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FINANCIAL CAPACITY AND OUTPUT FLUCTUATIONS IN AN ECONOMY WITH MULTIPERIOD FINANCIAL RELATIONSHIPS

BY

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ABSTRACT

This paper motivates a financial propagation mechanism in the context of an intertemporal production economy with asymmetric information where borrowers and lenders enter multiperiod financial relationships. A key feature is that aggregate output and borrowers' "financial capacity" - the maximum overhang of past debt they may feasibly carry - are determined jointly, much in the spirit of Gurley and Shaw (1955). Expectations of future economic conditons govern financial capacity which in turn may constrain current production, especially in bad times. A small but persistent shift in aggregate conditions may have a large impact on financial capacity, making the framework capable of motivating large endogenous fluctuations in financial constraints.

Key Words: asymmetric information, multiperiod contracts, financial capacity, business fluctuations.

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1. Introduction

Understanding how small disturbances can induce large output fluctuations is an on-going quest in macroeconomics. This paper develops a simple framework in which shifts in aggregate economic fundamentals are amplified through their impact on financial conditions. A distinguishing feature is that output and borrowers' "financial capacity" - the maximum overhang of past debt they may feasibly carry - are determined jointly, much in the spirit of Gurley and Shaw (1955). Expectations of future economic conditions govern financial capacity. And financial capacity may constrain current production, especially in bad times.

A number of recent papers have resurrected the view that financial factors may play a part in propagating business fluctuations.[1] The underlying theories exploit the idea that, with asymmetric information between borrowers and lenders, agency costs may drive the price of uncollateralized external funds above the price of internal funds. In this kind of setting, a borrower's financial position looms as a key determinant of the terms of credit she faces. In the aggregate, swings in borrower balance sheets over the cycle amplify fluctuations in investment and output. This simple mechanism is broadly consistent with informal descriptions of how financial and real variables interacted in the Great Depression [Bernanke (1983)] and in the postwar business cycles [Eckstein and Sinai (1986)]. In addition, numerous recent panel data studies, beginning with Fazzari, Hubbard and Peterson (1988), support the notion of a financial investment accelerator mechanism.

This paper enriches the description of the financial propagation

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[1]Examples include Farmer (1985), Greenwald and Stiglitz (1986), Williamson (1987), Bernanke and Gertler (1989) and Calomiris and Hubbard (1990).

mechanism in a way that makes it considerably easier to motivate large endogenous movements in financial constraints. The model here differs by permitting borrowers and lenders to enter on-going relationships. Opened up is the opportunity to reschedule unpaid debts. For this reason, the agency costs of investment finance become dependent not only on a borrower's financial assets but also on the present discounted value of her project earnings in subsequent periods. The possibility of rescheduling unpaid debts effectively permits expected earnings past the lifetime of the initial loan to serve as collateral. A ceiling quantity of debt that lenders are willing to reschedule emerges endogenously. This ceiling, interpretable as the borrower's financial capacity, reflects that maximum value of expected future earnings that the borrower can credibly offer as collateral. The paper demonstrates how, at the macro level, fluctuations in financial capacity can magnify fluctuations in output. It shows further how large movements in financial capacity may be induced by a small but persistent change in macroeconomic fundamentals.

Allowing for on-going borrower-lender relationships also ensures that conclusions do not hinge on arbitarily restricting the contract period. A lengthy literature has made the point that, in addition to capturing reality, multi-period financial arrangements introduce efficiency gains.[2] The analysis here incorporates some aspects of this literature. As in Stiglitz and Weiss (1983), the potential for borrowers to forfeit earnings in subsequent periods helps mitigate the agency problem. In that paper, though, lenders terminate credit to poorly performing borrowers, whereas here they obtain a claim on

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[2]Examples of multi-period financial contracting include include Townsend (1982), (1988), Stiglitz and Weiss (1983), Green (1987), Webb (1988) and Hart and Moore (1989).

their future revenue (by recheduling their debt).

The paper also uses insights from Green (1987) by formulating the contracting problem as one of repeated moral hazard, with the wealth of the agent serving as state variable. An important difference is the incorporation of non-negativity constraints on consumption - "limited liability constraints" in Sappington's (1983) terminology. These constraints make financial capacity a meaningful construct. In this vein, the model may be viewed as extending Sappington's analysis of moral hazard with limited liability to a dynamic setting.

It is finally useful to contrast the framework with Hart and Moore (1989), a setting where creditors have the right to seize a borrower's assets and where they decide ex post between doing so and rescheduling debt. Here rescheduling is the outcome of a preagreed state-contingent contract. A sufficient number of observable states exist to enforce the agreement ex post. Also, there are no Pareto improving gains from re-negotiation. Thus the ex post bargaining featured in Hart and Moore is absent and along with it a story about foreclosure and bankruptcy. These considerations are ignored to keep the model sufficiently tractable for addressing macroeconomic issues.

Section 2 presents the basic model, a multi-period production economy with borrowing and lending. The single period version is a variant of Grossman and Hart (1983) and Farmer (1985), settings in which asymmetric information makes underemployment possible in bad states of nature. The framework is distinct from Leach (1988) because it permits individuals to enter multiperiod contracts, in addition to differing in many other details. Section 3 derives the optimal multi-period financial arrangement and section 4 fleshes out the macroeconomic implications.

The basic model has just three periods, with individuals consuming only in the final period. Both these restrictions are relaxed in section 5. In the extended model it is more explicit how small but permanent shocks can induce large swings in financial capacity. There are as well some interesting implications for consumption and savings behavior.

2. The Basic Setting

There are three periods, numbered 0, 1, 2. A countable infinity of people exist, divided into "lenders" who wish to save and "entrepreneurs" who have access to a production technology but need to borrow to finance their capital needs. An individual drawn at random is a lender with probability $1-\eta$ and is an entrepreneur with probability η. Utility for both types is linear in period two consumption, implying that everyone acts to maximize expected final wealth.

Every person begins with W_0 units of a consumable good. There are two ways to save. Storing a unit of the good as inventory in period t-1 yields r_t units in period t, where the gross return r_t is exogenous. Alternatively, investing at t-1 yields capital for use as input in a technology that produces output of the good at t.

Entrepreneurs manage investment and production. Each entrepreneur operates a project which involves a sequence of investment and production that is repeated twice. Figure 1 lists the timing of events. In period t-1 (t = 1,2), the entrepreneur installs capital that is ready for use in period t. A unit of investment in t-1 yields one unit of period t capital, K_t, that fully depreciates at the end of t. The entrepreneur may finance this investment by borrowing from lenders, as well as by using her own resources.

Output, y_t, depends on a productivity disturbance, $\tilde{\psi}_t$, and on the quantity of capital employed, x_t.[3] There are increasing marginal costs of employing installed capital, implying y_t is concave in x_t:

$$y_t = \psi_t x_t - c(x_t) \tag{2.1}$$

$$K_t \geq x_t \tag{2.2}$$

The cost function $c(\cdot)$ is twice continuously differentiable, strictly increasing and convex, with $c(0) = 0$, $c'(0) = 0$, and $c'(z) \to \infty$ as $z \to \infty$. ψ_t is the realization of $\tilde{\psi}_t$. It becomes known after capacity K_t is installed but prior to the choice of x_t. Since employing installed capital is costly, the entrepreneur may choose to operate at less than full capacity in the event of a bad productivity realization.

The random productivity shock $\tilde{\psi}_t$ is independent and identically distributed over time and across projects. It has the following two point distribution:

$$\tilde{\psi}_t = \begin{cases} \psi^g = 1 + \Delta \text{ with probability } \pi_t^g \\ \psi^b = 1 \quad \text{" \quad "} \quad \pi_t^b = 1 - \pi_t^g \end{cases} \tag{2.3}$$

The "success" probability π_t^g, common to all entrepreneurs, is a measure of the economy-wide state of technology at time t. Also, the gap between productivity in the good and bad states, Δ, is large enough to satisfy,

$$\Delta > r_t/\pi_t^g \tag{2.4a}$$

$$\Delta > \pi_t^b/\pi_t^g \tag{2.4b}$$

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[3] While installation of capital occurs over an interval of time, production occurs at a point in time. Think of the production period as being short relative to the installation period.

Condition (2.4a) guarantees excess capacity in the bad productivity state. The significance of (2.4b) is taken up later.

After production in period t, capital fully depreciates. The entrepreneur then chooses how much capital to install for t+1, initiating a new sequence of investment and production. Investment is suspended in the final period as the project ends.

To motivate a meaningful interaction between financial variables and real activity, suppose that information is restricted in the following way: Output is not publicly observable; only the entrepreneur directly learns the realizations of $\tilde{\psi}_t$ and project output. Outsiders, including both lenders and third parties, are able to costlessly observe investment and capacity utilization. It is therefore possible to write and enforce contracts with contingencies based on these input variables; but it is not similarly possible to write and enforce contracts with contingencies based on output. These assumptions introduce an incentive problem connected with external finance. The entrepreneur may want to falsely claim hard times if by doing so she can substantially reduce her obligation to lenders.

The static version of this model, as noted earlier, is essentially the framework studied by Grossman and Hart (1983) and Farmer (1985). Underemployment of capital in the bad productivity state is possible, the reason being that the entrepreneur may have to reduce production below the optimum to make credible the claim of hard times. In the multi-period setting developed here, the degree of underemployment becomes interdependent over time. The effect at the aggregate level is to raise the sensitivity of output to exogenous disturbances. A channel - intrepretable as a financial propagation mechanism - emerges through which current shifts in either technology (π^g) or interest rates (r) persist into the future; and through which anticipated future shocks affect current output. The sections that follow derive and elaborate these results, beginning with a characterization of the optimal financial arrangement for this kind of environment.

3. The Optimal Multiperiod Financial Arrangement

It is convenient to introduce the fiction of competitive intermediaries which facilitate loans to entrepreneurs. In period 0, lenders allocate their wealth between deposits at these intermediaries and inventory storage. They repeat the process in period 1 and consume in the final period. The deposit rate each period equals the return on inventory storage, given inventories are held in equilibrium. Also in period zero, each entrepreneur enters a multiperiod financial arrangement with an intermediary. A key feature of the arrangement, distinguishing it from a one-shot agreement, is a contingency for rescheduling unpaid debts.

Solving for the optimal multi-period arrangement involves deriving a sequence of optimal one period contracts, working backwards. Following, Green (1987), imagine setting up an account balance which records the entrepreneur's financial asset position as it evolves over time. Among other things, the optimal contract specifies how the intermediary should adjust this account in response to the sequence of productivity outcomes the entrepreneur announces.[4] The optimal contract is found by beginning at the end of period 1 and deriving the optimal single period contract for period 2, assuming (an arbitrary value) W^j is in the entrepreneur's account as a result of state j having occurred in period 1. The solution yields a value function $V(W^j)$ that expresses the

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[4] The contract also specifies the capital employment levels contingent on the productivity announcements. Because employment is observable by the parties, the contract is enforceable, as in Grossman and Hart (1983) and Farmer (1985).

entrepreneur's expected period 2 payoff as a function of W^j. The complete solution is then found by moving back to period 0 and solving another single period contracting problem, one which picks W^j to maximize the entrepreneur's expected final wealth.

This section first characterizes the solution to the contracting problem for the terminal period - the period 1 problem. In the process, the concept of financial capacity is made precise and the value function is obtained. The section next turns to the period 0 problem and derives the overall multiperiod arrangement. The implications for macroeconomic behavior are then developed in the subsequent section.

Before proceeding, it is convenient to define the function $R_t(a,b)$,

$$R_t(a,b) \equiv \pi_t^g[(1+\Delta)a - c(a)] + \pi_t^b[b - c(b)] - r_t a \qquad (3.1)$$

as the t-1 expectation of the entrepreneur's net gain from operating her project in t, given a and b are the quantities of capital employed in the good and bad states, respectively. Eq. (3.1) is derived from eqs. (2.1) and (2.3). K_t is eliminated from the expression by embeding the result that capital employed in the good state always equals the quantity installed - installing more capital than is ever needed in the good state is simply wasteful.

3a. The period 1 problem. Suppose an entrepreneur ends period 1 with an account balance of W^j owing to the realization of state j. Further, let W^{jk} denote the entrepreneur's final wealth contingent on state j having occurred in period 1 and state k in period 2; and let x^{jk} similarly denote the state-contingent quantity of captial employed in period 2. The optimal arrangement with the intermediary for the remainder of the project is a vector of final payoffs and input allocations, $\{W^{jg}, W^{jb}, x^{jg}, x^{jb}\}$, that solves

$$\max \sum_k \pi_2^k W^{jk}/r_2 \quad , \; k = g,b \tag{3.2}$$

subject to,

$$\sum_k \pi_2^k W^{jk}/r_2 = R_2(x^{jg},x^{jb})/r_2 + W^j \tag{3.3}$$

$$W^{jg} \geq W^{jb} + \Delta x^{jb} \tag{3.4}$$

$$W^{jb} \geq 0 \tag{3.5}$$

The objective, eq. (3.2), is the entrepreneur's expected discounted final period wealth. This value, as eq. (3.3) states, must equal the sum of her expected discounted period two project earnings and her end-of-period-one wealth. Constraint (3.4) eliminates the entrepreneur's incentive to falsely claim the bad productivity state has been realized. An entrepreneur who falsely pleads hard times must set capacity utilization at x^{jb} in order to mimic the bad state. Her gain is the contractual payoff W^{jb} plus the unreported income Δx^{jb} (the difference bewteen $\psi^g x^{jb}$ and $\psi^b x^{jb}$). Eq. (3.4) requires that this gain from deception not exceed the payoff from honestly revealing the good state, W^{jg}.[5] Finally, eq. (3.5) together with eq. (3.6) requires that the entrepreneur's terminal wealth be non-negative. This limited liability condition simply reflects the fact that the entrepreneur can never honor any debt left at the end of the final period.

Let x_2^{g*} and x_2^{b*} denote the first-best values of x^{jg} and x^{jb} - the choices

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[5]It is not necessary to introduce a symmetric constraint to dissuade the entrepreneur from falsely claiming times are good; it is easy to show this constraint would never bind.

of x^{jg} and x^{jb} when the incentive constraint (3.4) is irrelevant[6]. Then under the optimal contract,

$$x^{jg} = x_2^{g*} \tag{3.6}$$

$$x^{jb} \leq x_2^{b*} < x_2^{g*} \tag{3.7}$$

In analogy to Grossman and Hart (1983) and Farmer (1985), the incentive problem distorts capital employment in the bad productivity state. x^{jg} always equals x_2^{g*} but x^{jb} may lie below x_2^{b*}. Operating at x^{b*} may not be feasible since the gain from falsely announcing bad times is increasing in x^{jb}. To credibly claim times are bad (and thus satisfy constraint (3.4)), the entrepreneur may have to set x^{jb} beneath x_2^{b*}.

Figure 2 illustrates how the incentive problem affects x^{jb}. The *ic'* curve reflects combinations of x^{jb} and the entrepreneur's bad state payoff, W^{jb}, which just satisfy the incentive condition (3.4):

$$ic' \qquad \underset{-}{W^{jg}}(\underset{-}{W^{jb}}, \underset{+}{x^{jb}}; \underset{+}{W^{j}}, \underset{+}{\pi_2^g}, \underset{-}{r_2}) - W^{jb} - \Delta x^{jb} = 0 \tag{3.8}$$

where the implicit function for W^{jg} is obtained from eq. (3.1). The *ic'* curve slopes downwards. A rise in x^{jb} increases the entrepreneur's incentive to falsely claim bad times, requiring that W^{jb} decline to offset the effect.[7] Since the limited liability condition precludes reducing W^{jb} below zero, the maximum feasible value of x^{jb} is the point on the *ic'* curve corresponding to $W^{jb} = 0$, denoted as $\hat{x}^{jb}$ in Figure 2. Thus, if $\hat{x}^{jb} < x_2^{b*}$, it is optimal to fix

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[6] x_t^{g*} is given by $\pi_t^g[(1+\Delta) - c'(x_t^{g*})] - r_t = 0$, and x_t^{b*} is given by $1 - c'(x^{b*}) = 0$. Condition (2.4a) guarantees $x_t^{g*} > x_t^{b*}$.

[7] The effect of a rise in x^{jb} on the entrepreneur's net gain from cheating, $\Delta x^{jb} + W^{jb} - W^{jg}$, is $\Delta - \pi_2^b[1-c'(x^{jb})]/\pi_2^g > 0$, given condition (2.4b).

W^{jb} at zero and x^{jb} at $\hat{x}^{jb}$.

Another key result is that the degree of underemployment depends inversely on the entrepreneur's intitial financial position. A rise in W^j enables the entrepreneur to commit a higher payment to the intermediary in the event of a bad productivity outcome, which entitles her to a higher payment in the good state. This reduces her incentive to dishonestly claim hard times. The *ic* curve shifts outward, allowing x^{jb} to rise.

An important financial threshhold for the entrepreneur is the value of her end-of-period-one account balance that makes her expected final wealth zero. From the budget constraint (3.3), this lower bound for W^j - call it $\underline{W}$ - satisfies,

$$\underline{W} = -R_2[x_2^{g*}, x^{jb}(\underline{W})]/r_2 \tag{3.9}$$

where the function $x^{jb}(\cdot)$ incorporates the influence of W^j on x^{jb}. Eq. (3.9) implies that $\underline{W}$ is negative. An entrepreneur with zero net worth has an overhang of debt equal to her expected discounted period two project earnings. The relationship is simultaneous, however, since expected future earnings depend on the entrepreneur's financial position. But it may be shown that

$$x^{jb}(\underline{W}) = 0 \tag{3.10}$$

x^{jb} must be fixed at zero in this case. Otherwise, the entrepreneur will always falsely claim hard times in the event of a good productivity realization. This is because she receives no payoff in the good state (as well as in the bad state) when her expected final wealth is zero. In Figure 2, $\underline{W}$ is the value of W^j which positions the *ic'* curve to intersect the horizontal axis at $x^{jb} = 0$.

The number - $\underline{W}$ is interpretable as the entrepreneur's "financial capacity". It is the maximum debt overhang that she can carry without having to suspend her project. At any level of indebtedness exceeding $-\underline{W}$ (i.e., when $W^j < \underline{W}$) the entrepreneur could not obtain fresh loans. Since her expected final wealth would be negative, her final wealth would have to be negative in at least one state, violating eq. (3.4). She would thus be unable offer a contract under which she could credibly commit to behave honestly. Financial considerations would force her to abandon her project even though it offered positive expected surplus.

For $W^j \geq \underline{W}$, the entrepreneur's expected discounted final wealth may be expressed in terms of the value function $V(W^j)$,

$$V(W^j) \equiv R_2[x_2^{g*}, x^{jb}(W^j)]/r_2 + W^j \tag{3.11}$$

As Figure 3 illustrates, $V(W^j)$ is initially concave with a slope exceeding unity.[8] In addition to increasing her financial wealth, the increment in W^j raises the entrepreneur's expected project return by allowing x^{jb} to rise. Concavity of the production function is responsible for the diminishing slope.

When W^j rises above an upper threshold, $\bar{W}$, $V(W^j)$ becomes linear with a slope equal to unity. $\bar{W}$ is the minimum value of wealth that enables the entrepreneur to avoid underemeployment in the bad productivity state. It corresponds to the value of W^j that positions the *ic'* curve in Figure 2 to intersect the horizontal axis at $x^{jb} = x_2^{b*}$.[9] Since it is never efficient to

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[8]For $W^j \in [\underline{W}, \bar{W})$; $V' = \{1 - (\pi_t^b/\Delta\pi_t^g)[1-c'(x^{jb})]\}^{-1} > 1$, and

$V'' = -(\pi_t^b/\Delta\pi_t^g)c''(x^{jb})(V')^2 \cdot \frac{dx^{jb}}{dW^j} < 0$.

[9] $\bar{W}$ equals $[\pi^g \Delta x_2^{b*} - R(x_2^{g*}, x_2^{b*})]/r$. At $\bar{W}$, the entrepreneur's expected payoff from acting honestly at the first best, $\bar{W} + R_2(x_2^{g*}, x_2^{b*})/r_2$, equals her

raise x^{jb} above x_2^{b*}, additional W^j beyond $\bar{W}$ simply adds to the entrepreneur's net worth without affecting her expected project return.

It is also useful to note the effects of changes in the interest rate r_2 and the economy-wide technology parameter π_2^g. In the first best optimum, shifts in these parameters affect capacity investment and hence the quantity of captial employed in the good state. In the incentive-constrained optimum, shifts in r_2 and π_2^g also affect capacity utilization in the bad state, and in a way that magnifies the overall affect on expected output.[10] A rise in r_2 shifts the *ic'* curve leftward, reducing x^{jb}. A larger value of r_2 lowers the entrepreneur's expected profits, which shrinks her good state payoff W^{jg}, forcing a cutback in x^{jb} to satisfy the incentive condition. Conversely, a rise in π_2^g increases the entrepreneur's expected profits, which shifts the *ic'* curve rightward, increasing x^{jb}.

3b. The period 0 problem. The multi-period contracting problem now collapses into a single period problem at time 0. The variables determined are W^j and the state-contingent quantity of capital employment in period one, x^j. Knowing W^j then pins down the complete multiperiod arrangement, given the results from the previous section.

The period 0 problem is to choose the vector $\{W^g, W^b, x^g, x^b\}$ to solve

$$\max \sum_j \pi^j V(W^j)/r_1 \quad , \; j = g,b \tag{3.12}$$

subject to,

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expected gain from cheating, $\pi^g \Delta x_2^{b*}$. $\bar{W}$ may be either positive or negative. But condition (2.4b) guarantees that it always exceeds $\underline{W}$.

[10]See Farmer (1985) for related results.

$$\sum_j \pi^j W^j / r_1 = R_1(x^g, x^b)/r_1 + W_0 \qquad (3.13)$$

$$V(W^g) \geq V(W^b) + \Delta x^b \qquad (3.14)$$

$$W^b \geq \underline{W} = - R_2(x_2^{g*}, 0)/r_2 \qquad (3.15)$$

The objective (3.12) is the entrepreneur's expected discounted final wealth expressed in terms of W^j, using the value function derived from the terminal period problem. Eq. (3.13), the period 0 budget constraint, relates the entrepreneur's expected discounted period 1 wealth to the sum of her expected discounted period 1 project rents and her initial wealth. Eq. (3.14), as eq. (3.4) before, is needed to eliminate any gain from misrepresenting the good productivity state as the bad one. In comparing the payoffs from honesty versus dishonesty, the value function is used to measure the gains from W^g and W^b. Unreported income Δx^b, however, does not enter $V(\cdot)$ in calculating the benefits from cheating. This is because the entrepreneur cannot use Δx^b to reduce agency costs in period two in the same way she can use W^b, else she would reveal her dishonesty.[11]

Eq. (3.15) places a lower bound on W^b. The entrepreneur may end period one with a net financial obligation. That is, W^b may be negative since debts unpaid at the end of period one can be rolled over to period two. There is a limit, however. The entrepreneur's debt overhang, $- W^b$, cannot be so large as to make her expected final wealth negative. Otherwise, it would not be possible to structure an incentive-compatible arrangement for the remainder of

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[11] The entrepreneur may secretly store unreported earnings. Her measured gain in period one wealth is thus $r_2 \Delta x^b / r_2 = \Delta x^b$.

the project, as the previous section makes clear. Eq. (3.15) accordingly constrains the debt overhang from exceeding the entrepreneur's financial capacity, $-\underline{W}$. From eqs. (3.9) and (3.10), $-\underline{W} = R_2(x_2^{g*},0)/r_2$; financial capacity equals expected discounted period two project earnings conditioned on the entrepreneur having zero expected final wealth (i.e., on $W^b = \underline{W}$).

At the optimum, x^g equals its first best value, x_1^{g*}, but x^b may lie below its respective first best value, x_1^{b*}. As in the non-repeated setting, the incentive problem may induce underemployment in the bad productivity state. The exact factors governing x^b, however, are somewhat different. Anticipated future economic conditions have a role here that is absent in the one shot setting.

If the incentive constraint binds, the following three conditions govern x^b and W^b, corresponding to the *ic*, *fc* and *ws* curves in Figures 4 and 5 (see the appendix):

$$ic \qquad V[W^g(\underset{-}{W^b},\underset{+}{x^b};\underset{+}{W_0},\underset{+}{\pi_1^g},\underset{-}{r_1})] = V(W^b) + \Delta x^b \qquad (3.16)$$

and either,

$$fc \qquad W^b = -R_2(x^{g*},0)/r_2 \qquad (3.17)$$

or,

$$ws \qquad \pi_1^b[V'(W^b)/V'(W^g) - 1] = (\partial R_1/\partial x^b)\left(-\frac{\partial x^b}{\partial W^b}\Big|_{ic}\right) \qquad (3.18)$$

The implicit function for W^g is obtained from eq. (3.13). In eq. (3.18), $\partial x^b/\partial W^b|_{ic}$ is the slope of the *ic* curve that eq. (3.15) defines.

The *ic* curve is the analogue of the *ic'* curve in Figure 2. It similarly is downward sloping, as Figure 4 illustrates, since x^b and W^b must move in opposite directions to satisfy the incentive constraint.[12] The *fc* curve portrays the minimum feasible value of W^b, the value at which the financial capacity constraint (3.14) is binding. The curve is perfectly horizontal and intersects the vertical axis below zero at $- R_2(x_2^{g*},0)/r_2$.

The maximum feasible level of capacity utilization in the bad state is $\hat{x}^b_{fc}$, corresponding to the intersection of the *ic* and *fc* curves. Permitting W^b to fall below zero increases $\hat{x}^b_{fc}$, as one may infer from comparing Figures 2 and 4. Contracting in advance to roll over a portion of unpaid debts commits the entrepreneur to sacrificing some of her future earnings stream in the event she announces a bad productivity state.[13] Effectively, her collateral base expands to include her expected discounted future project returns. As a consequence, her incentive to falsely claim hard times is reduced, relaxing the constraint on x^b.

Complicating matters is the dependency of future project revenues on the entrepreneur's current financial position, as captured by the value function [see eq. (3.11)]. Fixing W^b at its lower bound $\underline{W}$ may not always be desirable. Since $V(\cdot)$ is concave in wealth, the entrepreneur may prefer a contract that narrows the gap between W^g and W^b. The left side of eq. (3.18) reflects the marginal benefit from redistributing her payoff from the good state to the bad. It is increasing in the difference between the shadow values of wealth across states. The right side reflects the marginal cost of having to reduce

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[12]It is possible that the *ic* curve may slope upward at $x^b = 0$ before turning downward as x^b rises. It is straightforward but cumbersome to verify that the curve slopes downward at the equilibrium, regardless of whether that point is the intersection with *fc* curve or the intersection with the *ws* curve. See Gertler (1988).

[13]Note that the ex ante rescheduling agreement is enforceable ex post since it is conditioned only on input usage - which third parties may observe - and not on output.

x^b in order to satisfy the incentive constraint. If V(·) is sufficiently concave, then at $(W^b, x^b) = (\underline{W}, \hat{x}^b_{fc})$, the net marginal benefit from smoothing wealth across states may positive. In this case, the financial capacity constraint (3.16) is no longer binding; W^b and x^b instead adjust to satisfy the wealth smoothing condition (3.18). Figure 5 portrays this situation.

The *ws* curve defines combinations of W^b and x^b that satisfy eq. (3.18). The curve slopes upwards.[14] Given the concavity of V(·), a rise in W^b lowers the marginal benefit from smoothing wealth; x^b must rise to lower the marginal cost (by lowering the marginal product of x^b). The optimum corresponds to the intersection of the *ic* and *ws* curves.[15] Apparent from Figure 5 is the decision to accept lower employment in the bad state in return for a stronger ex post financial position. When the wealth smoothing optimum is likely to arise is taken up in the next section.

In either case, the optimal multi-period contract is interpretable as debt with a rollover provision. Specifically, it may be implimented with a sequence of one period debt contracts where the entreprenur finances a preagreed amount of any unpaid obligation with a new one period debt issue. The terminal period contract is standard debt without rollover. The only sense in which the arrangement differs under the wealth smoothing optimum is that the intermediary shares more risk with the entrepreneur. It does so by agreeing ex ante to forgive a higher percentage of unpaid debt in the event of a bad productivity outcome in return for a higher payment in the good state.

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[14] The *ws* curve intersects the *fc* curve between the origin and x_2^{b*}. It then slopes upward, reaching $\tilde{W}$ at $x^b = x^{b*}$. See Gertler (1988).

[15] The net marginal benefit from wealth smoothing is positive to the right of the *ws* curve, and is negative to the left. Accordingly, the *ws* curve is binding if it intersects the *fc* curve to left of the *ic* curve; otherwise the *fc* curve applies, as in Figure 3.

4. Macroeconomic Implications

At the macro level, a financial propagation mechanism emerges. The mechanism amplifies the effects of shocks to the economy by introducing simultaneous feedback between current and expected future output. This may be illustrated by considering how shifts in technology (π^g) and interest rates (r) affect the temporal pattern of per capita output. These exercises will also help flesh out the particular implications of allowing for multi-period financial relationships.

The intertemporal equilibrium is simple to characterize. Averaging across (the η per capita) entrepreneurs yields per capita output, $\hat{y}$, for periods one and two:

$$\hat{y}_1 = \eta[\pi_1^g y(x_1^{g*},\psi^g) + \pi_1^b y(\kappa^b,\psi^b)] \tag{4.1}$$

$$\hat{y}_2 = \eta[\pi_2^g y(x_2^{g*},\psi^g) + \sum_j \pi_1^j \pi_2^b y(\kappa^{jb},\psi^b)], \quad j = g,b \tag{4.2}$$

where the production function $y(\cdot,\cdot)$ comes from eq. (2.4). Eqs. (4.1) and (4.2) embed the result that x_t^g always equals its first-best value x_t^{g*}. The variables κ^b and κ^{jb} stand for the optimal values x^b and x^{jb}.

The propagation mechanism is absent in the benchmark case of perfect information; there is no interdependence between $\hat{y}_1$ and $\hat{y}_2$. Production at the individual level is uncorrelated over time since capital depreciates after one period and since the idiosyncratic productivity shock is i.i.d.. At the aggregate level, output each period depends only on contemporaneous factors. In this case, $x^b = x_1^{b*}$ and $x^{bb} = x^{gb} = x_2^{b*}$. Since both x_t^{b*} and x_t^{g*} depend only on current factors, the same holds for $\hat{y}_t$. Shocks to π_t^g and r_t thus only affect output at t.

Shocks may be propagated across time in the incentive-constrained setting, as $\hat{y}_1$ and $\hat{y}_2$ are determined jointly. This outcome stems from the temporal nature of the underemployment distortion, as manifested in the behavior of x^b and x^{jb}. The exact dynamics depend on whether the financial capacity constraint, eq. (3.17), or the wealth smoothing condition, eq. (3.18), governs behavior. Each case is discussed below. The effects of shifts in the period one and period two parameters are considered in turn.[16]

Case 1: $W^b = - R_2(x_2^{g*}, 0)$. The financial capacity constraint is binding in this circumstance and the intersection of the *ic* and *fc* curves in Figure 4 jointly determines x^b and W^b. This situation is more likely to arise the weaker the aggregate economy, as will become clear.

A rise in initial wealth W_0 increases x^b and therefore expands $\hat{y}_1$. Enhancing the entrepreneur's initial financial position relaxes the incentive constraint, as in the one shot setting (see section 3.1). The *ic* curve in Figure 4 shifts rightward, moving the equilibrium to a higher value of x^b. The impact, however, carries over into period two as the boom strengthens the average entrepreneurial balance sheet at the end period one. Though W^b remains fixed at its minimum feasible value, W^g rises. Expected period 2 production by entrepreneur's successful in period 1 goes up as result. In eq. (4.2), κ^{gb} increases, pushing up $\hat{y}_2$.

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[16]Think of these parameter shifts as occuring prior to contracting. As a matter of theory, the optimal contract will have the risk-neutral lenders insure borrowers' wealth against post-contracting shifts in economy-wide parameters. Reasons exist in practice, however, for why financial contracts may not be perfectly indexed to aggregate risks. It is possible that lenders are either too risk averse or have insufficient resources to perfectly insure borrower' wealth against non-diversifiable risks. Also, if the aggregate shocks are relatively infrequent or are measured with a large enough error, the overall costs of introducing contingencies could outweigh the expected benefits.

The same kind of financial mechanism transmits shocks to π_1^g and r_1 into period two. A positive shift in π_1^g increases both x_1^{g*} and x^b, raising $\hat{y}_1$. Improved balance sheets (of successful entrepreneurs) in turn push up x^{gb} and, therefore, $\hat{y}_2$. A positive shock to r_1 works in just the opposite way, contracting $\hat{y}_2$ as well as $\hat{y}_1$.

A financial channel also allows beliefs about future economic activity to influence the present. A positive shock to π_2^g raises expected period 2 profits. This expands each entrepreneur's financial capacity [equal to $R_2(x_2^{g*},0)$]. Unsuccessful entrepreneur's are now able roll over a larger quantity of debt into period 2, as the minimum feasible value of W^b declines. The *fc* curve in Figure 4 moves downward, lowering W^b and raising x^b. The anticipated boom thus stimulates output in period 1. It mitigates the incentive problem that constrains employment by allowing entrepreneurs to post larger bonds with creditors in terms of expected future earnings.

A positive shock to r_2 of course does just the reverse. The anticipated recession contracts the quantity of debt that lenders are willing to reschedule. The *fc* curve jumps up, and x^b declines as a consequence. Overall, the feasibility of debt-rollover in the multi-period environment provides an avenue through which beliefs about the future impact on current output.

Case 2: $W^b > -R_2(x_2^{g*},0)$. Improvements in the economic situation relax the financial capacity constraint. These improvements are mirrored in either outward shifts of the *ic* curve or downward shifts of the *fc* curve. The maximum feasible value of x^b expands, as Figures 4 and 5 indicate. Entrepreneurs accordingly become increasingly willing to reduce x^b below its maximum $\hat{x}_{fc}^b$ in return for an improved financial position that allows them to produce more in the future. This is because, at higher values of x^b, the marginal loss in output from contracting employment shrinks due to the concavity of the production function. As $\hat{x}^b_{fc}$ becomes sufficiently large, the financial capacity constraint is relaxed and the wealth smoothing optimum portrayed in Figure 5 arises instead.

Wealth smoothing behavior weakens the effect of period one disturbances on $\hat{y}_1$ but strengthens the effect on $\hat{y}_2$. A positive shock to π^g_1, for example, pushes up x^b though by less than the maximum amount possible, as some of the benefit is instead transfered to raising W^b. (The disturbance shifts the *ic* curve rightward but also moves the *ws* curve inward.) In this situation, entrepreneurs spread the expected improvement in their financial positions more evenly between W^g and W^b at the cost of dampening the rise in x^b. The gain is that the rise in period 2 output is larger than otherwise. This result emerges since expected production is concave in financial wealth when the incentive constraints bind [see eq. (3.11)].

Shifts in the period two parameters have an indeterminate effect on $\hat{y}_1$ in this case primarily because financial capacity no longer constrains production. Shocks to π^g_2 and r_2 do alter the shadow value of period one wealth but the net impact on incentives is indeterminate.

5. Extensions of the Basic Model.

This section briefly considers two modifications of the environment: first, lengthening the time horizon; and second, permitting entrepreneurs to enjoy consumption in interim periods. One consequence is of the former is that small but permanent disturbances may induce large swings in financial capacity. The latter change dampens entrepreneurs' propensity to save and may even tempt them to forgo ever accumulating enough wealth to relax the incentive constraints.

5a. Lenghthening time. Suppose the number of periods is extended to T+1, where T > 2. Time still begins with period 0 but now ends with period T. Investment occurs in periods zero through T-1 and production in periods one through T. The optimal multi-period arrangement is found, as before, by successively solving a sequence of single period problems backwards, this time beginning at period T-1.

The period zero problem that emerges is qualitatively similar to the one arising in the three period case, though it differs in two important details. First, the value function $V^{T-1}(\cdot)$, obtained from solving backwards recursively from periods T-1 to one, replaces $V(\cdot)$ in eqs. (3.12) and (3.14). $V^{T-1}(\cdot)$, however, has the same qualitative properties as $V(\cdot)$, including a region that is strictly concave. Second, eq. (3.15) is replaced by

$$W^b \geq \underline{W} = -\sum_{t=2}^{T} R_t(x_t^{g*},0)/\prod_{s=2}^{t} r_s \tag{5.1}$$

A lower minimum for W^b is feasible since extending the number of future production periods raises the entrepreneur's financial capacity. In analogy to eq. (3.15), the right side of eq. (5.1) is the present value of expected future project earnings conditioned on the entrepreneur having zero expected final wealth. Now included in this value are expected earnings from periods two through T.

Plainly, lengthening the horizon does not alter the qualitative aspects of the contracting problem. There are quantitative effects, of course. The expanded financial capacity that stems from larger expected future profits tends to mitigate the underemployment distortion. More pertinent to fluctuations, however, is that financial capacity becomes increasingly sensitive to permanent shifts in technology or interest rates. As T goes up, permanent shifts in either θ_t or r_t have a larger impact on expected future profits and therefore on financial capacity. Enhanced swings in financial capacity potentially transmit into magnified effects on employment.

A similar story may be told about $\bar{W}$, the threshhold end-of-period-one financial position that permits the entrepreneur to operate at the first best in period two. This value is given by

$$\bar{W} = \sum_{t=2}^{T} [\pi^g \Delta x_t^{b*} - R_t(x_t^{g*}, x_t^{b*})] / \prod_{s=2}^{t} r_s \tag{5.2}$$

$\bar{W}$ equals the expected discounted value of the difference betwen the gain from falsely claiming hard times and the return from operating the project at the first best.[17] Intuitively, to operate at the first best, the entrepreneur must have enough financial collateral to cover the gap between the present value gain from cheating and from running the project honestly. Permanent technology or interest rate shifts may clearly have large effects on the present value streams that determine $\bar{W}$. "Bad" shocks may accordingly induce sharp rises in $\bar{W}$, and "good" shocks may do the reverse.

5b. Consumption in interim periods. Suppose each entrepreneur's objective is now given by

$$\sum_{t=0}^{T} \beta^t c_t \quad , \qquad 0 < \beta < r_t, \ \forall \ t \tag{5.3}$$

where c_t is period t consumption and where β is a subjective discount factor. Preferences remain linear in consumption, which helps preserve the

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[17] Condition (2.4b) guarantees $\bar{W} > \underline{W}$, as it does in the three period case (see footenote 8). Also, it is interesting to note that $\bar{W}$ may rise as the time horizon increases. This will be the case if the expected gain from cheating each period, Δx_t^{b*}, exceeds the expected gain from honestly operating the project, $R_t(x_t^{g*}, x_t^{b*})$. Thus, the incentive problem does not necessarily vanish when T is large.

tractability of the model. Impatience is built in - β is assumed to be less than r_t - to keep entrepreneurs from simply saving all wealth until the final period.[18] Despite being impatient and risk neutral, however, entrepreneurs may still accumulate wealth in order to reduce subsequent agency costs of investing.

Consider the impact of allowing for interim consmption on the period zero problem. The objective (3.12) becomes $c_0 + \beta[\pi^g V^{T-1}(W^g) + \pi^b V^{T-1}(W^b)]$, where the value function $V^{T-1}(\cdot)$ now incorporates the impact of the nontrivial consumption/saving decisions made in subsequent periods. It is easy to verify that the key features of $V^{T-1}(\cdot)$ remain unchanged. In addition, c_0 is added to the left side of the period zero budget constraint (3.13); and, like before, V^{T-1} replaces $V(\cdot)$ in the incentive constraint (3.14) and eq. (5.1) replaces eq. (3.15) as the relevant financial capacity constraint.

The optimal contract is essentially the same as before, except that entrepreneurs no longer automatically save all of W_0, their period zero financial wealth. The marginal utility cost of saving is now unity, the quantity of foregone period zero consumption. The marginal benefit is the sum of the discounted gross return βr_1 - a number less than unity - and the reduction in subsequent agency costs owing to a strengthened period one financial position. If at $c_0 = 0$ the marginal benefit from saving is greater than or equal to unity, the entrepreneur consumes nothing and saves all of W_0. Otherwise, she reduces saving to the point where the marginal benefit equals unity, the marginal cost. Since she is impatient (i.e., since $\beta r_1 < 1$), she

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[18]For simplicity, assume that lenders still only care about consuming in the final period in order to ensure that inventory storage may still take place in equilibrium. The same objective could be (tractably) acccomplished allowing lenders to have concave preferences over consumption in interim periods. We ignore adding this complexity since it has no important bearing on the conclusions.

accordingly never saves enough to eliminate all subsequent agency costs.

The addition of interim consumption accordingly influences wealth dynamics but affects neither the qualitative nature of the optimal financial contract nor the basic macroeoconomic implications. Entrepreneurs with weak financial positions save a higher fraction of their assets since the shadow value of accumulated wealth is diminishing (when the incentive constraints are binding).[19] Further, in the special case presented here - with linear utility and impatience - entrepreneurs never save their way out of the incentive problem, even as the time horizon lengthens.

6. Conclusion.

Existing models of the financial propagation mechanism emphasize the role of borrowers' financial assets, or equivalently internal funds, in mitigating the agency costs of investment finance. In the multi-period contracting environment developed here, the terms of credit depend not only the supply of internal funds but also on beliefs about the borrower's unencumbered discounted future earnings. For this reason, the model is better able to motivate the kind of short run shifts in financial constraints needed to make the general story quantitatively significant in business fluctuations.

Pessimism about the future induced by an adverse shift in economic fundamentals anticipated down the road, for example, tightens financial constraints. The spread between the short term risky and safe interest rates rises[20]; and credit tightens as the quantity of debt that lenders are willing

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[19] See Holmes (1990) for a related story of retained earnings.

[20] A decline in π_2^g or a rise in r_2 raises $\underline{W}$ and therefore W^b. W^g must accordingly fall to satisfy the budget constraint (3.13). Either of these anticipated period two adverse shocks accordingly raises the period one risky

to reschedule contracts (i.e., as borrowers' financial capacity declines). This cyclical pattern in these symptoms of financial distress is broadly consistent with the evidence [see Mishkin (1990) and Eckstein and Sinai (1986)]. Further, since the financial capacity constraint is more likely to bind in bad times the model is also compatable with the empirical evidence suggesting that financial constraints have a greater impact in recessions [see Gertler and Hubbard (1989) and Hubbard and Kashyap (1990)].

A major limitation of this model is that it lies well short of a fully dynamic framework that can be matched to data. While allowing for multi-period contracts is a helpful step in this important direction, there is still a long way to go. A related extension, likely more tractable, would be to introduce endogenous cross-sectional variation in firms' expected horizons by allowing for entry and exit. Adding this kind of heterogenity might generate new testable cross-sectional implications, for example, on how financial constraints impact on young firms versus with less secure horizons versus mature established firms. By working along these lines, it may also be possible to capture a simultaneous relation between the evolution of industry and financial deepening in the form of expanding financial capacity, a phenomenon many development economists have cited as an important feature of growth.

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rate, equal to the ratio of the payment the lender receives in good state to the size of the loan, $[y(x_1^{g*},\psi^g) - W^g]/(x_1^{g*} - W_0)$.

Appendix: The Period Zero Problem

The formal contracting problem in period zero is to choose the vector $\{W^g, W^b, x^g, x^b\}$ to solve

$$\max \quad \sum \pi^j V(W^j)/r_1 \quad , \ j = g,b \tag{A.1}$$

subject to,

$$\sum \pi^j W^j/r_1 = R_1(x^g, x^b)/r_1 + W_0 \tag{A.2}$$

$$V(W^g) \geq V(W^b) + \Delta x^b \tag{A.3}$$

$$W^b \geq \underline{W} = -R_2(x_2^{g*}, 0)/r_2 \tag{A.4}$$

where (A.1) - (A.4) correspond to (3.13) to (3.17) in the text. Use (A.2) to eliminate W^g from from (A.1) and (A.3) and let λ and ν denote the (non-negative) multipliers for constraints (A.3) and (A.4). Then the first order conditions for x^g, x^b and W^b are, respectively;

$$\partial R_1/\partial x_1^g = 0 \tag{A.5}$$

$$(1 + \lambda/\pi_1^g)V'(W^g)\cdot\partial R_1/\partial x_1^b - \lambda\Delta = 0 \tag{A.6}$$

$$\pi_1^b[V'(W^b) - V'(W^g)] - \lambda[(\pi_1^b/\pi_1^g)\cdot V'(W^g) + V'(W^b)] + \nu = 0 \tag{A.7}$$

The second order conditions that guarantee an optimum are satisfied, as is straightforward to verify.

Note that x_1^g always equals its first-best value, given by eq. (A.5). If $\lambda = 0$ - i.e., if the incentive constraint is not binding - then x_1^b also equals its first-best value, as eq. (A.6) suggests. If $\lambda > 0$ then $\partial R_1/\partial x_1^b > 0$, implying x^b lies below its first-best value. There are two possibilities for the determination of x^b and W^b in this case. First, if $\nu > 0$ - i.e., if the financial capacity constraint is binding - then eqs. (A.3) and (A.4) jointly determine x^b and W^b [with W^g implicitly determined by eq. (A.2)]. If $\nu = 0$, then (A.6) and (A.7) imply

$$\pi_1^b[V'(W^b)/V'(W^g) - 1)] = (\partial R_1/\partial x^b)\left(-\frac{\partial x^b}{\partial W^b}\Big|_{A3}\right) \tag{A.8}$$

which corresponds to eq. (3.18) in the text. Eq. (A.8) then replaces eq. (A.4) in the determination of x^b and W^b. The factors determining which regime is relevant are characterized in section 4. Note that the possibility the financial capacity constraint may not be binding arises only because $V(\cdot)$ has a strictly concave region, as eq. (A.7) suggests. This explains why the solution for the period zero problem is somewhat more complicated than the solution to the period one problem of section 3.1.

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Figure 1: The Timing of Entrepreneurial Projects

Period 0	Period 1	Period 2
contract signed with lenders		
investment ⟶	capital (K_1)	
	ψ_1 is realized	
	capacity utilization (x_1) chosen	
	output (y_1) produced	
	K_1 depreciates	
	payment (possibly negative) credited to entrepreneur contingent on observable choice of x_1.	
	investment ⟶	capital (K_2)
		ψ_2 is realized
		capacity utilization (x_2) chosen
		output (y_2) produced
		K_2 depreciates
		final payment to entrepreneur contingent on observable choices of x_1 and x_2.

Figure 2: The Value Function $V(W^j)$

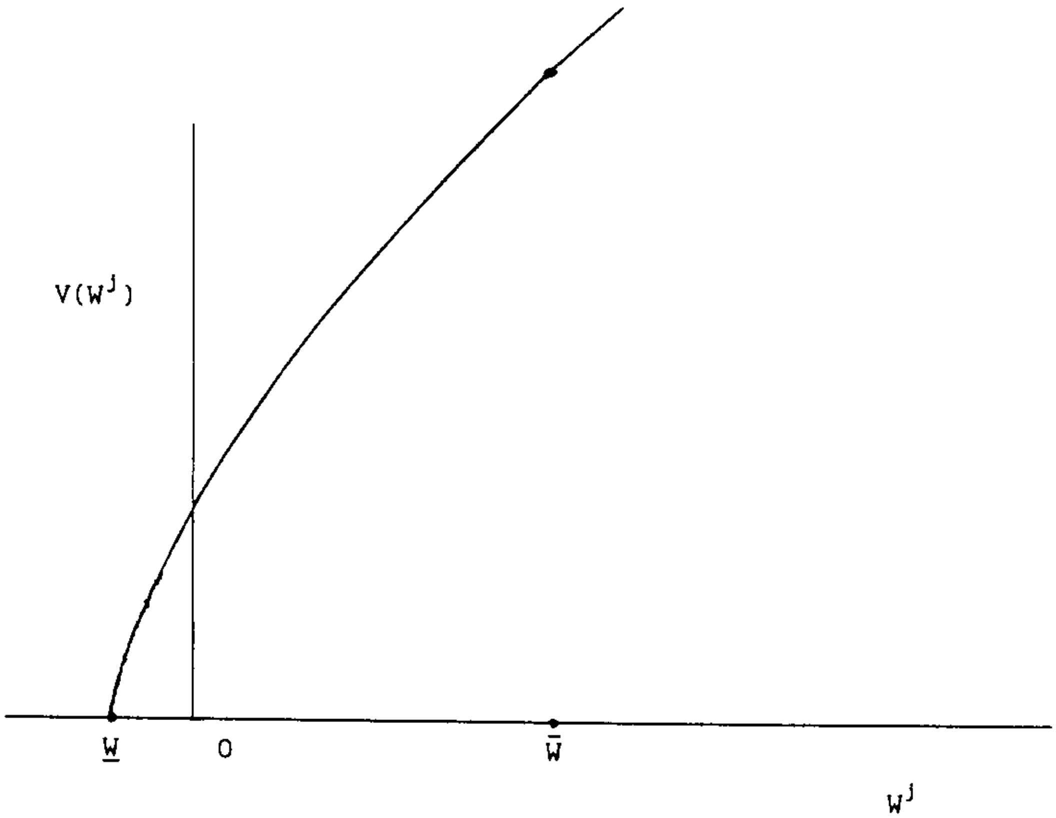

Figure 3: x^{jb} and W^{jb} Under the Period 1 Contract

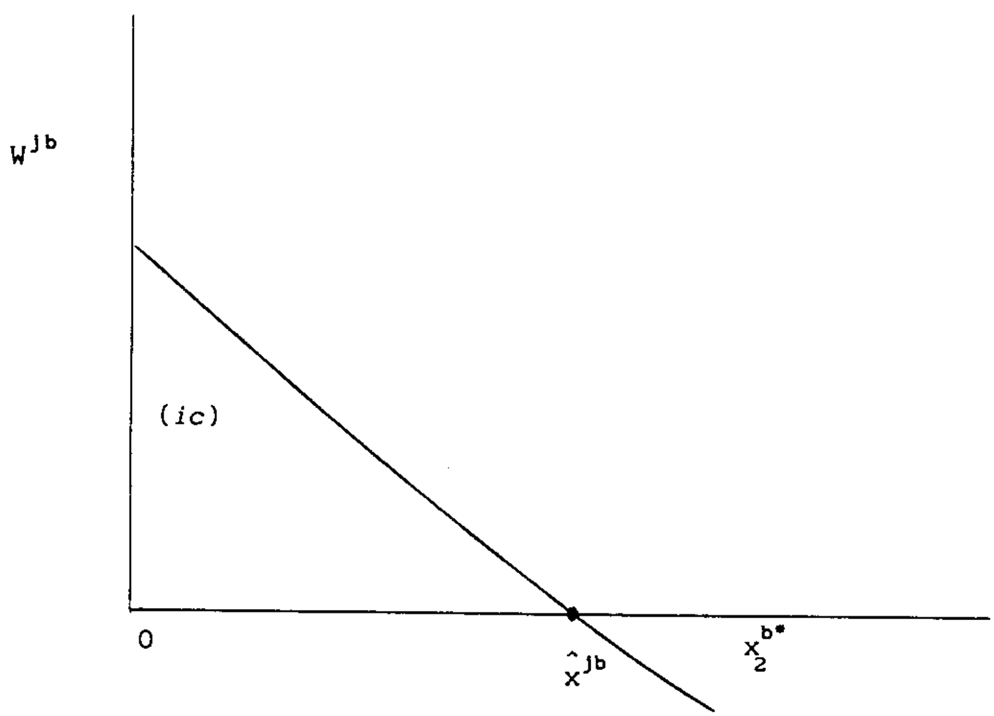

Figure 4: x^b and W^b under the Period 0 Contract - *fc* curve binding

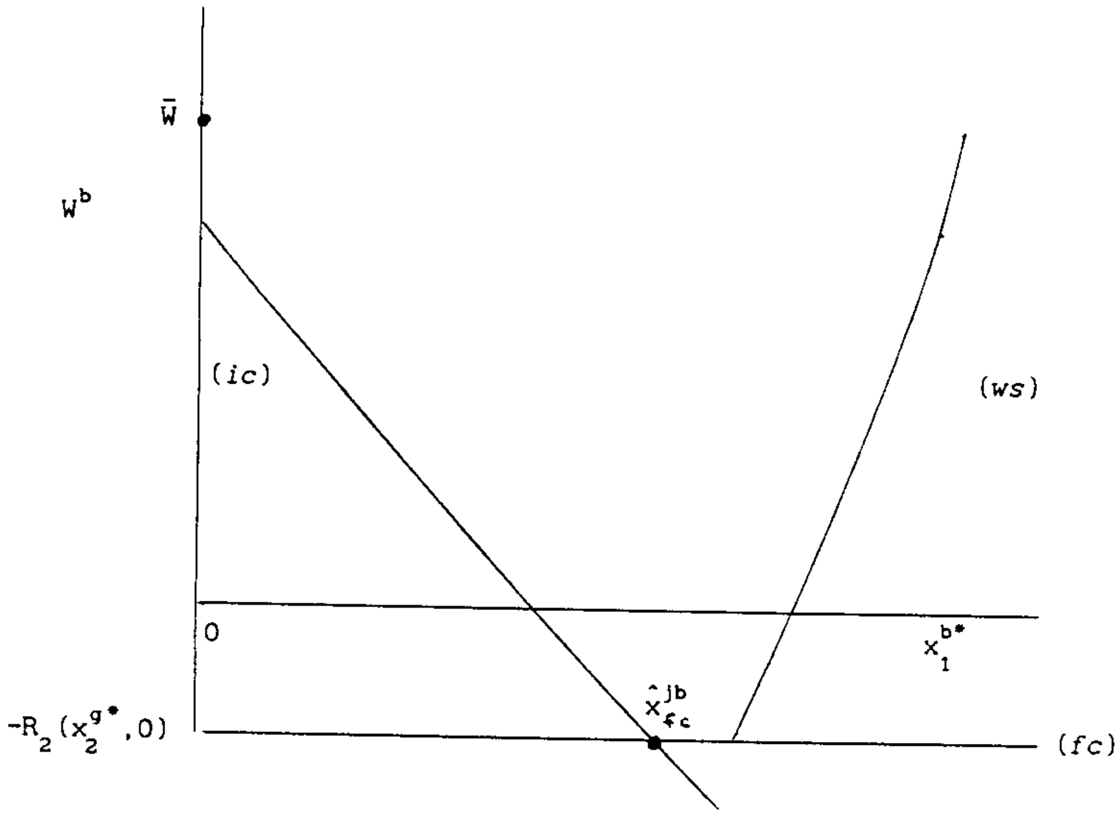

Figure 5: x^b and W^b under the Period 0 Contract - *ws* curve binding