ECONOMIC RESEARCH REPORTS

HOMEWORK IN MACROECONOMICS I: BASIC THEORY

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R.R. #: 90-17 May, 1990

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Revised: February 1990

We thank seminar participants at Northwestern, Stanford, Penn, UCLA, Berkeley, San Diego, and the Federal Reserve Bank of Minneapolis for their input into earlier work (Benhabib, Rogerson and Wright 1988) that gave rise to this paper. We acknowledge financial support from NSF grants SES-8704690 and SES-8821225, as well as technical support from the C.V. Starr Center for Applied Economics at New York University and the Federal Reserve Bank of Minneapolis. The usual disclaimer applies.

Abstract for Homework in Macroeconomics II: Basic Theory
and Homework in Macroeconomics II: Aggregate Fluctuations

These papers explore the implications of including home, or nonmarket production in an otherwise standard model of real business cycle fluctuations. Introducing home production significantly improves the quantitative performance of the standard model. In particular the correlation of productivity with output is closer to the data in our model.

I. Introduction

A standard assumption in many models of the labor market, and especially aggregate (macroeconomic) models, is that time has exactly two uses: market work and leisure.[1] This implies that individuals who do not work in the market, for whatever reason, must be enjoying leisure – which is patently false, as any homemaker could attest. The figures in Table 1, derived from Hill's (1985) analysis of the Michigan Time Use Survey, indicate that an average household consisting of a married couple spends about 57 hours per week in market work and 49 hours working in the home. As a fraction of "discretionary time" (market work plus homework plus leisure), market work amounts to 33 percent, while homework is only slightly less, at 28 percent. Notice also that leisure is roughly the same for married males and females, despite large differences in amount of market work.

Table 1: Time Use

Activity (hrs/wk)	Married Male	Married Female	Married Couple
Market work	40.18	16.73	56.91
Home work	14.25	34.85	49.10
Leisure	33.37	34.48	67.85
Sleep and other	80.20	81.94	162.14

Complementary to these data are studies that have attempted to measure the value of home produced output. Hawrylyshyn (1976), for example, estimates that the output of the household sector corresponding only to married women amounts to approximately 35% of measured GNP. Gronau (1980)

[1] An obvious exception is the extremely useful literature on job search; we simply have nothing to say about the search model in this paper.

estimates that the value of home production associated only with married women in 1973 can exceed 70% of a family's market income after taxes. These figures do not include the contributions of unmarried individuals or married males. Several researchers, including Nordhaus and Tobin (1972), Zolotas (1981), Fraumeni and Jorgenson (1987), and Eisner (1988, 1989), have studied the issue of modifying the existing National Income and Product Accounts to include household production (as well as a variety of other factors). Eisner (1988) provides an excellent summary of this literature, and reports a range of estimates for the value of home production relative to measured GNP of 20 - 50 percent.

Although the exact size of the household sector is difficult to measure, even a conservative estimate of 35% of measured GNP is a large amount of economic activity to ignore. By way of comparison, manufacturing output is only about 40% of GNP, and yet the manufacturing sector is heavily studied by macroeconomists. As an alternative comparison, consumption in the standard National Income and Product Accounts is approximately 70% of output (this figure counts expenditures on durables as investment rather than consumption, and excludes the public and foreign sectors). Therefore, by ignoring the consumption of home produced output, macroeconomists are excluding a category of consumption that is half as big as the one that they are including!

These facts lead us to conclude that home production is an empirically significant entity at the aggregate level, whether we measure it in terms of its labor input or its output.[2] In light of this, why is it conspicuously absent from existing macroeconomic models? One possible conjecture is that, although the home sector is large, its behavior is approximately independent

[2] Greenwood and Herkovitz (1990) argue that the home sector also uses a large amount of physical capital.

of the market sector. The data in Table 2, also from Hill (1985), suggest this conjecture is mistaken. The fact that individuals employed in the market sector spend much less time working in the home leads us to believe that there is, in fact, substantial substitutability between market and nonmarket activity. Notice, in particular, that individuals who are not employed in the market sector do enjoy more leisure, on average, but the difference in leisure is much less than the difference in time spent in market work.

Table 2: Time Use and Employment

Activity (hrs/wk)	Married Male		Married Female	
	Full Time Employed	Not Employed	Full Time Employed	Not Employed
Market work	48.62	6.60	39.08	3.22
Home work	12.70	20.01	24.58	40.90
Leisure	29.23	51.24	27.95	38.27
Sleep and other	77.45	89.34	76.39	85.61

Additional evidence on the substitutability between market and home production is provided by Rios-Rull's (1988) analysis of the Panel Study of Income Dynamics. He calculates hours of market and home work for a subsample of individuals in five wage groups, and some results from his study are shown in Table 3.[3] The important feature of these data is the way that individuals substitute between time in the market and in home production as the wage varies; for example, notice that especially for the

[3] The wage groups (in 1969 dollars) were:

$$1=[0,2),\ 2=[2,2.8),\ 3=[2.8,3.8),\ 4=[3.8,5.3),\ 5=[5.3,\infty).$$

Note that because his subsample excluded individuals who did not report positive hours in the market for at least four years, these numbers are not comparable to those in the previous tables.

upper wage groups, total work is roughly the same despite significant differences in the allocation of time to the two types of work. All of this taken together indicates that the home sector is not only large, but that there is a good deal of substitutability between it and the market.

Table 3: Time Use and Wages

Group Averages (work = hrs/wk)	Wage Group 1	2	3	4	5
Hourly wage	1.48	2.37	3.28	4.46	7.24
Years of education	11.18	11.97	12.73	13.00	14.30
Market work	21.38	29.92	34.52	36.92	38.63
Home work	12.46	11.19	8.94	6.73	5.02
Total work	33.85	41.12	43.46	43.65	43.65

The above evidence suggests that household production could be an important missing element in existing models of the aggregate economy. Our goal is to explore this possibility. Following Gronau (1977, 1985), we adopt a version of Becker's (1965) model in which each household has a home production function with time and (possibly) capital as inputs, and a nonmarket consumption good as output. Introducing this simple additional element into standard models will turn out to have fairly dramatic implications, in a variety of contexts.[4]

In this paper, we start by exploring some basic theoretical issues. We prove that any model with household production has a reduced form that is

[4] There has been some previous analyses of the implications of home production for macroeconomics, including Benhabib, Rogerson and Wright (1988), Rios-Rull (1988), and Greenwood and Herkovitz (1990). Becker's (1988) address to the American Economic Association also argues that home production is an important missing element in macroeconomics, although he stresses *family behavior*, while the focus in this paper is on the implications for standard macroeconomic variables, and we do not look at issues such as marriage, divorce, fertility, etc.

observationally equivalent to another model, without home production, but with agents having different preferences. Thus, it is always possible to replicate the behavior generated by the home production economy for market employment, market consumption, and so on, with an economy that has no home production sector, *if* preferences can be chosen arbitrarily. However, for a *given* set of preferences, the addition of household production can matter a lot. As an example, home production is incorporated into an economy with random layoffs resulting from nonconvexities, and we show how this affects the nature and interpretation of unemployment. One result is that we can have involuntary unemployment in reasonable specifications of this model, in contrast to models without home production where involuntary unemployment arises if and only if leisure is an inferior good (in a particular sense). We also discuss how home production affects the interpretation of recent empirical models that include consumer durables.

The project is organized as follows. In the next section, we introduce the basic assumptions and notation, discuss the mapping described above between models with and without home production, and work out some illustrative examples. In Section III, we show the basic points in these examples are fairly general. In Section IV, we discuss the economy with involuntary unemployment, while in Section V, we pursue dynamic issues, including consumer durables. Some conclusions are contained in Section VI. The basic message is that explicitly recognizing nonmarket economic activity changes qualitatively the way we think about a number of topics related to market activity. In a companion paper — Benhabib, Rogerson and Wright (1990) — we introduce home production into the stochastic growth model, or real business cycle model, in order to study how it affects the nature of aggregate fluctuations quantitatively.

II. The Basic Framework

We start with an underlying von Neumann - Morgenstern utility function, $U = U(c_m, c_n, h_m, h_n)$, defined over four objects: consumption of a market good (c_m), consumption of a home produced or nonmarket good (c_n), hours of work in the market sector (h_m), and hours of work in the home or nonmarket sector (h_n).[5] What makes this a model with home production is the assumption that c_n and h_n are nontradable. In particular, we impose the *home production constraint*, $c_n \le g(h_n)$, where $g(\cdot)$ is the home production function. This leads to the following decision problem

$$\max U(c_m, c_n, h_m, h_n)$$
$$\text{st } c_m \le x + wh_m, \; c_n \le g(h_n), \text{ and } h_n + h_m \le H \tag{2.1}$$

(ignoring nonnegativity constraints on c_j and h_j), where w is the real wage, x is exogenous endowment income, and H is the total endowment of time.[6]

Assume $U(\cdot)$ is strictly monotonically increasing in consumption and decreasing in labor, and that $U(\cdot)$ and $g(\cdot)$ are continuous. Then we can substitute the home production constraint into the utility function and maximize with respect to homework, taking as given the values of the market variables, to define the following function:

$$V(c_m, h_m) = \max_{h_n} U[c_m, g(h_n), h_m, h_n] \text{ st } h_n \in [0, H - h_m]. \tag{2.2}$$

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[5] A special case is when market and home variables are perfect substitutes, say $U = u(c_m + c_n, h_m + h_n)$; we would not want to assume this in general.

[6] According to Pollack and Wachter (1975), the "fundamental assumption" of the home production model is the imposition of the home production constraint, in addition to the standard budget constraint and the constraint that total time use cannot exceed the time endowment.

Additionally, as long as either $U(\cdot)$ or $g(\cdot)$ is strictly concave, we can define the *homework function* $h_n = h(c_m,h_m)$ to be the unique solution to the maximization problem in (2.2), and the *home consumption function* by $c_n = c(c_m,h_m) = g \circ h(c_m,h_m)$.

Substituting this into the underlying utility function, we have

$$V(c_m,h_m) = U[c_m,c(c_m,h_m),h_m,h(c_m,h_m)]. \tag{2.3}$$

One can think of $V(\cdot)$ as a *reduced form utility function*, defined over market quantities only. The following result demonstrates that $V(\cdot)$ inherits some basic properties of $U(\cdot)$, so that it in fact describes a well behaved preference ordering over c_m and h_m.[7]

Theorem 1: If $U(\cdot)$ and $g(\cdot)$ are continuous, strictly monotonic, and concave, then $V(\cdot)$ is continuous, strictly monotonic, and concave. If either $U(\cdot)$ or $g(\cdot)$ is strictly concave, then $V(\cdot)$ is strictly concave.

Proof: First, if $U(\cdot)$ and $g(\cdot)$ are continuous then so are $V(\cdot)$ and $h(\cdot)$, by the Theorem of the Maximum. Now choose $\hat{c}_m > \tilde{c}_m$ and $\hat{h}_m < \tilde{h}_m$, and define $\hat{h}_n = h(\hat{c}_m,\hat{h}_m)$ and $\tilde{h}_n = h(\tilde{c}_m,\tilde{h}_m)$. Then we have

$$V(\hat{c}_m,\hat{h}_m) = U[\hat{c}_m,g(\hat{h}_n),\hat{h}_m,\hat{h}_n] \geq U[\hat{c}_m,g(\tilde{h}_n),\hat{h}_m,\tilde{h}_n]$$

$$> U[\tilde{c}_m,g(\tilde{h}_n),\tilde{h}_m,\tilde{h}_n] = V(\tilde{c}_m,\tilde{h}_m).$$

Hence, V is monotonic. To check concavity, choose $\lambda \in (0,1)$, and let $\bar{c}_m = \lambda\hat{c}_m + (1-\lambda)\tilde{c}_m$, $\bar{h}_m = \lambda\hat{h}_m + (1-\lambda)\tilde{h}_m$, and $\bar{h}_n = \lambda\hat{h}_n + (1-\lambda)\tilde{h}_n$. Then we have

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7 Given differentiability, some of the results in this theorem can be derived in an alternative way. For example, using the Envelope Theorem, we have $V_1 = U_1$ and $V_2 = U_3$, establishing monotonicity immediately.

$$V(\bar{c}_m,\bar{h}_m) = U[\bar{c}_m,g(\bar{h}_n),\bar{h}_m,\bar{h}_n]$$

$$\geq U[\bar{c}_m,\lambda g(\hat{h}_n)+(1-\lambda)g(\tilde{h}_n),\bar{h}_m,\bar{h}_n]$$

$$\geq \lambda U[\hat{c}_m,g(\hat{h}_n),\hat{h}_m,\hat{h}_n] + (1-\lambda)U[\tilde{c}_m,g(\tilde{h}_n),\tilde{h}_m,\tilde{h}_n]$$

$$= \lambda V(\hat{c}_m,\hat{h}_m) + (1-\lambda)V(\tilde{c}_m,\tilde{h}_m).$$

Hence, V is concave. Furthermore, if either $U(\cdot)$ or $g(\cdot)$ is strictly concave, then one of the inequalities will be strict, so $V(\cdot)$ will be strictly concave. ■

The above discussion implies that decision problem (2.1) generates the same values of c_m and h_m as the problem without home production,

$$\max V(c_m,h_m) \text{ st } c_m \leq x + wh_m,\ h_m \leq H. \tag{2.4}$$

This is an important point. For example, consider a representative agent economy with home production function $g(h_n)$ and aggregate market production function $f(h_m)$. Its competitive equilibrium is characterized as the unique solution to the social planning problem

$$\max W = U[x+f(h_m),g(h_n),h_m,h_n] \text{ st } h_m+h_n \leq H \tag{2.5}$$

(again ignoring non-negativity constraints). The solution to (2.5) yields the same values for h_m and c_m as the solution to

$$\max W = V[x+f(h_m),h_m] \text{ st } h_m \leq H. \tag{2.6}$$

The economy with home production is therefore *observationally equivalent* to another economy, with no home production, but with different preferences.

Hence, there is a sense in which adding a home sector does not add to the set of outcomes that were possible without it. One might conclude, therefore, that the practice of ignoring nonmarket activity involves no loss in generality. Yet it is precisely because *preferences would have to be different if home production was excluded* that it turns out to be such a useful concept for understanding and interpreting economic phenomena. An obvious point is that it would be a mistake to interpret leisure as $H-h_m$, as specified in the reduced form, since in fact $H-h_m-h_n$ is the correct measure of leisure (and h_n may not be constant). A more subtle point is that the reduced form utility function is actually the offspring of an underlying utility function combined with a home production function, and this can lead to agents acting as if they had preferences quite different from their true preferences.

One important example of this principle is the following: the fact that leisure defined by $H-h_m-h_n$ is a normal good according to the underlying preference structure does not imply that leisure defined by $H-h_m$ is normal according to the reduced form structure. In other words, in contrast to the properties discussed in Theorem 1, a property of $U(\cdot)$ that does not carry over to $V(\cdot)$ is the wealth effect. Several interesting economic issues are known to hinge on this wealth effect.[8] By including home production, we are able to account for agents acting as if leisure is inferior, without violating the reasonable intuition or the long run evidence that it is normal. Even if the sign of the wealth effect is not necessarily reversed, we demonstrate in the next section that, under reasonable conditions, it is necessarily reduced. Hence, a model with home production can display a labor supply elasticity that would be difficult to generate using an empirically reasonable model in which home production is absent.

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[8] Examples include some perhaps surprising results, such as the effect of asymmetric information in implicit contract theory (see, e.g., Cooper 1987), or the issue of whether unemployment is voluntary or involuntary in a large class of models (see, e.g., Rogerson and Wright 1988, or Section IV below).

A second example of the principle is this: once we recognize that home production is important, we are forced to conclude that preferences defined over market variables should not be stationary, non-stochastic, functions. Consider the home production function $g(h_n) = s_n G(h_n)$, where s_n is a stochastic innovation to the household technology. The reduced form utility function then becomes

$$V(c_m, h_m, s_n) = \max_{h_n} U[c_m, s_n G(h_n), h_m, h_n] \text{ st } h_m + h_n \leq H. \tag{2.7}$$

Preferences over c_m and h_m as represented by $V(\cdot)$ now depend on s_n. Hence, it can appear in the reduced form economy as if there is a stochastic shock in the utility function, even though true preferences are stable. Similarly, to the extent that innovations to the home technology are accumulating over time, it will appear in the reduced form that there is trend drift in preferences, even if $U(\cdot)$ is stationary.

A third example is this: to the extent that *relative* productivity changes in the market and nonmarket sectors matter for the short run allocation of time, the observed relation between measured productivity and employment hours can be severely affected. Let s_m and s_n be shocks to the market and home technologies.[9] When s_m is relatively high, labor will flow into the market so that productivity and real wages (correctly measured) will rise along with market hours; thus, s_m shocks trace out a "labor supply" curve for the economy. On the other hand, when s_n is relatively high, labor will flow into the nonmarket sector, raising productivity and real wages as market employment falls; thus, s_n shocks trace out a "labor demand" curve for the economy. As long as both shocks are important at

[9] It is certainly the case that innovations to home and market technologies are not perfectly synchronized (think of the introductions of micro computers and microwave ovens).

different points in time, a scatter plot between market hours and productivity (or real wages) need not show any discernible pattern. By incorporating nonmarket activity, it evidently becomes possible in principle to reconcile the lack of empirical correlation between employment and productivity (or real wages) with theories based on technology shocks.[10]

To be clear, our intention is *not* to show that adding home production generates outcomes that were not possible without it; that would be a futile task. Our intention is to show that adding home production allows us to organize and intepret observations in a useful way. Macroeconomics is an empirical discipline, with the goals of accounting for existing regularities and helping to predict the consequences of changes in the underlying environment. As Becker (19xx, p. 5) writes, "The assumption of stable preferences provides a stable foundation for generating predictions about responses to various changes, and prevents the analyst from succumbing to the temptation of simply postulating the required shift in preferences to 'explain' all apparent contradictions to his predictions." However, as he also points out, "The preferences that are assumed to be stable do not refer to market goods and services, like oranges, automobiles, or medical care, but to the underlying objects of choice that are produced by each household

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10 The lack of a strong correlation in the data between hours and productivity or wages over the cycle is, of course, a classic conundrum for macroeconomists; see Geary and Kennan (198x) or Christiano and Eichenbaum,(1988) for up to date discussions and references. Of course, another way to reconcile theory and evidence is to include shocks to the marginal rate of substitution between consumption and leisure (which is exactly what Bencivenga (1988) does, and is also pretty close to the solution suggested by Christiano and Eichenbaum, who attempt to measure these shocks using government spending under the assumption that an increase in public consumption raises marginal utility of private consumption. This is another example of the principle that the equilibrium of a home production economy can always be replicated by a model in which home production is absent, if we are given enough latitude to play with preferences. We discuss this further in Benhabib, Rogerson and Wright (1990).

using market goods and services, their own time, and other inputs. These underlying preferences are defined over fundamental aspects of life ... that do not always bear a stable relation to market goods and services."

Before proceeding to general issues, we close this section with some illustrative examples.[11] We begin by defining preferences by

$$U = \ln(C) + A\cdot\ln(H-h_m-h_n) + Bh_m, \tag{2.8}$$

where $A > B \geq 0$, and C is a composite consumption good given by

$$C = \left(a_m c_m^e + a_n c_n^e\right)^{1/e}. \tag{2.9}$$

The composite good is defined by means of a fairly flexible CES aggregator, with constant elasticity of substitution $1/(1-e)$. On the other hand, market and nonmarket work are perfect substitutes if $B = 0$, whereas if $B > 0$, then for a given amount of total work and consumption the agent would rather work in the market than at home. One can show that leisure, given by $L = H-h_m-h_n$, is necessarily a normal good for this class of preferences.[12] For now we also assume home production is linear, $g(h_n) = s_n h_n$, which will allow us to easily derive closed form solutions.

First, consider the case of $e = 0$, so that (2.9) in fact defines a Cobb-Douglas function, and the elasticity of substitution between c_m and c_n is unity. Assuming an interior solution, the homework function is

$$h_n = h(c_m, h_m) = \left[\frac{a_n}{A+a_n}\right](H-h_m).$$

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[11] These examples are of particular interest, given the specifications in our quantitative analysis in Benhabib, Rogerson and Wright (1990).

[12] In the next section we prove that if the utility function is separable in consumption and hours, as (2.8) is, then leisure must be normal.

Substituting this into (2.8), the reduced form utility function becomes (after a linear transformation)

$$V = a_m \ln(c_m) + (A+a_n)\ln(H-h_m) + Bh_m. \tag{2.10}$$

In this case, home production adds nothing to the model, in the sense that if we had ignored it, or simply set c_n and h_n equal to constants in (2.8), then except for the constants in (2.10), nothing would have changed! To get any real effects with this specification, we therefore need to assume an elasticity of substitution different from unity.

Consider the case where c_m and c_n are perfect substitutes, $e = 1$, and assume for ease of notation that $a_n = a_m = H = 1$. Assuming an interior solution, the homework function in this case is

$$h_n = h(c_m,h_m) = \frac{s_n(1-h_m)-Ac_m}{s_n(1+A)} .$$

Substituting this into (2.8), the reduced form utility function now becomes (after a linear transformation)

$$V(c_m,h_m) = (1+A)\cdot\ln[c_m+s_n(1-h_m)] + Bh_m. \tag{2.11}$$

If $B = 0$, then (2.11) is of the special "zero wealth effect" class, $V(c_m,h_m) = v_1[c_m+v_2(1-h_m)]$, where v_1 and v_2 are increasing, concave functions; if $B > 0$, on the other hand, then leisure is actually inferior according to the reduced form utility function, even though it is normal according to the underlying utility function.[13]

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[13] Let $h(x)$ solve the labor supply problem, maximize $V(c,h)$ subject to $c = x + wh$. The standard result is that $h'(x)$ is proportional to $\eta \equiv wV_{11} + V_{12}$, so that leisure is normal if and only if $\eta < 0$. If $V = v_1[c_m+v_2(1-h_m)] + Bh_m$, we have $\eta = -Bv_1''/v_1'$; hence, the wealth effect for this class of utility

To see how this might be important for macroeconomics, consider the following "pseudo-dynamic" representative agent problem

$$\max E \sum \beta^t U[c_{mt}, c_{nt}, h_{mt}, h_{nt}]$$

$$\text{st } c_{mt} = \Gamma^t s_{mt} F(h_{mt})$$

$$c_{nt} = \Gamma^t s_{nt} G(h_{nt})$$

where $\beta \in (0,1)$, and $\Gamma > 1$ represents exogenous technological growth common to the two sectors.[14] In order to capture a long run stylized fact of actual economies, we impose the condition that market hours do not grow or shrink on average along a balanced growth path. This means that if the s_{jt} are constant over time then h_{mt} will be constant, too, which means wealth and substitution effects must cancel each other. In economies without home production, this means the utility function must be from either the class

$$u(c_m, h_m) = \left[\frac{c_m^{1-\rho}}{1-\rho}\right] \cdot v(h_m)$$

with $\rho > 0$ and $\rho \neq 1$, or

$$u(c_m, h_m) = \ln(c_m) + v(h_m),$$

where in either case $v(\cdot)$ is concave (see King, Plosser and Rebello 1987 for a proof).

Suppose that we also insist that h_m be constant along a balanced growth

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functions depends exclusively on the sign of B.

[14] The model is "pseudo-dynamic" in that there is no capital formulation, and so it really reduces to a sequence of static economies (we introduce capital in Section V, but for present purposes, the simpler structure will suffice).

path in the home production economy. One specification that satisfies this criterion is easily seen to be:

$$U = \frac{\left[C^b L^{1-b}\right]^{1-r} - 1}{1-r}, \quad C = \left(a_m c_m^e + a_n c_n^e\right)^{1/e} \text{ and } L = H - h_m - h_n$$

As a special case, consider $U = \ln(c_m + c_n) + A \cdot \ln(H - h_m - h_n)$ and linear home production, for which we have already derived the reduced form in (2.11). Thus, this home production model is observationally equivalent to a model without home production and the zero wealth effect utility function

$$V(c_{mt}, h_{mt}) = \ln\left[c_{mt} + \Gamma^t s_{nt}(H - h_{mt})\right]. \tag{2.12}$$

These preferences imply potentially large intratemporal substitution effects (in addition to the intertemporal substitution effects that would be present if we included capital) despite of the fact that h_{mt} does not change on average along the growth path.[15]

To illustrate things further, consider the market technology $f(\cdot) = s_{mt} h_{mt}^{\theta}$. Then it is straightforward to check that the equilibrium allocation involves:

$$h_{mt} = \left(\frac{\theta s_{mt}}{s_{nt}}\right)^{\frac{1}{1-\theta}} \qquad h_{mt} + h_{nt} = \frac{1}{1+A}$$

Total work is constant, while the mix of hours between the home and market fluctuates according to the ratio (s_{mt}/s_{nt}), with an elasticity $1/(1-\theta)$. It is also straightforward to show that the marginal product of labor in the

[15] One way to interpret this result is to note that the term Γ^t inside of the square brackets in (2.12) acts to increase the reduced form's marginal utility of leisure at the same rate as the marginal product of market labor, keeping the value of h_{mt} that equates the two constant on average.

market is proportional in equilibrium to s_{nt}, the nonmarket shock, and so productivity is necessarily negatively related to employment. Also, instantaneous utility in equilibrium is (a linear function of) $\ln(s_n)$, and hence it too is negatively related to employment. While this example is obviously simplistic in its functional form as well as its neglect for important factors such as capital, it does demonstrate how introducing home production can have some rather dramatic effects.

III. More General Results[16]

In this section, we derive some more general results concerning the way home production affects the mapping between the underlying utility function $U(\cdot)$ and the reduced form utility function $V(\cdot)$. We concentrate on the case of perfect substitutes up to a linear perturbation,

$$U = u(c_m+c_n, h_m+h_n) + Ac_m + Bh_m$$

where A and B are constants. This is not the most general case, of course, but it does deliver some sharp predictions. The interpretation of the linear terms is that $A > 0$ ($B > 0$) means that market consumption (market employment) is superior to its nonmarket alternative. We allow for general technology specifications, $g(h_n) = s_n G(h_n)$ and $f(h_m) = s_m F(h_m)$.

As a special case of problem (2.5), the unique competitive equilibrium in the representative agent version of this model has first order conditions

$$s_m F'(h_m)[u_1(\cdot)+A] + u_2(\cdot) + B = 0$$

$$s_n G'(h_n)u_1(\cdot) + u_2(\cdot) = 0.$$

Notice A or $B > 0$ implies $f' = s_m F' < s_n G' = g'$, and the marginal product is lower in the home than the market. Differentiating, we have

$$D \begin{bmatrix} dh_m \\ dh_n \end{bmatrix} = - \begin{bmatrix} (u_1+A)F'+F\eta_m \\ F\eta_m \end{bmatrix} ds_m - \begin{bmatrix} G\eta_m \\ u_1 G'+G\eta_n \end{bmatrix} ds_n - \begin{bmatrix} \eta_m \\ \eta_n \end{bmatrix} dx$$

where $\eta_m = f'u_{11}+u_{12}$, $\eta_n = g'u_{11}+u_{12}$, and D is a matrix given by:

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16 This section contains some messy derivations, designed to show our earlier discussion is fairly general; readers not interested in these details can skip to the section on unemployment with no loss in continuity.

$$D = \begin{bmatrix} (u_1+A)f''+f'^2u_{11}+2f'u_{12}+u_{22} & f'g'u_{11}+(f'+g')u_{12}+u_{22} \\ f'g'u_{11}+(f'+g')u_{12}+u_{22} & u_1g''+g'^2u_{11}+2g'u_{12}+u_{22} \end{bmatrix}$$

The determinant of D is positive:

$$|D| = (A+u_1)f''(g'^2u_{11}^2+2g'u_{12}+u_{22}) + (u_{11}u_{22}-u_{12})(f'-g')^2$$
$$+ u_1g''(f'^2u_{11}^2+2f'u_{12}+u_{22}) > 0.$$

Using Cramer's rule and simplifying, the pure wealth effect in general equilibrium on market hours is

$$|D|\partial h_m/\partial x = - u_1g''\eta_m + (g'-f')(u_{11}u_{22}-u_{12}^2).$$

With perfect substitutes (A = B = 0), we have $g' = f'$, and the second term vanishes; in this case, the condition for h_m to decrease with x is the standard (from models without home production) normal leisure condition, $\eta_m = f'u_{11} + u_{12} < 0$. If A, B > 0, however, then $g' > f'$, and the second term is positive; in this case, hours worked in the market may actually increase with x, even if $\eta_m < 0$.[17] A symmetric result holds for homework,

$$|D|\partial h_n/\partial x = - (u_1+A)f''\eta_n - (g'-f')(u_{11}u_{22}-u_{12}^2),$$

and these can be combined to yield the effect on total leisure, $L = 1-h_m-h_n$,

$$|D|\partial L/\partial x = u_1g''\eta_m - (u_1+A)f''\eta_n.$$

If η_m, $\eta_n < 0$ then leisure is unambiguously normal, even though h_m can

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[17]Note that linear home production implies the first term vanishes; therefore h_m increases with x *if and only if* $g' > f'$.

increase with x. In particular, $u_{12} = 0$ always implies $\partial L/\partial x < 0$.

We can also derive the effects of changes in the productivity parameters, s_m and s_n. For example,

$$\partial h_m/\partial s_m = - Q|D|^{-1}(u_1+A)F' + F\cdot\partial h_m/\partial x$$

where $Q = u_1 g''+g'^2 u_{11}+2g' u_{12}+u_{22} < 0$. The first term in this expression is the unambiguously positive substitution effect, while the second term is the wealth effect derived above. Finally, we can consider balanced technical progress by setting $s_m = s_n = s$. Then, assuming $A = u_{12} = 0$ to reduce the notation, it turns out that

$$\partial h_m/\partial s = |D|^{-1}u_1^2 sF' G'' + (F+G)\partial h_m/\partial x + |D|^{-1}(G' - F')u_1 u_{22}.$$

The key point here is that the third term is negative. Thus, in order to have h_m constant in response to balanced changes in technology, we do not need the wealth and substitution effects to cancel out, and we could easily have $\partial h_m/\partial x > 0$.

IV. Unemployment

In the previous sections, the competitive equilibrium involves everyone receiving exactly the same allocation. All agents spend the same amount of time in market work, and all agents spend the same amount of time in home work. There are a number of ways to amend the basic model to account for the fact that not all agents work in the market, and still maintain the tractability of a representative agent framework (e.g., any of a variety of fixed costs or other nonconvexities associated with market work could be modeled). For simplicity, we will assume directly that time allocated to the market can take on only two values, 0 or $\bar{h}$, where without loss in generality we set $\bar{h} = 1$ (renormalizing the total time endowment, H, if necessary). This is the indivisible labor assumption studied in Rogerson (1984, 1988), and subsequently employed in equilibrium macroeconomics by Hansen (1985), Greenwood and Huffman (1987), Hansen and Sargent (1988), Cho and Rogerson (1988), Christiano and Eichenbaum (1988), Cooley and Hansen (1989), and others.[18]

In nonconvex economies like this, it can be efficient to randomize the allocation. The relevant social planning problem is to choose a probability of employment for the representative agent, φ, and a consumption - homework package for both the employed and unemployed. Let c_m^j and h_n^j be consumption of the market good and hours of nonmarket work by an agent working j units of time in the market, j = 0 or 1. Then the planning problem is

$$\begin{aligned} &\max\ \varphi U[c_m^1, g(h_n^1), 1, h_n^1] + (1-\varphi)U[c_m^0, g(h_n^0), 0, h_n^0] \\ &\text{st } \varphi c_m^1 + (1-\varphi)c_m^0 \le x + f(\varphi) \text{ and } 0 \le \varphi \le 1, \end{aligned} \tag{4.1}$$

[18]See Prescott (1986) and Lucas (1987) for general discussions of the indivisible labor model in macroeconomics.

also subject to nonnegativity.[19] As always, we have substituted the home production constraints directly into the objective function. However, since individuals not employed in the market can still enjoy consumption of c_m, there is a single constraint concerning the market good. For robust specifications we can have $\varphi < 1$, and we assume this is the case in what follows. The fraction $(1-\varphi)$ of agents will be called *unemployed* (although they may well be working at home).

Let λ be the multiplier on the resource constraint in (4.1). Then the first order conditions are as follows:

$$U[c_m^1, g(h_n^1), 1, h_n^1] - U[c_m^0, g(h_n^0), 0, h_n^0] + \lambda[f'(\varphi) - c_m^1 + c_n^0] = 0 \tag{4.2}$$

$$U_2[c_m^1, g(h_n^1), 1, h_n^1] g'(h_n) + U_4[c_m^1, g(h_n^1), 1, h_n^1] = 0 \tag{4.3}$$

$$U_2[c_m^0, g(h_n^0), 0, h_n^0] g'(h_n) + U_4[c_m^0, g(h_n^0), 0, h_n^0] = 0 \tag{4.4}$$

$$U_1[c_m^1, g(h_n^1), 1, h_n^1] - \lambda = 0 \tag{4.5}$$

$$U_1[c_m^0, g(h_n^0), 0, h_n^0] - \lambda = 0 \tag{4.6}$$

$$x + f(\varphi) - \varphi c_m^1 - (1-\varphi) c_m^0 = 0. \tag{4.7}$$

These have straightforward interpretations. For example, let $g_j(\cdot)$ indicate that the home production function is being evaluated at h_n^j, and let $U^j(\cdot)$ indicate that the utility function is being evaluated at $[c_m^j, g(h_n^j), j, h_n^j]$, $j = 0$ or 1. Then (4.3) and (4.4) imply $g'_j = U_4^j/U_2^j$, so that the marginal product in home production is equated to the marginal rate of substitution

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[19] The solution to problem (4.1) is the optimal randomized allocation, which can be decentralized in a variety of ways. For example, Shell and Wright (1989) show formally how to support the the planner's randomized allocation as a standard competitive equilibrium with extrinsic uncertainty represented by "sunspots" (there is no home production in that construction, but it is clear how to extend the results to include it).

for both unemployed and employed workers.

Equations (4.5) and (4.6) imply the efficient risk sharing condition, equating the *marginal* utilities of the market good between employed and unemployed agents:

$$U_1^1 = U_1[c_m^1, g(h_n^1), 1, h_n^1] = U_1[c_m^0, g(h_n^0), 0, h_n^0] = U_1^0 \qquad (4.8)$$

In general, of course, (4.8) says nothing about total utility. Let z be the normalized difference between the total utilities of employed and unemployed agents: $z = (U^1 - U^0)/\lambda$. Then we define the case of $z > 0$ to be *involuntary unemployment*. In Rogerson and Wright (1988), in a model without home production, we found $z > 0$ if and only if $\partial\varphi/\partial x > 0$. Further, one can show that $\partial\varphi/\partial x > 0$ implies leisure is an inferior good, in the standard sense, over some region of commodity space, although not necessarily everywhere; see, e.g., Greenwood and Huffman (1988). Hence, it is impossible to have leisure everywhere normal and involuntary unemployment at the same time. We now show that in the model with home production, we still have $z > 0$ if and only if $\partial\varphi/\partial x > 0$, but the relation between this and normal leisure is broken.[20]

Begin by differentiating the first order conditions (4.2)-(4.7) and simplifying, to arrive at

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[20]Note that from (4.2) that we also have

$$z = c_m^1 - c_m^0 - f'(\varphi).$$

This tells us that unemployment is involuntary if and only if the difference in market consumption between employed and unemployed workers exceeds the marginal product of an employed worker, which in a sense could be interpreted as saying that the employed are being paid "too much." This result is true in models without home production, too, by the way.

$$
\begin{bmatrix}
\lambda f'' & 0 & 0 & 0 & 0 & -z \\
0 & Q_1 & 0 & \eta_1 & 0 & 0 \\
0 & 0 & Q_0 & 0 & \eta_0 & 0 \\
0 & \eta_1 & 0 & U^1_{11} & 0 & -1 \\
0 & 0 & \eta_0 & 0 & U^0_{11} & -1 \\
-z & 0 & 0 & -\varphi & \varphi-1 & 0
\end{bmatrix}
\begin{bmatrix}
d\varphi \\ dh^1_n \\ dh^0_n \\ dc^1_m \\ dc^0_m \\ d\lambda
\end{bmatrix}
=
\begin{bmatrix}
0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -dx
\end{bmatrix}
\tag{4.9}
$$

where we have used the notation

$$Q_j = g''_j U^j_2 + g'^2_j U^j_{22} + 2g'_j U^j_{24} + U^j_{44}$$

$$\eta_j = g'_j U^j_{21} + U^j_{41}.$$

Note that $Q_j < 0$, while η_j could be of either sign in general (one can show that $\eta_j < 0$ if and only if $\partial h^j_n/\partial x < 0$). In the Appendix, we prove the following result.

Lemma 1: $\Psi_j = Q_j U^j_{11} - \eta^2_j > 0$, $j = 0, 1$.

This lemma is also used in the Appendix to verify that the second order conditions for problem (4.1) hold; thus, if we let Δ be the determinant of the square matrix in (4.9), $\Delta < 0$. The next result is then straightforward.

Theorem 2: Unemployment is involuntary if and only if $\partial\varphi/\partial x > 0$.

Proof: Solving (4.9) for $\partial\varphi/\partial x$ and simplifying yields

$$\partial\varphi/\partial x = -\Delta^{-1}\Psi_1\Psi_0 z.$$

Now $\partial\varphi/\partial x > 0$ if and only if $z < 0$, which by definition means if and only if $U^1 > U^0$. ■

This extends the main result in Rogerson and Wright (1988) to the home production economy. However, we now argue that with home production there is no problem having involuntary unemployment and normal leisure at the same time.[21] We make this point by way of an example, using the utility function

$$U = v_1(c_m+c_n) + v_2(h_n+h_m) + Ac_m + Bh_m. \tag{4.10}$$

Again, the interpretation is that agents prefer market to home produced consumption if $A > 0$, and prefer market work to homework if $B > 0$. As shown in the previous section, such utility functions always entail normal leisure in the sense that $\partial L/\partial x > 0$, where $L = 1-h_m-h_n$, although at the same time, they potentially allow $\partial h_m/\partial x > 0$.

For utility function (4.10), the efficient risk sharing condition (4.8) implies that employed and unemployed agents enjoy the same total consumption, $c_m^1 + c_n^1 = c_m^0 + c_n^0 \equiv c$, although the employed get more of the market good and the unemployed get more of the home produced good. Suppose g is linear, $c_n = B \cdot h_n$; then the efficient hours conditions, $U_4^j/U_2^j = B$ for $j = 0$ or 1, imply that the employed and unemployed also work the same number of total hours, $1 + h_n^1 = h_n^0$. Hence, we have

$$U^1 - U^0 = A(c_m^1-c_n^0) + B.$$

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[21] In the special case of perfect substitutes and linear home production, one can show that employed and unemployed agents always get the same total consumption and hours,

$$c_m^1 + c_n^1 = c_m^0 + c_n^0 \quad \text{and} \quad 1 + h_n^1 = h_n^0.$$

In this case we can have normal leisure and $U^1 = U^0$. However, when the home technology is strictly concave, perfect substitutes, normal leisure and involuntary unemployment are not all possible at the same time. Thus, we need to assume less than perfect substitutes to get involuntary unemployment and normal leisure in the general case.

As long as either A or B is strictly positive, $U^1 > U^0$. Therefore it is possible to have involuntary unemployment simultaneously with normal leisure, at least if is linear. If $g'' < 0$, then one can show that the employed end up working fewer hours in the home but more hours in total, $1+h_n^1 > h_n^0 > h_n^1$. Nevertheless, simply by continuity, with $g'' < 0$ it is still possible to have involuntary unemployment simultaneously with normal leisure. In summary, we have:

Theorem 3: In the home production economy, involuntary unemployment does not imply inferior leisure.

We think that this result is important. It applies to not only the representative agent model with indivisible labor, but also to a variety of other models with random layoffs and efficient risk sharing. These include the standard Azariadis (1975) implicit contract model and versions of the Feldstein (1976) temporary layoff model of unemployment insurance (see Burdett and Wright 1989 for a discussion of these approaches). One reason such theories seem to have fallen into disfavor recently is that users were uncomfortable with the implication that laid off workers were happier than their employed colleagues, given normal leisure (see, e.g., the discussion in Rosen's 1985). Now it is obvious that these models should not be used to explain *all* types of unemployment, as they abstract from many relevant considerations for some types, such as frictional unemployment. Yet they do seem to be quite satisfactory for the analysis of other types, such as temporary layoff unemployment. It is perhaps comforting that versions that explicitly incorporate home production do allow the coexistence of efficient risk sharing, normal leisure, and involuntary unemployment.

To close this section, we briefly discuss how heterogeneity might enter the picture. Suppose individual types are indexed by i, and that

$$U^i = v_1(c_m+c_n) + v_2(h_m+h_n) + A^i c_m + B^i h_m.$$

Market labor is still indivisible, but now individuals also differ in terms of their productivities, say h_m and h_n hours of type i's time in the two sectors yield $h^i_m = \omega^i_m h_m$ efficiency units in the market and $h^i_n = \omega^i_n h_n$ in the home. The efficient allocation now determines a probability of employment in the market sector φ^i *for each type*. Any type with A^i, $B^i > 0$ and $h^i_m = 0$ is involuntarily unemployed, at least if g'' is close to 0, as shown above. However, if ω^i_m/ω^i_n is small, we expect there will be lots of type i workers unemployed. At the same time, A^i, $B^i < 0$ implies individual i would rather stay home, but if ω^i_m/ω^i_n is large, the efficient allocation will have him working in the market with positive probability. The point is that in a cross section it will be easy to find some agents not working in the market who wish they were, and at the same time, some who are employed but in a sense wish they were at home.

V. Dynamics

In this section we move to a genuinely dynamic formulation, in order to illustrate some other implications of home production. Consider the problem

$$\max E \sum \beta^t U(c_{mt}, c_{nt}, h_{mt}, h_{nt})$$

$$\begin{aligned} \text{s.t. } c_{mt} &= s_{mt}F(h_{mt}, k_{mt}) - i_t \\ c_{nt} &= s_{nt}G(h_{nt}, k_{nt}) \\ h_{mt} + h_{nt} &\leq H \\ k_{mt} + k_{nt} &\leq k_t \\ k_{t+1} &= (1-\delta)k_t + i_t \end{aligned} \tag{5.1}$$

where k_{jt} is capital in sector j, k_t is total capital, i_t is investment, and $\delta \in (0,1)$ is the depreciation rate.[22] The constraints hold at every date t. The maximization is over time paths $\{c_{mt}, c_{nt}, h_{mt}, h_{nt}, k_{mt}, k_{nt}, i_t\}$, given processes for the shocks $\{s_{mt}, s_{nt}\}$ and initial conditions.

Suppose that we are given $\{c_{mt}, h_{mt}, k_{mt}, k_{nt}\}$, and we are asked to choose a path for homework $\{h_{nt}\}$ to solve

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[22] We have set this up as an optimal growth problem, but of course the solution can be supported as a competitive equilibrium. We could have also illustrated essentially the same points with a single consumer decision problem. Also notice that although capital is an input into household production, we have assumed that it is produced exclusively in the market. Finally, we have assumed that capital can be freely moved between sectors. However, exactly the same message goes through if we alternatively assume two separate laws of motion,

$$k_{mt+1} = (1-\delta_m)k_{mt} + i_{mt} \quad \text{and} \quad k_{nt+1} = (1-\delta_n)k_{nt} + i_{nt},$$

where $i_{mt} + i_{nt} = i_t$.

$$\max E_0 \sum \beta^t U[c_{mt}, s_{nt}G(h_{nt}, k_{nt}), h_{mt}, h_{nt}]$$

$$\text{s.t. } h_{nt} \leq H - h_{mt} \text{ for all t.}$$

The first order conditions for this problem are $s_{nt}G_1 = -U_4/U_2$ for all t and for all realizations of $\{s_{mt}, s_{nt}\}$. This implies that, for all t, the *instantaneous homework function* is given by

$$h_{nt} = h(c_{mt}, h_{mt}, k_{nt}, s_{nt}).$$

Notice $h(\cdot)$ does not depend on k_{mt} or s_{mt}, or on t, or on variables at dates other than t. In the obvious way, we also have the *instantaneous home consumption function,*

$$c_{nt} = c(c_{mt}, h_{mt}, k_{nt}, s_{nt}),$$

and the *reduced form instantaneous utility function,*

$$V(c_m, h_m, k_n, s_n) \equiv U[c_m, c(\cdot), h_m, h(\cdot)].$$

We now have an equivalent alternative formulation of (5.1), in which we choose $\{c_{mt}, h_{mt}, k_{mt}, k_{nt}, i_t\}$ to solve

$$\max E_0 \sum \beta^t V(c_{mt}, h_{mt}, k_{nt}, s_{nt})$$

$$\text{s.t. } c_{mt} = s_{mt}F(h_{mt}, k_{mt}) - i_t$$

$$h_{mt} \leq H$$

$$k_{mt} + k_{nt} \leq k_t$$

$$k_{t+1} = (1-\delta)k_t + i_t.$$

In this problem, the home production variables c_n and h_n do not appear at all, although k_n and s_n do.[23] We interpret this by saying that the dynamic home production model is equivalent to a model without home production, but with different preferences, as well as a consumer durable good k_n. This is the natural extension of the static results in Section II.

For example, consider the utility function $U = \ln(C) + A \cdot \ln(L)$, where $C = C(c_m, c_n)$ and $L = 1-h_m-h_n$. Although it is obviously not possible to find an explicit solution for reduced form preferences, in general, it is possible for the following special cases.

Case i: $C = c_m + c_n$ (perfect substitutes) and $c_n = a_0 k_n + a_1 h_n$ (linear home production). In this case, the reduced form utility function is (after a linear transformation)

$$V = \ln\left[c_m + a_0 k_n + a_1(1-h_m)\right]$$

Case ii: $C = c_m^a c_n^{1-a}$ (Cobb-Douglas) and $c_n = a_0 k_n + a_1 h_n$. In this case,

$$V = a \cdot \ln(c_m) + (1-a+A) \cdot \ln\left[a_0 k_n + a_1(1-h_m)\right].$$

Case iii: $C = c_m^a c_n^{1-a}$ and $c_n = k_n^{\eta} h_n^{1-\eta}$. In this case,

$$V = a \cdot \ln(c_m) + (1-a)\eta \cdot \ln(k_n) + [(1-a)(1-\eta)+A] \cdot \ln(1-h_m).$$

The striking feature that emerges from the above examples is that one underlying utility function, $U = \ln(C) + A \cdot \ln(L)$, can give rise to such different reduced forms. In case i, the three commodities c_m, k_n, and $1-h_m$ are perfect substitutes, while in case iii, V is additively separable.

23 Again, we are assuming capital can be moved freely between the two sectors at t, but a similar result follows if we assume the two capital stocks evolve separately, as in the previous footnote.

Applied researchers are typically concerned with the choice of functional forms, including such issues as separability between variables or groups of variables. As shown by these examples, however, the assumption of separability in the true underlying preferences may or may not carry over to the reduced form that one takes to the data. In particular, we note that with this structure it is apparently difficult to obtain a reduced form in which c_m and k_n enter as perfect substitutes, but separable from h_m, a specification that is often used in empirical studies of durable goods and intertemporal consumption decisions. More generally, we note that the traditional distinction in economics between preferences and technology has become somewhat blurred here.

Eichenbaum and Hansen (1990) provide an interesting recent contribution to the literature on durables and intertemporal consumption. They posit preferences over *consumption services*, defined as flows derived from stocks of durable goods. Our framework is a special case of theirs in that consumers here have exactly two choices: purchase consumption services c_m directly from the market, or receive services c_n from the stock of home capital. However, the salient element of our approach, which is missing from Eichenbaum and Hansen's, is the time allocation decision (i.e., the choice of h_m and h_n). The use of time intuitively seems essential to producing a service flow from home capital or durables. Furthermore, modeling the allocation of time explicitly has many other implications, as we have attempted to demonstrate. It is hoped that future work in the area may benefit from some of these results.

VI. Conclusion

In this paper we have explored some implications of introducing home production into simple economic models. One result is that there is a mapping between models with home production and those without home production but with different preferences, with the property that the implications of the two models for market variables are identical. However, for fixed preferences the model with home production can generate very different implications. Further, the model without home production might require properties, such as nonnormal leisure, time varying utility, etc., that *ex ante* we may not be willing to entertain. If macroeconomics (or any other applied field) is to be an empirical science, research must ultimately proceed to functional forms, or at least to restrictions that specify certain classes of functional forms and parameters. One way to interpret our claim for the usefulness of including a nonmarket sector in models of market activity is that recognizing home production leads us to examine functional form and parameter issues in a new light. We think that this will have important implications for our ability to understand and interpret empirical observations in macroeconomics, and in other areas.

Appendix

Proof of Lemma 1: We make use of the following inequality:

$$U_{11}(U_{22}U_{44}-U_{24}^2) - U_{12}(U_{12}U_{44}-U_{14}U_{42}) + U_{14}(U_{12}U_{24}-U_{14}U_{22}) < 0 \qquad (A.1)$$

This can be verified by noting expression in question equals the determinant of the Hessian matrix of $\overline{U} \equiv U(c_m, c_n, \overline{h}_m, h_n)$, where $\overline{h}_m$ is fixed, which is a concave function. Now expanding Ψ_j (ignoring the subscript j), we have

$$\Psi = g''U_2U_{11} + g'^2(U_{11}U_{22}-U_{12}^2) + (U_{11}U_{44}-U_{14}^2) + 2g'(U_{11}U_{24}-U_{12}U_{14})$$

$$= g''U_2U_{11} + \left[g'(U_{11}U_{22}-U_{12}^2)^{.5} - (U_{11}U_{44}-U_{14}^2)^{.5}\right]^2$$

$$+ 2g'\left[(U_{11}U_{22}-U_{12}^2)^{.5}(U_{11}U_{44}-U_{14})^{.5} + (U_{11}U_{24}-U_{12}U_{14})\right]$$

after "completing the square." Since the first two terms are positive, it show the last term is, too. Suppose not; then

$$(U_{11}U_{22}-U_{12}^2)^{.5}(U_{11}U_{44}-U_{14})^{.5} < -(U_{11}U_{24}-U_{12}U_{14}).$$

But squaring both sides and simplifying contradicts (A.1). Hence, $\Psi > 0$, and this completes the proof. ■

Second order Conditions: We check the second order conditions for problem (4.1). Let H_k be the bordered Hessian matrix formed by deleting all but the last and the first k rows and columns the square matrix in (4.9). For a maximum, the determinants of these matrices must alternate in sign, starting with $|H_2| > 0$ (see, e.g., Takayama 1985). After a little algebra (rather a lot, actually), we find

$$|H_2| = -\,Q_1 z^2 > 0,$$

$$|H_3| = -\,Q_0 Q_1 z^2 < 0,$$

$$|H_4| = -\,Q_0(\varphi\lambda f'' Q_1 + z^2 \Psi_1) > 0,$$

$$|H_5| = -\,z^2 \Psi_0 \Psi_1 \;-\; \lambda f''[\varphi Q_1 \Psi_0 + (1-\varphi) Q_0 \Psi_1] < 0$$

using Ψ_0, $\Psi_1 > 0$, as shown in Lemma 1. In particular, $\Delta = |H_5| < 0$, as used in Theorem 2. ■

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