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PRIVATE INVESTMENT AND SOVEREIGN DEBT NEGOTIATIONS

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PRIVATE INVESTMENT AND SOVEREIGN DEBT NEGOTIATIONS*

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Abstract

I study a model of sovereign debt bargaining of the kind proposed by Bulow and Rogoff. I assume that all agents act rationally with perfect foresight and perfect information. The main departure from previous studies is that the government of the debtor country acts on behalf of but is not identical to its representative citizen. This seemingly minor change implies that (i) there is an indeterminacy of bargaining outcomes, including some of the sunspots type; (ii) agreement may be delayed for many periods; and (iii) marginal debt may not be worthless.

Running Title: Private Investment and Sovereign Debt Negotiations

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1. Introduction

Since its start in the early eighties, the debt crisis of less developed countries (LDCs) has been the subject of intense debate at all levels. Recent attempts to deal with the crisis have stressed debt relief in various ways [1]. Much of the impetus for this approach is based on the belief, in policy circles, that the current levels of debt have undesirable consequences for the debtor countries.

A dominant position in the academic literature, however, maintains that debt reduction schemes are worthless for debtor countries and only result in a windfall gain for creditors. The theoretical justification for such position was developed most convincingly in the pathbreaking paper of Bulow and Rogoff (1989a). Bulow and Rogoff argued that the present value of debt repayments, and therefore the market value of sovereign debt, is determined by a bargaining process between a country and its creditors. The outcome of this bargaining is independent of the face value of the debt; hence marginal debt reductions are worthless .

In spite of this academic argument, debt relief plans have continued and indeed accelerated. Policymakers, bankers, and ordinary people have welcomed new agreements incorporating more favorable terms for debtor governments. From the academic viewpoint described above, however, such gains are just illusory and the public euphoria is unwarranted.

The purpose of this paper is to show that it is the academic argument and not the public's belief that may be at fault. I will show that the academic argument relies on the unrealistic assumption that the governments of debtor countries make all decisions for their constituencies. Without such assumption, the academic argument does not survive. In particular, the

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[1] For a description of the different approaches, see Kenen (1990).

same bargaining models typically employed to support the academic argument imply that debt relief plans do benefit debtor governments under plausible circumstances.

More precisely, I study a model of sovereign debt renegotiation of the kind proposed by Bulow and Rogoff (1989a). Bulow and Rogoff assumed that there is no distinction between the government of the debtor country and its "representative" citizen. Under such "dictatorial" assumption, the model yields the following results, which are at the heart of the academic argument [2]: (i) There is a unique bargaining outcome, which is determined only by the debtor's country resources and the relative discount rates of the country and its creditors; (ii) The bargaining outcome is agreed upon with no delay, and therefore is Pareto efficient; (iii) The bargaining outcome is independent of the face value of the debt, and hence any nominal amount of debt in excess of the equilibrium value is worthless.

I reevaluate the model assuming instead that the government maximizes the utility of its representative citizen, but that it is <u>not</u> the representative citizen. I show that this seemingly trivial change implies that : (i) The model has a multiplicity of bargaining outcomes, some of which depend on variables that have no relevance for the fundamentals of the bargaining process; (ii) There are outcomes in which agreement is reached after long periods of negotiation, and hence are Pareto inefficient; and (iii) There are outcomes in which equilibrium payoffs depend on the face value of the debt, and in which a debt reduction benefits the debtor country even if in equilibrium the market value of the debt is less than its face value. These

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[2]These results are obtained by Bulow and Rogoff (1989a) and, for their Models 1 and 2, by Fernandez and Rosenthal (1990). Fernandez and Rosenthal do not examine the worth of marginal debt, although their results for Models 1 and 2 imply (iii). Note, however, that Fernandez and Rosenthal's Model 3 behaves very differently.

results follow in spite of the fact that all agents act rationally with perfect foresight and perfect information.

The intuition for my results is that eliminating the dictatorial assumption implies that the bargaining game played by the debtor government and the country's creditors interacts in a complex way with the expectations of the private sector about the outcomes of the bargaining. As negotiations continue, the savings of the representative household determine the evolution of the resources available for debt repayment and, therefore, of the bargaining situation. In turn, unless the dictatorial assumption is imposed, the savings decisions of the household while the negotiations are still in progress depend on its expectations about the bargaining outcome, which affects the rate of return.

In the light of my results, readers may wonder why so many studies have imposed analogous "dictatorial" assumptions in the first place. No doubt part of the explanation is that such assumptions enable us to reduce the bargaining process to a well defined game with a small number of players; for instance, under the dictatorial assumption the bargaining model in this paper reduces to a version of the celebrated Rubinstein's (1982) bargaining game. A technical contribution of this paper is to illustrate how to eliminate the dictatorial assumption and still have a manageable model: one can analyze this and similar situations by employing the concept of sustainable bargaining equilibrium (SBE) which I have developed elsewhere [3].

The paper proceeds as follows: Section 2 sets up the debt renegotiation problem under study. Section 3 analyzes its solution under the dictatorial assumption, showing that the bargaining outcomes are identical to those

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[3] I developed the SBE concept in Chang (1991), extending the concept of sustainable plans advanced by Barro and Gordon (1983) and Chari and Kehoe (1989) in the literature of time consistency.

obtained by Bulow and Rogoff (1989a). The remaining sections discuss the outcomes of the same model when the dictatorial assumption is eliminated. Section 4 defines and discusses the concept of sustainable bargaining equilibrium that I use to characterize the bargaining outcomes. Section 5 shows three simple SBEs. One of them results in the Bulow-Rogoff outcome. In the second SBE, the bank captures all of the gains from trade. In the third SBE, the bank recovers almost nothing. Section 6 shows that the model has sunspots equilibria and also equilibria in which negotiations last for a long time before agreement is reached. Section 7 shows that there are SBEs in which marginal debt reductions benefit the debtor country, and also discusses the distinction between the "reputational" and "direct sanctions" approaches to sovereign lending. Section 8 concludes with some further remarks and questions for future work. Some technical details are delayed to an Appendix.

2. The Model

I will study a very simple model of sovereign debt renegotiation. In spite of its simplicity, it will become apparent that the model captures most of the essential features of other prominent models in the literature.

Consider a small open country that takes world prices and interest rates as given. This country is populated by a "representative" agent. The government of this country maximizes the welfare of the representative household. Hence my arguments will not rely on any divergence between the government's and the private interests in the debtor country.

At the start of time the representative agent is endowed with k_0 units of a domestic good ("capital"). Each unit of capital can be consumed, stored, or exchanged in the world market for $P > 1$ units of a foreign good. Capital and the foreign good are perfect substitutes in consumption. The discounted utility of the representative household is given by $\sum_{t=0}^{\infty} \beta^t c_t$, where $\beta \in (0,1)$ is the agent's subjective discount factor, $c_t = c_{Ht} + c_{Ft}$, and c_{Ht} and c_{Ft} denote, respectively, consumption of capital and of the foreign good in period t. The assumption of risk neutrality simplifies our computations, but is not essential for any of my results.

The world interest rate is constant and given by $r > 0$. For simplicity, we assume that the country's subjective discount rate exceeds r. In the absence of debt, the representative household would just export all of its capital in period zero and consume Pk_0.

I assume, however, that at the start of time the household owes an amount D to a foreign bank. For simplicity, I will assume until Section 7 that D is equal to Pk_0 (that is, the debt is very large but not impossible to repay) and that the debt is guaranteed by the government [4]. The creditor bank is assumed to negotiate with the government of the debtor country about the amount of the debt which will be repaid.

In the absence of legal rules to enforce international commitments the outcome of this negotiation would be trivial: the government of the debtor country would just declare a default on the debt, and the household would consume the full world value of the capital Pk_0. But Bulow and Rogoff (1989a) have argued forcefully that creditor banks do have, in reality, some power to retaliate against a country in default. For our discussion, I will accept Bulow and Rogoff's arguments and assume that the international legal environment prevents residents of the debtor country from exporting the domestic good as long as its government has not reached some agreement with the creditor bank about the debt. Hence the bank and the government will bargain about how much the debtor country has to pay in exchange for its

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[4] Alternatively, we can assume that D is public debt.

freedom to export.

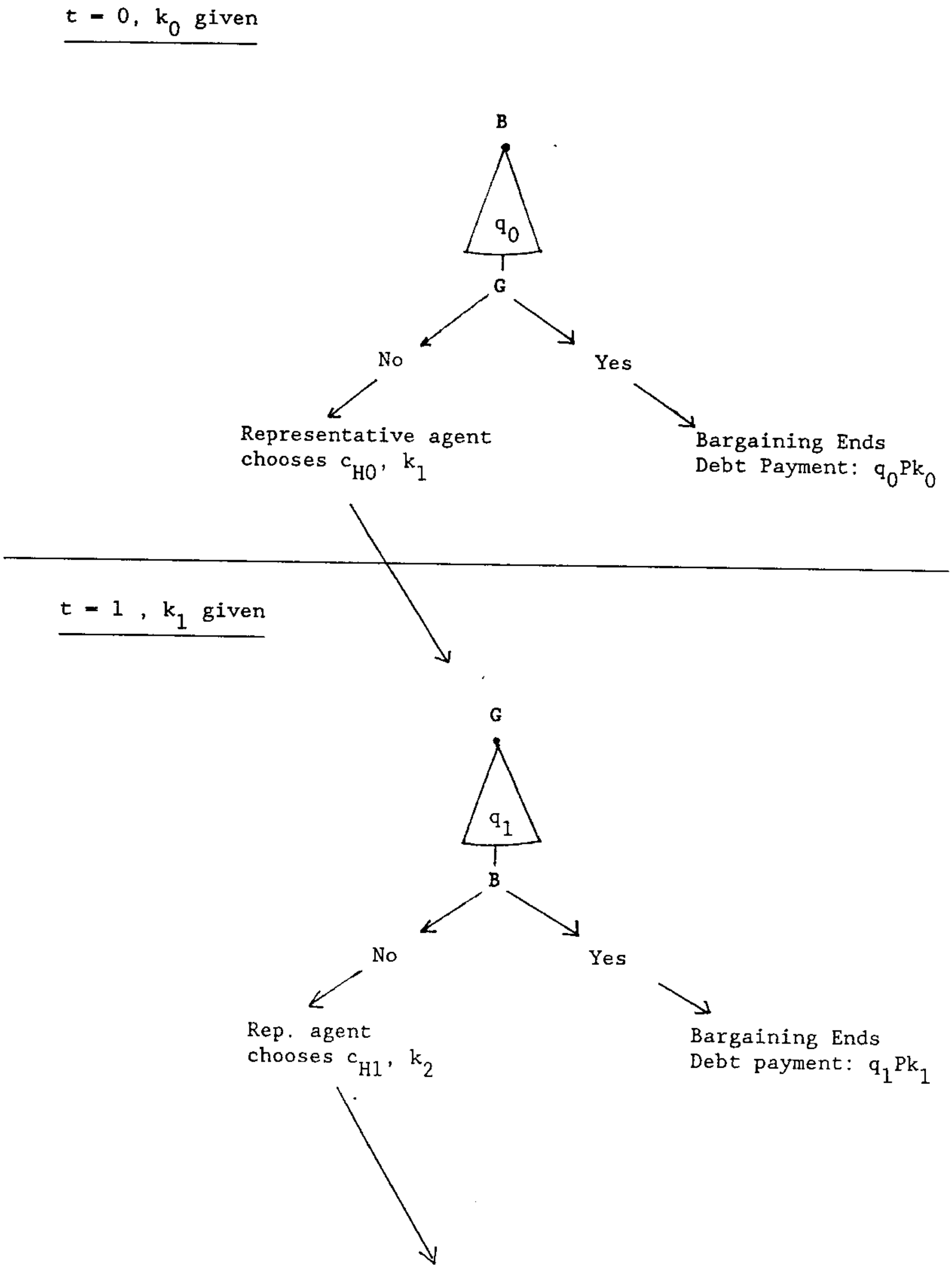
I will assume that the bank and the government play a bargaining game similar to that in Rubinstein (1982). At the beginning of each period t, the bank and the government in turn propose an agreement $q_t \in [0,1]$ to the other, where q_t is the proposed fraction of the country's resources to be transferred to the bank for a debt settlement.

The bargaining process is as follows: At $t = 0$, the bank (player "B") makes an offer $q_0 \in [0,1]$. If the debtor government (player "G") accepts the offer, the negotiation ends, the bank is paid q_0Pk_0 and the representative household consumes $(1-q_0)Pk_0$; the remaining debt is forgiven. If the government does not agree, the household may consume some amount $c_0 \in [0,k_0]$, and store $k_1 = k_0 - c_0$. Period one then starts with G offering $q_1 \in [0,1]$ to B. If B agrees, bargaining ends, B is paid q_1Pk_1, and the representative household consumes $(1-q_1)Pk_1$. If B rejects the offer, the household may consume $c_1 \in [0,k_1]$ and store $k_2 = (k_1 - c_1)$. The bargaining continues, with the bank making offers in even periods and the government in odd periods, until an agreement is reached, or forever. Figure 1 depicts the first two periods of the bargaining process.

In this negotiation, the bank's objective is to maximize the value, discounted by $(1/1+r) \equiv \theta$, of the debt repayment. As stated before, the government maximizes the welfare of the representative household.

This completes the description of the environment. This setup captures, I believe, most of the essential elements of other debt bargaining models: the bargaining parties exchange offers and counteroffers over time, the parties are impatient, and delaying agreement is costly because of legal sanctions or other exogenous factors. In fact, most readers will probably recognize that this model is a simple version of that postulated by Bulow and Rogoff (1989a) for the bargaining process starting in period T. It does not, however,

FIGURE 1



t = 0, k0 given
B
q0
G
No
Yes
Representative agent chooses cH0, k1
Bargaining Ends
Debt Payment: q0Pk0
t = 1 , k1 given
G
q1
B
No
Yes
Rep. agent chooses cH1, k2
Bargaining Ends
Debt payment: q1Pk1

incorporate their "constant recontracting" before period T; I will comment on this at the end.

Given the model of this section, we will address the following questions: (1) How can we characterize the outcomes of the model?; (2) How much of the debt will be repaid and how much forgiven?; (3) Will the outcome of this process be Pareto efficient?

3. The Dictatorial Case

It is typical in the literature to assume that the debtor government and the representative household of the debtor country are one and the same. I will call this the "dictatorial" assumption. In this section I will show that, under this assumption, one obtains the following three results, which are typical in the literature: (a) There is a unique outcome of the negotiation; (b) this outcome is efficient; (c) marginal debt is worthless.

In our model, the dictatorial assumption implies that the debtor government both negotiates with the bank and decides upon consumption and savings in the debtor country while the negotiation proceeds. Then our model becomes a version of the Rubinstein (1982) bargaining game, which has been extensively studied.

In this game, a strategy for the bank is what offer to make at $t = 0, 2, 4, \ldots$ contingent on the development of the economy up to each decision point, and whether to accept (Y) or reject (N) an offer from the government at $t = 1, 3, 5, \ldots$

A strategy for the debtor government is a little more complex: it must prescribe what offer to make at $t = 1, 3, 5, \ldots$ and what offers to accept at $t = 0, 2, 4, \ldots$, but also it must prescribe, for each period t, how much to consume (c_t) and invest (k_{t+1}).

Rubinstein (1982) has shown that bargaining games of this kind have a unique subgame perfect equilibrium. Subgame perfection is a sensible requirement on the equilibrium concept: it rules out "incredible" threats. Thus we have the following:

<u>Proposition 1 (Bulow and Rogoff (1989a)</u>

Assume that $\beta^2(1-\theta)P \geq (1-\beta\theta)$. Let $x = (1-\beta)/(1-\beta\theta)$, $y = \theta(1-\beta)/(1-\beta\theta)$. Then, under the dictatorial assumption, the following is the only subgame perfect equilibrium of the bargaining game:

The bank offers x in every period $t = 0,2,4,\ldots$, and in $t = 1,3,5,\ldots$ accepts any offer q such that $q \geq \theta x$

The debtor government offers y in every period $t = 1,3,5,\ldots$, in $t = 0,2,4,\ldots$ accepts any offer q such that $(1-q) \geq \beta(1-y)$, and, in every period t and after any history, it chooses $k_{t+1} = k_0$. ∎

Note that, construction, this result is exactly the same as in Bulow and Rogoff (1989a, pages 164-165).

Proposition 1 has three increasingly surprising consequences. First, this bargaining game has only one possible outcome: in period zero xPk_0 is repaid and the rest is forgiven. Equilibrium repayment as a fraction of the debtor's resources depends only on θ and β, that is, only on the relative "impatience" of the bank and the country. Subgame perfection plays an important role in ensuring the uniqueness of the solution. It implies that "take it or leave it" offers by the bank are not credible. Likewise, the country cannot credibly threaten the bank to avoid any repayment by consuming all of its resources.

Second, the bargaining outcome is reached without any delay and therefore is Pareto efficient. This is widely regarded as a consequence of symmetric information and of the structure of the Rubinstein game. Hence the natural response has been to change the model (as in Fernandez and Rosenthal (1990)) or to introduce asymmetric information .

Third, equilibrium debt repayments are equal to xPk_0 independently of the nominal face value of the debt (as long as $D \geq xPk_0$). Thus marginal debt, that is, debt in excess of xPk_0, is worthless. This argument, stressed in particular by Bulow and Rogoff (1988, 1989a, 1990), has important policy consequences. It implies that debt forgiveness schemes, debt-equity swaps, or debt repurchasing schemes are of little benefit to debtor countries and result only in a transfer of creditor banks. In the words of Bulow and Rogoff (1989, p. 157):

> When the market value of a country's debt lies far below its face value, marginal increases in the face value of the debt have little effect on its aggregate market value...By the same token, today's "problem" debtors would probably benefit very little from widely discussed schemes to forgive 10 or 20 percent of their debts, or from debt-for-equity swaps (see Bulow and Rogoff 1988). Marginal decreases in the debt's face value have only a second order effect on eventual repayments.

However useful, the results in this section are based on the questionable "dictatorial" assumption. This assumption eliminates any interesting interaction between debt negotiations and the expectations of the representative citizen. Both that, in reality, citizens of debtor countries are continuously trying to predict the outcome of debt negotiations, and that debtor governments usually attempt to influence the expectations of their constituencies to obtain support for their negotiation procedures, are then implicitly assumed to be irrelevant.

The dictatorial assumption is not only at odds with real world; it is also the case, as I will show in the next sections, that eliminating it changes our results dramatically.

4. Sustainable Bargaining Equilibria: Definition

For the remainder of this paper, I will assume that the government of the debtor country acts on behalf of but is not identical to the representative household. More precisely, assume that at the beginning of each period t an offer q_t is made by one of the players. The offer $q_t \in [0,1]$ will be now interpreted as the rate of a proportional tax on the representative agent's income Pk_t [5]. If the offer is accepted, bargaining ends, the bank is repaid $q_t Pk_t$, and the representative household consumes $(1-q_t)Pk_t$. If the offer is rejected, the representative household chooses how much capital to save $k_{t+1} \in [0,k_t]$ and and how much to consume $c_{Ht} = k_t - k_{t+1}$.

Thus the only change with respect to the previous section is that it is the representative household and not the debtor government who chooses savings and consumption in each period while the bargaining lasts. This looks like a trivial change, but it is not. Now the game played by the bank and the debtor government is affected by the behavior of the representative household who decides on the evolution of capital. Conversely, if the representative household is rational its savings-investment decisions will depend on its expectations about the bargaining outcome, which determines the real return on savings.

Clearly, eliminating the dictatorial assumption implies much more realistic and complex interactions between the bank, the government and the private sector of the debtor country. Predicting the outcome of this

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[5] It will become clear that the debtor government may have no incentive to substitute this tax with a lump-sum tax. See footnote 10.

interaction becomes, correspondingly, more difficult than in Section 3. Part of the difficulty is that we no longer have a standard game: Although the bank and the government can reasonably be assumed to act strategically, the private sector cannot. Hence the bargaining outcome cannot be simply characterized by the subgame perfect Nash equilibria of a game with a small number of players, as in Section 3.

I have argued elsewhere (Chang (1991)) that the appropriate equilibrium concept for this kind of bargaining model is sustainable bargaining equilibrium. In this context, a SBE is a pair of strategies, one for the bank and one for the government, and an allocation rule prescribing the evolution of capital such that (i) Given the allocation rule, the pair of strategies is a subgame perfect Nash equilibrium of the game played by the bank and the government, and (ii) Following the allocation rule is optimal for the representative household after any history, given the continuation of the proposed strategy pair.

In order to make these ideas precise we need some additional notation [6]. An allocation rule $F = \{F_t\}_{t=0}^{\infty}$ is a sequence of functions $F_t = (c_{Ht}, k_{t+1})$ each of which describes, for each period t, the consumption and savings of the representative household contingent on the history of the negotiation up to that period, provided no agreement has been yet reached. Formally, let $h^t = (q_0, q_1, \ldots, q_t)$ denote the history of rejected offers [7]. An allocation rule is then simply a sequence of functions $F = \{F_t\}_{t=0}^{\infty}$ such that, for each t, F_t maps h^t to $\mathbb{R}_+^2$.

Given an allocation rule F, the bank (B) and the debtor government

[6] The rest of this section follows closely Section III of my (1991) paper.

[7] To simplify the exposition, I assume in this section that the only relevant information is the sequence of offers and counteroffers. In Section 6 I discuss how extrinsic uncertainty or "sunspots" may affect the bargaining process.

(G) can be viewed as players in a well defined dynamic game. A strategy σ_i for player i = B,G is a sequence of functions that specify i's move at each node in which i is called upon to play.

A strategy pair $\sigma = (\sigma_B, \sigma_G)$ determines an outcome, which can be an agreement $q = Q(\sigma)$ in some period $t = \tau(\sigma)$, or perpetual disagreement. It also determines a history of rejected offers h^t, for each $t = 0, 1, .., \tau(\sigma)-1$.

Finally, given an allocation rule F and a strategy pair σ, the payoff for the government is the discounted welfare of the representative household, given by:

$$W_G(k_0,\sigma) = (1-Q)Pk_0 \qquad \text{if } \tau(\sigma) = 0$$

$$= \sum_{t=0}^{\tau-1} \beta^t\, c_{Ht}(h^t) + \beta^\tau\, (1-Q)\, Pk_\tau(h^{\tau-1}) \qquad \text{if not}$$

where $Q = Q(\sigma)$, $\tau = \tau(\sigma)$, and h^t, $t = 0,1,\ldots, \tau-1$ are determined by σ.

The interpretation of W_G is simple. If $\tau(\sigma) = 0$, there is an immediate agreement $Q = Q(\sigma)$; the country exports its capital, obtains Pk_0 units of the foreign good, pays QPk_0 to the bank, and consumes the rest. If $\tau(\sigma) > 0$, the allocation rule determines c_{Ht}, the country's consumption in each period before the agreement. The representative household's welfare is then equal to the discounted value of this interim consumption plus the discounted value of the agreement in period τ.

Analogously, B's payoff is simply the discounted value of the payment B receives at the end of the bargaining, that is, $W_B(k_0,\sigma) = \theta^\tau\ Q\ Pk_\tau$.

Given an allocation rule F, the strategy pair $\sigma = (\sigma_B, \sigma_G)$ is a subgame perfect Nash equilibrium if σ_B maximizes W_B given σ_G and k_0, and vice versa, and if at each node in which B or G are called upon to play, the continuations of σ_B and σ_G are a Nash equilibrium of the subgame starting at that node. For the reasons as in Section 3, subgame perfection is a natural requirement to impose on the equilibrium concept of our bargaining model.

What requirements should one impose on the allocation rule F? It is natural to assume that the representative agent be rational and behave competitively. To see what these assumptions imply, imagine the choice problem of the representative agent in any period t. If no agreement has been reached, he faces a nontrivial decision about how much to consume and save. Now, if he knows the strategies of the bank and the government he can calculate when and how much tax he will have to pay on his capital savings. Suppose that the continuation strategies imply an agreement at t+k with a corresponding tax q. Then the representative agent will consume all of his capital immediately if he expects a large tax, that is, if $\beta^k(1-q)P < 1$, and save it all if he expects a low tax. Rationality requires that the allocation rule prescribe such behavior after any possible history.

Formally, consider any period t and history h^{t-1}; capital is then given by $k_t(h^{t-1})$. If an offer q_t is made and rejected, there will be a new history $h^t = (h^{t-1}, q_t)$. Given a strategy pair σ, the continuations of σ_B and σ_G determine an outcome (τ,Q) in the subgame starting at (t+1). I will say that the allocation rule is <u>competitive</u> if for all t and h^t, $c_{Ht}(h^t) + k_{t+1}(h^t) = k_t(h^{t-1})$, and

$$
\begin{aligned}
k_{t+1}(h^t) &= k_t(h^{t-1}) && \text{if } \beta^{\tau+1}(1-Q)P > 1 \\
&= 0 && \text{if } \beta^{\tau+1}(1-Q)P < 1 \\
&\in [0,\ k_t(h^{t-1})] && \text{if } \beta^{\tau+1}(1-Q)P = 1
\end{aligned}
$$

where τ and Q are determined by the continuation of σ after h^t.

An allocation rule F and a strategy pair σ form a <u>sustainable bargaining equilibrium</u> (SBE) if (i) given the allocation rule F, the strategy pair σ is a subgame perfect Nash equilibrium, and (ii) given the strategy pair σ, the allocation rule F is competitive.

The rest of the paper examines the existence and characteristics of the SBEs of our bargaining model, and discusses how they relate to the conventional wisdom on sovereign debt renegotiations.

5. Sustainable Bargaining Equilibria: Three Simple Examples

In this section I show the existence of three simple SBEs. The three SBEs result in immediate agreement, but in very different payoffs for the bank and the debtor country. The SBE described by Proposition 2 results in the Bulow-Rogoff payoffs described by Proposition 1. In the SBE of Proposition 3, the bank captures all of the gains from trade. In the SBE of Proposition 4, the bank is repaid almost nothing.

I first describe the SBE that results in the Bulow-Rogoff outcome:

Proposition 2: Consider the following allocation rule:

$$c_{Ht}(h^t) = 0, \ k_{t+1}(h^t) = k_0 \qquad \text{any } t, \text{ any } h^t$$

and the following strategy pair, where x and y are as in Proposition 1:

σ_B: offer x in even periods, and accept q iff $q \geq \theta x$ in odd periods

σ_G: offer y in odd periods, and accept q iff $(1-q) \geq \beta(1-y)$ in even periods.

Then, if $\beta^2(1-\theta)P \geq (1-\beta\theta)$, (F, σ) is a SBE. ▮

Proof: That σ is a subgame perfect equilibrium given F is a straightforward application of Rubinstein (1982) [8]. So it remains to show

[8] Given the allocation rule F, the bank and the government play a Rubinstein

that, given σ, the allocation rule F is competitive.

Consider any period t and history h^t. By construction, the continuation of σ prescribes an agreement Q = x or Q = y in (t+1). Hence the allocation rule is competitive if both $\beta(1-x)P \geq 1$ and $\beta(1-y)P \geq 1$. But these two inequalities hold if $\beta^2(1-\theta)P \geq (1-\beta\theta)$. ∎

Proposition 2 shows that if the representative household believes in the Bulow-Rogoff outcome, it will behave in such a way that it is optimal for the bank and the government to agree on such outcome.

However, this is not the only outcome, as the following Proposition demonstrates:

<u>Proposition</u> <u>3</u>. The following allocation rule F and strategy pair σ is a SBE:

F: $c_{H0} = k_0$, $c_{Ht} = 0$ for $t \geq 1$, $k_{t+1} = 0$ all t.

σ_B: offer $q_0 = (P-1)/P$ at t = 0; offer $q_t = 1$ at t = 2,4,...; accept any offer in odd periods.

σ_G: offer $q_t = 1$ in odd periods; in period zero, accept an offer q if and only if $(1-q)P \geq 1$; in periods t = 2,4,.. accept any offer. ∎

<u>Proof:</u> Given the allocation rule, it is obvious that the proposed strategies are optimal after period t = 1 because the bank and the government have nothing to bargain about. In period zero, the government obtains $P(1-q)k_0$ if it accepts the offer q, and k_0 if not; hence its strategy is optimal. Given the government strategy, it is obviously optimal for the bank to offer

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bargaining game where the size of the surplus is Pk_0, and the discount factors are β and θ.

(P-1)/P.

Now we show that the allocation rule F is competitive. Suppose that an offer q_0 is made and rejected in period zero. Then, the continuation of σ prescribes an agreement Q = 1 in period one. Hence it is optimal for the representative consumer to consume all of the capital immediately. The representative consumer's decision problem is trivial from period one on. ∎

Note that agreement is reached immediately, and that the bank obtains all the gains from trade. The resulting payoffs are remarkably different from the Bulow-Rogoff payoffs: in particular, they do not depend on discount rates.

Loosely speaking, in the SBE of Proposition 3 the government's position is weakened by the possibility of a "panic" : should the players not reach agreement immediately, the representative agent fears a very unfavorable debt settlement and a large tax on capital. So he would rather eat its capital. Knowing this, the bank can in effect make a take-it-or-leave it offer that the government cannot reject [9].

The SBE of Proposition 3 is such that the government's bargaining position is weakened by the expectations of its own people. A benevolent dictator would do better for the debtor country than in Proposition 3. However, the opposite may hold, as shown next:

<u>Proposition</u> <u>4.</u> Suppose that $\beta P \geq 1$. Then the following is a SBE:

F: $c_{H0} = 0$, $c_{H1} = k_0$, $c_{Ht} = 0$ for $t \geq 2$; $k_1 = k_0$, $k_{t+1} = 0$ for $t \geq 1$

σ_B: Offer $q_0 = (1-\beta)$ at $t = 0$; offer $q_t = 1$ at $t = 2,4,...$; accept any offer in odd periods.

σ_G: Offer $q_1 = 0$ at $t = 1$; offer $q_t = 1$ at $t = 3,5,...$; accept an offer q

[9] This argument has the same flavor as that of Eaton (1987) in his analysis of capital flight.

at $t = 0$ iff $q \leq (1-\beta)$; accept any offer at $t = 2,4,\ldots$ ∎

The proof is similar to that of Proposition 3 and left to the reader. In the SBE of Proposition 4, there is a panic if agreement has not been reached after the second round of negotiations. This possibility weakens the bank's position in period zero, because the bank knows that in period one the government will successfully demand complete forgiveness. In equilibrium, the bank is paid almost nothing.

The separation between the government and the representative agent, in the SBE of Proposition 4, helps the government commit itself to the threat of consuming all of the capital. Under the dictatorial assumption, such threat is ruled out by subgame perfection [10].

Note that $\beta P \geq 1$ is implied by $\beta(1-x)P \geq 1$, and therefore the SBE of Proposition 4 exists if the SBE of Proposition 2 exists.

Finally, note also that as the length of the time intervals between offers go to zero, the three SBEs in this section converge to very different payoff partitions.

It not difficult to construct other SBEs of the kind presented in this section. However, readers may still conjecture that the set of all SBEs is discrete, that they may not depend on extrinsic uncertainty, or that they always result in efficient agreements. The next section shows that none of this conjectures is valid [11].

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[10] Note here that the government does not have an incentive to use lump sum taxes.

[11] Since the next section is technically more difficult than the others, readers may prefer to skop it in a first reading and go directly to the discussion of policy implications in Section 7.

6. Sunspots and Inefficiency

One purpose of this section is to show that without the dictatorial assumption the outcome of sovereign debt renegotiation may be affected in an essential way by seemingly irrelevant uncertainty. This happens because, as shown in the previous section, the bargaining outcome depends on the expectations of the representative agent, which may be a function of extrinsic uncertainty, as in Azariadis (1981) or Cass and Shell (1983).

In this section I also show that there may be sustainable bargaining equilibria in which the government and the bank negotiate for a long period of time before reaching an agreement. Since negotiating is costly, such SBEs are inefficient.

I show the existence of "sunspots" equilibria first. Suppose that sunspots may occur at the end of each period, after the representative agent has made its consumption-savings decision, with probability $p \in [0,1]$. Sunspots are identically and independently distributed across periods, and are observed by all agents. Although sunspots do not affect any of the fundamentals of the economy, they may affect the bargaining outcome because agents may condition their behavior on the history of observed sunspots.

Modifying the definitions of Section 3 to take into account the existence of sunspots is straightforward [12]. Instead of going through the details, I will just state and prove the following:

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[12] Let $s_t = 1$ if sunspots occur in period t, and = 0 if they do not. Define a history up to period t by $h^t = (q_0, s_0, \ldots, q_t, s_t)$. That is, now we assume that the relevant information at the end of period t includes both the sequence of rejected offers and the observed history of sunspots. With this change, the notions of strategies for the players and of allocation rule do not need any additional change. The notions of subgame perfect Nash equilibria, competitive allocation rule, and SBE can be modified in a straightforward way by using expected values instead of certain values.

Proposition 5. Define $\bar{p} = (\beta P-(1/\beta))/(\beta P-1)$ and $\hat{q}(p) = p(1-1/P) + (1-p)(1-\beta)$. For any $p \in [0,\bar{p}]$, there is a SBE with sunspots in which in period zero the bank offers $(1-\beta) + \beta\theta\hat{q}(p)$ which the government accepts. ∎

The proof is in the Appendix. Although the strategies and allocation rule described in the proof seem complicated, their interpretation is simple: if at the start of period two the bank and the government are still bargaining, the SBE "switches" to the SBE of Proposition 3 if sunspots occurred in period one, and to the SBE in Proposition 4 if they did not. The rest of the argument is completed by simple backwards induction for periods one and zero.

If $p = 0$, the payoff for the bank in the corresponding SBE of Proposition 5 is $\{(1-\beta)(1+\beta\theta)\}Pk_0$. When β is close to one, this payoff is very small, as in Proposition 4 of the previous section. If $p = \bar{p}$, the corresponding SBE from Proposition 5 implies that the bank's payoff is $\{(1-\beta) + \theta\ (\beta - 1/P)\}\ Pk_0$. When θ and β are small, the bank's payoff is very close to $(P-1)k_0$, which is the bank's payoff in Proposition 3.

Hence, sunspots SBEs may result in payoffs that are very similar to those of Propositions 3 and 4. But, in addition, they may result in <u>any</u> agreement in which the bank's payoff is between $(1-\beta)Pk_0$ and $\{(1-\beta) + \theta\ (\beta - 1/P)\}\ Pk_0$. That is, <u>there is an indeterminacy of SBEs</u>.

The following Proposition shows that there may be inefficient SBEs:

Proposition 6: Let $T \geq 2$ be even. Let x be defined as in Proposition 1 and $\hat{q}(\bar{p})$ as in Proposition 5. If the following conditions are satisfied:

$$\beta^T\ P\ (1-x) \geq 1$$
$$\beta^{T-1}(1-x) \geq 1 - \theta\ \hat{q}(\bar{p})$$

$$\theta^T x \geq (1-\beta)$$

then there is a SBE in which the agreement x is reached in period T. ∎

In the Appendix I describe the allocation rule and strategy pair underlying the Proposition. Showing that they are in fact a SBE is not difficult but very messy and therefore left to the interested reader. Instead, I will provide some intuition for the Proposition.

The SBE of Proposition 6 results in an equilibrium path in which, in each period before T, the bank requests full repayment and the government requests full forgiveness. In period T, they agree on the Bulow-Rogoff solution.

Should the bank deviate from this equilibrium path and request less than full repayment in some period $t < T$, the SBE "switches" to a very unfavorable continuation for the bank. In this continuation, failure to agree in period (t+1) causes a "panic" in (t+1), as in Proposition 4. If the government receives an offer $q_t < 1$ in period t, it knows that the bank will be "punished" and therefore becomes "tougher": it accepts q_t if it is equal to or below $(1-\beta)$.

Analogously, should the government demand less than full forgiveness in period $t < T$, the SBE "switches" to a very favorable continuation for the bank. This continuation, which is constructed as in Proposition 5, is such that the bank will accept the government's offer q_t if and only if $q_t \geq \theta\hat{q}(\bar{p})$.

The meaning of the three conditions in the statement of the Proposition is the following: for the postulated allocation rule to be competitive, it is necessary that in all periods $t = 0,1,\ldots,T-1$ saving all capital be optimal for the representative agent, given that the agreement x is expected in period T. For such optimality it is necessary and sufficient that $k_0 \leq \beta^T(1-x)Pk_0$, i.e., $1 \le \beta^T P(1-x)$.

Likewise, given G's strategy it is necessary for equilibrium that B prefers to wait for the agreement x in period T instead of making an acceptable offer at $t = 0$. At $t = 0$, the minimum acceptable offer for the government is $q = (1-\beta)$. Hence B will not make such offer if $(1-\beta)Pk_0 \le \theta^T xPk_0$, i.e., $\theta^T x \ge (1-\beta)$.

Finally, given B's strategy it is necessary for equilibrium that in period one G prefers to wait for the agreement x in period T instead of offering $\theta\hat{q}(\bar{p})$ to the bank and reach immediate agreement. Hence it is necessary that $(1-\theta\hat{q}(\bar{p}))Pk_0 \le \beta^{T-1}(1-x)Pk_0$, i.e., $1-\theta\hat{q}(\bar{p}) \le \beta^{T-1}(1-x)$.

Other equilibria with delay can be constructed in a similar way. Note that inefficient SBEs do not rely on the existence of sunspots, although I use them in Proposition 6.

7. The Worth of Marginal Debt and The Incentives for Repayment

So far we have established that, in the absence of a dictatorial assumption, there may be an indeterminacy of bargaining outcomes, some of which depend on extrinsic uncertainty, and some of which are inefficient. This happens because the volatile but rational expectations of the representative citizen may affect the negotiation in a nontrivial way.

I should be clear now why and how marginal debt may not be worthless. A reduction in the face value of the debt may affect the expectations of the representative household and affect the bargaining outcome even if without such reduction the aggregate value of the debt is strictly less than the face value.

Consider the following simple example: Suppose that at the beginning of time the face value of the debt may be any $D \in [(1-\beta)Pk_0, Pk_0]$. Suppose that there are two values, D_1 and D_2, such that $Pk_0 > D_1 > (P-1)k_0 > D_2 > xpK_0$, and define a SBE by the following:

if $(1-\beta)Pk_0, \leq D \leq D_2$, the allocation rule and strategies are as in Prop. 4

if $D_2 < D \leq D_1$, the allocation rule and strategies are as in Prop. 2

if $D_1 < D \leq Pk_0$, the allocation rule and strategies are as in Prop. 3

This is a perfectly reasonable SBE: if the face value of the debt is large, everybody (rationally!) expects a favorable outcome for the bank; if it is small, everybody expects a favorable outcome for the government; if intermediate, everybody expects the Bulow-Rogoff type of solution.

The example is such that, as long as $D > (1-\beta)Pk_0$, the face value of the debt is greater than the equilibrium repayment. However, marginal debt is hardly worthless. Suppose that D was initially equal to Pk_0. The expected repayment then is $(P-1)k_0$. According to the Bulow-Rogoff argument, any reduction of the debt below Pk_0 would be meaningless as long as the post-reduction face value is still above $(P-1)k_0$. But in this case, such view is not correct. If the debt is reduced to D_1, which is greater than $(P-1)k_0$, the value of debt repayments decreases in equilibrium. And since D_1 may be arbitrarily close to Pk_0, even a small debt reduction can benefit the debtors greatly.

The example is based on the fact that the bargaining strength of the bank and the debtor government depend on of private sector expectations, which may in turn depend on the face value of the debt. Such dependence is totally rational. A debt reduction affects private investment behavior and enables the government to become credibly tougher. As a result, the bargaining outcome changes in favor of the debtor country, confirming expectations.

The policy corollary is that debt reduction schemes or debt-for-equity swaps may be beneficial for debtor countries, as opposed to earlier contentions by Bulow and Rogoff.

The SBE of Proposition 6 illustrates that debt reduction schemes can be, in fact, Pareto improving. That SBE exhibits a "debt overhang" of the kind emphasized by Krugman (1989) and Sachs (1986). The existence of the debt problem prevents the country from using its resources in the most efficient way, which is to export them immediately. As a result, the bargaining outcome is Pareto inefficient. But neither the government nor the bank have a unilateral incentive to reduce the debt before period T. If a third institution, say the US government or the IMF, could in period zero force the bank to write off the debt to xPk_0 and the government to repay immediately that amount, both debtors and creditors would be better off.

I will close this section with a remark on the debate about why sovereign countries pay their debts. A prominent position, started by Eaton and Gersovitz (1981), maintains that legal sanctions against sovereign borrowers are irrelevant. Instead, sovereign borrowers make repayments to preserve a "reputation" that enables them to keep borrowing in the world market. The alternative position, sponsored mainly by Bulow and Rogoff, maintains that the debtor's primary motivation for repayments is the threat of legal sanctions imposed by the lenders. Bulow and Rogoff (1989b) went further to argue that the "legal sanctions" approach is the only legitimate one because reputation alone cannot sustain any positive level of sovereign lending.

Our results imply that legal sanctions by themselves cannot sustain sovereign lending either. There is a SBE, described in Proposition 4, in which the bank is repaid only $(1-\beta)Pk_0$, which becomes arbitrarily close to zero as the interval between offers becomes small. If this SBE was selected, no bank would have lent any money to the government in the first place.

8. Final Remarks

This paper elaborates on a single theme: the conventional wisdom about sovereign debt negotiations derived from game-theoretic bargaining models depends crucially on a "dictatorial" assumption. Absent such assumption, bargaining models do not support the conventional wisdom.

I have showed this to be the case for a simple model of debt bargaining. True, the model examined in this paper does not capture all aspects of reality; for example, I have abstracted from the "constant recontracting" feature of Bulow and Rogoff (1989a) model. But more complicated models are likely to yield more complicated SBEs. For conventional wisdom to be restored, one has to show that all of these SBEs result in the same outcome. Such demonstration is, in my view, unlikely.

Our investigation has confirmed what bankers and debtor governments knew all along: LDC debt negotiations depend in a complex way on the internal economic situation in debtor countries. Banks and governments typically need to avert confidence crises, and have to convince the private sector that everything is fine. But they can also attempt to use the danger of confidence crises to their advantage. Such interaction is neglected when one imposes a dictatorial assumption.

I have emphasized that debt bargaining models imply that marginal debt is worthless only under the dictatorial assumption. It is this assumption that guarantees that the bargaining outcome is unique and independent of the face value of the debt. Absent a dictatorial assumption, one can easily construct equilibria in which the debt's face value affects private expectations, investment in the debtor country, and hence the bargaining outcome. The practical consequence is that debt relief plans may be of great benefit to debtor countries, contrary to Bulow and Rogoff's (1989a, 1990) assertions.

What about the Bolivian evidence of Bulow and Rogoff (1988)? It is not clear at all how such evidence supports the conventional wisdom. For one thing, the Bolivian case is very special, as pointed out by Kenen (1990). A second reason is that the Bolivian case is only one: the next buyback may have a different outcome. Finally, the Bolivian evidence does not support the hypothesis that, of all the SBE outcomes, the Bulow-Rogoff one is the most reasonable. The same evidence is also consistent, for instance, with the SBEs of Propositions 3 and 4.

Given all of these results, the most important task for further research is to find a theory about how a particular equilibrium is selected. Short of a theory of equilibrium selection, too many things can happen, and our debt renegotiation models lack predictive power. Such a theory will probably have to describe how bankers and governments may affect the expectations of the private sector with public announcements, threats, or seemingly unreasonable stances. Perhaps the need for an intervention from creditor governments or multilateral organizations can be understood in the same light.

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Appendix

Proof of Proposition 5: I claim that the following is a SBE:

F: $k_1 = k_2 = k_0$; $k_3 = 0$ if $s_1 = 1$, and $= k_0$ if $s_1 = 0$; $k_{t+1} = 0$ for $t \geq 3$

σ_B: Offer $q_0 = (1-\beta) + \beta\theta\hat{q}(p)$; at $t = 1$, accept an offer q iff $q \geq \theta\hat{q}(p)$; at $t = 2$, offer $q_2 = 1 - 1/P$ if $s_1 = 1$, and $q_2 = (1-\beta)$ if $s_1 = 0$; at $t = 3,5,\ldots$ accept any offer; at $t = 4,6,\ldots$ offer $q_t = 1$.

σ_G: At $t = 0$, accept q if $q \leq (1-\beta) + \beta\theta\hat{q}(p)$; at $t = 1$, offer $\theta\hat{q}(p)$; at $t = 2$, if $s_1 = 1$ (resp. if $s_1 = 0$) accept q iff $(1-q)P \geq 1$ (resp. iff $q \leq (1-\beta)$); at $t = 3$, offer $q_3 = 0$ (resp. $q_3 = 1$) if $s_1 = 1$ (resp. if $s_1 = 0$); at $t = 4,6,\ldots$ accept any offer; at $t = 5,7,\ldots$ offer $q = 1$.

I will show next that, given F, the pair σ is a subgame perfect equilibrium. That the continuation of σ is a subgame perfect equilibrium for the subgames starting at $t = 2$ follows immediately by construction and Propositions 3 and 4. Hence it is sufficient to check the optimality of σ_B and σ_G in periods one and zero.

Consider B's problem in period one, after receiving an offer q from G. If B takes the offer, it obtains qPk_0. It it rejects the offer, it obtains $(P-1)k_0$ with probability p and $(1-\beta)Pk_0$ with probability (1-p) in period two. Thus it is optimal for B to accept G's offer if $qPk_0 \geq \theta\ \{\ p(P-1) + (1-p)(1-\beta)P\}\ k_0$, that is, if $q \geq \theta\ \hat{q}(p)$, as claimed.

Now look at G's problem in period one. If it offers $\theta\hat{q}(p)$, then agreement is reached and G obtains $(1-\ \theta\hat{q}(p))Pk_0$. If it offers less, G obtains k_0 with probability p and βPk_0 with probability (1-p). Hence offering $\theta\hat{q}(p)$ is optimal for G if $(1-\theta\hat{q}(p))Pk_0 \geq \beta\ \{p + (1-p)\ \beta P\}k_0$. It is easy to check that, under the conditions of the Proposition, the inequality holds. Hence G's strategy is optimal in period one.

Checking that σ_B and σ_G are optimal in period zero is straightforward and left to the reader.

It remains to check that, given σ, the allocation rule F is competitive. By construction, the continuation of F after period two is competitive, given that $k_2 = k_0$. Hence it is sufficient to check that $k_2 = k_1 = k_0$ is optimal for the representative agent in periods one and zero.

Consider the decision problem of the representative agent in period one. The continuation of σ after period two prescribes immediate agreement with $q_2 = 1 - 1/P$ with probability p and $q_2 = (1-\beta)$ with probability (1-q). Thus saving all of its capital is optimal for the representative household if $1 \leq \beta P \{p(1-(1-1/P)) + (1-p)(1-(1-\beta))\} = \beta \{p + (1-p)\beta P\}$. It is easy to check that this inequality holds if $p \leq \tilde{p}$.

Finally, consider the problem of the representative agent in period zero. The continuation of σ after period one prescribes the immediate agreement with $q_1 = \theta\hat{q}(p)$. Thus it is optimal for the representative agent to save all of its capital if $1 \leq \beta\,(1-\theta\hat{q}(p))\,P$. One can check that this inequality holds if $p \leq \tilde{p}$. The proof is complete. ▮

<u>Proof of Proposition 6:</u> Let $q_0 = q_2 = \ldots = q_{T-2} = 1$ and $q_1 = q_3 = \ldots = q_{T-1} = 0$ be a postulated equilibrium path of offers before period T. I construct an equilibrium allocation rule as follows:

If, as of period t, neither B or G have deviated from the proposed path, $k_{t+1} = k_t = k_0$. If B was the first player to deviate, and did so in period t-j, $k_{t-j+1} = k_0$, and $k_{n+1} = 0$ for $n \geq t-j+1$. If G was the first player to deviate and did so in period t-j, $k_{t-j+1} = k_0$, $k_{t-j+2} = 0$ if $s_{t-j} = 1$, $k_{t-j+2} = k_0$ if $s_{t-j} = 0$, and $k_{n+1} = 0$ for $n \geq t-j+2$.

Thus the allocation rule "switches" to an unfavorable continuation of the process for the deviant.

The equilibrium strategy for the bank is then the following:

In any period t in which B has to make an offer:

if $t < T$, and no deviation has occurred, offer $q_t = 1$; if $t \geq T$, and no deviation occurred before T, offer x; if G was the first to deviate and did so in period t-1, offer $q_t = (P-1)/P$ if $s_{t-1} = 1$, and $q_t = (1-\beta)$ if $s_{t-1} = 0$; if G deviated in period $t-j < t-1$, offer $q_t = 1$; if B was the first to deviate offer $q_t = 1$.

In any period t in which B has to respond to an offer q:

if $t < T$ and no deviation has occurred up to period (t-1), accept q if and only if $q \geq \theta\hat{q}(\bar{p})$; if $t \geq T$ and no deviation occurred before T, accept q if and only if $q \geq \theta x$; if a deviation occurred before t, accept any offer.

Finally, the equilibrium strategy for the government is the following:

In any period t in which the government has to make an offer:

if $t < T$, and no deviation has occurred, offer $q_t = 0$; if $t \geq T$, and no deviation occurred before T, offer y; if B was the first to deviate and did so in period t-1, offer $q_t = 0$; if B deviated in period $t-j < t-1$, offer $q_t = 1$; if G was the first to deviate offer $q_t = 1$.

In any period t in which G has to respond to an offer q:

if $t < T$ and no deviation has occurred up to period (t-1), accept q if and only if $(1-q) \geq \beta$; if $t \geq T$ and no deviation occurred before T, accept q if and only if $(1-q) \geq \beta(1-y)$; if G deviated in (t-1) and $s_{t-1} = 1$, accept q iff $q \leq (P-1)/P$; if G deviated in (t-1) and $s_{t-1} = 0$, accept q iff $q \leq (1-\beta)$; if there was a deviation before (t-1), accept any offer. ▮