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FISCAL INCREASING RETURNS, INVESTMENT AND INTERNATIONAL CAPITAL MOVEMENTS

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Abstract This paper introduces the possibility of fiscal increasing returns to domestic capital into a dynamic model of international portfolio selection. The resulting model displays multiple steady states and surprising dynamic behavior. Cycles of any length can occur around the welfare-inferior equilibrium. Initial conditions and expectations both matter in selecting the equilibrium (or cycle) on which the economy converges.

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I. Introduction

In the aftermath of the debt crisis, developing countries are endeavoring to repatriate flight capital and to resume domestic private investment. But the performance of investment and the associated capital inflows continues to be disappointing in most countries.[1] This poses a puzzle for standard theory, which suggests that, in the presence of a low capital-labor ratio (and hence a high marginal product of domestic capital), capital should flow into such countries.

This lackluster investment performance is often attributed to macroeconomic disarray in the countries in question. Yet undertaking structural reforms and setting the macroeconomy in order are necessary but not sufficient conditions for the resumption of investment. Mexico and Chile have stabilized and opened up their economies and, after some years of hesitation, domestic investment is picking up strongly. Bolivia, on the other hand, undertook the most drastic adjustment program of them all, and yet capital has remained outside the country and investment continues to stagnate.[2]

A second feature of capital inflows (and investment rates) in developing countries is their volatility. In the second half of the 1980s, for instance, Mexico witnessed vast movements of capital in and out of the country, while macroeconomic policies and the associated returns on domestic capital remained reasonably stable throughout. This has led some economists to assert that decisions on investment and capital movements may have been driven by rumors and arbitrary changes in expectations, and not by "fundamentals", however defined.[3]

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[1]See World Bank (1989).

[2]See Rodrik and Ozler (1991).

[3]On this point see especially Tornell (1990).

This paper presents a model that generates patterns of investment and international capital movements which have both of these features.[4] Key to the model is the difference in the tax status of domestic and foreign capital. The latter cannot be taxed, while the former's after-tax rate of return depends on an endogenous tax rate.[5] This rate is set by the local fiscal authorities to ensure that a constant amount of spending is financed --hence, the equilibrium tax rate is inversely related to the size of the tax base. In this setup, domestic capital can enjoy what Blanchard and Summers (1987) label "fiscal increasing returns." In their view, these are "for macroeconomics, the most important type of increasing returns." One aim of this paper is to show that the presence of such increasing returns can produce new and interesting results in open economy growth theory.

Embedding such a fiscal sector in a standard, optimal portfolio selection model with adjustment costs in the fitting of domestic capital yields two main results. The first is that there are multiple steady-state equilibria, with different associated values for the stock of domestic capital held by agents. This suggests an answer to the question of how it is possible for apparently similar economies to display very different investment performances. The intuition for the result is simple. When the tax base is large (there is a large amount of capital at home) the tax rate is low --hence the after tax return on domestic capital is high, and it pays off for agents to keep their capital at home. Exactly the opposite happens when there is little capital at home.[6]

[4]For other explanations --that focus on the roles of uncertainty and credibility-- for the dismal investment performance of many countries undergoing structural reform, see Van Wijnbergen (1985) and Rodrik (1990).

[5]Assume also that taxes on domestic capital are the only source of revenue for the government.

[6]This mechanism is similar to that explored by Eaton (1987).

Second, the dynamics generated by the model are congruent with the perceived volatility in the behavior of capital movements. For a range of initial capital stocks, exogenous changes in expectations can lead the economy to jump across different equilibrium trajectories, possibly leading to different steady state equilibria. In addition, we show that regular cycles of any length can occur around a low-domestic-capital steady state. As a result, the stocks of foreign and domestic capital held by agents and the relative price between the two can display endogenous and regular fluctuations.

From a technical point of view, the model is an example of a Hamiltonian system of non-linear differential equations. Non-linear systems involving multiple equilibria typically have very complicated dynamics which cannot be solved for explicitly.[7] By contrast, Hamiltonian systems are a rare case where dynamics have a simple diagrammatic representation. To my knowledge, the only other recent application of Hamiltonian theory to economic growth problems is Matsuyama (1989).

The paper is organized as follows. Section II describes the static conditions necessary for the existence of multiple equilibria. Section III develops the associated dynamics, and shows the effects of exogenous changes in expectations as well as the possibility of cycles. Finally, Section IV suggests some conclusions.

II. The Role of Fiscal Increasing Returns

Consider an economy with two assets : foreign capital (b) and domestic capital (k). The foreign asset has the fixed rate of

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[7]For a general discussion, see Guckenheimer and Holmes (1983).

return r,[8] while domestic production takes place according to the function f(k), with f'(k)>0 and f"(k)<0. A crucial feature of the model is that the fruits of domestic capital are taxed at the rate τ, while foreign capital is always beyond the reach of the taxman. The tax rate τ is endogenous but is regarded as given by the individual. If the two assets are perfect substitutes, arbitrage will ensure that in any steady state equilibrium the following relationship must hold:

$$r = f'(k)(1-\tau) \tag{1}$$

The equilibrium level of taxes is determined in the following way. Assume that the government must finance an exogenous level of spending g by using only taxes on domestic output. The constant g should be interpreted as any type of government spending that is independent of the level of economic activity --for instance, payments on foreign debt. Then, the government budget constraint is

$$\tau_t f(k_t) = g \tag{2}$$

Solving (7) for τ and substituting into (5) we have

$$r = f'(k_t)\left[1-\frac{g}{f(k_t)}\right] \tag{3}$$

The R.H.S. of (3) appears in Figure 1, plotted as a function of k. Up to the point k^M, where the function reaches a maximum, domestic capital enjoys increasing fiscal returns --in that the after-tax return to this asset is increasing in k-- even if f(k) is a constant-returns-to-scale production function. To the right of k^M, the marginal return on k earned by the individual is decreasing in

[8]This reflects the "small country" assumption.

k, as usual.[9]

In equilibrium, the after-tax return on domestic capital must equal the world interest rate, as indicated by (3). Figure 1 shows that there are therefore two equilibria --either at a low level of domestic capital (labeled k_L) or at a high level (labeled k_H). The equilibria can be Pareto ranked unambiguously, with k_H preferred to k_L. Since in the presence of distortionary taxes the equilibrium level of domestic capital is always too low, the lower the tax rate (and the higher the resulting capital stock) the higher the resulting level of welfare.

III. Dynamics

The presence of multiple equilibria raises a number of questions. First, which equilibria are either locally or globally stable? In particular, will the economy ever converge to k_L? Many multiple equilibria models involve welfare-inferior equilibria that are unstable and hence probably unobservable in reality.[10] If that were so, the existence of a possible equilibrium at k_L need not be a source of concern to the policymaker.

We must also ask what forces place the economy on an equilibrium trajectory leading to a given steady state. If initial conditions alone determine which path is feasible then, in

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[9]If f(k) is Cobb-Douglas, with capital share equal to α, k^M is given by $(g/(k^M)^\alpha)=1-\alpha$. Assume production has constant returns to scale, so that the labor share equals $1-\alpha$. Then, for the economy to be in the increasing returns range it is sufficient for the share of government spending in output of the domestic good to be greater than labor's share. This is not entirely farfetched if one considers the ratios of government spending over GDP --often above 60%-- recently observed in Latin American countries. See Larraín and Selowsky (1991).

[10]This point is discussed exhaustively by Howitt and McAffee (1988), who also provide conditions under which inferior equilibria can be stable in a model of trading externalities.

Krugman's (1989) terminology, "history" determines the outcome. By contrast, if more than one path is feasible from given initial conditions, then "expectations" determines the outcome.

To answer such questions it is necessary to add dynamics to the model, being more explicit about the underlying structure of the economy. Suppose there are two goods, one foreign and one domestic. The foreign good is produced with a linear technology with constant marginal and average product r (of course, we can still think of it as the world interest rate, fixed from the point of view of a small country) and the domestic good is produced with technology f(k).[11] Define e as the relative price of foreign goods in terms of domestic goods (or of the asset b in terms of the asset k --it is the same thing) faced by agents in our economy. This relative price can be referred to (with only slight abuse of language) as "the real exchange rate."

Let π_I be the total instantaneous profits earned by an investor who holds a portfolio of both technologies, written using the foreign good as the numeraire. We then have

$$\pi_I = \frac{f(k)\,(1-\tau)}{e} + rb - \dot{b} - \frac{\dot{k}}{e} \qquad (4)$$

where the dot over a variable indicates a rate of change with respect to time, as usual. Each investor faces the constraint that total holdings of both assets do not exceed his constant total wealth w, expressed in terms of domestic goods:

$$w = \frac{k}{e} + b \qquad (5)$$

Differentiating (5) with respect to time we have

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[11]It is best to think of these as "corn" technologies, in that the output from each can be either consumed or reinvested.

$$\frac{\dot{k}}{e}+\dot{b} = \frac{k}{e}\,\frac{\dot{e}}{e} \tag{6}$$

Substituting (5) and (6) into (4) yields

$$\pi_I = \frac{f(k)\,(1-\tau)}{e}+r\left(w-\frac{k}{e}\right)-\frac{k}{e}\,\frac{\dot{e}}{e} \tag{7}$$

Each domestic investor must maximize (7) with respect to k, taking e, de/dt and τ as given.[12] The result is

$$f'(k)\left(1-\frac{g}{f(k)}\right) = r+\frac{\dot{e}}{e} \tag{8}$$

where we have also substituted for the value of τ from (2). Equation (8) has an obvious interpretation: the after-tax marginal return on domestic capital must equal the international rate of interest plus the capital gain from holding the foreign asset.

Finally, domestic capital is subject to installation costs, as is usual in much of the optimal investment literature. I assume there is a "retrofit sector" (similar to that in Mussa, 1976) that can transform foreign capital into domestic capital. If we assume, for simplicity, that such costs are quadratic, then the instantaneous profits of the representative firm in the retrofit sector (expressed, as usual, in terms of domestic goods) are

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[12] Notice that (a) there is no aggregate asset accumulation; and (b) from the point of view of the individual investor, there are no adjustment costs and therefore no slow-moving state variables. Therefore, instant-by-instant maximization is sufficient to satisfy intertemporal optimality.

$$\pi_R = \left(\frac{1}{e}-1\right)\dot{k}-\frac{1}{2\theta}(\dot{k})^2 \qquad (9)$$

where θ is a positive parameter that serves as an index of how costly the installation process is. The first order condition for these firms can be rewritten as

$$\dot{k}=\theta\left(\frac{1}{e}-1\right) \qquad (10)$$

Hence, the retrofit firms will be in operation as long as the relative price is different from one.

Equations (8) and (10) represent a system of two differential equations in two variables --the real exchange rate and the domestic stock of capital --which can be solved independently of the equilibrium stock of foreign capital.[13] To solve for the latter, is sufficient to take the solution of the system (8) and (10) and substitute it into (6).

Steady state conditions are clearly e=1 and $f'(k)(1-\tau)=r$.[14] Hence, the long run equilibria of the dynamic model correspond to the static equilibria discussed above.

One more step is necessary before we can solve explicitly for the dynamics of the system. Define x=log(e). Then, using this transformation on (8) and (10) yields

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[13] Notice that the real exchange rate is a "jump" variable, while the capital stock is a state (sluggish) variable.

[14] In turn, the steady state condition for the stock of foreign capital is simply given by the wealth constraint in (5).

$$\dot{x} = f'(k)\left(1-\frac{g}{f(k)}\right)-r \qquad (11a)$$

$$\dot{k} = \theta(\exp^{-x}-1) \qquad (11b)$$

System (11) is the one we will work with. It has the convenient feature that dx/dt=F(k) and dk/dt=G(x) --that is, the rate of change of one variable with respect to time depends on the other variable only.

The phase diagram of (11) appears in Figure 2. The dk/dt=0 schedule coincides with the horizontal axis. The dx/dt=0 schedule is composed of two vertical segments, at k_L and k_H. The corresponding intersections give rise to the two steady states we have already observed. It is clear from the arrows that the steady state at k_H is a saddle path, and convergence to k_H seems assured, at least from some initial values of k. Behavior around the steady state at k_L, on the other hand, is more difficult to determine. The usual techniques would suggest that convergence to k_L should depend on the sign of the real parts of the characteristic roots of the system linearized around k_L.

That is as much as one can typically say about the local dynamics (around the various steady states) in a non-linear system such as this one. In this case, however, we can actually characterize the full global dynamics of the system.[15] It is easier to work with a functional example, so let $f(k)=k^{\alpha}$, where α is capital's share in output. Define the following function H:

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[15]By characterizing the global dynamics we mean characterizing the behavior of the system on the whole of the Cartesian plane, no matter how far from steady state. By contrast, our usual analysis of non-linear system is confined to local dynamics, typically by working with linear approximations around a steady state. For a detailed treatment of this distinction, see Guckenheimer and Holmes (1983).

$$H = k^{\alpha} - \alpha\, g \log(k) - rk + \theta(\exp^{-x} + x) \tag{12}$$

H has the property that $\partial H/\partial k = dx/dt$ and $\partial H/\partial x = -dk/dt$. Hence, it must be the case that H is constant through time, for $dH/dt = (\partial H/\partial k)(dk/dt) + (\partial H/\partial x)(dx/dt) = (dx/dt)(dk/dt) - (dk/dt)(dx/dt) = 0$. Therefore, any solution to (11) must lie on a level curve of H; or, alternatively, the full phase diagram of (11) is the contour map of H. Such a system is called Hamiltonian.[16] It is also important to note that in a Hamiltonian system all steady states can only be either centers or saddle-path stable --sinks or sources cannot exist.

The corresponding phase diagram appears in Figure 3, where the $dk/dt=0$ and $dx/dt=0$ schedules have been omitted. The unstable trajectory that leaves k_L to the SW bends around and becomes the saddle-path that approaches k_L from the NW. Such an orbit, which connects a steady state to itself, is called a homoclinic orbit.[17]

This dynamic structure has two main implications. The first can be summarized in the following proposition:

__Proposition 1__: **There is a continuum of cycles around k_L in the region bounded by the homoclinic orbit.**

Within that region (the shaded area) the level curves of H give rise to infinitely many possible cycles, which are not shown

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[16]See Guckenheimer and Holmes (1983); see also Matsuyama (1989) for an application.

[17]Orbits are homoclinic if they connect a steady state to itself, and heteroclinic if they connect two different steady states. See Guckenheimer and Holmes (1983).

in the figure. Thus, there exist possible cycles of any length![18] In particular, while the system cannot converge to k_L, it can oscillate arbitrarily close to it. This means that the existence of a second (inefficient) steady state cannot be dismissed (from a policy point of view) by appealing to lack of convergence.

Notice that, by (6), if k and log(e) oscillate, b will normally oscillate as well. Hence, the endogenous cycles the system can display suggest an explanation for the waves of capital inflows and outflows observed in some of the countries mentioned at the outset.

The second main result is that in this model we can specify precisely the role of initial conditions in selecting the path followed by the economy. This result is summarized by:

Proposition 2: Beginning to the left of the point labeled k_0, the system cannot jump to either k_H or to a cycle around k_L. To the right of k_H, the system can only converge to k_H by jumping onto the relevant saddle path. For initial conditions between k_0 and k_H, the system can converge to k_H by jumping onto the relevant saddle path or it can settle on one of the infinitely many cycles.

Hence, we can usefully divide the plane in three areas. As intuition would suggest, economies that start with much domestic capital[19] (to the right of k_H) can only converge to the better equilibrium at k_H. This suggests why "capital-rich" economies cannot display pathological behavior, in the sense of cycling

[18] For other recent macro models that display endogenous fluctuations, see Drazen (1989) and Diamond and Fudemberg (1989). In both of those cases the cycles arise from a local bifurcation, unlike the situation here. A useful survey of some of the endogenous fluctuations literature is found in Boldrin and Woodford (1990).

[19] Relative to government spending.

around the inferior equilibrium. On the other hand, an economy that starts out with little capital (to the left of k_0) must be prone to instability, being unable to jump to the saddle path. The best it can do is to place itself on an unstable trajectory moving to the NE or SE until it reaches k_0 and is in a position to jump to either the saddle path or a cycle. How expectations could be coordinated to ensure such a trajectory, however, is far from clear.

Finally, starting from the intermediate region (between k_0 and k_H), the system can move instantaneously to either the saddle path or a cycle --though of course only some cycles are reachable from an arbitrary starting position. It will be enough for expectations to become coordinated (somehow) on one of these outcomes for it to materialize. In that case the real exchange rate will simply jump up or down to place the system on the expected trajectory. In turn, such expectations can change at any time, changing the course of the system. If expectations are volatile, so will the real exchange rate and the stock of domestic capital --another feature of the model that is reminiscent of the country experiences discussed at the start.

Notice that while these reversals of expectations may be arbitrary, there is nothing irrational about them. Expectations of future returns are always governed by fundamentals, for these expectations become self-fulfilling. In this sense, the indeterminacy observed here is very different from that present in bubbles. Furthermore, trajectories on the saddle path or the cycles are stable, in that they will not eventually violate transversality conditions.

IV. Conclusions

The notion that the return from investing in a given country is a function of how many agents are investing there as well is part of the folk wisdom of foreign investment decisions --

typically, foreign investment flows go largely to countries that are already enjoying substantial flows of that sort. This logic --which implicitly assumes an externality associated with capital flows-- applies not only to foreign investors, but to any agent planning to acquire capital in a given economy. This paper formalizes one mechanism --a fiscal externality that can give rise to increasing returns to domestic capital-- and links it to the decision faced by domestic residents choosing whether to hold capital at home or abroad. The model shows that the existence of this externality has important implications for the dynamic behavior of investment, asset prices and capital flows. The system has two steady states, which can be Pareto-ranked. Most strikingly, the economy can display regular oscillations around the more inefficient steady state.

The resulting multiplicity of equilibria and associated indeterminacy pose serious questions for the conduct of policy. Future research should focus on the possible role of government in mitigating the many inefficiencies present in this situation.

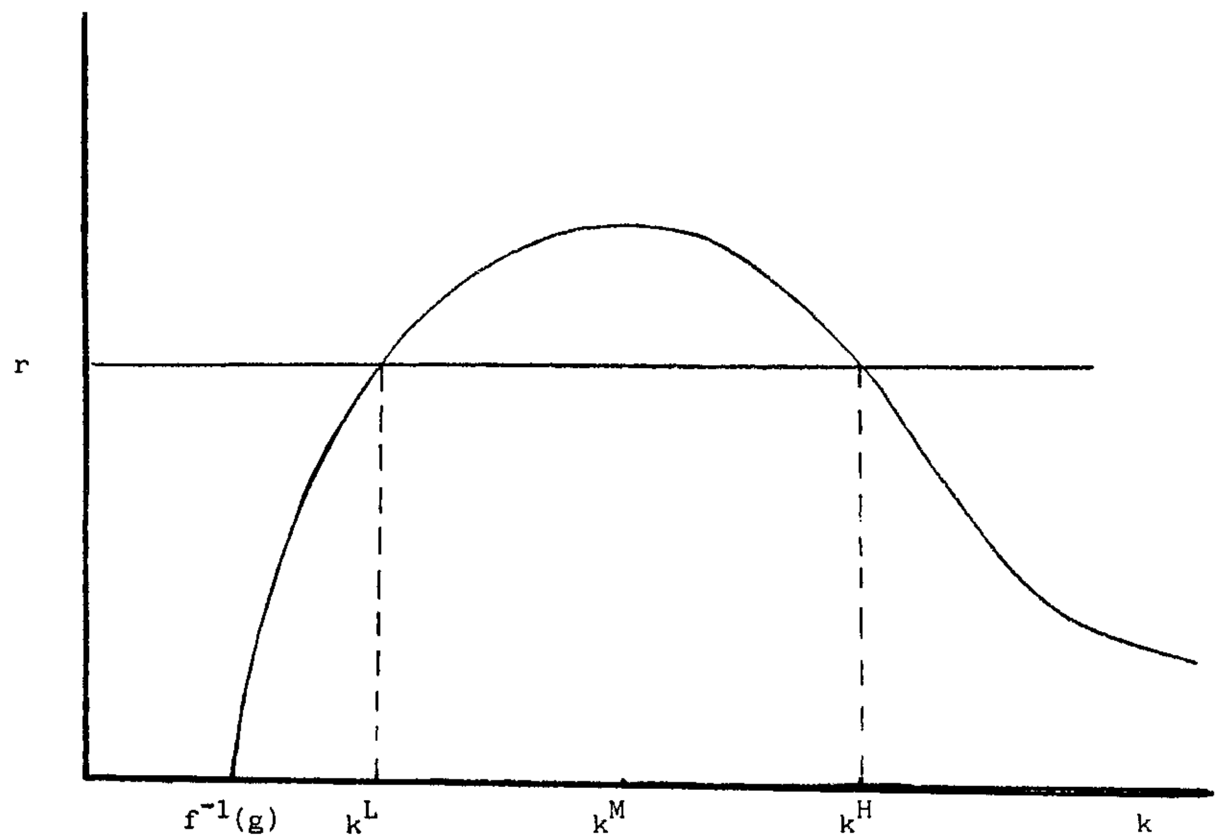

FIGURE 1

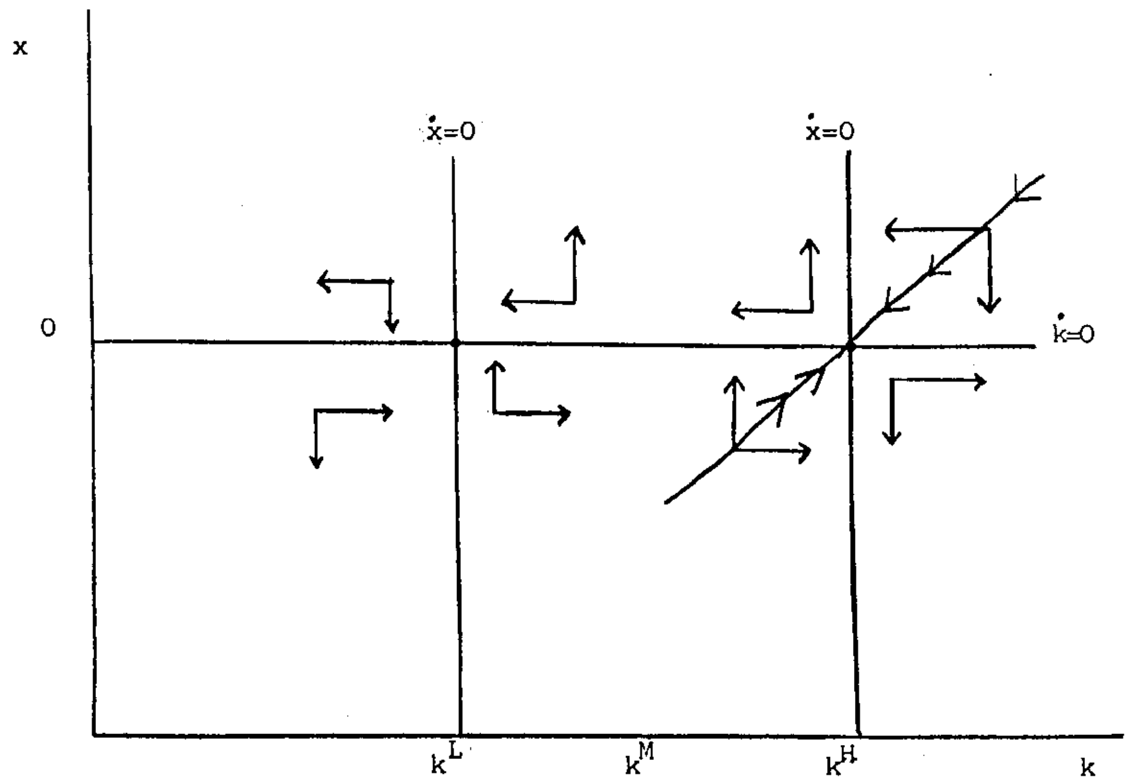

FIGURE 2

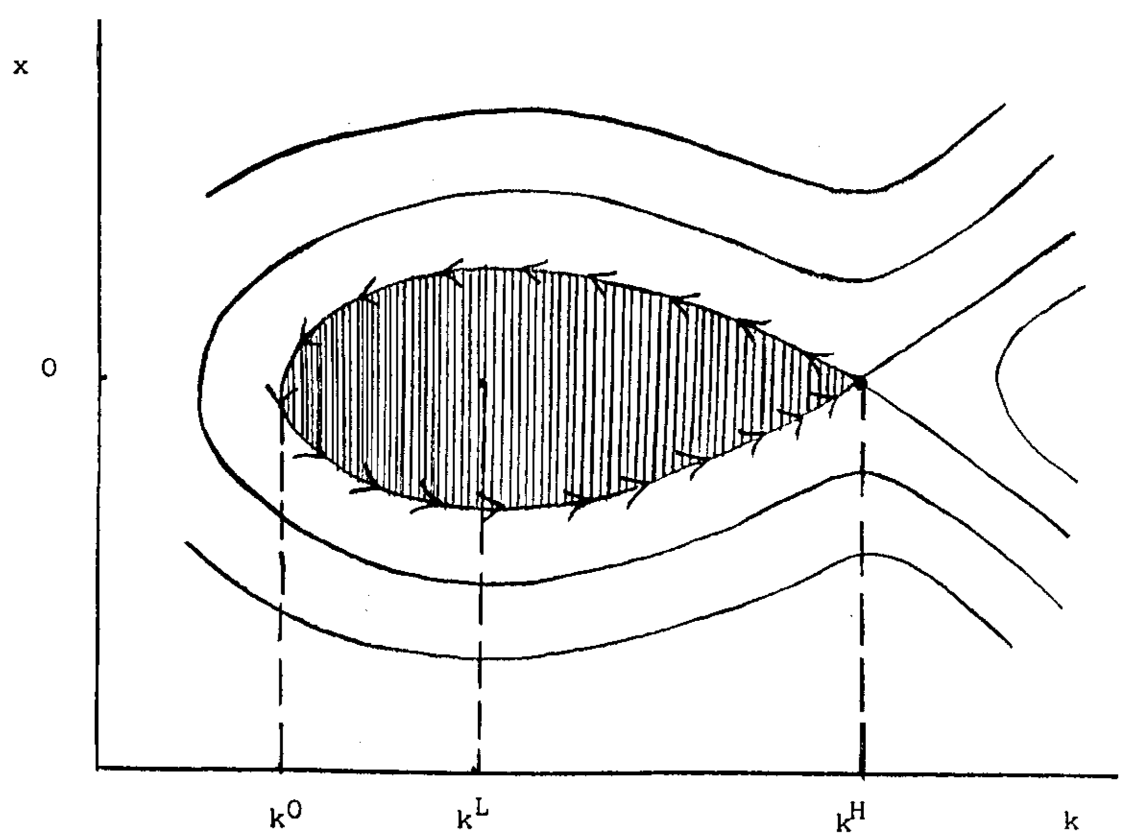

FIGURE 3

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