section.

The log likelihood value of the first model is higher than that of the second, though as was noted in Section 3, the models are not nested. The accuracy of prediction of both behavioral models is approximately the same.

5. The Prediction of Child Support Orders and Custody Arrangements

Using the (point) estimates of the equilibrium level of expenditures on the child as a function of custody and support orders, we are able to say something about optimal custody arrangements and support orders under various assumptions regarding the objective of the court. We will approach this question in two stages. First, we look at the optimal order *conditional* on the custody arrangement. This is a reasonable approach if the court's preferences are as given in [8]. In this case, the court "cares" about the allocation of time between the parents. Whatever it chooses in terms of τ, however, it should set child support levels so as to maximize the expected value of a monotone function of total child expenditures conditional on τ. Second, we

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to such objectives are referred to as maximum score estimators [see Manski (1975,1985)]. Thus we should not make too much of the "superiority" of the behavioral model over the probit model in regards to this one predictive criterion.

[32]This probability is computed using the point estimates of α_f and δ.

TABLE 4

Estimates of Preference Parameters from the Father's Compliance Decision with Heterogeneous Value of the Mother's Preference Parameter

Parameter	*Point Est.*	*Asy. S.E.*
δ	.701	.399
α_f	.507	.224
ψ	10.836	2.482

Log Likelihood = -110.572

Comparison of Predicted and Actual Compliance Decisions

		Predicted: Comply	*Predicted: Don't Comply*	
Actual	*Comply*	44	29	73
	Don't Comply	30	53	83
		74	82	

Proportion of Correct Classifications: .622

will look at the optimal custody arrangement *and* child support order given the assumption that the court's preferences are defined as the expected value of a monotone function of total child expenditures.

In our sample, the average child support order is \$159.77, and the standard deviation is \$96.35.[33] In looking at the optimal order conditional on the custody arrangement, we performed the calculation on a case by case basis under four different sets of assumptions. In terms of the court's objective function, we considered two cases:

$$[25a] \qquad V_a(T|\tau) = E_x\{\bar{e}_f(x|\tau)+\bar{e}_m(x|\tau)\}$$

$$[25b] \qquad V_b(T|\tau) = E_x\{\ell n[\bar{e}_f(x|\tau)+\bar{e}_m(x|\tau)]\},$$

where x denotes the random variables of the model. In the first case [25a], the court's objective is defined in terms of the total expected expenditures made on the child, while in the second case [25b], the court's objective is defined as the expected value of the logarithm of total expenditures on the child, so that an element of risk-aversion is present in the second case but not in the first. Note that standard results on choices under risk neutrality and risk aversion do not hold for this problem, since the judge's decisions affect the probabilities of the states as well as the value of the states; consequently, we provide only heuristic discussions of the affect of risk aversion on orders.

We have computed the optimal order under the two different models estimated in the previous Section. What we refer to as Model I below is the case in which the mother's and father's preference parameters are known, while Model II refers to the situation in which the mother's preference parameter is distributed according to a power function distribution. In both cases, the point estimates of all population parameters from Tables 3 and 4 are used in the computations.

In solving the court's problem, we assumed throughout that only the father would ever be asked to transfer income. When we computed optimal child support orders conditional on the custody arrangement, we searched for the optimal order over the interval $[0,I_f]$. We used the simple procedure of com-

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[33]All monetary values are denominated in 1980 dollars.

puting [25a] and [25b] at a variety of points $T_0, T_1, \ldots, T_n$, and then selected that value associated with the maximum value of the objective.[34] For each sample point, the values of T_i satisfied $T_i = i[I_f/n]$, $i=0,\ldots,n$. In obtaining all the results reported here, n has been set at 100.

In computing the value of the optimal custody arrangement and child support order, we restricted the choice set of the court severely in making custody arrangements. The court was only allowed to choose between mother's custody, $\tau=.1$, and joint custody, $\tau=.5$, which is an assumption consistent with our sample observations. Then we define the modified objective functions of the court to be

$$[25a'] \quad V_a(T,\tau) = \max\{V_a(T|\tau=.1)\ ;\ V_a(T|\tau=.5)\}$$

$$[25b'] \quad V_b(T,\tau) = \max\{V_b(T|\tau=.1)\ ;\ V_b(T|\tau=.5)\}$$

For each custody arrangment, we computed the optimal child support order in the manner described in the preceding paragraph. We then selected the custody arrangement-child support pair associated with the maximum value of the objective, [25a'] or [25b'].

Some description of the distribution of "optimal" child support orders conditional on custody arrangments is given in Table 5. When the court seeks to maximize the expected expenditures on the child, under the model in which the mother's preference parameter is constant in the population, the mean optimal order is $240 per month, which is 50% greater than the mean of the actual orders. There is also much more variation in optimal orders than in actual orders; the coefficient of variation for actual orders is .603 while for optimal orders it is .875. Finally, note that the sample correlation between the actual order and the optimal order is .354, indicating a moderate degree of association between actual orders and those produced optimally under the quite stringent assumptions of our model.

When the court maximizes the expected value of the logarithm of total expenditures on the child, under Model I, the average optimal order is reduced to $224.78 due to risk aversion on the part of the court. The variance in the

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[34] Alternatively, we could have searched for solutions using the method of successive approximation.

TABLE 5

Optimal Child Support Orders For Two Judicial Objective Functions Given Predetermined Custody Arrangements[1]

Child Support Order	*Mean*	*Std.Dev.*	*Corr. with Actual*
Actual	159.770	96.35	...
Predicted Values from Model I[2] *using Court's Objective with Property:*			
Linear	240.527	210.393	.354
Logarithmic	224.780	200.912	.319
Predicted Values from Model II using Court's Objective with Property:			
Linear	213.115	153.340	.497
Logarithmic	199.128	140.399	.470

Notes for Table 5:

1 Predicted values are obtained using the point estimates of behavioral parameters reported in Tables 3 and 4 in solving the decision problem of the court.

2 Model I refers to the fixed α_m case and Model II refers to the heterogeneous preference parameter case.

orders is also slightly smaller than under risk neutrality. The predicted orders under this objective are less correlated with the actual orders than are the orders produced under the linear objective.

When issuing orders in the case where the judge is uncertain as to the mother's preferences, an additional element of risk is present. This risk accounts for the substantial drop in average optimal orders under both specifications of the court's objective function. Under the logarithmic objective, the average optimal order is only 25% greater than the average order in the sample, though the standard deviation of the optimal order is about 50% greater than the standard deviation of observed orders. The correlation between predicted and actual orders is quite a bit higher under Model II than under Model I.

In Table 6, we report regressions of child support orders, both actual and "optimal," on a set of regressors which include the custody arrangement, parental incomes, and parental education levels.[35] In line with intuition and the results of our analysis, the actual child support order is an increasing function of the father's income and a decreasing function of the mother's. The coefficients associated with parental education are of the correct sign if these variables measure permanent income. Though all coefficients have the correct signs, only father's income is statistically significant; this variable accounts for virtually all of the explained variance in orders. When joint custody was included as a regressor (Column II), its coefficient was found to be negative, though the absolute value of the point estimate was only slightly greater than the corresponding standard error.

We performed the same regressions using as the dependent variable the optimal predicted orders produced under Model II with a linear objective for the court. Since these orders are determistic functions, although nonlinear ones, of the custody arrangement and parental incomes, we should expect our regression model to fit the data very well, which is the case. We see in Column III that the coefficients associated with parental income are much larger in absolute value than those obtained when the actual order was used as

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[35]Other variables, such as the ages of the parents, age of the child, and the length of the marriage, were initially included in the regressions, but their associated coefficients never approached levels of statistical significance in any of the specifications estimated.

TABLE 6

Child Support Order Regressions Using Both Actual and "Optimal" Orders

(Standard Errors in Parentheses)

Variable	*Actual Order* I	II	*"Optimal" Order*[1] III	IV
Constant	46.490 (62.573)	50.205 (62.445)	-43.760 (53.276)	-63.802 (37.728)
Joint Custody		-26.494 (19.209)		142.925 (11.606)
Mother's Income	-.004 (.021)	-.006 (.021)	-.106 (.018)	-.096 (.013)
Father's Income	.095 (.010)	.093 (.010)	.221 (.009)	.231 (.006)
Mother's Education	-1.881 (5.523)	-2.452 (5.522)	3.960 (4.702)	7.036 (3.336)
Father's Education	3.044 (4.226)	3.823 (4.251)	2.779 (3.598)	-1.423 (2.568)
R^2	.375	.383	.821	.911

Notes for Table 6:

1 Optimal child support orders are computed assuming the court's objective function is linear and using the specification of the model in which the mother's preference parameter is heterogeneous in the population.

the dependent variable. The interpretation of the coefficients associated with parental education is problematic, since they are nonzero only due to their association with child custody and the nonlinear functions of parental income used in determining the optimal order.

In Column IV we include joint custody in the regression. The coefficient is positive and large, in marked contrast to its coefficient in the regression using actual orders. In our model, joint custody leads to a higher degree of willingness on the part of the father to transfer income to the mother. Since the mother, on average, has a higher propensity to allocate resources to the child, the judge optimally has the father essentially buy custody rights. In practice, even working within the general framework of our model, this may not be observed if their are substantial fixed costs of providing a more or less permanent home for the child, and where both parents would have to bear this cost under joint custody as opposed to only one bearing the cost under sole custody. Such costs would presumably reduce the expenditure advantages of joint custody over mother custody, and would be reflected in the amounts the judge would "charge" the father to purchase custody rights (see Lazear and Michael (1988, Chapter 8)).

In Table 7, we present a description of our results on optimal orders when the court chooses both custody arrangements (joint or mother) and the child support order. The average optimal orders for the four models range from $363 to $559, which are from 225% to 350% of the average actual order. The standard deviations of the actual orders are from 300% to over 500% of the size of the standard deviation of the actual orders. Somewhat surprisingly perhaps, while the optimal orders are enormously different from the actual orders in terms of location and scale, the correlation between them is quite high.

If a judge is only concerned with expenditures on the child, and if the preferences of mothers and fathers are not too dissimiliar, it will be optimal to transfer custody to the individual controlling the greater amount of income or income-generating assets. Since the fathers in this sample, and in the general population, have substantially greater income than mothers, judges will almost invariably award joint custody.[36] This pattern is what we observe

[36] In fact, when we expanded the choice set of judges to include father's custody (τ=.9), this choice was made in the majority of cases in the sample.

TABLE 7

Optimal Child Support Orders and Custody Arrangements For Two Judicial Objective Functions[1]

Child Support Order	*Mean*	*Std.Dev.*	*Corr. with Actual*
Actual	159.770	96.350	...
Predicted Values from Model I using Court's Objective with Property:			
Linear	559.025	565.476	.570
Logarithmic	556.372	564.702	.569
Predicted Values from Model II using Court's Objective with Property:			
Linear	363.411	308.618	.586
Logarithmic	374.112	297.884	.595

Predicted Custody Arrangments[2]

Model 1:

		Linear: Mother	*Linear: Joint*	*Logarithmic: Mother*	*Logarithmic: Joint*	
Actual	*Mother*	58	79	56	81	137
	Joint	7	12	7	12	19
		65	91	63	93	

Model 2:

		Linear: Mother	*Linear: Joint*	*Logarithmic: Mother*	*Logarithmic: Joint*	
Actual	*Mother*	33	104	14	123	137
	Joint	5	14	2	17	19
		38	118	16	140	

Notes for Table 7:

1 See note 1, Table 5.

2 See note 2, Table 5.

in the bottom portion of Table 7. The proportion of joint custody awards varies from 58% to 90% across the four models; the actual percentage of joint custody awards is only 12% in this sample. It seems clear from this evidence that in setting custody arrangements and child support orders, the institutional actor is not seeking simply to maximize the objective functions (defined over distributions of total expenditures on the child) which we have examined here.

The probit estimates reported in Table 8 make clear the fact that none of the parental characteristics we have access to accounts for much variation in the probability of a joint custody arrangement, aside from father's income and education. In most of the specifications estimated, father's income is negatively related to the probability of joint custody, which is at odds with the prediction of our model (since we are conditioning on mother's income as well). Father's education is positively related to the probability of joint custody, which is consistent with the predictions of our model if it is a measure of permanent income. It is also consistent with a model in which the level of education of the father is associated with the father's preferences for being with their children, though such an interpretation is something of a tautology.

6. Conclusion

This paper marks an initial attempt to examine the post-divorce behavior of mothers, fathers, and institutional actors within an integrated framework. We view our attempt to formulate and implement an optimizing model of the father's decision to comply with child support orders as moderately successful. It is an open question as to the degree to which our inferences are sensitive to the particular functional form assumptions adopted and the sample used in the estimation; we intend to pursue some of these issues in future work.

Our analysis of the determination of custody arrangements and child support orders was considerably less successful, particularly from an empirical point of view. Our assumptions regarding the objectives of institutional actors and the values of the behavioral parameters characterizing the decisions of fathers and mothers produced optimal divorce settlements markedly different from the actual ones. On the positive side, we

TABLE 8

Probit Estimates of the Custody Arrangement
(Joint Custody = 1 and Mother's Custody = 0)

(Asymptotic Standard Errors in Parentheses)

Variable	*Specification* I	II	III	IV
Constant	-.475 (.393)	-1.179 (1.402)	-1.139 (1.396)	-1.312 (1.428)
Mother's Income (x 10^{-3})	-.468 (.487)	-.491 (.526)		-.404 (.537)
Father's Income (x 10^{-3})	-.435 (.282)	-.477 (.292)		-.395 (.309)
Mother's Education		-.103 (.128)	-.151 (.125)	-.107 (.129)
Father's Education		.163 (.097)	.170 (.103)	.183 (.104)
Length of Marriage			-.056 (.064)	-.022 (.067)
Child's Age			.021 (.075)	-.005 (.079)
$\mathcal{L}$	-55.818	-54.360	-54.998	-53.857

have provided a framework in which the objectives of the institutional actor can be empirically analyzed in a coherent manner.

In terms of developing models in which there is closer agreement between predicted and observed insititutional behavior, there are at least three approaches to take. First, more institutional knowledge regarding the rules employed by judges and lawyers in the adjudication and advising process could be fit into the analytic model. While there exists a limited degree of such information,[37] the interpretation of it is not clear, and how it would be incorporated in any analytical model is even less so.

A second approach would be to postulate objective functions for institutional agents which are parameteric, and to define estimates of these parameters. For example, the objective of the institutional agent could be represented as a parameteric function of total expenditures on the child and the custody arrangement, $U_{\gamma}(\bar{e}_f+\bar{e}_m,\tau)$, and γ could then be estimated. We could then assess the success of the model both in terms of its ability to fit the data and the reasonableness (in light of institutional evidence) of the estimated γ. This approach is being pursued in our current research.

Finally, institutional agents, even those with the preferences such as those posited in this paper, may be able to employ mechanisms which offer welfare improvements over those examined here. In essence, the judge faces a difficult principal-agent problem. He or she relies on the parents to provide resources to the child, and the judge cannot observe the actual levels of expenditures. In the case in which there exists population heterogeneity in parental preferences, the judge's decision is made significantly more complex. We defined a procedure by which the judge chose the settlement without first attempting to gather any information from the father and/or mother regarding their preferences. Of course, the judge cannot simply ask them the values of their own and their ex-spouse's preference parameters, since in this adversarial setting with no truth-telling constraints, each would claim to be altuistic, while accusing the other of complete selfishness. The question arises as to whether choices can be offered to the mothers and/or fathers which would truthfully reveal their private information, in so doing increasing the value of the institutitonal agent's problem. If such mechanisms exist, the

[37]As was mentioned above, Weitzman (1985) reports this type of information, which was acquired from interviews with judges and family law lawyers.

interesting policy question is whether it would be feasible to implement them, thereby improving the welfare of some or all of the parties affected by divorce.

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