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A GENERIC NEGOTIATION GAME

BY

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Abstract

A generic Negotiation Game (NG) is used to model a conflict between two parties seeking to resolve their differences and reach a settlement. NG is a 2 x 2 nonconstant-sum symmetric game that is "generic" in the sense that its payoffs, which are assumed to be cardinal, are only incompletely specified. Consequently, conclusions drawn about it are applicable to all games with which it is consistent, including such well-known games as Prisoners' Dilemma, Chicken, Deadlock, and Stag Hunt.

NG may be in exactly one of four mutually exclusive "states," with different pure-strategy Nash equilibria associated with each state: (1) conflict alone; (2) cooperation alone; (3) a combination of conflict and cooperation; and (4) two win-lose outcomes, each of which favors a different player. The most potent force driving NG toward state (2) is reducing the difference between winning and losing; increasing the rewards of cooperation and the penalties of conflict are not as helpful unless they occur in tandem. Normative implications of these results for encouraging negotiated settlements are discussed.

Key Words: Negotiation; game theory, Nash equilibrium, incomplete information, inducement

A Generic Negotiation Game

I. Introduction

In this paper I posit a generic Negotiation Game (NG) to model a conflict between two parties seeking to resolve their differences and reach a settlement. It is applicable to negotiations at all levels, from interpersonal to international, and other conflicts as well (e.g., arms races and crises). I analyze different solutions to this game, including its Nash equilibria in pure and mixed strategies and minimax and maximin solutions.

The innovation in this paper is not in the solution concepts used but rather in the generic 2 x 2 nonconstant-sum symmetric game I propose as a model of negotiations. NG is "generic" in the sense that its payoffs are only incompletely specified. Consequently, conclusions drawn about it are applicable to all games with which it it consistent, including such well-known games as Prisoners' Dilemma, Chicken, Deadlock, and Stag Hunt.

Unlike these specific games, which are usually analyzed only at an ordinal level (i.e., the players are assumed only to rank the outcomes from best to worst), I assume that the payoffs in NG are cardinal. Thereby I am able to analyze how much they need to be altered to produce certain effects and also to make expected-payoff calculations.

I show that NG may be in exactly one of four mutually exclusive "states," with different pure-strategy Nash equilibria associated with each state: (1) conflict alone; (2) cooperation alone; (3) a combination of conflict and cooperation; and (4) two win-lose outcomes, each of which favors a different player. Coincident with the latter two states is a Nash equilibrium in mixed strategies, which is also the minimax solution and Pareto-inferior to the pure-strategy Nash equilibria in each state. On the other hand, the maximin solution is always Pareto-superior but may not be a Nash equilibrium. Inducement strategies are also investigated.

NG is consistent with exactly five different symmetric ordinal games, wherein the diagonal outcomes are ranked the same by the players and the off-diagonal outcomes are mirror images of each other. If it is refined to allow asymmetric payoffs, it is consistent with ten additional games; this refinement is not analyzed here (see Appendix for a listing of both the symmetric and asymmetric games). Thus, despite its simple structure, NG--and its asymmetric relative--has considerable richness. At the same time, one can identify and readily interpret the key parameters that determine the shift of NG from one state to another, and what specific ordinal games are associated with each state.

The most potent force driving NG toward the state in which cooperation is a dominant-strategy Nash equilibrium is to reduce the difference between winning and losing. Increasing the rewards of cooperation and the penalties of conflict are not as helpful unless they occur in tandem. Normative implications of these results for encouraging negotiated settlements are discussed.

2. The Negotiation Game (NG)

NG, which is played between players P1 and P2, is shown in Figure 1.

Figure 1 about here

Figure 1

Payoff Matrix of NG

		P2		
		C	$\bar{C}$	
P1	C	(b+y, b+y) Cooperation	(b, a) P2 wins, P1 loses	p
	$\bar{C}$	(a, b) P1 wins, P2 loses	(a-z, a-z) Conflict	1-p
		q	1-q	

Key: C = compromise; $\bar{C}$ = noncompromise

p = probability P1 chooses C; q = probability P2 chooses C

Assumptions: $a > b$; $y > 0$; $z > 0$

Assume that each player can take either a compromise (C) or a noncompromise ($\bar{C}$) position. I make the following assumptions about the payoffs to the players:

- $a > b$: P1 does better at $\bar{C}C$ (P1 wins, P2 loses) than $C\bar{C}$, whereas P2 does better at $C\bar{C}$ (P2 wins, P1 loses) than $\bar{C}C$. Interpretation: the player who does not compromise wins because its inflexibility forces the other player, who does compromise, toward its position and thereby makes this other player the loser.
- $y > 0$: both players, by choosing C and achieving cooperation, do better at CC ($b+y$, $b+y$) than losing (b). Interpretation: when each player indicates flexibility by moving toward the position of the other player, an agreement is reached, which is better for each than had it lost.
- $z > 0$: both players, by choosing $\bar{C}$ and being in conflict, do worse at $\overline{CC}$ ($a-z$, $a-z$) than winning (a). Interpretation: when each player indicates inflexibility by holding out at its position, no agreement is reached, which is worse for each than had it won.

"Winning" and "losing" are perhaps somewhat misnomers here, because NG is nonconstant-sum: whereas the payoffs to the players at the two off-diagonal outcomes sum to the constant $a+b$, the sum of the players' payoffs at the two diagonal outcomes ($2b+2y$ and $2a-2z$) are not in general equal, much less $a+b$.

The foregoing assumptions imply the following partial ordering of payoffs of each player: winning (a) is better than losing (b) and also better than conflict ($a-z$); cooperation ($b+y$) is better than losing (b). But because no functional relationship is assumed between the parameters y and z, conflict (a-z) may be better or worse than cooperation (b+y) or losing (b), and winning (a) may be better or worse than cooperation (b+y).

The assumption that the two players value comparable outcomes exactly the same is obviously a simplification. It probably best mirrors situations in which two negotiators have more or less equal bargaining resources. In these situations, a compromise is likely to wring equal concessions from each player, a conflict to impose equal costs on them, and winning or losing to be equally good or bad for the players. When this is not the case, it would be appropriate to refine the game by specifying an a, b, y, and z for each player and analyze the asymmetric version of NG.

3. Pure-Strategy Nash Equilibria in NG

In this section I categorize all pure-strategy Nash equilibria--wherein each player chooses either C or $\bar{C}$ with certainty--in NG. These equilibria fall into four mutually exclusive classes (i.e., the existence of equilibria in one class precludes the existence of equilibria in the other three classes):

I. Conflict dominant. A strategy is dominant for a player if it results in an outcome at least as good as, and in at least one case better than, any other strategy, whatever the choice of the other player. (Except for Nash equilibria in mixed strategies, I shall analyze only conditions under which strategies are strictly dominant and Nash equilibria are strictly preferred, which is to say that these strategies and equilibria are always better than other strategies or outcomes to which they are compared.) Thus, $\overline{C}\overline{C}$ is the (unique) dominant-strategy Nash equilibrium for both players if and only if (iff)

- $\bar{C}$ for P1 is dominant: $a > b+y$ and $a-z > b$; and
- $\bar{C}$ for P2 is dominant: identical conditions.

Rewriting these conditions,

$$a-b > y \text{ and } a-b > z.$$

II. Cooperation dominant. An analogous argument demonstrates that CC is the (unique) dominant-strategy Nash equilibrium iff the class I inequalities are reversed:

$$a-b < y \text{ and } a-b < z.$$

III. Both conflict and cooperation are Nash equilibria. Both $\overline{CC}$ and CC are Nash equilibria iff $b+y > a$ and $a-z > b$, which can be rewritten:

$$a-b < y \text{ and } a-b > z \Rightarrow z < a-b < y$$

IV. Both win-lose outcomes are Nash equilibria. Both $\bar{C}C$ and $C\bar{C}$ are Nash equilibria iff the class III inequalities are reversed:

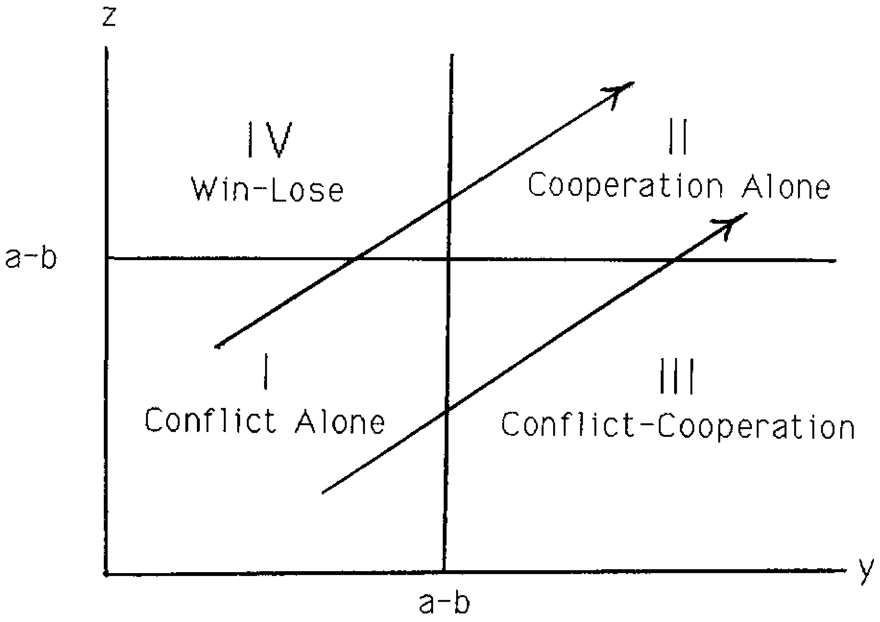
$$a-b > y \text{ and } a-b < z \Rightarrow y < a-b < z.$$

Because the conditions for no two classes of Nash equilibria are consistent (i.e., can be satisfied simultaneously), the four classes are mutually exclusive. Furthermore, these classes exhaust all pure-strategy Nash equilibria in NG. Graphically, the four classes are depicted in Figure 2

Figure 2 about here

Figure 2

Four Different States of NG, with Two Paths from I to II



(ignore the diagonal arrows for now), which might be thought of as the the four states of NG, depending on whether y or z is less than or greater than a-b.

Observe that neither (a, b) nor (b, a)--or, more properly, the strategy pairs associated with each outcome--can be a Nash equilibrium by itself, because the conditions that make one outcome a Nash equilibrium (given for class IV) also make the other outcome a Nash equilibrium. Consequently, even if NG is a game of incomplete information (i.e., the players do not know the exact values of the payoffs, though they do know NG is symmetric), the players can rule out either of these outcomes as the unique equilibrium: no values of a, b, y, and z can render only one of these outcomes stable.

Another fact about NG that players with incomplete information can surmise is that it always has at least one Nash equilibrium in pure strategies. If it did not, it would be possible to "cycle" from any outcome back to itself. Thus, clockwise cycling from (b, a) to (a-z, a-z) to (a, b) to (b+y, b+y), and then back to (b, a), is rational iff

$$b < a-z < b; \qquad a < b+y < a,$$

so there is a contradiction. The conditions for counterclockwise cycling lead to the same contradiction, illustrating Nash's (1951) theorem that every game has at least one equilibrium in pure or mixed strategies.

Pure-strategy equilibria in NG may be Pareto-inferior (i.e., worse for both players than some other outcome), whereas nonequilibrium outcomes may be Pareto-optimal (i.e., not Pareto-inferior).[1] The Pareto-optimality--or, more stringently, Pareto-superiority--of outcomes in NG depends on the class:

I. Dominance comparisons are insufficient to say whether the single conflict equilibrium (a-z, a-z) is Pareto-superior or Pareto-inferior to outcome (b+y, b+y); outcomes (a, b) and (b, a) are Pareto-optimal.

II. The single cooperation equilibrium (b+y, b+y) is the only Pareto-superior outcome, because the players' payoffs at this outcome are greater than their payoffs at the other three outcomes, based on dominance comparison b+y > a.

III. The cooperation equilibrium (b+y, b+y) is the only Pareto-superior outcome, because it is Pareto-superior to (a, b) and (b, a), based on equilibrium comparison b+y > a; hence, it is also Pareto-superior to the conflict equilibrium (a-z, a-z).

IV. The win-lose equilibria (a, b) and (b, a) are Pareto-superior to outcome (a-z, a-z), based on equilibrium comparison b > a-z. Outcome (b+y, b+y) is also Pareto-superior to (a-z, a-z), based on the same comparison; but (b+y, b+y) is not Pareto-superior to (a, b) or (b, a), based on equilibrium comparison a > b+y. Hence, all outcomes are Pareto-superior except Pareto-inferior outcome (a-z, a-z).

In summary, equilibria in classes II, III, and IV are always Pareto-superior, but the equilibrium in class I may or may not be. Nonequilibrium outcomes are always Pareto-inferior in classes II and III, but they are Pareto-optimal or Pareto-superior in classes I and IV. As illustrations of games whose ordinal rankings are consistent with the NG, Prisoners'

Dilemma and Deadlock are class I games, Stag Hunt is a class III game, and Chicken is a class IV game (see the Appendix).[2]

4. Other Solutions to NG[3]

Although there is at least one pure-strategy Nash equilibrium in every state of NG, the existence of these equilibria do not preclude the existence of other Nash equilibria. In fact, there is a fifth class of Nash equilibria in mixed strategies, whereby the players choose their pure strategies according to some probability distribution.

To determine these strategies, assume that P1 chooses C with probability p and $\bar{C}$ with probability 1-p (see Figure 1). Then P1 can render P2 indifferent between choosing C and $\bar{C}$ if P2's expected payoff from choosing C is equal to P2's expected payoff from choosing $\bar{C}$:

$$p(b+y) + (1-p)b = pa + (1-p)(a-z), \tag{1}$$

from which one can solve for p:

$$p = (a-b-z)/(y-z). \tag{2}$$

Substituting p and $1-p = (b-a+y)/(y-z)$ into (1), P2 receives the expected payoff,

$$EP_{P2} = (ay-bz-yz)/(y-z), \tag{3}$$

whatever strategy (C or $\bar{C}$) it chooses. (Because of the symmetry of NG, P2's choice of $q = p$ makes P1 indifferent between C and $\bar{C}$ and also renders $EP_{P1} = EP_{P2}$.) Depending on whether $y > z$ or $y < z$ [$y = z$ is ruled out, because the denominator or the right-hand side of (2) would be 0], each of the following conditions is necessary and sufficient to ensure that p and 1-p are nonzero probabilities (i.e., $0 < p < 1$):

(i) $y > z$: $p > 0$ iff $a-b > z$ and $p < 1$ iff $a-b < y \Rightarrow z < a-b < y$;

(ii) $y < z$: $p > 0$ iff $a-b < z$ and $p < 1$ iff $a-b > y \Rightarrow y < a-b < z$.

Because $q = p$, exactly the same conditions ensure that $0 < q < 1$.

But (i) is the condition for the existence of class III Nash equilibria, and (ii) is the condition for the existence of class IV Nash equilibria. In other words, iff (b+y, b+y) and (a-z, a-z) are both Nash equilibria, or (a, b) and (b, a) are both Nash equilibria, there is a third mixed-strategy Nash equilibrium. At the mixed-strategy equilibrium, each player's mixed strategy [i.e., P1's choice of (p, 1-p) over its pure strategies, and P2's choice of (q, 1-q) over its pure strategies] robs the other player of the incentive to switch to a different strategy, because this switch has no effect on the switcher's expected payoff.

This makes the mixed-strategy equilibrium weak: it renders each player indifferent between choosing one or the other (or any mix) of its two pure strategies. On the other hand, the strict inequalities that characterize the four classes of pure-strategy equilibria render these equilibria strong, because each player's departure from such an equilibrium results in a worse outcome.

But the weakness of the mixed-strategy equilibrium is not its main problem, given the class III or class IV conditions are met. More serious, this equilibrium is Pareto-inferior to both the pure-strategy Nash equilibria in each class.

To demonstrate this result when the class IV conditions are met, assume that the mixed-strategy expected payoffs, $EP_{P1} = EP_{P2}$, are Pareto-superior to (a, b) [and (b, a)]:

$(ay-bz-yz)/(y-z) > a;$ $(ay-bz-yz)/(y-z) > b.$

The first inequality is true iff

(iii) $y > z$ and $a-b > y$ ($\Rightarrow z < y < a-b$); or $y < z$ and $a-b < y$ ($\Rightarrow a-b < y < z$),

and the second inequality is true iff

(iv) $y > z$ and $a-b > z$; or $y < z$ and $a-b < z$.

But each of (iii) and (iv) are consistent only with class I and class II equilibria (see Figure 2). Because a mixed-strategy equilibrium exists in this case iff the conditions for a class IV equilibrium obtain, there is a contradiction. Hence, the mixed-strategy equilibrium must pay less than (a, b) and (b, a). By an analogous argument, one can show that the mixed-strategy equilibrium when the conditions for a class III equilibrium are met must pay less than both (b+y, b+y) and (a-z, a-z). In general, then, <u>the mixed-strategy Nash equilibrium is Pareto-inferior to each of the two pure-strategy equilibria in classes III and IV</u>.

In 2 x 2 games, Nash-equilibrium mixed strategies coincide with <u>minimax strategies</u>, because each player minimizes the maximum that the other player can receive by making the other player indifferent between its two pure-strategy choices.[4] Not only are these mixed strategies not compelling in NG because they lead to a Pareto-inferior Nash equilibrium, but it is also difficult to understand what would motivate each player to want to minimize the other player's maximum payoff--by making it indifferent between its two pure strategies--if both players can always do better at each of the Pareto-superior pure-strategy Nash equilibria in classes III and IV.

On the other hand, there may be a rationale for a player to alter slightly its (mixed) minimax strategy. For example, by slightly perturbing its minimax strategy so that P2 favors C [i.e., choosing p so that the left-hand side of (1) is greater than the right-hand side], P1 can induce P2 always to choose C, which would then provide P1 with a mix of payoff b+y (cooperation) and payoff a (A's winning).

Inducement mixed strategies have been proposed in different contexts. For example, to induce compliance to an arms-control treaty, one side might announce in advance a mixed inspection strategy of random inspections (Brams, 1985; Avenhaus, 1986; Fichtner, 1986; Brams and Kilgour, 1988). In negotiations, mixed strategies may reflect the often-observed "carrot-and-stick" approach, which prescribes making concessions and using threats in some artful combination--to signal both flexibility and firmness--that pushes an opponent in a desired direction. If NG is repeated, an inducement mix might take the form of a conciliatory approach at one time and a belligerent approach at another, which communicates a willingness to compromise but not to capitulate.

Yet an inducement strategy is not an equilibrium strategy in NG. If inducement works, the inducer immediately has an incentive to switch to its own pure-strategy best response to the inducee's choice of its preferred (pure) strategy. Because there is a Nash equilibrium in mixed strategies iff there are class III or IV pure-strategy Nash equilibria, any perturbation of the mixed-strategy equilibrium in these classes is likely to end up at one of the two pure-strategy equilibria in each class, depending on which player is doing the inducing.

As a final possible solution to NG, consider how each player can maximize a minimum payoff for itself, whatever strategy the other player chooses. (The mixed-strategy Nash equilibrium, which exists under class III or class IV conditions, ensures that the other player receives the same expected payoff whatever strategy it chooses; however, it does not maximize a player's minimum payoff.) Because P1's certain choice of $\bar{C}$ minimizes P2's payoff (either b or a-z), P1, by choosing its strategy associated with the greater of b (C) or a-z ($\bar{C}$), can guarantee at least this amount.

This guarantee strategy is called a maximin strategy, because it maximizes the minimum that a player can guarantee for itself. A similar choice by P2 would lead the players to choose either outcome (b+y, b+y) or (a-z, a-z), depending on whether b+y or a-z gives them the greater payoff. If (b+y, b+y) is the players' choice, it is Pareto-superior but not necessarily a Nash equilibrium (see Prisoners' Dilemma in the Appendix); if (a-z, a-z) is their choice, it is a Pareto-superior Nash equilibrium (see Deadlock in the Appendix).[5]

The aforementioned problems with the alternative solution concepts considered--Nash equilibria in mixed (minimax) strategies, inducement strategies, and maximin strategies--suggest that pure-strategy Nash equilibria, associated with single outcomes, are most likely to be chosen in NG. I shall next compare and interpret the conditions under which each of the four classes of equilibria are likely to arise. Then I shall indicate how the players might, in order to foster negotiated settlements, move NG from states I, III, and IV into state II (compromise dominant) by changing the values of certain parameters.

5. Possibilities of Cooperation in NG

While conflict is certain in state I, and cooperation in state II, what will occur in states IV and III, each with two Nash equilibria, is less clear-cut:

I. Conflict dominant. Conflict is a dominant-strategy Nash equilibrium, but it may or may not be a Pareto-superior outcome in this game. This unique equilibrium occurs when the differential between winning and losing,

$$d = a-b,$$

is more than both the rewards of cooperation over losing (y) and the penalties of conflict over winning (z) (i.e., $d > y$ and $d > z$). In other words, when the value of winning is greater than both the rewards of cooperation and the penalties of conflict, conflict will be hard to avoid, even if cooperation may give both players still greater benefits.

IV. Both win-lose outcomes are Nash equilibria. Cooperation is impeded in this case because the differential for winning is greater than the rewards of cooperation ($d > y$); however, this differential is less than the penalties of conflict ($d < z$). The fact that these two inequalities imply $z > y$ (the penalties of conflict are greater than the rewards of cooperation) suggests that competition between the players to try to achieve their winning outcomes will dominate play. This competition is reinforced by the fact that the two win-lose outcomes are Pareto-superior Nash equilibria.

III. Both cooperation and conflict are Nash equilibria. Cooperation is more promising than in either of the previous two states because it is a Nash equilibrium, though it competes with conflict, which is also a Nash equilibrium. Although the rewards of cooperation are greater than the differential between winning and losing ($y > d$), the penalties of conflict are less than this differential ($z < d$). The fact that these two inequalities imply $y > z$ (the rewards of cooperation are greater than the penalties of conflict) seems fairly auspicious.

II. Cooperation dominant. The rewards of cooperation are greater than the differential between winning and losing ($y > d$), as are the penalties of conflict ($z > d$). In this case, cooperation is not only the dominant-strategy Nash equilibrium but also the only Pareto-superior outcome in this game. These attributes of cooperation give it impeccable credentials as the obvious rational choice of the players.

The progression of states of NG from conflict to cooperation are summarized by the following inequalities:

Conflict	Win-Lose	Conflict-Cooperation	Cooperation
I ⟶	IV ⟶	III ⟶	II
$d > y, d > z$	$y < d < z$	$z < d < y$	$d < y, d < z$

There are two efficient ways to go from I to II:

- via IV, shown by the top arrow in Figure 2, which requires first reversing $d > z$ and then $d > y$; or
- via III, shown by the bottom arrow in Figure 2, which requires first

reversing $d > y$ and then $d > z$.

In the first case, one makes an intermediate stop at the Win-Lose state, and in the second case at the Conflict-Cooperation state, given both changes are not made at once. Of course, the shift from either IV or III to II requires the reversal of only one inequality.

Interpreted, these changes mean that the differential d between winning and losing must fall below both the rewards y of cooperation and the penalties z of conflict in order to single out cooperation as the dominant-strategy Nash equilibrium. What facilitates both changes is a decrease in d; in addition, the switch to $d < z$ is helped by making conflict more burdensome, and the switch to $d < y$ is helped by making cooperation more rewarding. The latter two factors are probably better recognized as conducive to cooperation, but a reduction in the differential between winning and losing is, in fact, more significant because of its two-pronged effect.

6. Conclusions

NG models a generic game between two more or less equal players who may choose to compromise or not compromise--or possibly select some mixture of each--in negotiating an agreement. Both players attribute the same value to winning versus losing, with winning naturally preferred to losing. The costs of conflict--as compared with winning--and the benefits of cooperation--as compared with losing--are also valued in the same way by the players. Despite this simple symmetrical structure, NG admits four mutually exclusive sets of pure-strategy Nash equilibria, depending on relationships among the differential between winning and losing (d = a-b), the rewards of cooperation (y), and the penalties of conflict (z).

Conflict is the only outcome at which the players fail to reach agreement. At the other three outcomes, one player capitulates, the other does, or they achieve cooperation that benefits both players equally--just as the conflict outcome hurts both equally.

I do not assume that conflict is necessarily inferior to cooperation. Indeed, conflict may be useful for the players to set a precedent of resolve, especially for bargaining purposes if play is to be repeated. However, conflict is always inferior to winning. On the other hand, cooperation may be superior to winning, under the presumption that it may lead to a more permanent solution than a (temporary) win for one player that might later unravel because it is viewed as unfair.

Depending on the state of NG, every pure-strategy outcome may be in equilibrium. Either (I) conflict is the dominant-strategy Nash equilibrium, (II) cooperation is, (III) both conflict and cooperation are Nash equilibria, or (IV) both win-lose outcomes are Nash equilibria. In the latter two cases, there is also a mixed-strategy Nash equilibrium, but it is always Pareto-inferior to the two pure-strategy equilibria and hence not likely to be selected, though conceivably it might be perturbed for purposes of inducement, especially if play is repeated. By contrast, the players' maximin strategies in NG are pure and lead to a Pareto-superior outcome (conflict or cooperation), but cooperation may not be a Nash equilibrium.

Both rewards and penalties are helpful in moving the players away from the conflict outcome. But they are beneficial in different ways: rewards make cooperation more appealing, moving the process toward a competition with conflict; penalties make conflict less appealing, moving the process toward a competition between the two win-lose outcomes.

The most potent force in favor of cooperation, however, is to lower the differential between winning and losing. This is because a reduced differential has a two-fold impact: fewer rewards are required to make cooperation attractive, and fewer penalties are needed to make conflict unattractive. Also, because a reduced differential undermines the appeal of winning and diminishes the fear of losing, it pushes players in the direction of cooperation not just to escape conflict but also to avoid a win-lose competition, the outcome of which is no longer so consequential.

When the differential is less than both the rewards of cooperation and the penalties of conflict, cooperation becomes well-nigh irresistable. To be sure, there are almost always incentives for players to posture in bargaining (Brams, 1990), so it may not be easy for them to refrain from exaggerating the importance of winning, or being less than magnanimous in victory (Brams and Mor, 1991).

International relations is one arena in which winning is becoming increasingly anachronistic (Mueller, 1989). This is especially true in nuclear conflicts, given their possibility of precipitating a nuclear catastrophe. In conventional conflicts, the seven-year Iran-Iraq war (1980-87) is a good recent example of the ambiguity of winning and losing, suggesting how the payoffs of the players may change over time so as to favor a negotiated settlement. Even the 1991 Persian Gulf War, which resulted in a decisive win for the United States and its allies, has had as an aftermath mixed consequences.

As a generic game, NG offers a parsimonious structure for analyzing conflict and cooperation, and possible paths between them, in several different ordinal games with which it is consistent. Moreover, the parameters of NG are palpable: negotiators, it seems, think in terms of rewards, penalties, and winning and losing. Operationalizing these concepts and testing some of the effects predicted by the model is the next step.

Appendix

Five Ordinal Games Consistent with Negotiation Game (with classes shown)

Ia	Ib	II	III	IV
$\begin{array}{\|cc}(3,3) & (1,4) \\ (4,1) & \underline{(2,2)}\end{array}$	$\begin{array}{\|cc}(2,2) & (1,4) \\ (4,1) & \underline{(3,3)}\end{array}$	$\begin{array}{\|cc}\underline{(4,4)} & (2,3) \\ (3,2) & (1,1)\end{array}$	$\begin{array}{\|cc}\underline{(4,4)} & (1,3) \\ (3,1) & \underline{(2,2)}\end{array}$	$\begin{array}{\|cc}(3,3) & \underline{(2,4)} \\ \underline{(4,2)} & (1,1)\end{array}$
Prisoners' Dilemma	Deadlock		Stag Hunt	Chicken

Ten Ordinal Games Consistent with Asymmetric Negotiaton Game (with classes for row and column player shown)

Ia-Ib	Ia-II	Ia-III	Ia-IV	Ib-II
$\begin{array}{\|cc}(3,2) & (1,4) \\ (4,1) & \underline{(2,3)}\end{array}$	$\begin{array}{\|cc}(3,4) & (1,3) \\ \underline{(4,2)} & (2,1)\end{array}$	$\begin{array}{\|cc}(3,4) & (1,3) \\ (4,1) & \underline{(2,2)}\end{array}$	$\begin{array}{\|cc}(3,3) & (1,4) \\ \underline{(4,2)} & (2,1)\end{array}$	$\begin{array}{\|cc}(2,4) & (1,3) \\ \underline{(4,2)} & (3,1)\end{array}$

Ib-III	Ib-IV	II-III	II-IV	III-IV
$\begin{array}{\|cc}(2,4) & (1,3) \\ (4,1) & \underline{(3,2)}\end{array}$	$\begin{array}{\|cc}(2,3) & (1,4) \\ \underline{(4,2)} & (3,1)\end{array}$	$\begin{array}{\|cc}\underline{(4,4)} & (2,3) \\ (3,1) & (1,2)\end{array}$	$\begin{array}{\|cc}(4,3) & \underline{(2,4)} \\ (3,2) & (1,1)\end{array}$	$\begin{array}{\|cc}(4,3) & (1,4) \\ (3,2) & (2,1)\end{array}$

Key: a and b distinguish two different class I games

4 = best; 3 = next best; 2 = next worst; 1 = worst

Pure-strategy Nash equilibria are underscored

Notes

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[1] Not all Pareto-optimal outcomes are Pareto-superior, or better for both players than all the Pareto-inferior outcomes in a game. In Prisoners' Dilemma, for example (see the Appendix), (4,1), (1,4), and (3,3) are all Pareto-optimal: there is no other outcome better for both players. But only (3,3) is Pareto-superior--namely, to the unique Pareto-inferior outcome, (2,2). For a comprehensive analysis of 2 x 2 games, see Rapoport, Guyer, and Gordon (1976).

[2] Consistency does not mean these games are subsumed by NG: receiving equal cardinal payffs at comparable outcomes in NG is more restrictive than ranking these outcomes the same. Given this restriction, however, NG is general in the sense that findings about it apply to all five ordinal games that it characterizes. This relieves one of the necessity of having to single out which one of the five games is most descriptive of a conflict in order to draw conclusions--of the kind described in this paper--about it.

[3] This section is more technical than other sections. Because the "other solutions" in this section are not used subsequently, the section can be skipped without loss of continuity.

[4] To see this in the case of NG, suppose that P1 chose some p' different from the p given by (2). Then either the left-hand or the right-hand side of (1) would yield P2 a greater expected payoff than that given by (3). Thus, by choosing one of C or $\bar{C}$, P2 can do better than EP_{P2},

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so P1's choice of p' $\neq$ p does not minimize P2's maximum possible expected payoff. It follows that p is P1's unique minimax strategy.

[5] In 2 x 2 games in which maximin strategies are mixed (unlike NG), Holler (1987, 1988) showed that the players' expected payoffs from choosing their maximin strategies are exactly the same as their expected payoffs from choosing their Nash-equilibrium mixed (minimax) strategies. Moreover, if games in which both maximin and minimax strategies are mixed contain no pure-strategy equilibra (also unlike NG), the players' expected payoffs from choosing these strategies are Pareto-optimal.

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