ECONOMIC RESEARCH REPORTS

ENDOGENOUS FERTILITY AND GROWTH

BY

Jess Benhabib

and

Kazuo Nishimura

R.R. # 90-20 May, 1990

C. V. STARR CENTER FOR APPLIED ECONOMICS

NEW YORK UNIVERSITY
FACULTY OF ARTS AND SCIENCE
DEPARTMENT OF ECONOMICS
WASHINGTON SQUARE
NEW YORK, N.Y. 10003

Endogenous Fertility and Growth*

by

Jess Benhabib
Department of Economics
New York University

and

Kazuo Nishimura
Institute of Economic Research
Kyoto University
and
Australian National University

February 1990
Revised March 1990

* Kazuo Nishimura's research was supported by grants from The Murata Science Foundation and the Ministry of Education in Japan. The authors also acknowledge the technical support of the C.V. Starr Center for Applied Economics at New York University.

Abstract

We extend the Barro-Becker (1989) model by introducing technical progress and a general altruism function towards children. We characterize the dynamics of accumulation in terms of the parameters of the altrusim function. In particular we are interested in the correlation of wealth and fertility across multiple steady states and along the optimal path, which may in some cases be oscillatory. We provide characterizations in terms of the parameters of the altruism function. We also discuss a variant of the model with human capital and with an endogenous choice of the amount of schooling for children.

JEL Classification Numbers: 020, 100, 800.

Key Words: Fertility, Growth.

1. Introduction

Traditional approaches to growth theory have modeled the process of growth as arising from decisions to invest in physical capital and from technological progress (see McKenzie (1986)). A recent model by Barro and Becker (1989) has incorporated fertility decisions, or decisions about the number of children to have, into a growth theoretic context. The welfare of children (through their per capita consumption) is incorporated into the utility of their parents (also see Kemp and Kondo (1986,1989) and Benhabib and Nishimura (1989)). Since consumption must be sacrificed to educate children, the investment decisions become more complex in such a setting. The model can further be enriched by allowing parents to invest in the human capital of their children so as to raise their productivity. We propose to study the implications of introducing technological progress and human capital into the Barro–Becker model of endogenous fertility.

Harrod–neutral technical progress, apart from human capital, is incorporated into our analysis. One of the implications of technical progress is that steady states are no longer in levels; they represent growth paths of per capita variables like output, consumption on capital. The next section spells out the model. We then show that the dynamic behavior of physical capital will be monotonic or oscillatory, depending on the elasticity of a certain "altruism" function which may also be viewed as the elasticity of an inverse demand curve for children. This elasticity, which is always equal to unity in the Barro and Becker model, also plays a major role in determining the existence of multiple steady states. From an empirical perspective multiple steady states may be useful to explain persistent differentials in wealth between some poor and rich countries.

Another important issue is the characterization of the correlation between wealth and fertility. In Section 3, we give conditions, again in terms of preference elasticities, for wealth and fertility to be positively correlated. When these conditions fail, wealth and fertility may be negatively correlated. Empirically, while studies using short-run data indicate a positive relation between income and fertility, long-run data show a negative relationship, possibly because of the correlation of income with education and the costs of raising children, both of which have a negative impact on fertility (See Easterlin (1973, 1987) and Simon (1977), Chapters 14, 15 and 16). Our model also incorporates the increasing opportunity cost of raising children in terms of forgone output.

Finally in section 4, we explicitly introduce investment decisions in the schooling of children. Higher investment in schooling makes children more productive in adulthood. To keep the analysis manageable we abstract from physical capital, which simplifies the portfolio decisions of parents. We show that the results of the previous section that apply to physical capital, that is to the properties of monotonicity or oscillation, multiplicity of steady states and positive or negative correlation with wealth, also apply to investments in schooling.

2. The Model and Assumptions

Becker and Barro [1988] consider a model where parents derive utility from their own consumption as well as from the utility of their children. Given the utility attained by each child V^{t+1}, the utility of each adult is

$$V^t = U(c_t) + \alpha(n_t)n_t V^{t+1} \tag{1}$$

where c_t is the consumption of parents and n_t is one plus the endogenous population growth rate. By substituting out for V^{t+1} and V^{t+2} , etc., in the equation (1), we get the dynastic utility function

$$V^0 = U(c_0) + \sum_{t=1}^{\infty} \prod_{w=1}^{t} \alpha(n_w)n_w U(c_t) \tag{2}$$

We introduce the production function with Harrod-neutral technical process.

$$Y_t = F(K_t, \lambda^t L_t) \tag{3}$$

where Y_t is output of goods, K_t is capital stock, L_t is labor and λ is exogenous factor of technological progress and $\lambda > 1$. The production function satisfies

F is differentiable on $\mathbb{R}_+^2 = \{x \in \mathbb{R}^2 \mid x_1 \geq 0,\ x_2 \geq 0\}$ with $F_K > 0$ and $F_L > 0$. A.1

F is concave and linearly homogenous. A.2

Each adult earns the wage w_t, has a capital stock $(1-\delta)k_t$ which yields a rent $r_t k_t$ where r_t is the interest rate and δ is the depreciation rate. His wealth is $w_t + (1-\delta+r_t)k_t$. He spends it on his own consumption c_t, on bequests to children $n_t k_{t+1}$, and on costs of raising and educating children $n_t B_t$, where $B_t = \lambda^t \beta$. Here we assume that the cost of raising children increases with technological progress which is confined to the goods sector. Otherwise the cost of raising and educating children would eventually be negligible in terms of output. The budget constraint for each adult is

$$w_t + (1-\delta+r_t)k_t = c_t + n_t(B_t + k_{t+1}) \tag{4}$$

Total capital stocks K_t and K_{t+1} satisfy

$$K_t = L_t k_t, \quad K_{t+1} = L_{t+1} k_{t+1} \tag{5}$$

The production function may be rewritten as the following

$$Y_t = \lambda^t F(\lambda^{-t} K_t, L_t) \tag{6}$$

Using per capita variables $y_t = Y_t/L_t$ and $k_t = K_t/L_t$,

$$y_t = \lambda^t F(\lambda^{-t} k_t, 1) \tag{7}$$

Set $x_t = \lambda^{-t} k_t$ and $h(x) = F(x, 1)$. Then the profit maximizing conditions imply

$$r_t = F_k(K_t, \lambda^t L_t) = h'(x_t)$$

$$w_t = \lambda^t F_L(K_t, \lambda^t L_t) = \lambda^t [h(x_t) - x_t h'(x_t)] \tag{9}$$

By substituting (8) and (9) and $B_t = \lambda^t \beta$ into the budget constraint (4) we get

$$h(x_t) + (1-\delta)x_t = \lambda^{-t} c_t + n_t(\beta + \lambda x_{t+1}) \tag{10}$$

Given x_0, each adult maximizes the dynastic utility function (2) subject to the constraints (10) for $t \geq 0$. We specify the utility function and assume

$$U(c) = c^\sigma, \ 0<\sigma\leq 1 \qquad \text{A.3}$$

If $\sigma = 1$, then utility is linear and if $0 < \sigma < 1$, it is strictly concave. Then using $V(k_0)$ of the dynastic utility (2), we can rewrite the maximizing problem into the following:

$$V(x_0) = \max_{c_0, n_0, x_1} \left(u(c_0) + a(n_0)V(x_1)\right)$$

s.t. $f(x_0) = c_0 + n_0(\lambda x_1 + \beta) \quad c_0 \geq 0,\ n_0 \geq 0,\ x_1 \geq 0$

where $a(n_0) = \lambda^{\sigma}\alpha(n_0)n_0$ and $f(x) = h(x_0) + (1-\delta)x_0$.

Here $a(n_0)$ is assumed to be increasing and strictly concave, which assumes that the utility of the parents is increasing at a diminishing rate with the number of children, for a given level of well-being $V(x_1)$ per child.

$a(n)$ is defined on $[0,\overline{n}]$ with $a(0) = 0$ and $a(\overline{n}) < 1$. A.4

$a(n)$ is differentiable on $(0,\overline{n}]$ with $a'(n) > 0$ and $a''(n) < 0$. A.5

In order to interpret the conditions that characterize the solutions, we confine our analysis to the restricted non-empty convex domain $D\times(0,\overline{n}) \subset \mathbb{R}^3_+$ on which

$$W(k_0, k_1, n) \equiv U(f(x) - n_0(\lambda x_1 + \beta)) + a(n_0)V(x_1) \tag{12}$$

is differentiable.

After choosing n_0 optimally as a function of $(x_0,\ x_1)$, the above problem (11) can also be written as

$$V(x_0) = \underset{x_1}{\text{Max}}\, W(x_0, x_1, n(x_0, x_1)) \tag{13}$$

Define $\overline{W}(x_0, x_1) \equiv W(x_0, x_1, n(x_0, x_1))$ and $\overline{W}_1 = \partial\overline{W}/\partial k_0$. Let x^* be a steady state satisfying $V(x^*) = \overline{W}(x^*, x^*) = \underset{x_1}{\text{Max}}\, \overline{W}(x^*, x_1)$. We assume that a non-trivial steady state $x^* > 0$ exists, and we let E be the set of non trivial steady states.

$E \neq \phi$ and $E \subset D$. A.6

Then the non-trivial steady state values x^* and $n(x^*, x^*)$ are the solutions of the following equations;

$$W_n = -(\lambda x + \beta)U'(c) + a'(n)V(x) = 0 \tag{14}$$

$$W_{x_1} = -\lambda n U'(c) + a(n)V'(x) = 0 \tag{15}$$

In addition, by the envelope theorem we have

$$V'(x) = U'(c)f'(x) \tag{16}$$

Hence from (15) and (16)

$$f'(x) = \frac{\lambda n}{a(n)} \tag{17}$$

must be satisfied at steady states.

3. Characterization of Fertility Rates Changes

We will use the following lemma which holds on the relevant domain D.

Lemma 1: (i) If $\overline{W}_{12}(x_0, x_1) > 0$, then an optimal path $\{x_t\}$ from any $x_0 > 0$, $x_0 \notin E$, is strictly monotone, i.e., $(\hat{x}_{t+1}-\hat{x}_t)(\hat{x}_t-\hat{x}_{t-1}) > 0$, as long as it remains in D.

(ii) If $\overline{W}_{12}(x_0, x_1) = 0$, the capital stock jumps to its steady state value in one period, i.e., $\hat{x}_2 = \hat{x}^*$.

(iii) If $\overline{W}_{12}(x_0, x_1) < 0$, then an optimal path from any $x_0 > 0$, $k_0 \notin E$, fluctuates, i.e., $(\hat{x}_{t+1}-\hat{x}_t)(\hat{x}_t-\hat{x}_{t-1}) < 0$, as long as it remains in D.

Proof of Lemma 1; (1) Consider optimal paths $(\hat{x}_t)$, $(\hat{x}_t')$ from $\hat{x}_0$, $\hat{x}_0'$ respectively, where $\hat{x}_0' > \hat{x}_0$, $(\hat{x}_0, \hat{x}_1) \in D$ and $(\hat{x}_0', \hat{x}_1') \in D$.

Then

$$\overline{W}(\hat{x}_0, \hat{x}_1) \geq \overline{W}(\hat{x}_0, \hat{x}_1') \tag{18}$$

$$\overline{W}(\hat{x}_0', \hat{x}_1') \geq \overline{W}(\hat{x}_0', \hat{x}_1) \tag{19}$$

Hence

$$W(\hat{x}_0', \hat{x}_1') - \overline{W}(\hat{x}_0, \hat{x}_1') + \overline{W}(\hat{x}_0, \hat{x}_1) - \overline{W}(\hat{x}_0', \hat{x}_1) \geq 0. \tag{20}$$

This implies

$$\int_{\hat{x}_0}^{\hat{x}_0'} [\overline{W}_1(s, \hat{x}_1') - \overline{W}(s, \hat{x}_1)]\, ds \geq 0$$

$$\int_{\hat{x}_1}^{\hat{x}_1'} \int_{\hat{x}_0}^{\hat{x}_0'} [\overline{W}_{12}(s, t)\, ds\, dt \geq 0 \tag{21}$$

Since $\overline{W}_{12} > 0$ and $\hat{x}_0' > \hat{x}_0$, $\hat{x}_1' \geq \hat{x}_1$ must hold. We note that along the optimal paths,

$$\overline{W}_2(\hat{x}_0, \hat{x}_1) = W_{x_1} + W_n(\partial n/\partial x_1) = 0. \tag{22}$$

$\overline{W}_{12} > 0$ implies

$$0 = \overline{W}_2(\hat{x}_0, \hat{x}_1) < \overline{W}_2(\hat{x}_0', \hat{x}_1) \tag{23}$$

for $\hat{x}_0' > \hat{x}_0$. Hence $(\hat{x}_0', \hat{x}_1, \ldots)$ cannot be an optimal path and $\hat{x}_1'$ must differ from $\hat{x}_1$. We have shown that $\hat{x}_0' > \hat{x}_0$ implies $\hat{x}_1' > \hat{x}_1$. Therefore $\hat{x}_0 \underset{(<)}{>} \hat{x}$ implies $\hat{x}_t \underset{(<)}{>} \hat{x}$.

(ii) Let x^* be a steady state. It satisfies

$$\overline{W}_2(x^*, x^*) = 0 \tag{24}$$

Since $\overline{W}_{12} = 0$, $\overline{W}_{12}$ is independent of the value of $\hat{x}_0$. Hence $(\hat{x}_0, x^*)$ for any $\hat{x}_0 > 0$ satisfies

$$\overline{W}_2(\hat{x}_0, x^*) = 0 \tag{25}$$

Therefore $(\hat{x}_0, x^*, x^*, \ldots)$ is an optimal path from any $\hat{x}_0 > 0$.

(iii) The inequality (17) and $\overline{W}_{12}$ are used to get $\hat{x}_0' > \hat{x}_0 \to \hat{x}_0' \le \hat{x}_1$. $\hat{x}_1' = \hat{x}_1$ is excluded by the same argument as in the proof of (i). Q.E.D.

At this stage we consider the marginal evaluation of children's utility by each adult, that is

$$p(n) = \frac{a'(n_0)V(x_1)}{a(n_0)V(x_1)} = \frac{a'(n_0)}{a}(n_0) \tag{26}$$

This gives a kind of inverse demand function for children (see Figure 1 below). We introduce the elasticity of this inverse demand curve.

$$e = -\frac{n}{a'/a}\cdot\frac{d(a'/a)}{dn}\cdot = \frac{n}{a/a'}\cdot\frac{d(a/a')}{dn} \tag{27}$$

If $e < 1$ (>1), then the inverse demand is inelastic (elastic) with respect to the number of children. This corresponds to the case where the demand for children is elastic (inelastic) in the usual sense. Note that Barro and Becker (1989) use a Cobb–Douglas form for $a(n)$ which forces e to be equal to unity. The next theorem shows that the elasticity determines the behavior of the optimum paths of capital stocks within the set D.

<u>Theorem 1</u>: If $e < 1$ (>1) and $0 < \sigma < 1$, then the capital stock oscillates (is monotonic). If $e = 1$ (or $\sigma = 1$), the capital stock jumps to its steady state value in the first period.

<u>Proof</u>: The theorem follows from Lemma 1 if we can establish that the sign of $\overline{W}_{12}$ is the same as that of $e-1$. We set

$$W(x_0, x_1, n_0) = U(f(x_0)) - n(\lambda x_1 + \beta)) + a(n_0)V(x_1) \tag{28}$$

Maximizing $W(x_0, x_1, n_0)$ with respect to n and x_1 yields

$$W_n = (\lambda x_1 + \beta)U'(c_0) + a'(n_0)V(x_1) = 0 \tag{29}$$

and

$$W_{x_1} = -\lambda n_0 U'(c_0) + a(n_0)V'(x_1) = 0 . \tag{30}$$

Using (29), we can obtain the optimal value of n_0 as $n(x_0, x_1)$ with the following derivatives:

$$\frac{\partial n}{\partial x_1} = \frac{\lambda U'(c_0) - a'(n_0)V'(x_1) - \lambda(\lambda x_1 + \beta)n_0 U''(c_0)}{a''(n_0)V(x_1) + (\lambda x_1 + \beta)^2 U''(c_0)} . \tag{32}$$

Using (28) and (29), we can evaluate $\overline{W}_{12}$ as follows:

$$\overline{W}_{12} = W_{x_0 x_1} + W_{x_0 n_0}\left(\frac{\partial n_0}{\partial x_1}\right) + W_{n_0 x_1}\left(\frac{\partial n_0}{\partial x_0}\right) + W_{n_0 n_0}\left(\frac{\partial n_o}{\partial x_1}\right)\cdot\left(\frac{\partial n_0}{\partial x_0}\right) \tag{33}$$

where $W_{x_0 x_1} = -\lambda n_0 f'(x_0)U''(c_0)$, $W_{x_0 n_1} = -(\lambda x + \beta)U''(c_0)f'(x_0)$,

$W_{n_0 n_0} = (\lambda x_1 + \beta)^2 U''(c_0) + a''(n_0)V(x_1)$,

$W_{x_1 n_0} = -\lambda U'(c_0) + \lambda n_0(\lambda x_1 + \beta)U''(c_0) + a'(n_0)V'(k_1)$. Substituting into (33) and canceling, we obtain.

$$\overline{W}_{12} = \frac{f'(x_0)U''(c_0)}{a''(n_0)V(x_1)+(\lambda x_1+\beta)^2U''(c_0)}\Big((\lambda x_1+\beta)(a'(n_0)V'(x_1)-\lambda U''(c_0))-\lambda n_0 a''(n_0)V(x_1)\Big)$$

Solving for $V(x_1)$ and $V'(x_1)$ from (29) and (30) and substituting, we obtain

$$\overline{W}_{12} = \frac{f'(x_0)U''(c_0)}{a'(n_0)V(x_1)+(\lambda x_1+\beta)^2U''(c_0)}\left[(\lambda x_1+\beta)\left[\frac{\lambda n_0 a'(n_0)U'(c_0)}{a(n_0)}-\lambda U'(c_0)\right]-\frac{\lambda n_0 a''(n_0)(\lambda x_1+\beta)U'(c_0)}{a'(n_0)}\right]$$

which reduces to

$$\overline{W}_{12} = \frac{\lambda f'(x_0)U''(c_0)(\lambda x_1+\beta)U'(c_0)}{a''(n_0)V(x_1)+(\lambda x_1+\beta)^2U''(c_0)}\left[\frac{n_0 a'(n_0)}{a(n_0)}-1-\frac{n_0 a''(n_0)}{a'}\right] \tag{34}$$

If $1 > \sigma > 0$, then the first square bracket on the right is positive by strict concavity. The second can be further simplified so that it equals

$$\left[\left[\frac{n_0 a'}{a}\right]\frac{(a'(n_0))^2-a(n_0)a''(n_0)}{(a'(n_0))^2}-1\right] = \left[\frac{n_0 a'}{a}\right]\frac{d(a/a')}{dn}-1 = e-1 \tag{35}$$

Therefore the sign of $\overline{W}_{12}$ is the same as that of $e - 1$. If $\sigma = 0$, then $U''(c_0)$

$= 0$ and $\overline{W}_{12}$ is always equal to 0. Q.E.D.

Theorem 1 gives conditions under which the capital stock is oscillatory or monotonic. We now turn to the analysis of how the fertility rate n changes with the capital stock.

<u>Theorem 2</u>: (i) If $e < 1$ and $1 > \sigma > 0$, the fertility rate n oscillates in phase with the per capita stock k, that is $dn_0/dx_0 > 0$.

(ii) If $e = 1$ or $\sigma = 1$, then the fertility rate n jumps to its steady state value in one period.

<u>Proof</u>: (i) We have $dn_0/dx_0 = \partial n_0/\partial x_0 + (\partial n_0/\partial x_1)dx_1/dx_0$. From the proof of Lemma 1, we know that $e < 1$ implies $dx_1/dx_0 < 0$. From (30), (31) and (32) we can compute how the optimal value of n_0 changes with x_0:

$$\frac{dn_0}{dx_0} = \frac{\lambda U'(c_0)(1-\frac{n_0a'}{a})\frac{dx_1}{dx_0} + \lambda(\lambda x_1+\beta)U''(c_0)(f'(x_0)-\lambda n_0\frac{dx_1}{dx_0})}{a''(n_0)V(x_1)+(\lambda x_1+\beta)^2U''(c_0)} \tag{36}$$

However, we also have

$$1 - \frac{n_0a'}{a} = \frac{a-n_0a'}{a} > 0 \tag{37}$$

by A4. Thus, $dn_0/dx_0 > 0$ follows under our concavity assumptions and $e < 1$. Since the capital stock oscillates when $e < 1$, so does n.

(ii) In this case $dx_1/dx_0 = 0$. Hence $dn_0/dx_0 = \partial n_0/\partial x_0 > 0$. Q.E.D.

We will show how previous results are related to the uniqueness of the steady state.

Theorem 3: If $0 < e \leq 1$ or $\sigma = 0$, then a steady state is unique.

Proof: From (17) the steady state condition is

$$f'(x) = \frac{\lambda n}{a(n)} \tag{38}$$

where $n = n(x, x)$. The left hand side of the equation is decreasing in x as $f''(x) < 0$. The derivative of the right hand side of the equation is

$$\frac{\lambda[a - a'n]}{a^2} \frac{dn}{dx} \tag{39}$$

This is increasing in x by the strict concavity of a(n) and $dn/dx > 0$ for the cases of $e \leq 1$ or $\sigma = 0$. Hence a solution of (38) must be unique. Q.E.D.

Remark 1: When $e > 1$, the relationship between fertility and wealth may become negative, since the sign of (36) becomes indeterminate. This seems to be supported by long-run data (see Easterlin (1973, 1987)) while a positive relation seems to be observed with short-run data (see Simon (1977), Chapters 14-17). If we choose an altruism function of the form $a(n) = \delta(n+z)^{A}$ (for $n+z \geq 0$), then $e = n/(n+z)$. Note that $e > 1$ (<1) if $z < 0$ ($z>0$) and that e may vary positively with n. Barro and Becker (1988, 1989) use a Cobb-Douglas altruism function for which $e = 1$.

Remark 2: When $e > 1$ there may also be multiple steady states. Note that Theorem 3, which guarantees the uniqueness of the steady state, assumes $e < 1$. Therefore the multiplicity of steady states arises when the inverse demand function for children is elastic. This may be useful to explain persistent wealth differentials between certain rich and poor countries (see Figure 2 below).

When there are multiple steady states, we can compare the values of the fertility rate at the different steady state levels of the capital stocks.

Theorem 4: Let x_j^* and $n_j^* = n(x_j^*, x_j^*)$ be steady state values for $j = 1, 2$. If $x_1^* < x_2^*$, then $n_1^* > n_2^*$.

Proof: From the left hand side of (38), $f'(x_1^*) > f'(x_2^*)$. But the right hand side of (38) is an increasing function of n. This means that $n_1^* > n_2^*$. Q.E.D.

Remark 3: If multiple steady states can explain the persistent wealth differentials between some rich and poor countries, Theorem 4 then can also account for the observed inverse relationship between wealth and fertility rates.

4. The Model with Schooling

We can incorporate the decision of a parent to invest in the human capital of her children with a simple modification. We denote the cost of raising children by $B_t = s_t\lambda^t\beta$ where s_t is a choice variable. The productivity of labor in period t+1 is affected by the choice of s_t, so that output becomes $Y_{t+1} = F(K_{t+1}, s_t\lambda^{t+1}L_t)$. Defining $x_t = s_{t-1}\lambda^{-t}k_t$, we obtain a budget constraint

$$s_{t-1}\big(h(x_t) + (1-\delta)x_t\big) = \lambda^{-t}c_t + n_t s_t(\beta + \lambda x_{t+1}) \qquad (10)'$$

This constraint generates a model with two state variables, human and physical capital, that is difficult to analyze and can involve complex dynamics. We may substantially simplify the analysis by eliminating physical capital and adopting a technology linear in labor. Let output be denoted by $aL\lambda^t s_{t-1}$. Then the budget constraint for the parent is

$$as_{t-1}\lambda^t = c_t + n_t(\beta_0\lambda^t s_t)$$

or, defining $x_t = s_{t-1}\lambda^t$,

$$ax_t = c_t + n_t\big((\beta_0/\lambda)x_{t+1}\big)$$

In the terminology of the previous section if we now let $f(x) = ax$, set $\delta = 1$, and redefine λ as β_0/λ, we have the problem described in equation (11). Therefore, the results of the previous section immediately apply. In particular, the dynamic behavior of x_t and therefore of s_t is determined by e, as described in Theorems 1 and 2. All this implies that investment in children's schooling may be monotonic or oscillatory, depending on the sign of $1 - e$, that there may be multiple steady states if $e > 1$, and that investment in human capital may be positively or negatively correlated with wealth.

REFERENCES

Barro, R., and C. Becker, "Fertility Choice in a Model of Economic Growth," Econometrica, 57, (1989), 481-501.

Becker, C., A Treatise on the Family, Harvard University Press, (1981).

Becker, C., "A Family Economic and Macro Behavior," American Economic Review, 78, (1988), 1-13.

Becker, C., and R. Barro, "A Reformulation of the Economic Theory of Fertility," Quarterly Journal of Econmics, 103, (1988), 1-25.

Benhabib, J., and K. Nishimura, "Competitive Equilibrium Cycles," Journal of Economic Theory, 35, (1985), 284-307.

Benhahib, J., and K. Nishimura, "Endogenous Fluctuations in the Barro-Becker Theory of Fertility," in Demographic Change and Economic Development, Alois Wening and Klaus Zimmermann (eds.), 1989, Springer-Verlag.

Easterlin, R., "Relative Economic Status and the American Fertility Swing," in Family Economic Behavior: Problems and Prospects, E.B. Sheldon, ed., Philadelphia: Lippincott, 1973.

Easterlin, R., Birth and Fortune, 2nd edition, University of Chicago press, 1987.

Kemp, M., and H. Kondo, "Overlapping Generations, Competitive Efficiency and Optimal Population," Journal of Public Economics, (1986), 236-247.

Kemp, M., and H. Kondo " Analysis of International Migration: The Unilateral Case," in Economic Theory and Optimal Population, K. Zimmerman (ed.), Springer-Verlag, 1989.

McKenzie, L., "Optimal Economic Growth and Turnpike Theorems," Handbook of Mathematical Economics, K. Arrow and M. lntrilligator (eds.), 1986, North-Holland.

Simon, J., The Economics Population Growth, 1977, Princeton University Press.