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# SEQUENTIAL DECISIONS BY A SINGLE TORTFEASOR

by

Lewis A. Kornhauser
and
Richard L. Revesz

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C. V. STARR CENTER
FOR APPLIED ECONOMICS

NEW YORK UNIVERSITY
FACULTY OF ARTS AND SCIENCE
DEPARTMENT OF ECONOMICS
WASHINGTON SQUARE
NEW YORK, N.Y. 10003

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ABSTRACT

This paper studies the incentives created by liability rules on an actor who chooses a care level in several periods. An accident may occur, and harm result, in any period. If an accident does not occur, the effects of prior actions on the risk in the current period are cumulative in the sense that expected harm increases over time. If an accident occurs, the actor starts fresh in the next period.

Strict liability obviously induces the actor to choose levels of care identical to those that maximize social welfare. In this context, in an infinite period problem, the actor grows more careful over time; over a finite horizon, conditions under which care increases with time are identified.

"Forgiving" and "unforgiving" negligence rules are then defined and compared. Under a forgiving rule, responsibility for harm occurs only if the actor fails to meet her standard in the period in which the harm occurs. Many forgiving rules fail to induce socially optimal actions, though one optimal rule is identified. One may more easily design optimal, unforgiving rules but, over long time horizons, these rules approximate strict liability rules.

The model is motivated by concern over the generation of hazardous waste and over liability rules like those imposed by the Comprehensive Environmental Response, Compensation and Liability Act.

# Sequential Decisions by a Single Tortfeasor

**Lewis A. Kornhauser* and Richard L. Revesz****

This article studies the effect of liability rules in cases in which a single injurer makes sequential decisions. Our work is motivated in large part by our interest in the Comprehensive Environmental Response, Compensation, and Liability Act (CERCLA),[1] also known as the federal Superfund statute. Under CERCLA, liability for the cleanup of hazardous waste sites and for damage to natural resources falls upon the owner of the hazardous waste site, certain prior owners of the site, and transporters and generators of the waste.[2]

The legal regime appears to apportion the bulk of the damages to generators,[3] and the incentives that liability rules transmit to generators forms the focus of this article. Generators typically send wastes to a site on an ongoing basis. Thus, in analyzing the incentives of liability rules, one requires a model of sequential decisionmaking, in which a generator's

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*Professor of Law, New York University.

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[1]42 U.S.C §§9601-9675 (1982 & Supp. IV 1986).

[2]Id. §9607(a) (Supp. IV 1986).

[3]See Non-Binding Preliminary Allocations of Responsibility, 52 Fed. Reg. 19919 (1987).

decision to send wastes to a site at any particular time is dependent upon the prior stream of wastes that she has already sent to the site as well as upon the wastes that she intends to send to that site in the future.

A full treatment of this subject would not only model an actor's decisions as sequential, but would also study the implications of the presence of multiple generators at a Superfund site. Instead, to abstract the features that result from the sequential nature of the decisions, we study here the actions of a single actor.

Another feature of Superfund cases is the potential insolvency of the actors. The average cleanup cost of a Superfund site is currently $25,000,000;[4] some cleanups cost more than $100,000,000.[5] Even though the cost of cleanup is generally apportioned among many actors, individual actors are often responsible for large amounts. Thus, with some regularity, an actor may be unable to pay the share of the damages that the legal regime allocates to her. In this article, we restrict attention to the incentives of liability rules on actors who are infinitely solvent; we hope subsequently to address the more complex problems posed by potentially insolvency.

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[4]See Environmental Protection Agency, A Clear Strategy for Superfund (1989) (cited in 5 Toxics Law Reporter 78 (1990)).

[5]See EPA Proposes GM Spend $138 Million to Clean Up Site, Wall St. J., March 22, 1990, at B2.

This article builds upon prior work that we have done in the area. We first studied the incentives of liability rules upon joint, infinitely solvent, tortfeasors acting in a single period.[6] We then relaxed the assumption of infinite solvency and studied the incentives of liability rules upon potentially insolvent tortfeasors.[7] In this article, we complicate the model by providing for sequential decisions but simplify it by dealing only with a single, solvent actor.

Our work on infinitely solvent tortfeasors acting in a single period concluded that negligence rules are efficient under joint and several liability as long as the standards of care for each of the actors are set at the socially optimal level, but that negligence rules are not generally efficient in the absence of joint and several liability.[8] We also determined that strict liability rules are not efficient regardless of whether there is joint and several liability.[9]

Next, in our article on potential insolvency among joint tortfeasors acting in a single period, we determined that it is not possible to draw any general conclusion about whether, on

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[6]Lewis A. Kornhauser & Richard L. Revesz, Sharing Damages Among Multiple Tortfeasors, 98 Yale L.J. 831 (1989).

[7]Lewis A. Kornhauser & Richard L. Revesz, Apportioning Damages Among Potentially Insolvent Actors, 19 J. Legal Stud. 617 (1990).

[8]See Kornhauser & Revesz, supra note 6, at 846-55.

[9]See id. at 856-58.

efficiency grounds, negligence is preferable to strict liability, or whether joint and several liability is preferable to non-joint (several only) liability.[10] The relative efficiency of these rules depends on the characteristics of the joint tortfeasors: the benefits that they derive from the economic activity, the costs that their activity imposes on society, and their levels of solvency.

In this study of sequential decisionmaking, we conclude that, for solvent actors, strict liability leads to the maximization of social welfare.[11] We also analyze the conditions under which an actor would increase or decrease the amount of waste dumped in subsequent periods.

We then distinguish various types of negligence rules. Specifically we contrast "forgiving" and "unforgiving" rules: under the latter, an actor's negligence in one period makes her liable for accidents that occur in subsequent periods in which she is not negligent. We then analyze three plausible negligence rules. We show that one produces underdeterrence and the remaining two rules lead to the maximization of social welfare. We classify one of these two rules as "forgiving" and the other

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[10] Kornhauser & Revesz, supra note 7, at __.

[11] Even though CERCLA is a strict liability statute, we analyze the effects on the actor's incentives of both negligence and strict liability because it is relevant inquire whether the choice of strict liability was wise. See Kornhauser & Revesz, supra note 6, at 836, n. 27.

as "unforgiving," and discuss their relative merits.

In analyzing all of the rules, we focus primarily on a model in which an actor makes sequential decisions over two periods. We also consider, however, an infinite-period problem. We can thus determine the extent to which our conclusions depend upon "end-period" effects.

While the example used throughout the article concerns the disposal of hazardous wastes, the analysis has a broader scope. Problems of sequential decisionmaking are not restricted to the hazardous waste context. For example, a manufacturing firm deciding the level of toxic hazard to which its workers should be exposed faces exactly the same issues. At any given time, the firm's decision will be a function of the past exposure of the workers as well as an estimate of future exposure.

The article is organized as follows. Section I sets forth the model that underlies our analysis. Section II analyzes the effects of strict liability. Section III analyzes the effects of the several plausible negligence rules. A mathematical Appendix provides proofs to various of the claims that are made informally in the text.

## I. The Model

In our model, a single, infinitely solvent actor generates hazardous wastes and dumps them at a landfill. The actor benefits from this dumping, because the wastes are the by-product of profitable economic activity. In each of two periods, the actor makes decisions about the level of wastes to generate. At the end of each period, there can be a release of hazardous wastes into the environment.

The loss caused by such a release is equal to the cost of the cleanup and any damage to natural resources.[12] This loss is a "social" loss because it does not fall directly on the dumper absent a legal provision shifting the liability to her. Under strict liability, the actor is responsible for the full loss. Under negligence, in contrast, she is responsible for the full loss only if she violates the standard of care imposed by the legal regime; otherwise, she is liable for no loss at all.[13]

We label as $\underline{x}$ the amount of waste that the actor chooses to

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[12] See 42 U.S.C. §9607(a) (Supp. IV 1986).

[13] The standard of care in our example is the maximum permissible amount of waste that an actor can dump without incurring responsibility for the loss. Under this formulation, more dumping implies less care. In a sense, what is being chosen here is an activity level rather than a level of care. However, neither the model nor the results are conceptually different from the formulation in which the actor chooses more traditional levels of care.

In a prior article, we distinguished between full liability and partial liability rules. Under a full liability rule, when a single actor is negligent, she is responsible for the full social loss. Under a partial liability rule, the actor is responsible only for losses that would have been prevented through due care. See Kornhauser & Revesz, supra note 6, at 837-40. Here, we focus on full liability rules; we consider some features of partial liability rules at notes 30, 33, and 39 infra.

dump in period 1.[14] In period 2, the actor's decision of the level of waste to dump depends on whether there was a release at the end of period 1. We label as y the amount of waste the actor chooses to dump in period 2, given that there has been no release in period 1. In turn, we label as z the amount of waste the actor chooses to dump in period 2, given that there has been a release in period 1. Under rules of negligence, the standards of care that correspond to the choices x, y, and z, are x^, y^, and z^, respectively.

At the end of period 1, the probability of a release is $p_1$. If there is a release at the end of period 1, the probability of a release at the end of period 2 is also $p_1$. If there is no release at the end of period 1, the probability of a release at the end of period 2 is $p_2$. To capture the idea that the probability of a release increases with the time that the wastes remain in a landfill, we assume that $p_2$ is greater than $p_1$.[15]

The benefits that the actor receives in each period from dumping a level of wastes r is B(r); we assume that this function is concave. The cleanup cost for r in the event of a release is L(r); we assume that this function is convex. We assume further that either B(r) is strictly concave or L(r) is strictly convex.

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[14] We assume that all of the wastes generated in one period are dumped in that period.

[15] All of our conclusions, however, hold even if $p_2$ equals $p_1$.

Thus, if the release occurs in period 1, the cleanup costs are L(x); then, if there is a second release in period 2 there are additional cleanup costs equal to L(z). If there is no release at the end of period 1, but there is a release at the end of period 2, the cleanup costs are L(x + y).

Finally, there is a discount factor k that reflects the time value of money. The benefits and costs in period 2 are discounted by a factor k.[16]

Thus,

x = amount dumped in period 1.

y = amount dumped in period 2, given no release in period 1.

z = amount dumped in period 2, given a release in period 1.

$p_1$ = probability of a release in period 1, or of a release in period 2 given a release in period 1.

$p_2$ = probability of a release in period 2, given no release in period 1; $0 < p_1 < p_2 < 1$.

k = discount factor; $0 < k \leq 1$.

B(r) = benefit that accrues to the actor from dumping r.

L(r) = cleanup costs for r in the event of a release.

$x^\wedge$, $y^\wedge$, $z^\wedge$ = standards of care, under rules of negligence, that correspond to the choices x, y, and z, respectively.

The various possible outcomes are illustrated in Figure I. Under outcome A, there is a release in both periods; the actor

[16]We assume that there is no divergence between the private discount factor and the social discount factor.

therefore causes a loss of $L(x) + L(z)$. The probability of this outcome is $(p_1)^2$. Under outcome B, there is a release in period 1 but no release in period 2; the actor therefore causes a loss of $L(x)$. The probability of this outcome is $p_1(1 - p_1)$. Under outcome C, there is a no release in period 1 but there is a release in period 2; the actor therefore causes a loss of $L(x + y)$. The probability of this outcome is $(1 - p_1)p_2$. Finally, under outcome D, there is no release in either period; therefore the actor does not cause any loss. The probability of this outcome is $(1 - p_1)(1 - p_2)$.

Throughout the article, we will proceed by backward induction. Thus, we will look first at the actor's decision in period 2, define an optimal response in period 2 for a given action in period 1, and then solve the actor's maximization problem in period 1.

## II. Strict Liability

Under strict liability, the actor is responsible for the full cleanup costs. In period 2, the actor faces the following objective function if there has been no release in period 1:

$$\mathrm{Max}_y \; B(y) - p_2L(x + y)$$

If there has been a release in period 1, the objective function is

$$\mathrm{Max}_z \; B(z) - p_1L(z)$$

Let $y(x)$ and $z^*$ satisfy these conditions; $y(x)$ is the optimal response in period 2 to the dumping of x in period 1. Note that, if there has been no release in period 1, the choice of y in period 2 is dependent upon the choice of x in period 1 because the actor's liability in the event of a release is $L(x + y)$. In contrast, if there has been a release in period 1, the choice of z in period 2 is independent of the choice of x in period 1 because the actor's liability in the event of a release is $L(z)$, which is not a function of x.

In period 1, the actor faces the following objective function:

$$\text{Max}_x\ B(x) - p_1L(x) + k\{(1 - p_1)[B(y(x)) - p_2L(x + y(x))] + p_1[B(z^*) - p_1L(z^*)]\}$$

The solution to this expression defines $x^*$ and $y(x^*)$; we will label the latter term as $y^*$--the optimal response in period 2 to the dumping of $x^*$ in period 1.

These objective functions show that strict liability induces the socially optimal outcome. Indeed, in each of the periods, and regardless of when a release occur, the actor captures the full benefits of the activity but must also bear the full social loss. Thus, the actor's private objective function is equal to the social objective function.

Turning to the relationships among $x^*$, $y^*$, and $z^*$, one can draw the following conclusions.[17] First, z* is greater than y*.[18] The actor faces a smaller loss function in period 2 when there has been a release in period 1 for two reasons. In the face of such a release, the probability of a further release is $p_1$. In the absence of a release in period 1, the probability of a release in period 2 is $p_2$, which is greater than $p_1$. Moreover, for a given amount of waste dumped in period 2, say <u>u</u>, the potential liability in period 2, given that a release occurred in period 1, is L(u), whereas, if no release has occurred in period 1, the potential liability in period 2 is L(x + u), which is larger. The actor derives the same benefit from a given level of dumping in period 2, regardless of whether there has been a release in period 1. Thus, the smaller loss function in period 2 when there has been a release in the period 1 leads to z* being greater than y*.

Second, z* is also greater than or equal to x*, with

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[17]Because "negative" dumping does not have a physical interpretation, it is necessary that $x \geq 0$, $y \geq 0$, $z \geq 0$. In addition, the description of the model requires that $x > 0$. If the actor chooses not to dump at all in period 1, there can be no release at the end of this period. Thus, the actor can never face the question of how much to dump in period 2 following a release in period 1; the distinction between <u>y</u> and <u>z</u> loses its meaning.

Thus, we place certain constraints on the benefit and damage function to ensure that $x^* > 0$. If $x^* > 0$, it follows that $z^* > 0$. It is possible, however, that absent an additional constraint, $y^* < 0$. The shadow price associated with $y^* < 0$ measures the "cost" of the inability to perform a cleanup in advance of a release. See Appendix, equations (2)-(4).

[18]See Appendix, Lemma 1(b).

equality only if k = 0.[19] If k = 0, the actor is completely myopic; she does not consider the consequences of her actions in period 1 on her situation in period 2.[20] Consequently, in period 1, she acts as if she faces only a risk $p_1$ of a release in that period; she ignores the additional risk of $(1 - p_1)p_2$ of a release in period 2 of the waste dumped in period 1. But, given a release in period 1, the actor, in choosing z*, in fact faces only a risk $p_1$ of a release. Hence, the actor would dump the same amount in each period, as the sequential features of the model would disappear. But when k is positive, the actor understands that the dumping in period 1 can lead to a release at the end of period 1 with probability $p_1$ and a release at the end of period 2 with probability $p_2$. In contrast, the dumping in period 2, given a release in period 1, can lead only to a release at the end of period 2 with probability $p_1$. Once again, the smaller loss function faced by the actor in period 2 when there has been a release in period 1 leads to z* being greater than x*.

Third, the relationship between x* and y* is dependent upon the functions B and L and the parameters $p_1$, $p_2$, and k.[21] At first glance, it would appear that dumping in period 2 would be

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[19]See Appendix, Lemma 1(a).

[20]We do not wish to imply, however, that the actor's private discount factor differs from the social discount factor. See note 15 *supra*.

[21]See Appendix, Lemma 2.

more desirable because it exposes the actor only to a single loss, with probability $p_2$, rather than two losses, one with probability $p_1$ and the other with probability $p_2$.

This incentive to dump more in period 2 is counteracted in several ways. The net benefits that accrue from dumping in period 2 must be discounted by a factor k, thereby creating a preference for dumping in period 1. Also, if the benefit function is strictly concave, an actor would not choose to do all of its dumping in period 2, because she would derive a greater benefit from an additional unit of dumping in a period in which she is dumping fewer units.

In addition, the convexity of the damage function and the fact that $p_2$ is greater than $p_1$ push the actor in the direction of dumping more in period 1. Assume that there is going to be a release at the end of period 2. For a given total amount of wastes dumped over the two periods, if the damage function is strictly convex, the amount paid where there is a single release at the end of period 2 will be greater than that paid where there is a release at the end of each period. Thus, the absence of a release at the end of period 1 induces the actor to dump less in period 2. This result is further exacerbated because $p_2$ is greater than $p_1$.

With one modification, the results of our model are generally consistent with a case in which the actor dumps in an infinite number of periods rather than just in two. In the two-period case, the actor faces an "end-period" in period 2 in which her current decisions have no future consequences. This factor is most evident in the event of a release in period 1. In that instance, the actor will choose z* greater than x* because that choice exposes her to liability in one period only. But in an infinite-period problem, this result does not hold; dumping in period 2 exposes the actor to liability not only in that period, but also in all subsequent periods until there is a release. Consequently, in an infinite period model, after a release, the actor would once more choose x*.

For similar reasons, when there is no release at the end of period 1, the amount dumped in period 2 will be less in the infinite period model than in the two-period model. In the infinite period model, the lack of a release at the end of two periods does not preclude a release in subsequent periods. Once again, the strict convexity of the damage function implies that the absence of a release at the end of one period induces the actor to dump less in the subsequent period. We show that the amount dumped starts at x* and then decreases in each period until there is a release, and is x* once again in the period following a release.[22]

## III. Negligence

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[22]See Appendix, Proposition 2. Of course, if the optimal amount dumped in one a given period is zero, the actor will not dump again until after a release.

In a single-actor, single-period model, a rule of negligence is simple both to define and to analyze. Under a rule of negligence, the actor is responsible for the full social loss if she violates the standard of care. Otherwise, she is responsible for no loss at all. Moreover, when the standard of care is set at the optimal level of care, it will induce the actor to adopt that level of care.[23] The extension of the model to two (or more) periods complicates not only the analysis of the model but also the definition of negligence and the assignment of appropriate standards of care.

In particular, we need to draw a distinction between "forgiving" and "unforgiving" rules. The two rules assign responsibility for releases in period $\underline{t}$ to an actor who is negligent in that period. They differ, however, in their treatment of actors who, though non-negligent in period $\underline{t}$, when a release occurs, were negligent in at least one period following the prior release. A forgiving rule does not assign responsibility for releases in period $\underline{t}$ to an actor who is non-negli-

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[23]If the law establishes an extremely stringent standard of care, negligence will function like strict liability and also induce the single actor to adopt the optimal level of care.

For the traditional law-and-economics analysis of negligence rules in single-actor, single-period models, see John Prather Brown, Toward an Economic Theory of Liability, 2 J. Legal Stud. 323 (1973); Peter A. Diamond, Single Activity Accidents, 3 J. Legal Stud. 107 (1974); Steven Shavell, Strict Liability versus Negligence, 9 J. Legal Stud. 1 (1980).

gent in that period, regardless of her prior behavior. In contrast, an unforgiving rule assigns liability to such an actor if the actor was negligent in at least one period following the prior release.

Consider, in our model, a situation in which there is no release in period 1 but there is a release in period 2, and where the actor is negligent in period 1 but non-negligent in period 2. Under a forgiving rule, the actor would not face liability in period 2, but under an unforgiving rule she would.

We analyze first a forgiving rule that sets the following standards of care: $\hat{x} = x^*$, $\hat{y} = y^*$, and $\hat{z} = z^*$. This rule is suggested by the single-period problem. In essence, the single-period problem is no different from a multi-period problem in which an actor's actions in one period have no effect on the social loss in other periods. In that case, the optimal standard of care for each period is set at the level that maximizes social welfare and an actor is immune from liability if there is a release in a period in which she meets the standard of care. By analogy, one might think that where an actor's decisions in one period have an effect on the social loss in other periods, the efficient result will be generated by a rule that sets the standards of care for each period at the socially optimal level and immunizes the actor from liability for a release in periods in which she meets the standard of care. We show below that despite this surface plausibility, the analogy does not hold; we then analyze two reformulations of the standards of care.

It is easy to see under this rule, as well as under all the other rules that we analyze below, that the actor will choose z* in period 2, contingent on a release in period 1.[24] Indeed, as we pointed out in our discussion of strict liability, the choice of z in period 2 is independent of the choice of x in period 1. Thus, in the event of a release in period 1, the problem the actor faces in period 2 is a single-period problem. If she chooses a level greater than z*, she bears the full cost of her actions. The net benefits to her of this decision are less than if she chose z*, the level of waste which maximizes net social welfare. Since the other rules that we analyze also induce the optimal choice of z, we focus in the remainder of the discussion, only on the choices of x and y.

We now analyze the rule's performance with respect to the choice of x and y. The actor faces the following objective function in period 2, given no release in period 1:

$$\text{Max}_y \begin{cases} B(y) - p_2L(x + y) & y > y^* \\ B(y) & y \le y^* \end{cases}$$

This expression shows that the actor will not be negligent in both periods.[25] For, if $x > x^*$, the $y(x)$ that maximizes $B(y)$ -

[24]See Appendix, Lemma 4.

[25]See Appendix, Lemma 5.

$p_2L(x + y)$ is less than or equal to $y^*$.[26] Thus, if the actor is negligent in period 1, she will choose y equal to $y^*$ in period 2; the actor would not choose a smaller amount because she can escape all liability by choosing $y^*$. Conversely, she will choose y greater than $y^*$ only if x is smaller than $x^*$ in period 1.[27] The actor thus faces the following objective function in period 1:

$$\text{Max}_x \begin{cases} \{B(x) - p_1L(x) + k(1-p_1)B(y^*) & x > x^* \quad (1) \\ \{B(x) + k(1 - p_1)[B(y(x)) - p_2L(x+y(x))] & x < x^* \quad (2) \\ \{B(x^*) + k(1 - p_1)B(y^*) & x = x^* \quad (3) \end{cases}$$

But over the range $x \leq x^*$, the expression

$$B(x) + k(1 - p_1)[B(y) - p_2L(x + y)]$$

is maximized at $x^*$.[28] This objective function differs from the one that arises under strict liability in that the actor does not face liability for $p_1L(x)$, as she meets the standard of care in period 1. Thus, if the choice of x where unconstrained, the actor would choose a value larger than $x^*$. But given the constraint, she would choose $x^*$. Hence, the actor will never choose an x that is smaller than $x^*$. A corollary is that she will never be negligent in period 2 and will therefore choose to dump $y^*$,[29] as we have indicated that she will choose y greater

[26] See Appendix, Lemma 3(a).

[27] See Appendix, Lemma 3(b).

[28] See Appendix, Lemma 7.

[29] See Appendix, Lemma 6.

than y* only if she chose an x smaller than x*.

Note, however, that for some functions B and L, the actor will dump more than x* in the period 1, but for other functions she will dump x*.[30] If the actor is negligent in period 1, the level of dumping that maximizes her objective function is greater than x*. In contrast to the objective function that the actor faces under strict liability, here she is not responsible for the expected damage associated with the period 2 discharge, which is $k(1 - p_1)p_2L(x + y)$. For some functions the actor will prefer to dump at this higher level and be liable for the expected damage $p_1L(x)$, rather than dump at x* and be exempted from this liability.

In summary, the rule fails to create the correct incentives because, when no release occurs in period 1, it permits the actor to escape the consequences of her negligence in this period.

This underdeterrence is not a product of an "end-period" problem. It exists as well in the infinite-period problem, where an actor can avoid the consequences of a violation of the standard of care in any period in which there is no release, even though the violation has consequences in subsequent periods.[31]

This underdeterrence can be corrected by means of an

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[30]See Appendix, Proposition 1. In contrast, a partial liability rule always produces underdeterrence in period 1, regardless of the functions B and L.

[31]See Appendix, Lemma 13.

unforgiving rule. Consider such a rule, with the standards of care set, once again, at $x\hat{} = x*$, $y\hat{} = y*$, and $z\hat{} = z*$. Now, if the actor dumps an amount greater than $x*$ in period 1 but there is no release in that period, the actor will be liable for a release in period 2, even if she meets the standard of care in this period by dumping $y*$.

This unforgiving rule is suggested by the literature on sequential decisions by two actors, each operating in one period.[32] For example, under a rule of negligence with contributory negligence, if an injurer acts first and violates her standard of care, and the victim acts next and meets her standard of care, the injurer is liable for the full loss, even for the portion attributable to the victim. Here, under an unforgiving rule, if the actor violates the standard of care in period 1 but meets it in period 2, and the release occurs in period 2, the actor is liable for the full social loss, even for the portion of the social loss attributable to her actions in period 2.

Under this unforgiving rule, instead of equation (1), the

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[32]See Mark F. Grady, Multiple Tortfeasors and the Economy of Prevention, 19 J. Legal Stud. __ (1990); William M. Landes & Richard A. Posner, The Positive Economic Theory of Tort Law, 15 Ga. L. Rev. 851, 889-91 (1981); Steven Shavell, Torts in Which Victim and Injurer Act Sequentially, 26 J.L. & Econ. 589 (1983); Donald Wittman, Optimal Pricing of Sequential Inputs: Last Clear Chance, Mitigation of Damages, and Related Doctrines in the Law, 10 J. Legal Stud. 65 (1981).

The models in these papers adopt $y\hat{} = y(x)$ as the "second-period" standard of care. For a discussion of this standard of care in the context of a single actor operating in two periods, see note 38 <u>infra</u>.

actor faces the following objective function in period 1, for $x > x^*$:

$$B(x) - p_1L(x) + k(1 - p_1)[B(y^*) - p_2L(x + y^*)]$$

This function is identical to the function that the actor faces under strict liability, and is therefore maximized at $x^*$, rather than at a level higher than $x^*$. Moreover, if the actor chooses $x^*$ in period 1 she receives, in fact, the greater net benefits of equation (3), namely

$$B(x^*) + k(1 - p_1)B(y^*)$$

Thus, this unforgiving rule corrects the flaw of its forgiving counterpart and induces the actor to choose $(x^*,y^*,z^*)$. The unforgiving rule produces the socially optimal results in the infinite-period problem as well.[33]

While the unforgiving rule with the standards of care set at the socially optimal level in each period leads to the efficient outcome, it has some troubling features. Suppose, for example, that the actor is negligent in period 1 and generates a level $x = (x^* + \epsilon)$ of waste. Suppose further that no release occurs in period 1 and that in period 2, not only is the actor non-negligent but that she "compensates" for her negligence in period 1 by generating $y = (y^* - \epsilon)$. Thus, by the end of period 2, the actor

[33]See Appendix, Lemma 14. A partial liability, unforgiving rule with the same standards of care would also induce the efficient outcome. Under such a rule, the actor is responsible only for the cleanup cost attributable to the "excess" amount of waste dumped in period 1, which is $L(x + y^*) - L(x^* + y^*)$.

has generated the optimal amount of waste: $x^* + y^*$. The unforgiving rule will still not absolve the actor of responsibility for a release in period 2. In contrast, if the actor had dumped $x^*$ in period 1 and $y^*$ in period 2, she would not have been liable, even though the aggregate amount dumped would have been the same. To some, this result seems unfair.[34]

This "unfairness" may be exacerbated in a model with more than two periods. For example, the actor might dump $(x^* + \epsilon)$ in period 1, $(y^* - \epsilon)$ in period 2, and then meet the standard of care, say, for the next 18 periods. At the end of period 20, the first release might occur. The actor would be liable for the release even though the effects of her negligence would have been cured many periods earlier, and this negligence would therefore have been of no continuing social consequence. This effect is common to all unforgiving negligence rules.

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[34]The sense of unfairness has two related sources. First, if an observer restricts attention to period 2, the two cases are "alike," but they are treated differently. Of course, the actor behaved differently in period 1 in the two cases; moreover that difference in period 1 behavior has different incentive effects on the actor that, from an economic perspective, justify the different treatment in period 2. For a more extensive discussion of the problems in the idea of "equivalent cases," see Kornhauser, An Economic Perspective on Stare Decisis, 65 Chicago-Kent L. Rev. 63 (1989).

Second, economic models of negligence do not adequately capture the legal concept of a standard of care. In our model, the actor's negligence in period 1 is intentional when, in reality, it may have been "inadvertent." Thus, the actor may have tried to meet the standard but, "through no fault of her own," failed. If she then compensates for her period 1 error by restricting her output in period 2, the unforgiving rule seems overly harsh.

This example highlights a second, general problem of unforgiving negligence rules. If the actor is negligent in one period, in all subsequent period before a release the rule is, in effect, a strict liability rule. Indeed, regardless of what the actor does in the periods following her negligence, she will be liable for a release. Given that strict liability induces the efficient response, it is not surprising that the unforgiving rule does so as well. But, in some sense, the unforgiving rule can be seen as not being a true negligence rule, and to the extent that one prefers a rule of negligence over one of strict liability, one has reason to dislike an unforgiving rule.

Third, our prior analysis of two infinitely solvent actors in a single period showed that strict liability induced under-deterrence.[35] Plausibly, a similar problem might arise under an unforgiving rule. Suppose the actors dump over many periods and, early on, at least two actors inadvertently exceed their standard of care. Until the time of the first release, these actors face a rule of strict liability and presumably that rule offers socially undesirable incentives.

We therefore search for a forgiving rule that induces the socially optimal outcome. Consistent with our prior discussion, a natural rule is one that sets the standard of care in period 2, contingent on no release in period 1, so that it is violated if

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[35]See Kornhauser & Revesz, supra note 6, at 856-58.

and only if the total amount dumped in periods 1 and 2 is greater than $x^* + y^*$. Thus, $\hat{x} = x^*$; $\hat{y} = c(x) = x^* + y^* - x$ ; $\hat{z} = z^*$.[36]

Under this rule, the actor will meet the standard of care in period 2. She will dump no less than the amount permitted by her standard of care because, until she exceeds her standard, additional waste imposes no additional cost on her.

To understand why she will not dump more than $\hat{y}$, we must consider two cases. Recall, from the discussion of strict liability, that if the actor faces liability in period 2, her optimal response to the dumping of $\underline{x}$ in period 1 is $y(x)$. If, in period 1, the actor chose $x < x^*$ and hence was non-negligent, her optimal response, if she faced liability in period 2, would be $y(x) > y^*$. But, for $x < x^*$, it follows that $x + y(x) < x^* + y^*$.[37] Thus, if the actor chose to be non-negligent in period 1, she would choose to dump less than $c(x)$ in period 2 even if she faced liability in period 2. It follows, <u>a fortiori</u>, that she will choose to dump no more than $c(x)$ where this choice exempts her from liability.

If, in contrast, the actor chooses to dump more than $x^*$ in period 1, she will not choose to be negligent in period 2 as

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[36] If the actor dumps more than $(x^* + y^*)$ in period 1, the standard of care in period 2 is negative. Thus, regardless of how much the actor dumps in period 2, even if she dumps nothing at all, she will have violated the standard of care.

[37] See Appendix, Lemma 3(b).

well. We know from the analysis of strict liability, that if the actor is responsible for the full social loss, she will choose to dump x* and y*, rather than larger amounts.

Consider now the actor's choice in period 1. We have established that if the actor dumps more than x* in period 1, she will have to dump less than y* in order not to violate the standard of care in period 2. Indeed, because $x + y(x) > x^* + y^*$ for $x > x^*$, the actor will have to dump $y < y(x)$ in order to avoid violating the standard of care in period 2. This restriction removes the incentive the actor had under the rule discussed above to be negligent in period 1.[38] Nor does the actor have an incentive to choose $x < x^*$ in period 1. If the actor is non-negligent in both periods, she faces the objective function:

$$\text{Max}_x\ B(x) + k(1 - p_1)B(x^* + y^* - x) \qquad x \leq x^*$$

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[38]See Appendix, Lemma 8. The restriction that $\hat{y} = c(x) < y(x)$ for $x > x^*$ is necessary to remove the incentive.

Another plausible, forgiving rule would set $\hat{y} = y(x)$. In a two-actor model, if the first actor violates her standard of care, the standard for care for the second actor is set at the optimal response to the first actor's choice. This standard is more stringent than the optimal standard where the first actor is non-negligent. See Wittman, supra note 32.

In our one-actor, two-period model, if $\hat{x} = x^*$ and $\hat{y} = y(x)$, the incentive for negligence in period 1 would remain. That is, for some functions B and L, the actor will prefer to dump at a level higher than x*, and be liable for the expected damage $p_1L(x)$ as well as for the "penalty" that attaches in period 2. For other functions, she will prefer to dump at x*.

This underdeterrence could be corrected by making the rule unforgiving. Such a rule, however, would share all of the negative characteristics of the unforgiving rule discussed in the text. Moreover, courts would require more information in order to implement it. See our discussion of the informational requirements of various rules, infra.

which is maximized at x*.[39]

Thus, this forgiving rule, like the unforgiving rule discussed above, will lead to the maximization of social welfare.[40] It will do so, however, without raising the potential negative effects of unforgiveness. Perhaps most importantly, it is a truer negligence rule.

In terms of the information that a court must have in order to adjudicate claims, the two rules do not differ significantly; to the extent that they do, however, the unforgiving rule is somewhat more parsimonious. Two potential differences arise when the first release occurs in period 2. First, under the unforgiving rule, once a court determines that the actor violated the standard of care in period 1, it does not need to ascertain the level of dumping or the standard of care in period 2. Regardless of what the actor does in this period, she will be liable. In contrast, under the forgiving rule, the court will need to ascertain the actor's dumping in period 2, to determine whether the aggregate amount dumped is greater than (x* + y*).

Second, under the unforgiving rule, once a court determines that the actor has violated the standard of care in period 1, it does not need to determine the extent of the violation, as this

[39]See Appendix, Lemma 9. A partial liability, forgiving rule with the same standards of care would also induce the efficient outcome.

[40]We believe, but have not proven, that this result also holds in the infinite-period problem.

information would have no legal significance. In contrast, under the forgiving rule, the extent of the violation is relevant to determine whether the aggregate amount dumped is greater than $(x^* + y^*)$.

## Conclusion

We show that strict liability induces the socially optimal outcome. The analysis of negligence is more complex because of the multiplicity of plausible negligence rules. Out of three such rules, only two lead to the maximization of social welfare. The third produces underdeterrence. The two socially desirable rules have different characteristics. Most importantly, one is forgiving whereas the other is unforgiving. We explain that the forgiving rule is a more traditional negligence rule, but that the unforgiving rule may impose somewhat lesser informational burdens on courts.

**Appendix**

In this appendix, we outline in somewhat greater detail the logic of the arguments made in the text. A complete exposition would be overly tedious.

A. The Two-Period Model

Recall that we assume B(r) is concave, L(r) is convex and that B(r) - L(r) is strictly concave: $B'(r) > 0$ for $r < r^H$; $L'(r) > 0$ and either $B''(r) < 0$ or $L''(r) > 0$. In addition, we assume that $p_2 > p_1$. The social objective function is given by

$$V(x,y,z) = B(x) - p_1L(x) + k(1 - p_1)[B(y)-p_2L(x + y)] + kp_1[B(z)-p_1L(z)] \quad (1)$$

Let $(x^*,y^*,z^*)$ maximize (1).

The actor, of course, chooses y or z in period 2 optimally given her choice of x in period 1. She thus maximizes V(x,y,z) over (x,y(x),z(x)). Now $z(x) = z^*$ is independent of the choice of x because if a release occurs in period 1, decisions in period 1 influence neither the costs nor benefits of decisions in period 2. Consequently, analysis of the model reduces to analysis of the choices of x and y(x).

Before we proceed to that study, we first must state conditions necessary for non-negativity of $x^*$, $y^*$, and $z^*$. Clearly, from (1), a sufficient condition for non-negativity is $B'(0) = \infty$. This condition is hardly necessary. The three necessary conditions are: $z^* > 0$ if and only if

$$B'(0) > p_1L'(0) \quad (2)$$

In turn, $x^* > 0$ if and only if

$$B'(0) > p_1L'(0) + k(1 - p_1)L'(y(0)) \quad (3)$$

where $y(0)$ satisfies $\max\{0, \arg[B'(y) = p_2L'(y)]\}$. Finally, $y^* > 0$ if and only if

$$B'(0) > p_2L(x(0)) \quad (4)$$

where $x(0) = \max\{0, \arg[B'(x) = [p_1 + k(1 - p_1)p_2L'(x)]\}$. Note that if B and L do not satisfy (2), they do not satisfy (3) or (4). Moreover if (3) is not satisfied, (4) is irrelevant as the actor would not dump in period 1 and hence there could be no release in period 1. In situations where (3) is satisfied but (4) is not, the constrained maximization problem determines a shadow price for the inability to "clean-up" prior to a release.

Henceforth, unless otherwise specified, we assume, as does the text, that B and L satisfy (2)-(4). We now prove:

Lemma 1: (a) for $k > 0$, $z^* > x^*$ and (b) $z^* > y^*$.

Proof: Consider (1). The first order conditions for $x^*$, $y^*$, and $z^*$ are respectively:

$$B'(x^*) = p_1L'(x^*) + k(1 - p_1)p_2L'(x^* + y^*) \quad (5)$$

$$B'(y^*) = p_2L'(x^* + y^*) \quad (6)$$

$$B'(z^*) = p_1L'(z^*) \quad (7)$$

Clearly $p_1L'(x) + k(1 - p_1)p_2L'(x+y^*) > p_1L'(x)$ which, with the convexity of L and concavity of B imply (a). Similarly, (b) follows from the concavity and convexity assumptions and the observations that $p_2 > p_1$ and $x^* > 0$. Q.E.D.

<u>Lemma 2</u>: $y^* > x^*$ if and only if

$$L'(x^*)/L'(x^* + y^*) > (p_2/p_1)[1 - k(1 - p_1)]$$

<u>Proof</u>: The first order conditions for $x^*$ and $y^*$ imply

$$B'(x^*) - B'(y^*) = p_1 L'(x^*) + p_2 L'(x^* + y^*)[k(1 - p_1) - 1] \quad (8)$$

The left-hand side of (8) is positive for $y^* > x^*$. Thus, $y^* > x^*$ if and only if

$$p_1 L'(x^*) + p_2 L'(x^* + y^*)[k(1 - p_1) - 1] > 0$$

which implies the lemma. Q.E.D.

In addition, the concavity of <u>B</u> and convexity of <u>L</u> establish the following:

<u>Lemma 3</u>: (a) if $x > x^*$, $y(x) \leq y^*$ and $x + y(x) \geq x^* + y^*$; (b) if $x < x^*$, $y(x) \geq y^*$ and $x + y(x) < x^* + y^*$.

The equalities hold only if <u>L</u> is linear; strict convexity of <u>L</u> establishes the inequalities.

We now consider more formally the analysis of a forgiving negligence rule where $\hat{x} = x^*$, $\hat{y} = y^*$, and $\hat{z} = z^*$.

<u>Lemma 4</u>: The actor chooses $z = z^*$.

<u>Proof</u>: The actor faces the objective function:

$$\text{Max}_z \begin{cases} B(z) - p_1 L(z) & z > z^* \\ B(z) & z \leq z^* \end{cases}$$

But, by definition, $z^*$ maximizes the top function over an unrestricted domain and, as $B'(z) > 0$, it maximizes the lower function over the interval $[0,z^*]$. Q.E.D.

Lemma 5: The actor will never choose $x > x^*$ and $y > y^*$.

Proof: Recall that

$$B'(y^*) = p_2 L'(x^* + y^*)$$

For $y > y^*$,

$$B'(y) = p_2 L'(x + y)$$

Assume that $x > x^*$. Then $L'(x + y) > L'(x^* + y^*)$. Therefore $B'(y) > B'(y^*)$, which implies that $y < y^*$, and contradicts the assumption. Q.E.D.

Lemma 6: $y = y^*$.

Proof: Assume that $y > y^*$. Lemma 5 establishes that $x \leq x^*$. Thus, the relevant objective functions are

$$\text{Max}_x\ B(x) + k(1 - p_1)[B(y(x)) - p_2 L(x + y(x))] \qquad x \leq x^*$$

$$\text{Max}_y\ B(y) - p_2 L(x + y) \qquad y > y^*$$

The first-order conditions are given by

$$B'(y) = p_2 L'(x + y)$$

$$B'(x) = k(1 - p_1) p_2 L'(x + y) \quad .$$

Recall that

$$B'(y^*) = p_2 L'(x^* + y^*)$$

$$B'(x^*) = p_1 L'(x^*) + k(1 - p_1) p_2 L'(x^* + y^*)$$

Thus, $y > y^*$ implies that $B'(y) < B'(y^*)$, which in turn implies that $L'(x + y) < L'(x^* + y^*)$ and that $(x + y) < (x^* + y^*)$.

But $(x + y) < (x^* + y^*)$ implies that $B'(x) < B'(x^*)$ and therefore that $x > x^*$. The contradiction establishes that $y \leq y^*$ and that the actor is non-negligent in period 2. But clearly $y^*$ maximizes $B(y)$ over the interval $[0,y^*]$. Q.E.D.

<u>Lemma 7</u>: $x \geq x^*$

<u>Proof</u>: $x = x^*$ maximizes $B(x) + k(1 - p_1)B(y)$ where $x \leq x^*$. Q.E.D.

<u>Proposition 1</u>: For some functions <u>B</u> and <u>L</u>, the actor is negligent in period 1, but the actor is never negligent in period 2.

<u>Proof</u>: Lemma 6 establishes non-negligence in period 2. In period 1, therefore, the actor faces the objective function

$$\text{Max}_x \begin{cases} B(x) - p_1 L(x) & x > x^* \\ B(x) & x \leq x^* \end{cases}$$

The top function is maximized over an unrestricted domain by $z^* > x^*$. The bottom function is maximized over the interval at $x^*$. Thus the actor always chooses $x \geq x^*$. The following example shows that the actor may choose $x = z^*$.

Let $B(r) = r$ and $L(r) = r^2$. Then, from the relevant first-order conditions $z^* = 1/(2p_1)$, $(x^* + y^*) = 1/(2p_2)$, and $x^* = (1/2p_1)[1 - k(1 - p_1)]$.

Then, $x > x^*$ for

$$1/(2p_1) - p_1[1/(2p_1)]^2 > (1/2p_1)[1 - k(1 - p_1)]$$

or, equivalently,

$$k(1 - p_1) > 1/2$$

For $k = 1$, $p_1 < 1/2$ satisfies the inequality. Q.E.D.

For the forgiving negligence rule with the standards of care set at $\hat{x} = x^*$; $\hat{y} = c(x) = x^* + y^* - x$; $\hat{z} = z^*$, the analysis is similar.

Lemma 8: $y = c(x)$

Proof: If $x \le x^*$, from Lemma 3(b), $y(x) \le c(x)$ so the actor chooses $y = c(x)$. If $x > x^*$, $y > c(x)$ implies negligence in both periods which cannot be optimal. Q.E.D.

Lemma 9: $x = x^*$

Proof: We first show, by contradiction, that $x \le x^*$. So assume that $x > x^*$. From Lemma 8, the actor chooses $y = c(x) < y(x)$. Assume that her choice of x is unconstrained so that she chooses $x_u$ to maximize

$$B(x) - p_1L(x) + k(1 - p_1)B(x^* + y^* - x)$$

which has first-order condition

$$B'(x) = p_1L'(x) + k(1 - p_1)B'(x^* + y^* - x)$$

This condition is satisfied at $x = x^*$. But then the actor chooses to be non-negligent. Hence $x \le x^*$.

To see that $x = x^*$, note that, for $x \le x^*$, $y = c(x)$, the actor faces the following objective function in period 1:

$$\text{Max}_x\ B(x) + k(1 - p_1)B(x^* + y^* - x) \qquad x \le x^*$$

The first-order condition for the unconstrained function, where $x_u$ is the unconstrained maximum, is given by

$$B'(x_u) - k(1 - p_1)B'(x^* + y^* - x_u) = 0$$

From (5) and (6),

$$B'(x^*) = p_1L'(x^*) + k(1 - p_1)p_2B'(y^*)$$

Thus,

$$B'(x_u) - B'(x^*) =$$

$$k(1 - p_1)[B'(x^* + y^* - x_u) - B'(y^*)] - p_1 L'(x^*)$$

Assume that $x_u \leq x^*$. Then, the left-hand side of the expression is positive but the right-hand is negative. The contradiction establishes that $x_u > x^*$. Q.E.D.

## B. Infinite-Period Model

Some additional definitions are necessary.

$p_i$ = probability of a release in period i, conditional on no prior release, and probability of a release in the ith period following the prior release.

$q_i = 1 - p_i$.

$x = (x_1, x_2, x_3, \ldots)$ = vector of waste levels, with $x_i$ chosen in period i, conditional on no release, or in the ith period following the prior release. We require $x_i \geq 0$, for all i.

$X_t = \Sigma^t x_i = \Sigma_{i \epsilon T} x_i$, where $T = \{1, 2, \ldots, t\}$.[41]

$V(x)$ = discounted present value of x.

$\sigma_\tau = p_\tau(q_1 q_2 \cdot \cdot \cdot q_{\tau-1}) = p_\tau \pi^{\tau-1} q_i$ = probability that the first release occurs in period $\tau$, or that a subsequent release occurs in the $\tau$th period following the prior release.

$s_j = \Sigma_j \sigma_\tau$

We may now calculate $V(x)$, the expected value of any dumping strategy x. Suppose no release occurs for $\tau$-1 periods but does

[41] We use the following other summation conventions:
$\Sigma x_i = \Sigma_{i \epsilon S} x_i$, where $S = \{ 1, 2, \ldots, \infty)$
$\Sigma_u x_i = \Sigma_{i \epsilon U} x_i$, where $U = \{u, u + 1, \ldots, \infty)$

occur in period $\tau$. Then, in each period $i \leq \tau$, the actor receives $B(x_i)$, and in period $\tau$ she pays $L(X_\tau)$. Thus, the actor receives the following payoff:

$$V_\tau(x) = k^\tau V(x) - k^{\tau-1}L(X_\tau) + \Sigma^\tau k^{i-1}B(x_i)$$

But, a first release must occur at some period $\tau$ so that we may write

$$V(x) = \Sigma\sigma_\tau V_\tau(x)$$
$$= 1/[1 - \Sigma\sigma_\tau k^\tau]\Sigma\sigma_\tau[- k^{\tau-1}L(X_\tau) + \Sigma^\tau k^{i-1}B(x_i)]$$

The actor maximizes $V(x)$ over $x_i$, $i = 1, 2, \ldots, \infty$. Thus, for each $j$, the first-order condition is given by

$$F_j(x_j) = k^{j-1}B'(x_j)\Sigma_j\sigma_\tau = \Sigma_j\sigma\tau k^{\tau-1}L'(X_\tau)] \quad (9)$$

Let $\Sigma_j\sigma_\tau = s_j$. $s_j = 1 - G(t)$ where $G(t)$ is the cumulative distribution function that gives the probability that a release will occur by period t. Thus, $s_j \leq s_1 = 1$ for all positive j.

Rewriting (2) yields

$$B'(x_j) = \Sigma_j(\sigma_\tau/s_j)k^{\tau-j}L'(X_\tau) \quad . \quad (9')$$

<u>Lemma 10</u>: Suppose $\sigma_t = p(1-p)^{t-1}$ and $M_t(p) = \Sigma_t(\sigma_t/s_t)k^{t-1}$. Then $d[M_t(p)]/dp = 1 - k > 0$ if $0 \leq k < 1$.

<u>Proof</u>: $M_t(p) = p/[1-k(1-p)]$. Differentiating with respect to p yields $d[M_t(p)]/dp = 1 - k$. Q.E.D.

<u>Lemma 11</u>: $p_t \leq p_{t+1}$ for all t implies

$$\Sigma_t(\sigma_\tau/s_t)k^{\tau-t} \geq p_t/[1 - k(1-p_t)] \quad (10)$$

<u>Proof</u>: By induction on the number of "steps" in the sequence $\{p_t\}$. (10) holds with equality if there are 0 steps. Assume there is single step (and that it occurs after t in (10)). Thus,

for some n, $\{p_t\}$ has the form $p_\tau = p$, $\tau \le t + n$, and $p_\tau = q > p$ for $\tau > t + n$. (10) will thus hold if

$$\Sigma_t(\sigma_\tau/s_t)k^{\tau-t} \ge p/[1 - k(1-p)]$$

as $p_t = p$. Now

$$\Sigma_t(\sigma_\tau/s_t)k^{\tau-t} - M_t(p)$$
$$= k^{n-1}(1-p)^{t+n}\Sigma_{t+n}\{q[k(1-q)]^{\tau-1} - p[k(1-p)]^{\tau-1}\}$$
$$\ge 0$$

if and only if

$$q/[1-k(1-q)] \ge p/[1-k(1-p)]$$

which is true by Lemma 10 and the hypothesis that $q > p$.

Now suppose that (10) holds for any $\{p_t\}$ with T steps and consider a sequence which has T+1 steps. A problem arises only if all T+1 steps occur after t. So assume that $\{p_t\}$ has the form $p_\tau = r_1$, $\tau \le t + n_1$, $p_\tau = r_2$, $t+n_1 < \tau \le t+n_1+n_2$, . . . $p\tau = r_{T+1}$ for $\tau > t + \Sigma^T n_i = t + N_T$.

Consider the sequence $\{q_t\}$ defined as $q_t = p_t$ for $t \le t+N_T$ and $q_t = r_T$ otherwise. Let $\Phi_\tau$ and $w_t$ be defined for $q_t$ as $\sigma\tau$ and $s_t$ are defined for $p_t$. By the induction hypothesis, (10) holds for $\{q_t\}$ as it has only T steps. Now consider

$$D(p,q) = \Sigma_t[(\sigma_\tau/s_t)-(\Phi_\tau/w_t)]k^{\tau-t} \quad (11)$$

The terms of (11) are non-zero only for $\tau > t + N_T$ which defines a new series with only one step. Thus $D(p,q) > 0$ and

$$\Sigma_t(\sigma_\tau/s_t)k^{\tau-t} \ge \Sigma_t(\Phi_\tau/w_t)]k^{\tau-t} \ge p_t/[1 - k(1-p_t)] \quad (12)$$

This completes the induction. Therefore (10) holds for any non-decreasing sequence of positive numbers bounded above by 1.

Q.E.D.

<u>Proposition 2</u>: $p_t \le p_{t+1}$ for all t implies $x_{t+1} \le x_t$ (with equality only if $x_t = 0$).

<u>Proof</u>: Note first that $x_{t+1} \le x_t$ if and only if

$$B'(x_t) - B'(x_{t+1}) \le 0 \quad (13)$$

From (9') we may write

$$B'(x_t) - B'(x_{t+1}) = \Sigma_t k^{\tau-t}[(\sigma_\tau/s_t)L'(X_\tau) - (\sigma_{\tau+1}/s_{t+1})L'(X_{\tau+1})]$$

$$= (\sigma_t/s_t)L'(X_t) + [(k/s_t)-(1/s_{t+1})]\Sigma_{t+1}k^{\tau-t-1}\sigma_\tau L'(X_\tau) \quad (14)$$

$$< L'(X_t)\{(\sigma_t/s_t) + [(k/s_t)-(1/s_{t+1})]\Sigma_{t+1}\sigma_\tau k^{\tau-t-1}\} \quad (15)$$

where (15) holds because $(k/s_t) - (1/s_{t+1}) < 0$ (and when $x_{t+1} > 0$).

Now (15) is non-positive if and only if

$$(\sigma_t/s_t) \le [1-(s_{t+1}k/s_t)]\Sigma_{t+1}(\sigma_\tau/s_{t+1})k^{\tau-t-1} \quad (16)$$

But $(\sigma_t/s_t) = p_t$ and $s_{t+1}/s_t = 1 - (\sigma_t/s_t) = 1 - p_t$. We may thus rewrite (16) as

$$p_t/[1 - k(1-p_t)] \le \Sigma_{t+1}(\sigma_\tau/s_{t+1})k^{\tau-t-1} \quad (17)$$

But by Lemma 11, the right hand side of (17) is larger than $p_{t+1}/[1 - k(1-p_{t+1})]$ which, by Lemma 10 and the hypothesis that $p_{t+1} \ge p_t$, is at least as great as the left-hand side of (17).
Q.E.D.

Now consider the actor's behavior under negligence. Let $x_t^*$ solve (9'). It will be helpful to prove:

1. The Forgiving Rule with standards of care $\hat{x_t} = x_t^*$.

<u>Lemma 12</u>: Not-$[x_t > x_t^*$ for all t].

<u>Proof</u>: Suppose $x_t > x_t^*$ for all $t > 1$. Then, the actor's

objective function conditional on being negligent in period 1 is identical to the social objective function but the $x_1$ that solves the first-order condition, given $x_t > x_t^*$ for $t > 1$ must be less than $x_1^*$. Q.E.D.

Lemma 13: For some functions B and L, $x_t > x_t^*$, for some t.

Proof: Suppose $x_t = x_t^*$ for all $t > 1$. Then the actor maximizes over $x_1$ the objective function

$$V(x_1) = \begin{cases} B(x_1) - p_1 L(x_1) & x_1 > x_1^* \\ B(x_1) & x_1 \leq x_1^* \end{cases}$$

The example in the proof to Proposition 1 establishes this lemma as well. Q.E.D.

2. The Unforgiving Rule with $\hat{x_t} = x_t^*$.

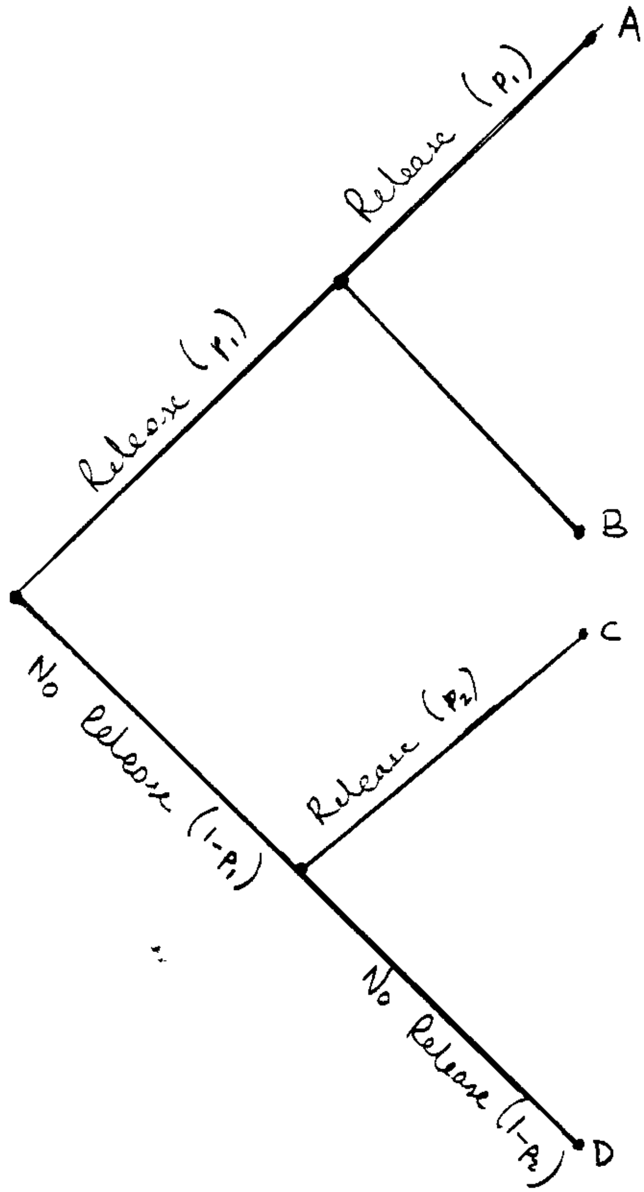
Lemma 14: Under an unforgiving rule with $\hat{x_t} = x_t^*$ for all t, the actor meets her standard of care in every period.

Proof: Suppose that there has been no release through period t-1 and that the actor has met her standard in each prior period. Then, she chooses $x_t$ to maximize

$$Wt(x_t) = \begin{cases} B(x_t) - \Sigma_t(\sigma_\tau/s_t)k^{\tau-t}L(X_\tau) & x_t > x_t^* \\ B(x_t) & x_t \leq x_t^* \end{cases}$$

But, given that the actor has met her standard in all prior periods, $x_t^*$ maximizes (by definition) the upper function over an unrestricted domain. The lemma thus follows by induction on $x_t$. Q.E.D.

Figure 1


A
Release ($p_1$)
Release ($p_1$)
B
C
Release ($p_2$)
No Release ($1-p_1$)
No Release ($1-p_2$)
D