ECONOMIC RESEARCH REPORTS

# THE INEQUITY OF AN EFFICIENT INDIRECT TAX STRUCTURE

BY

Charles A.M. de Bartolome

R.R. # 90-32 July 1990

**C. V. STARR CENTER FOR APPLIED ECONOMICS**

NEW YORK UNIVERSITY
FACULTY OF ARTS AND SCIENCE
DEPARTMENT OF ECONOMICS
WASHINGTON SQUARE
NEW YORK, N.Y. 10003

# THE INEQUITY OF AN EFFICIENT INDIRECT TAX STRUCTURE

by

Charles A. M. de Bartolome*

## ABSTRACT

Households differ only in their productivities, and household demands may be exactly aggregated. The indirect tax structure which minimizes the aggregate excess burden (efficiency) is the same indirect tax structure which maximizes a Bergson-Samuelson social welfare function. I show that the implicit social weight of a household increases (decreases) as the productivity of the household increases, if leisure is a normal (inferior) good. I develop further necessary conditions on the social welfare function, if household demands may be linearly aggregated.

* New York University. The technical assistance of the C.V. Starr Center for Applied Economics is gratefully acknowledged.

# 1. INTRODUCTION

An indirect tax system is made efficient by choosing the tax rates to minimize the aggregate excess burden. It is made equitable by choosing the tax rates to maximize a social welfare function. If the welfare function is exogenous, it will not in general be maximized at the efficient tax structure. A change in the tax rates will redistribute the tax burden between households; social welfare will rise or fall depending on the relative weights of the benefiting households and of the households which are made worse off. I ask the question: what welfare function is maximized at the efficient tax structure? In my model, households have identical tastes but differ in their productivities, household demands may be exactly aggregated, and leisure is a normal good. I show that the efficient tax structure maximizes a social welfare function in which households of high productivity have high social weight. Equity however requires that households of high productivity have low weights in the social welfare function (Atkinson (1970)). The implicit welfare function is therefore inequitable and the result shows the inherent tension between "efficiency" and "equity."

The excess burden of a tax instrument is the resource cost of using the tax instrument to collect tax revenue. It is measured by the additional resources which households are willing to pay to have the same tax revenue collected by a lump-sum levy. At least since Dupuit (1844), it has been loosely associated with the triangular area between the demand and supply curves. The modern formulation of the excess burden in terms of the expenditure function is due to Mohring (1971) and Diamond and McFadden

(1974). Efficiency requires that the tax structure minimizes the excess burden.

Instead of minimizing the excess burden, the tax structure may be chosen to maximize a social welfare function. The early formulation (most importantly by Ramsey (1927)) ignored the distributional issues by considering the representative household. For a representative household, the two criteria coincide: the efficient tax structure maximizes the utility of thw household (Auerbach (1985)). Diamond and Mirrlees (1971) extended the analysis to many households, using a Bergson-Samuelson social welfare function to make inter-household comparisons. The advantage of this methodlogy is that it makes the prescribed tax structure depend on the social value assigned by the planner to the different households. Diamond (1975) and Atkinson and Stiglitz (1976) show that the reduction in the compensated demand of a commodity is smaller (and presumably the commodity tax rate is reduced) as the social value of the high-consuming households is increased. If a lump-sum levy is an available instrument and is set optimally, Mirrlees (1975) and Diamond (1975) show that less tax revenue is collected from households of high social value.

Efficiency may be contrasted with equity by comparing the prescribed tax rates. Most importantly, efficiency suggests that tax rates should be higher on goods which have few substitutes and inelastic demand. Since there is a general tendency for price inelasticity to be associated with income inelasticity (Deaton (1974)), efficiency seems to require that necessities should be taxed more highly than luxuries. From an equity perspective, however, the tax burden should be shifted to the wealthy, or the tax rate on luxuries should be raised as the social weighting of the poor increases (Deaton (1977) and Muellbauer (1981)).

I change the focus. Instead of taking the social welfare function as exogenous and comparing tax prescriptions, I characterize the social welfare function which is maximized at the efficient tax structure. I assume that household demands may be exactly aggregated: this is an implicit assumption if the excess burden is estimated using aggregate data, e.g., Harberger (1964). The associated expenditure functions are those given by Simmons (1979) and Muellbauer (1981). I show that, when the planner chooses the indirect tax structure to minimize the aggregate excess burden, it is "as if" he were maximizing a social welfare function in which the weights increase monotonically with household productivity. The prediction of a monotonic increase in the weights is dependant on leisure being a normal good. In setting the efficient tax structure, the planner implicitly favors households of high productivity.

The change in household utility, associated with a tax rate change, is converted into units of excess burden by multiplying with a conversion factor, measured as units of expenditure per unit of utility. If leisure is a normal good, an increase in utility is associated with an increase in leisure, which is more expensive for the more productive (high wage) household. The conversion factor is therefore larger for a more productive household, implying that a planner seeking to minimize the aggregate excess burden will place greater weight on the utility loss of these households.

The above argument corresponds to a sufficient condition for the weights in the social welfare function, which is being "as if" maximized when the planner chooses the efficient indirect tax structure. In fact, the tax structure which miminizes the aggregate excess burden may maximize many social welfare functions with different weight distributions. If household demands may be linearly aggregated, I establish necessary conditions on the weight distributions of the implicit social welfare functions. If the proportion of earned income spent on each commodity varies across households, the weights are higher "on average" (in a sense defined in the text) for more productive households. If the proportion of earned income spent on each commodity is constant (as happens if households have Cobb-Douglas utility), the efficient tax structure maximizes the utility of all households, there is no inter-household trade-off, and the minimization of aggregate excess burden is equivalent to the maximization of a social welfare function with any distribution of weights.

Throughout this paper I assume that the indirect taxation of commodities is the only available tax instrument. This follows a strong tradition in the literature, and Deaton (1977) suggests that it may be realistic for undeveloped countries. More generally, a proportional wage tax would be a redundant instrument in the model (Atkinson and Stiglitz (1980)), but the results would be expected to change if a progressive income tax were an available instrument. Similarly, I assume that a lump-sum tax is infeasible: if it were available, it would be the only instrument used by a planner seeking efficiency.

This paper is organized as follows. Section 2 presents the model. Sections 3 and 4 establish the main results on the household weights. Section 5 concludes.

## 2. THE MODEL

The economy consists of H households, who are identical except for their productivity. The productivity of household h is $w^h$. The economy is competitive and the unit of labor is chosen so that household h receives wage $w^h$. There are n commodities. Labor is the only input and the production of each commodity shows constant returns to scale: the production of one unit of good i requires $p_i$ units of labor. The competitive producer price of good i is therefore $p_i$, independent of the quantity of production. Total tax revenue R must be collected by the indirect taxation of commodities; labor is, by assumption, untaxed. If the commodity i is taxed at rate $t_i$, its consumer price is $q_i = p_i(1+t_i)$. It is convenient to represent the producer and consumer price by vectors, $\mathbf{p} = (p_1, \ldots, p_n)$ and $\mathbf{q} = (q_1, \ldots, q_n)$. Because $\mathbf{p}$ is fixed by the production technology, $\mathbf{q}$ is a representation of the tax structure.

The time endowment of each household is T, and labor is the only source of income. Households have identical tastes. Each household is price-taking, and takes it wage $w^h$ and the after-tax price $\mathbf{q}$ of commdities as given when choosing its commodity purchases and labor supply. The consumption by household h of commodity i is $x_i^h = x_i(\mathbf{q}, w^h)$, and its labor supply is $L^h(\mathbf{q}, w^h)$. The commodity demand of household h is represented by the vector $\mathbf{x}^h \equiv (x_1^h, \ldots, x_n^h) \equiv (x_1(\mathbf{q}, w^h), \ldots, x_n(\mathbf{q}, w^h)) \equiv \mathbf{x}(\mathbf{q}, w^h)$. The concern is about how the tax revenue should be collected, and not about expenditure; the public project, on which the tax revenue is spent, is therefore ignored. The household receives utility $U(\mathbf{x}^h, L^h)$,[1] and the indirect utility function is $V(\mathbf{q}, w^h) \equiv U(\mathbf{x}(\mathbf{q}, w^h), L(\mathbf{q}, w^h))$.

The tax revenue collected from household h is $(\mathbf{q}-\mathbf{p})\cdot\mathbf{x}^h$. In the terminology of Mohring (1971),[2] the excess burden of household h under the tax structure $\mathbf{q}$ is the willingness to pay of the household to have the tax structure replaced by a lump-sum of equal revenue, or

$$EB(\mathbf{q};\ w^h) \equiv e(\mathbf{q},\ w^h;\ V(\mathbf{q},\ w^h)) - e(\mathbf{p},\ w^h;\ V(\mathbf{q},\ w^h)) - (\mathbf{q}-\mathbf{p})\cdot\mathbf{x}^h(\mathbf{q},\ w^h)$$

where $e(\mathbf{q},\ w^h;\ U)$ is the full expenditure function, $e(\mathbf{q},\ w^h;\ V(\mathbf{q},\ w^h)) \equiv w^hT$.

The planner seeking to minimize the aggregate excess burden chooses tax rates as

$$\min_{\mathbf{q}} \sum_{h=1}^{H} w^hH - \sum_{h=1}^{H} e(\mathbf{p},\ w^h;\ V(\mathbf{q},w^h)) - \sum_{h=1}^{H} (\mathbf{q}-\mathbf{p})\cdot\mathbf{x}^h(\mathbf{q},w^h) \ \text{ s.t. } \sum_{h=1}^{H} (\mathbf{q}-\mathbf{p})\cdot\mathbf{x}^h(\mathbf{q},w^h) = R.$$

The revenue constraint must hold with equality.[3]

Differentiating the Lagrangian function with respect to the consumer prices, using Roy's Identity $\partial V(\mathbf{q};\ w^h)/\partial q_k = -\alpha^h x_k^h$ where $\alpha \equiv \partial V(\mathbf{q};\ w^h)/\partial M$ is the household's marginal utility of income, and rearranging, the first-order conditions become

$$\frac{\displaystyle\sum_{h=1}^{H} \frac{\partial e(\mathbf{p},\ w^h;\ V^h)}{\partial U}\cdot\alpha^h x_k^h}{\displaystyle\sum_{h=1}^{H} \Big[\sum_{i=1}^{n} (q_i-p_i)\ \frac{\partial x_i^h}{\partial q_k} + x_k^h\Big]} = \frac{\displaystyle\sum_{h=1}^{H} \frac{\partial e(\mathbf{p},\ w^h;\ V^h)}{\partial U}\cdot\alpha^h x_1^h}{\displaystyle\sum_{h=1}^{H} \Big[\sum_{i=1}^{n} (q_i-p_i)\ \frac{\partial x_i^h}{\partial q_1} + x_1^h\Big]}, \qquad k=(2,\ldots,n). \quad (1)$$

where $V^h \equiv V(\mathbf{q},\ w^h)$. The efficient indirect tax structure $\mathbf{q}^*$ minimizes the aggregate excess burden, and is defined by the Equation (1) and the revenue constraint.

A possible alternative objective of the planner is to choose the tax structure to maximize a Bergson-Samuelson social welfare function W. I confine attention to social welfare functions which are additively separable

$$W = \beta^1 V^1 + \ldots + \beta^H V^H, \qquad \text{with } \sum_{h=1}^{H} \beta^h = 1, \tag{2}$$

where $\beta^h \equiv \beta(w^h)$ is the social weight attached to household h.[4] The planner problem would be

$$\max_{\mathbf{q}} \sum_{h=1}^{H} \beta^h V(\mathbf{q}, w^h) \qquad \text{s.t.} \sum_{h=1}^{H} (\mathbf{q}-\mathbf{p}) \cdot \mathbf{x}^h(\mathbf{q}, w^h) = R.$$

Differentiating the Lagrangian function with respect to the consumer prices, using Roy's Identity and rearranging, the first-order conditions become

$$\frac{\sum_{h=1}^{H} \beta^h \alpha^h x_k^h}{\sum_{h=1}^{H} [\sum_{i=1}^{n} (q_i - p_i) \frac{\partial x_i^h}{\partial q_k} + x_k^h]} = \frac{\sum_{h=1}^{H} \beta^h \alpha^h x_1^h}{\sum_{h=1}^{H} [\sum_{i=1}^{n} (q_i - p_i) \frac{\partial x_i^h}{\partial q_1} + x_1^h]}, \qquad k=(2, \ldots, n). \tag{3}$$

Equation (3) and the revenue constraint defines the indirect tax structure which maximizes the social welfare function.

This paper is concerned with the social weights $(\beta^h, \ldots, \beta^H)$, for which the solution to the social welfare maximization problem is $\mathbf{q}^*$, the efficient indirect tax structure.

## 3. SUFFICIENT CONDITIONS ON THE SOCIAL WEIGHTS

Attention is confined to the case in which the full expenditure function of the household can be written as

$$e(\mathbf{q}, w; U) = A(\mathbf{q}, w) + B(\mathbf{q}, w)\, U \;. \tag{4}$$

The important property of this expenditure system is that $\partial^2 e/\partial U^2 = 0$. It is a reasonably general form, which includes the important case in which household demands may be exactly aggregated under maximizing behaviour. This seems the empirically relevant case when the aggregate excess burden is to be estimated using aggregate data.[5]

The solutions to Equations (1) and (3) are the same if the weights of the social welfare function are

$$\beta^h = \frac{\dfrac{\partial e(\mathbf{p}, w^h; V(\mathbf{q}^*, w^h))}{\partial U}}{\displaystyle\sum_{h=1}^{H} \frac{\partial e(\mathbf{p}, w^{h'}; V(\mathbf{q}^*, w^{h'}))}{\partial U}} \;. \tag{5}$$

Appendix A shows that the second-order conditions for a maximum are also satisfied at this choice of $\beta^h$.[6] This establishes Proposition 1.

PROPOSITION 1: *When household preferences may be characterized by the Full Expenditure Function (4), minimizing the aggregate excess burden is equivalent to maximizing a social welfare function* $W = \sum_{h=1}^{H} \beta^h U^h$, *where the social weights are given by Equation (5).*

The adjustment in instrument $q_k$ changes the utility of household h by $dV^h = \partial V^h/\partial q_k \cdot dq_k = - \alpha^h x_k^h \cdot dq_k$ , (using Roy's Identity). The excess burden evaluates the utility change by the income change, at consumer prices **p** and pre-existing income $e(\mathbf{p}, w^h; V^h)$, which would give an equal utility change. To obtain the change in the excess burden, the utility change is therefore converted into a resource equivalent by multiplying by $\partial e(\mathbf{p}, w^h; V^h)/\partial U$. In contrast, the effect on social welfare is obtained by multiplying by the social weight $\beta^h$. Minimization of the excess burden is equivalent to the maximization of social welfare when the two conversion factors are the same.

The variation of the social weight $\beta^h$ with productivity is obtained by differentiating, (remembering that $\partial^2 e/\partial U^2=0$),

$$\frac{d\beta^h}{dw^h} = \frac{d}{dw^h} \frac{\partial e(\mathbf{p}, w^h; V(\mathbf{q}^*, w^h))}{\partial U} = \frac{\partial}{\partial w^h} \frac{\partial e}{\partial U} + \frac{\partial^2 e}{\partial U^2} \frac{\partial V^h}{\partial w} = \frac{\partial}{\partial U} \frac{\partial e}{\partial w^h} = \frac{\partial h_0^h}{\partial U} ,$$

where $h_0^h$ is the compensated demand of household h for leisure. If leisure is a normal good, $\partial h_0^h/\partial U$ is positive and the social weights $\beta^h$ increase with household productivity.[7] If leisure is an inferior good, $\partial h_0^h/\partial U$ is negative and the social weights $\beta^h$ decrease with household productivity. This establishes Proposition 2.

PROPOSITION 2: *The minimization of the aggregate excess burden is equivalent to the maximization of a social welfare function in which the social weights increase (decrease) with household productivity if leisure is a normal (inferior) good.*

In the likely case that leisure is a normal good, Proposition 2 shows the inevitable conflict between "efficiency" and "equity." In a conventional formulation of a social welfare function, households with *lower* productivity are given higher social weight (e.g., Atkinson (1970)). When a planner maximizes the aggregate excess burden, Proposition 2 shows that it is "as if" he were maximizing a social welfare function in which households of *higher* productivity are given a higher weight.

The intuition for Proposition 2 is that, with leisure being a normal good, a small increase in household utility is associated with an increase in leisure. The purchase of this leisure is more expensive at higher wage rates, or $\partial/\partial w\ \partial e/\partial U$ is positive. The conversion factor, with which the change in household utility is multiplied to obtain the excess burden, is therefore higher for a more productive household. When calculating the effect of a tax rate change on the aggregate excess burden, the planner therefore assigns more weight to the utility changes of the more productive households.[8]

## 4. NECESSARY CONDITIONS ON THE SOCIAL WEIGHTS.

Equation (5) is a sufficient condition for the social welfare function to be maximized at the efficient tax structure. In this section I consider the necessary conditions, and further restrict the full expenditure function to the form

$$e(\mathbf{q},\ w;\ U) = F(\mathbf{q}) + G(\mathbf{q})w + C(\mathbf{q})w^{\delta}U, \tag{6}$$

where $F(\mathbf{q})$ is homogeneous of degree 1, $G(\mathbf{q})$ is homogeneous of degree 0 and $B(\mathbf{q})$ is homogeneous of degree $(1-\delta)$. This is a necessary and sufficient expenditure function for linear aggregation (Muellbauer (1981)).[9] Leisure is a normal good if $0<\delta$.[10]

By comparison of Equations (1) and (3), the minimization of the aggregate excess burden is equivalent to the maximization of a social welfare function for which the social weights $\beta^h$ satisfy

$$\sum_{h=1}^{H} \beta^h \alpha^h x_k^h = K \sum_{h=1}^{H} \frac{\partial e(\mathbf{p},\ w^h;\ V^h)}{\partial U} \cdot \alpha^h x_k^h\ , \qquad k=(1,\ldots,n),$$

for each commodity k and some constant K. Using the Full Expenditure Function (6), the demand of commodity k by household h is

$$x_k^h = \left(F_k - \frac{C_k}{C} F\right) + \left(G_k - \frac{C_k}{C} (G-T)\right) w^h \equiv r_k(\mathbf{q}^*) + s_k(\mathbf{q}^*) w^h, \qquad (7)$$

where $F_k \equiv \partial F/\partial q_k$, $G_k \equiv \partial G/\partial q_k$, $C_k \equiv \partial C/\partial q_k$, and all functions are evaluated at consumer prices $\mathbf{q}^*$. The equivalence condition therefore becomes

$$\sum_{h=1}^{H} \left[ \frac{\beta^h}{(w^h)^\delta} - KC(\mathbf{p}) \right] \cdot [r_k(\mathbf{q}^*) + s_k(\mathbf{q}^*) w^h] = 0 \qquad k = (1,\ldots,n). \quad (8)$$

A sufficient condition is that $\beta^h/(w^h)^\delta$ is constant, as shown in Section 3.

Equation (8) is k simultaneous equations. There are two possible cases. In the first case, the ratio $r_k(\mathbf{q}^*)/s_k(\mathbf{q}^*)$ is not constant across all commodities. In this case, Equation (8) requires that $(\beta^1,\ldots,\beta^H)$ satisfy

$$\sum_{h=1}^{H} \left[ \frac{\beta^h}{(w^h)^\delta} - KC(\mathbf{p}) \right] = 0 \quad \text{and} \quad \sum_{h=1}^{H} \left[ \frac{\beta^h}{(w^h)^\delta} - KC(\mathbf{p}) \right] w^h = 0 ,$$

or

$$\sum_{h=1}^{H} \left( \frac{\beta^h}{(w^h)^\delta} - E\left[\frac{\beta^h}{(w^h)^\delta}\right] \right) \cdot w^h = 0 .$$

The social weights must be chosen so that $\beta^h/(w^h)^\delta$ is uncorrelated with household productivity $w^h$. If leisure is a normal good, $0 < \delta$, $(w^h)^\delta$ is increasing in $w^h$, and there is a strong presumption that $\beta^h$ is an increasing function of $w^h$ over most of the productivity range.

The second case arises when $r_k(\mathbf{q}^*)/s_k(\mathbf{q}^*) \equiv r/s$, constant across all commodities. In this case, Equation (8) becomes the single equation

$$K = \frac{\sum_{h=1}^{H} \frac{\beta^h}{(w^h)^\delta} \left( \frac{r}{s} + w^h \right)}{HC(\mathbf{p}) \left( \frac{r}{s} + \bar{w} \right)} .$$

where $\bar{w}$ is mean productivity. In this case, for any values $(\beta^1, \ldots, \beta^H)$, a value of K can be found to satisfy Equation (8), and minimization of the aggregate excess burden is equivalent to the maximization of *all* social welfare functions.

If $r_k(\mathbf{q}^*)/s_k(\mathbf{q}^*)$ is a constant, the expenditure share of commodity k for household h is

$$\frac{q_k x_k^h}{w^h L^h} = \frac{q_k x_k^h}{\sum_{i=1}^{n} q_i x_i^h} = \frac{q_k (r_k + s_k w^h)}{\sum_{i=1}^{n} q_i (r_i + s_i w^h)} = \frac{q_k s_k (\frac{r}{s} + w^h)}{\sum_{i=1}^{n} q_i s_i (\frac{r}{s} + w^h)} = \frac{q_k s_k}{\sum_{i=1}^{n} q_i s_i},$$

which is constant for all households. This establishes Proposition 3.

PROPOSITION 3: *Social welfare is restricted to be additive separable in household utilities,* $\beta^h$ *is the social weight of household h and* $w^h$ *is the productivity of household h. Household demands may be exactly aggregated. Expenditure shares are measured at the efficient tax structure* $\mathbf{q}^*$.

*(a) If the expenditure share of each commodity varies across households, minimization of the aggregate excess burden is equivalent to the maximization of a social welfare function in which* $\beta^h/(w^h)^\delta$ *is uncorrelated with* $w^h$.

*(b) If the expenditure share of each commodity is constant across households, minimization of the aggregate excess burden is equivalent to the maximization of a social welfare function with any distribution of social weights.*

A speical case of Proposition 3(b) is if households have Cobb-Douglas utility: expenditure shares are constant at all prices (and not just $\mathbf{q}^*$). When commodity expenditure shares are constant across households, the utility of every household is maximized at $\mathbf{q}^*$. There is therefore no inter-household trade-off, and the distribution of weights is irrelevant. This is formally established in Appendix B.

## 5. CONCLUSIONS

This paper reexamines the inherent tension between efficiency and equity in indirect taxation. If households differ only in their productivity and household demands may be exactly aggregated, I show that a planner setting tax rates efficiently is behaving "as if" he were maximizing a social welfare function which places higher weight on the households of higher productivity. The efficient tax structure maximizes many social welfare functions, but there is a strong bias in all such functions towards favoring the households of higher productivity. If commodity demands are proportional to earned income, the efficient tax structure maximizes any welfare function with any distribution of weights.

## APPENDIX A: THE SECOND ORDER CONDITIONS

This appendix shows that $\mathbf{q}^*$ satisfies the second order conditions for the constrained maximization of social welfare, if the weights $\beta^h$ satisfy Equation (5). The minimization of excess burden is equivalent to the maximization of negative excess burden. The Lagrangian is

$$L_1 = -\sum_{h=1}^{H} w^h T + \sum_{h=1}^{H} e(\mathbf{p},\ w^h;\ V(\mathbf{q},w^h)) + \sum_{h=1}^{H} (\mathbf{q}-\mathbf{p})\cdot\mathbf{x}^h(\mathbf{q},w^h) - \mu[R - \sum_{h=1}^{H} (\mathbf{q}-\mathbf{p})\cdot\mathbf{x}^h(\mathbf{q},w^h)].$$

The bordered Hessian for the second-order conditions is, (remembering that $\partial^2 e/\partial U^2 = 0$),

$$\overline{H}_1 = \begin{vmatrix} 0 & R_i \\ R_j & Y_{ji} \end{vmatrix},$$

where $R_i \equiv \frac{\partial}{\partial q_i}\left(\sum_{h=1}^{H}\sum_{i=1}^{n}(q_i-p_i)\cdot x_i^h\right)$, and $Y_{ji} = \sum_{h=1}^{H} \frac{\partial e}{\partial U} \frac{\partial^2 V^h}{\partial q_j \partial q_i} + (1+\mu) \frac{\partial R_i}{\partial q_j}$.

The Lagrangian for the constrained maximization of social welfare is

$$L_2 = \sum_{h=1}^{H} \beta^h V(\mathbf{q},\ w^h) + \Lambda \left(\sum_{h=1}^{H} (\mathbf{q}-\mathbf{p})\cdot\mathbf{x}^h(\mathbf{q},\ w^h) - R\right).$$

The first order conditions are equivalent if $\beta^h$ is given by Equation (5). In this case

$$(1+\mu) = \sum_{h'=1}^{H} \frac{\partial e(\mathbf{p},\ w^{h'};\ V^{h'})}{\partial U} \cdot \Lambda . \tag{A.1}$$

The bordered Hessian for the second-order conditions is

$$\overline{H}_2 = \begin{vmatrix} 0 & R_i \\ R_j & Z_{ji} \end{vmatrix} ,$$

where $R_i$ is as before and $Z_{ji} = \sum_{h=1}^{H} \beta^h \frac{\partial V^h}{\partial q_j \partial q_i} + \Lambda \frac{\partial R_i}{\partial q_j}$ .

Replacing $\beta^h$ and $\Lambda$ by the respective values (from Equations (5) and (A.1)), multiplying each row ( 2, ..., n) by $\sum_{h=1}^{H} \partial e/\partial U$ and dividing the first column by the same, leaves a bordered Hessian whose principal minors are identical to $\overline{H}_1$.

## APPENDIX B: THE MAXIMIZATION OF HOUSEHOLD UTILITY

This appendix shows that the optimal indirect tax structure for every household is $\mathbf{q}^*$ when $r_k(\mathbf{q}^*)/s_k(\mathbf{q}^*) = r/s$ for all k. The optimal tax structure for household h, given that total tax revenue R is collected, is the solution to the problem

$$\max_{\mathbf{q}} \quad V(\mathbf{q}, w^h) \quad \text{s.t.} \quad \sum_{h'=1}^{H} (\mathbf{q}-\mathbf{p})\cdot\mathbf{x}^{h'}(\mathbf{q}, w^{h'}) = R.$$

Setting up the Lagrangian and taking the first order conditions, the optimal tax structure $(q_1, \ldots, q_n)$ for household h is the solution to the revenue constraint and the (n-1) equations

$$\frac{\alpha^h x_k^h}{\sum_{h'=1}^{H}[\sum_{i=1}^{n}(q_i-p_i)\frac{\partial x_i^{h'}}{\partial q_k} + x_k^{h'}]} = \frac{\alpha^h x_1^h}{\sum_{h'=1}^{H}[\sum_{i=1}^{n}(q_i-p_i)\frac{\partial x_i^{h'}}{\partial q_1} + x_1^{h'}]}, \quad k = (2,\ldots,n), \quad (B.1)$$

$\mathbf{q}^*$, efficient tax structure, satisfies the revenue constraint by construction. At $\mathbf{q}^*$, rearranging Equation (1), and noting that $\partial e/\partial U^h \cdot \alpha^h = C(\mathbf{p})/C(\mathbf{q}^*)$,

$$\frac{\sum_{h'=1}^{H}[\sum_{i=1}^{n}(q_i^*-p_i)\frac{\partial x_i^{h'}}{\partial q_k} + x_k^{h'}]}{\sum_{h'=1}^{H}[\sum_{i=1}^{n}(q_i^*-p_i)\frac{\partial x_i^{h'}}{\partial q_1} + x_1^{h'}]} = \frac{\sum_{h'=1}^{H}[\frac{\partial e(\mathbf{p}, w^{h'}, V^{h'})}{\partial U}\cdot\alpha^{h'}x_k^{h'}]}{\sum_{h'=1}^{H}[\frac{\partial e(\mathbf{p}, w^{h'}, V^{h'})}{\partial U}\cdot\alpha^{h'}x_1^{h'}]} = \frac{\bar{x}_k}{\bar{x}_1}.$$

where $\bar{\mathbf{x}}$ is the mean demand. Hence $\mathbf{q}^*$ satisfies Equation (B.1) if

$$\frac{x_k^h}{x_1^h} = \frac{\bar{x}_k}{\bar{x}_1} . \qquad \text{(B.2)}$$

Equation (B.2) is satisfied if the expenditure share of commodity k is the same for all households, or if $r_k(\mathbf{q}^*)/s_k(\mathbf{q}^*)$ is independent of k.

## REFERENCES


Atkinson, A.B., (1970), "On the measurement of inequality," *Journal of Economic Theory*, 2, 244-263.

Atkinson, A.B., and J.E. Stiglitz, (1976), "The design of tax structure: direct versus indirect taxation," *Journal of Public Economics*, 6, 55-75.

____________, and __________, (1980), *Lectures in Public Economics*. New York: McGraw Hill.

Auerbach, A.J., (1985), "The theory of excess burden and optimal taxation," in *The Handbook of Public Economics, Volume 1*, edited by A.J. Auerbach and M. Feldstein. New York: Elsevier Science.

Deaton, A., (1974), "A reconsideration of the empirical implications of additive preferences," *Economic Journal*, 84, 338-48.

________ , (1977), "Equity, efficiency, and the structure of indirect taxation," *Journal of Public Economics*, 8, 299-312.

Diamond, P.A., (1975), "A many-person Ramsey tax rule," *Journal of Public Economics*, 4, 335-342.

Diamond, P.A., and D.L. McFadden, (1974), "Some uses of the expenditure function in public finance," *Journal of Public Economics*, 3, 3-21.

Diamond, P.A. and J.A. Mirrlees, (1971), "Optimal taxation and public production 2: tax rules," *American Economic Review*, 61, 261-278.

Dupuit, J., (1844), "De la mesure de l'utilité de traveaux publics," *Annals des Ponts et Chaussees*, 8. Translated and reprinted in *Readings in Welfare Economics*, editerd by K. Arrow and T. Scitovsky. Homewood, Ill: Irwin.

Harberger, A.C., (1964), "Principles of efficiency: the measurement of waste," *American Economic Association Papers and Proceedings*, 64, 58-76.

Mirrlees, J.A., (1975), "Optimal commodity taxation in a two-class economy," *Journal of Public Economics*, 4, 27-33.

Mohring, H., (1971), "Alternative welfare gain and loss measures," *Western Economic Journal*, 9, 349-368.

Muellbauer, J., (1981), "Linear aggregation in neoclassical labor supply," *Review of Economic Studies*, 48, 21-36.

Ramsey, F.P., (1927), "A contribution to the theory of taxation," *Economic Journal*, 37, 47-61.

Simmons, P. (1979), "A theorem on aggregation across consumers in neoclassical labor supply," *Review of Economic Studies*, 46, 737-740.

FOOTNOTES

[1] Strictly, it may be necessary that U is a monotonic transformation of the household utility function, to enable the expenditure function to be written in the forms of Equation (4) and (6).

[2] Mohring bases his formulation on the equivalent variation, whereas Diamond and McFadden (1974) use the compensating variation. For the representative household, minimizing the excess burden is equivalent to maximizing its utility only if the excess burden is based on the equivalent variation (Auerbach (1985)). It seems natural in this paper to use therefore the formulation of Mohring.

[3] Higher tax revenue may be associated with a smaller excess burden. The total welfare cost of the tax is the equivalent variation $R(\mathbf{q}) + \sum_{h=1}^{H} EB(\mathbf{q}, w^h)$. A representative household will acheive lower utility under tax structure $\mathbf{q}_1$ then under $\mathbf{q}_2$ iff $R(\mathbf{q}_2) + \sum_{h=1}^{H} EB(\mathbf{q}_2, w^h) < R(\mathbf{q}_1) + \sum_{h=1}^{H} EB(\mathbf{q}_1, w^h)$. It is only correct to use the excess burden to compare the goodness of two tax codes of *equal* revenue $R(\mathbf{q}_1) = R(\mathbf{q}_2)$.

[4] Section 3 compares the two problems. Comparison of the first order conditions can be made without requiring additive separability, but the second order conditions are more problematic. Because my interest is in weights in the social welfare functions which are maximized at the efficient indirect tax structure, the restriction does not seem too serious.

[5] Exact linear aggregation under maximizing behaviour requires that the expenditure function be of the form of Equation (6), (Muellbauer (1981)). More generally, if the macro wage $w_*$ is allowed to depend on the wage distribution, $w_*=w_*(w^1,...,w^H)$, the micro expenditure function must be of the

form $e(\mathbf{q}, w; U) = F(\mathbf{q}) + G(\mathbf{q})w^{\epsilon} + C(\mathbf{q})w^{\delta}U$, (Simmons (1979)). For exact aggregation under optimizing behavior, with the macro wage $w_*$ being a function of the wage distribution and of prices, $w_* = w_*(\mathbf{w}, \mathbf{q})$, the micro expenditure function must be of the form

$e(\mathbf{q}, w; U) = F(\mathbf{q}) + G(\mathbf{q})((w/G(\mathbf{q}))^{\epsilon}-1)/\epsilon + C(\mathbf{q})w^{\delta}U$, (Muellbauer (1981)).

[6] It is because of the second-order conditions that I require the expenditure function to have the property $\partial^2 e/\partial U^2 = 0$.

[7] The normality of leisure should be distinguished from whether more productive households work more. For example, using the important special case of Equation (6), labor supply is $L(\mathbf{q}, w; M) = (1-\delta)(T-G) - \delta(M-F)/w$, where M is exogenous income. Evaluating at M=0, leisure is a normal good if $\partial L/\partial M < 0$ or $0 < \delta$ . Labor supply increases as the wage increases if $0 < \partial L/\partial w = -\delta F/w^2$, of which the sign depends on the sign of F.

[8] Conversely, if leisure is an inferior good, an increase in utility is associated with a decrease in leisure. As the wage rises, this decrease is more valuable, or $\partial/\partial w\ \partial e/\partial U$ is negative.

[9] Similar conditions can be derived for expenditure functions which allow exact (but non-linear) aggregation. See Footnote 5.

[10] See Footnote 7.