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Abstract

I examine the existence of a subgame perfect equilibrium in a duopoly where the products are differentiated in quality, and derive the conditions under which an interior solution for the optimal qualities is obtained. The effect of government subsidy is investigated. I find that, in equilibrium, the subsidization of the high quality good may cause the quality of both products to fall, while subsidization of the low quality good unambiguously improves the quality of both products. If both products are subsidized, the effect is ambiguous.

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I. INTRODUCTION

In this paper, I examine the existence of a subgame perfect equilibrium in a duopoly where the products are differentiated in quality, and derive the conditions under which an interior solution for the optimal qualities is obtained. The effect of government subsidy is investigated. I find that, in equilibrium, the subsidization of the high quality good may cause the quality of both products to fall, while subsidization of the low quality good unambiguously improves the quality of both products. If both products are subsidized, the effect is ambiguous.

In some developing countries, the government has often provided subsidies to the producer of the highest quality good in an industry in order to encourage the firm to raise the quality of its product even further, presumably to "export" quality. However, these programs have had little success. Therefore, in this paper, I explain why they have not been effective and seek for an alternative to the policy.

Consumers are ranked by their willingness to pay for quality, and the model is a two-stage game with two firms. In the first stage, firms choose qualities, and in the second stage, firms set prices. According to Shaked and Sutton (1982), under the assumption of zero production costs, each firm wants to produce a higher quality product because a higher quality attracts more consumers if ceteris paribus. In addition, the firm faces a trade-off: if it chooses a product quality close to its competitor, it will gain market share but will face more intense price competition. Therefore, each firm would like to produce a more differentiated product. As a consequence, the high quality firm always produces at the upper bound, while the optimal choice of the low quality firm can be the lower bound or within the boundaries, depending on the heterogeneity of consumers.

However, to investigate a policy effect on quality, we need to have an interior equilibrium qualities. For that, I introduce a positive unit variable production cost which is increasing in quality and is independent of the level of output. This captures the idea that the unit variable cost to produce a higher quality good is greater. Furthermore, if the unit variable cost is assumed to rise rapidly with increases in quality, i.e., the cost curve is convex, then increasing the quality differential will diminish its relative cost advantage, which is defined as the ratio of the cost difference to quality difference. Therefore, firms are more likely to choose quality levels inside the boundaries. Thus, if the exogeneously given available quality range is sufficiently wide, the convexity of the unit variable cost curve guarantees the existence of an interior quality equilibrium. Also, if the response of each firm is less than its rival's initial movement, the uniqueness and stability of the equilibrium is ensured. In addition, a minimum level of heterogeneity in consumer tastes must exist in order to justify the presence of two firms in the industry. When consumers are very similar, each firm can attract all the consumers in the market by lowering its price slightly, so that each firm keeps cutting its price. Finally, the firm which has a relative cost disadvantage is eliminated. However, when consumer heterogeneity is great, price competition is not profitable since a firm has to lower its price greatly in order to capture the entire market.

After establishing the existence of the internal equilibrium, I extend the model to consider the effect of subsidies. In particular, a subsidy targeted at the firm producing the high quality product makes the firm less likely to eliminated from the industry in price competition. Thus, in order to gain a larger market share, the high quality firm may lower its quality and charge a lower price. As for the low quality firm, with an increased risk of being eliminated, it wants to lower its quality further in order to avoid increased price competition. After the interactions between the firms are considered, the qualities can move either way.

In contrast, a subsidy targeted at the firm producing the low quality product increases the competitiveness of the producer, who responds by improving the product in order to gain a larger share of the market. The optimal response of the high quality producer in this case is to improve its product to avoid intense price competition. Therefore, the subsidy has the effect of raising the quality produced by both firms. If government subsidizes both products, its effect on quality is ambiguous.

In Section II, I derive the conditions for a duopolistic non-cooperative price equilibrium, and then find the conditions under which an interior solution for the optimal qualities is obtained. In Section III, I show an example of a cost function which satisfies the conditions for the interior quality equilibrium. In Section IV, I analyze the effect of government subsidy. Section V concludes the paper.

II. THE MODEL

The present analysis is based on a two-stage non-cooperative game; in the first stage, each firm chooses the quality level of its product; in the second stage, having observed the choices, each firm sets its price. I use the concept of the Perfect Equilibrium (Selten (1975)). A two-tuple of strategies is a Perfect Equilibrium in the two-stage game, if a Nash Equilibrium occurs in the first stage as a result of predictions formed by agents with perfect foresight about the outcome of a Nash Equilibrium in the second stage.

II-1. Price Equilibrium

To seek a perfect equilibrium, we analyze the second stage and work backwards.

Consider two firms producing distinct, substitute goods. We label their respective products by an index $k = 1,2$ where firm k sells product k at price p_k.

Assume that each consumer is different in quality valuation, say t, which is uniformly distributed over $[a,b]$. A consumer of type t derives the following (indirect) utility from buying one unit of product k of quality u_k:

$$U(u_k;t) = tu_k - p_k \qquad \cdots\cdots\cdots \ (1)$$

We suppose that consumers buy exactly one unit of the differentiated commodity. Then they will select the product for which utility (1) is higher.

Now we derive from (1) the consumer who is just indifferent between product 1 and 2. Its valuation t^* is obtained from

$$t^*u_1 - p_1 = t^*u_2 - p_2,$$

where $u_2 > u_1$; hence

$$t^* = (p_2-p_1)/(u_2-u_1)$$

Thus the (necessary and sufficient) condition for two firms to share the market is $a < t^* < b$; that is,

$$a(u_2-u_1) < p_2-p_1 < b(u_2-u_1). \qquad \cdots\cdots\cdots (2)$$

Then consumers with values of t higher than t^* strictly prefer good 2 at price p_2 to good 1 at price p_1, and the converse is true for persons with $t < t^*$. Hence consumers are partitioned into segments corresponding to the market share of each firm.

Let c(u) represent the level of unit variable cost as a function of quality. It is assumed independent of the level of output. We also assume that c(u) is continuously differentiable, and that all firms have the identical unit variable cost functions.

The profit of each firm now becomes

$$\Pi_1(p_1;p_2,u_1,u_2) = (p_1-c(u_1))(t^*-a), \text{ and} \qquad \cdots\cdots\cdots (3)$$

$$\Pi_2(p_2;p_1,u_1,u_2) = (p_2-c(u_2))(b-t^*). \qquad \cdots\cdots\cdots (4)$$

Firm 1 chooses its price to maximize Π_1, given its rival's price. From (2), we find that, when the price of product 2 is low, i.e., $c(u_2) < p_2 \leq$

$c(u_1)+b(u_2-u_1)$, firm 1 has no chance to expel the rival from the industry as long as it remains profitable, i.e., $p_1 > c(u_1)$. Thus its optimal price p_1^* is selected to satisfy $\partial\Pi_1/\partial p_1 = 0$. On the other hand, if $p_2 > c(u_1)+b(u_2-u_1)$, firm 1 can eliminate its rival. However, the elimination may be less profitable than sharing the market with its rival. That is, when firm 2 charges $c(u_1)+b(u_2-u_1) < p_2 < c(u_1)+(2b-a)(u_2-u_1)$, it is more profitable for firm 1 to charge p_1^* and share the market. If its rival's price is $p_2 \geq c(u_1)+(2b-a)(u_2-u_1)$, it will be optimal for firm 1 to charge $p_1^d = p_2-b(u_2-u_1)$ and win the entire market.[1] Therefore, we find that, if $c(u_2)$ is greater than or equal to $c(u_1)+(2b-a)(u_2-u_1)$, firm 2 can not remain profitable in the industry. In other words,

$$2b-a > \frac{c(u_2)-c(u_1)}{u_2-u_1} \qquad \cdots\cdots\cdots (5)$$

must be satisfied in order for firm 2 to make a positive profit in the industry.

From condition (5), we derive the reaction function of firm 1:

$$p_1^R = \begin{cases} p_1^* = \{p_2-a(u_2-u_1)+c(u_1)\}/2 & \text{when } c(u_2)<p_2<c(u_1)+(2b-a)(u_2-u_1) \\ p_1^d = p_2-b(u_2-u_1) & \text{when } p_2 \geq c(u_1)+(2b-a)(u_2-u_1) \end{cases}$$

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[1] Consider a tangent line Q_i at p_i^d to an imaginary profit function of firm i which is defined as if its market can increase without a bound as its price is lowered. If the slope of line Q_1 is negative, i.e., $p_2 \geq c(u_1)+(2b-a)(u_2-u_1)$, it will be optimal for firm 1 to charge p_1^d and occupy the market. If the slope is positive, i.e., $p_2 < c(u_1)+(2b-a)(u_2-u_1)$, the optimal price $p_1^* = \{p_2-a(u_2-u_1)+c(u_1)\}/2$, will be selected.

Similarly, firm 2 chooses its price to maximize Π_2, given p_1. When its rival's price is $c(u_1) < p_1 \leq c(u_2)-a(u_2-u_1)$, firm 2 has no chance to profitably occupy the whole market. Thus it chooses p_2^* to satisfy $\partial\Pi_2/\partial p_2 = 0$. On the other hand, if $p_1 > c(u_2)-a(u_2-u_1)$, firm 2 can eliminate its rival. However, When firm 1 charges $c(u_2)-a(u_2-u_1) < p_1 < c(u_2)+(b-2a)(u_2-u_1)$, it is more profitable for firm 2 to charge p_2^* and share the market. If its rival's price is $p_1 \geq c(u_2)+(b-2a)(u_2-u_1)$, it will be optimal for firm 2 to charge $p_2^d = p_1+a(u_2-u_1)$ and force its opponent out of the market.[2] Thus we find that, if $c(u_1) \geq c(u_2)+(b-2a)(u_2-u_1)$, firm 1 will be expelled from the market. Therefore,

$$b-2a > -\frac{c(u_2)-c(u_1)}{u_2-u_1} \qquad \cdots\cdots\cdots \quad (6)$$

is the condition under which firm 1 can make a positive profit.

From condition (6), we derive the reaction function of firm 2:

$$p_2^R = \begin{cases} p_2^* = \{p_1+b(u_2-u_1)+c(u_2)\}/2 & \text{when } c(u_1)<p_1<c(u_2)+(b-2a)(u_2-u_1) \\ p_2^d = p_1+a(u_2-u_1) & \text{when } p_1 \geq c(u_2)+(b-2a)(u_2-u_1). \end{cases}$$

From conditions (5) and (6), we obtain the equilibrium prices for firms 1 and 2:

$$p_1^E = \frac{1}{3}\{(b-2a)(u_2-u_1)+c(u_2)+2c(u_1)\}$$

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[2] If the slope of Q_2 is negative, i.e., $p_1 \geq c(u_2)+(b-2a)(u_2-u_1)$, p_2^d is profit maximizing. However, if the slope is positive, i.e., $p_1 < c(u_2)+(b-2a)(u_2-u_1)$, $p_2^* = \{p_1+b(u_2-u_1)+c(u_2)\}/2$ will be chosen.

$$p_2^E = \frac{1}{3}\{(2b-a)(u_2-u_1)+2c(u_2)+c(u_1)\}$$

Point E in Figure 1 is one such equilibrium. In other words, when conditions (5) and (6) are satisfied, there exists a unique price equilibrium (p_1^E, p_2^E), where the profits of the firms are both positive. This equilibrium is globally stable.[3]

Assuming zero production costs, Shaked and Sutton (1982) find that the condition for two firms to share the market is $b-2a > 0$. That is, when consumers are very different, firm 2 has to lower its price greatly in order to occupy the whole market. This threatens to reduce profits. However, when consumers are very similar, $b-2a \le 0$, firm 2 can obtain a higher profit by lowering its price slightly, and so attracting all consumers. In this paper, however, we have introduced a positive unit variable cost, which increases with quality, so that firm 2 may not be able to take the whole market because it becomes to more difficult to profitably reduce its price sufficiently to eliminate its rival. Thus the condition $b-2a \le 0$ does not guarantee that firm 2 will occupy the entire market, but a more strict condition

$$b-2a \le -\frac{c(u_2)-c(u_1)}{u_2-u_1}$$

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[3] To check the stability conditions for the price equilibrium, we have

$$\partial^2\Pi_1/\partial p_1{}^2 = -2/(u_2-u_1) < 0,\ \partial^2\Pi_1/\partial p_2\partial p_1 = 1/(u_2-u_1) > 0,$$
$$\partial^2\Pi_2/\partial p_1\partial p_2 = 1/(u_2-u_1) > 0,\ \partial^2\Pi_2/\partial p_2{}^2 = -2/(u_2-u_1) < 0, \text{ and}$$
$$(\partial^2\Pi_1/\partial p_1{}^2)(\partial^2\Pi_2/\partial p_2{}^2)-(\partial^2\Pi_1/\partial p_2\partial p_1)(\partial^2\Pi_2/\partial p_1\partial p_2) = 3/(u_2-u_1) > 0.$$

Therefore, all the conditions for global stability are satisfied.

does. Therefore, condition (6) must hold in order to avoid the possibility that firm 2 occupies the market.

On the other hand, there is no way for the firm with the low quality product to take the whole market in the Shaked and Sutton model. Whenever there is a threat of elimination by a low p_1, firm 2 can protect its market by lowering p_2 close to p_1. However, when a positive unit variable cost is introduced, firm 2 may no longer be able to lower its price close enough to p_1 to remain in the market. Condition (5) constrains the cost difference to be less than $(2b-a)(u_2-u_1)$, and enables firm 2 to remain in the industry.

There are two types of heterogeneity which may affect the existence of a price equilibrium; namely, consumers' heterogeneity in quality valuation t, and the difference in quality of the products u_2-u_1, selected at the first stage of the game. When the quality differential becomes very small, $\{c(u_2)-c(u_1)\}/(u_2-u_1)$ approaches a certain value, i.e., the marginal cost in terms of quality, $c'(u)$, at the quality level where u_1 and u_2 converge. In fact, the value is independent of the quality difference, even though it depends on the quality level to which u_1 and u_2 approach. Thus, from conditions (5) and (6), we find that the existence of the price equilibrium is not significantly threatened by the similarities of the products as long as consumers' heterogeneity is sufficiently large.

On the other hand, when consumer heterogeneity is very small, even if the quality difference is great, conditions (5) and (6) are less likely to hold, so that we may end up with only one firm staying in the industry with a positive profit. In other words, the consumers' heterogeneity is indispensable to the maintenance of a duopoly, while product differentiation does not significantly threaten the existence of the price equilibrium.

II-2 Quality Equilibrium

Now we turn to the next stage of the process, in which each firm chooses the optimal quality level of its product. Substituting p_1^E and p_2^E into the profit functions (3) and (4) yields

$$\Pi_1^E = \frac{X^2}{9(u_2-u_1)},$$

$$\Pi_2^E = \frac{Y^2}{9(u_2-u_1)},$$

where $X = (b-2a)(u_2-u_1)+c(u_2)-c(u_1) > 0$ and $Y = (2b-a)(u_2-u_1)+c(u_1)-c(u_2) > 0$ from conditions (5) and (6).

Now we define an exogeneously given upper bound on quality $\bar{u}$ and a lower bound $\underline{u}$. We refer to $m(u_1,u_2) \equiv \{c(u_2)-c(u_1)\}/(u_2-u_1)$ as the relative cost disadvantage, the increase in cost resulting from a given increase in quality. We define u_1^m as the quality level satisfying $m(u_1^m,\bar{u}) = a-2b+2c'(\bar{u})$; that is, as we see in Figure 2, if u_1 is greater than u_1^m, it is optimal for firm 2 to choose the upper bound. Likewise, u_2^m is the quality level satisfying $m(\underline{u},u_2^m) = b-2a+2c'(\underline{u})$; that is, if u_2 is lower that u_2^m, it is optimal for firm 1 to choose the lower bound. Next, we obtain the reaction functions under the assumption that c(u) is convex: (See the appendix for the derivation.)

$$R_1(u_1;u_2) = \begin{cases} u_1-\underline{u} = 0 & \text{when } u_1<u_2\le u_2^m \\ (2a-b)(u_2-u_1)+c(u_2)-c(u_1)-2(u_2-u_1)c'(u_1)=0 & \text{when } u_2^m<u_2<\bar{u} \end{cases}$$

$$R_2(u_2;u_1) = \begin{cases} u_2-\bar{u} = 0 & \text{when } u_1^m\le u_1<u_2 \\ (2b-a)(u_2-u_1)+c(u_2)-c(u_1)-2(u_2-u_1)c'(u_2)=0 & \text{when } \underline{u}<u_1<u_1^m \end{cases}$$

Thus

$$\underline{u} < u_1 < u_1^m \text{ and } u_2^m < u_2 < \bar{u} \qquad \cdots\cdots\cdots (7)$$

are necessary conditions for an interior solution.

Proposition: Under condition (7), if $\underline{u}$ and $\bar{u}$ are selected to satisfy

$$\frac{3}{2}(b-a) > c'(\bar{u})-c'(u_1^m), \text{ and} \qquad \cdots\cdots\cdots (8)$$

$$\frac{3}{2}(b-a) > c'(u_2^m)-c'(\underline{u}), \qquad \cdots\cdots\cdots (9)$$

and a convex c(u) satisfy

$$2c''(u_1) > \frac{c'(u_2)-c'(u_1)}{u_2-u_1}, \text{ and} \qquad \cdots\cdots\cdots (10)$$

$$2c''(u_2) > \frac{c'(u_2)-c'(u_1)}{u_2-u_1}. \qquad \cdots\cdots\cdots (11)$$

there exists a unique and globally stable interior solution (u_1^E, u_2^E).

The proof of this proposition is in the appendix. From Figure 2, we find that conditions (8) and (9) guarantee that the value of u_2 at point A is greater than $\bar{u}$, and the value of u_1 at point C is less than $\underline{u}$. Thus, there exists at least one intersection of the reaction curves exists where $\underline{u} < u_1 < u_1^m$ and $u_2^m < u_2 < \bar{u}$. Furthermore, conditions (10) and (11) imply that marginal cost in terms of quality, $c'(u)$, rises rapidly as quality improves. Thus the response of each firm is less than its rival's initial movement; that is, the slope of $R_1(u_1;u_2) = 0$ is greater than one and the slope of $R_2(u_2;u_1) = 0$ is less than one in the range denoted by (7). Therefore, the uniqueness of equilibrium is obtained. Under the second order conditions, global stability is guaranteed.

We have examined a unique interior solution for the optimal qualities, under the assumption of convexity of $c(u)$. However, if $c(u)$ is concave, we can not expect an interior solution. Intuitively, when $c(u)$ is concave, as u_1 decreases, firm 2's disadvantage in relative costs, defined as $m(u_1,u_2)$, becomes greater. In order to minimize the impact, firm 2 will increase u_2 since $m(u_1,u_2)$ is decreasing in u_2. Firm 1 lowers u_1 in response in order to achieve greater cost advantage. This process continues until at least one firm chooses a bound of the available qualities. This is shown in the appendix.

III. AN EXAMPLE

To provide an example of a cost function which satisfies all the above conditions, I let $c(u) = u^2$. Then

$$p_1^E = \frac{1}{3}\{(b-2a)(u_2-u_1)+u_2^2+2u_1^2\}, \text{ and}$$

$$p_2^E = \frac{1}{3}((2b-a)(u_2-u_1)+2u_2^2+u_1^2).$$

That is, under conditions (5) and (6),

$$2a-b < u_1+u_2 < 2b-a,$$

there exists a unique and stable price equilibrium (p_1^E, p_2^E).

The quality reaction functions are then:

$$u_1^R = \begin{cases} \underline{u} & \text{when } u_1 < u_2 \leq b-2a+3\underline{u} \\ (2a-b+u_2)/3 & \text{when } b-2a+3\underline{u} < u_2 < \bar{u} \end{cases}$$

$$u_2^R = \begin{cases} \bar{u} & \text{when } a-2b+3\bar{u} \leq u_1 < u_2 \\ (2b-a+u_1)/3 & \text{when } \underline{u} < u_1 < a-2b+3\bar{u} \end{cases}$$

We find that where the conditions $\underline{u} < u_1 < a-2b+3\bar{u}$ and $b-2a+3\underline{u} < u_2 < \bar{u}$ are satisfied:

$$u_1^E = \frac{5a-b}{8}, \text{ and } u_2^E = \frac{5b-a}{8}.$$

Then we see that conditions (10) and (11) are satisfied. If $\underline{u}$ and $\bar{u}$ are chosen to satisfy conditions (8) and (9), we will have $\underline{u} < u_1^E < u_2^E < \bar{u}$.

IV. EFFECT OF GOVERNMENT SUBSIDY

In this section, we will examine the effect of government subsidy on the equilibrium qualities. If the subsidy is offered proportionally to the unit variable cost of product 2, the production cost will be $(1-\delta)c(u_2)$, where $0 \leq \delta < 1$. Thus we will see how u_1^E and u_2^E change as δ increases from zero.

We have

$$\frac{du_1^E}{d\delta} = \frac{|H_1|}{|R|} \text{ and } \frac{du_2^E}{d\delta} = \frac{|H_2|}{|R|},$$

where

$$R = \begin{pmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{pmatrix}, \; H_1 = \begin{pmatrix} h_1 & R_{12} \\ h_2 & R_{22} \end{pmatrix}, \; H_2 = \begin{pmatrix} R_{11} & h_1 \\ R_{21} & h_2 \end{pmatrix},$$

$$h_1 = c(u_2) > 0, \text{ and}$$

$$h_2 = c(u_2) - 2(u_2-u_1)c'(u_2).$$

Since we know that $|R|$ is positive by conditions (8) and (9), the sign of $|H_1|$ and $|H_2|$ depends on h_2. When h_2 is negative, the effect on the optimal qualities is not clear. However, if h_2 is positive, $|H_1|$ and $|H_2|$ are both negative; that is, u_1^E and u_2^E will decrease as δ increases. Therefore, the government subsidy on the high quality product may result in the deterioration of the qualities of both commodities in the industry.

To see how the reaction curves shift in the diagram, we find $\partial u_1/\partial\delta|_{R1=0} = h_1/R_{11} < 0$ and $\partial u_2/\partial\delta|_{R2=0} = h_2/R_{22}$ whose sign depends on h_2. As a consequence of an increase in δ, $R_1(u_1;u_2) = 0$ will shift to the left and $R_2(u_2;u_1) = 0$

will shift downward when h_2 is positive and upward when h_2 is negative. This is illustrated in Figures 3 and 4.

The intuition behind the quality movements is as follows: When the high quality good is subsidized, firm 1 becomes less competitive in the sense that it is more likely to be eliminated from the industry due to an intense price competition. Thus u_1^E will be lowered to avoid the price competition. For firm 2, the subsidy ameliorates its relative cost situation, so that product 2 becomes more competitive with product 1. In consequence, u_2^E will be lowered to take a greater market share. However, as u_2^E falls, the size of the subsidy is reduced. In other words, there are two opposite forces on u_2^E. Therefore, depending on their relative magnitude, u_2^E might be higher or lower. After the interactions between the two firms are considered, the final outcome of this subsidy is not guaranteed to be quality-improving.

Now, if the government gives a subsidy proportional to the unit variable cost of product 1, the production cost will be $(1-\gamma)c(u_1)$, where $0 \leq \gamma < 1$. Thus we examine how the equilibrium quality changes as γ increases from zero:

$$\frac{du_1^E}{d\gamma} = \frac{|F_1|}{|R|} \text{ and } \frac{du_2^E}{d\gamma} = \frac{|F_2|}{|R|},$$

where

$$F_1 = \begin{pmatrix} f_1 & R_{12} \\ f_2 & R_{22} \end{pmatrix}, \quad F_2 = \begin{pmatrix} R_{11} & f_1 \\ R_{21} & f_2 \end{pmatrix},$$

$$f_1 = -c(u_1) - 2(u_2-u_1)c'(u_1) < 0, \text{ and}$$

$$f_2 = -c(u_1) < 0.$$

It follows that $|F_1| > 0$ and $|F_2| > 0$, so that the effects of γ on u_1^E and u_2^E are both positive. That is, government subsidization of the low quality product gives each firm greater incentive to improve its product.

Now we will demonstrate this effect in Figure 5. When u_2 is given, the effect of γ on u_1 is positive because $\partial u_1/\partial\gamma|_{R1=0} = f_1/R_{11} > 0$. Consequently, the reaction curve of firm 1 shifts to the right as γ increases. Similarly, when u_1 is given, $\partial u_2/\partial\gamma|_{R2=0} = f_2/R_{22} > 0$, so that the reaction curve of firm 2 shifts upward as γ increases. Therefore, the equilibrium quality levels (u_1^E, u_2^E) will both be raised.

Intuitively, subsidization lowers the unit variable cost of product 1 making relatively cheap to produce; hence the competitiveness of good 1 is improved. It is, therefore, optimal for firm 1 to raise u_1^E to capture a greater market share. Moreover, the additional cost due to the improvement of u_1^E is reduced by the government. In consequence, firm 1 has two incentives to increase the quality of its product. Meanwhile, the competitiveness of product 2 is weakened, so that it needs to be more differentiated from product 1 in order to avoid intense price competition. Furthermore, the interaction between the two firms accentuates the movements. Therefore, subsidization of the low quality good raises both u_1^E and u_2^E.

If the government offers a subsidy for both products, proportional to each cost, the unit variable costs becomes $(1-\lambda)c(u_1)$ and $(1-\lambda)c(u_2)$, respectively. Then

$$\frac{du_1^E}{d\lambda} = \frac{|J_1|}{|R|} \quad \text{and} \quad \frac{du_2^E}{d\lambda} = \frac{|J_2|}{|R|} \ ,$$

where

$$J_1 = \begin{pmatrix} j_1 & R_{12} \\ j_2 & R_{22} \end{pmatrix}, \quad J_2 = \begin{pmatrix} R_{11} & j_1 \\ R_{21} & j_2 \end{pmatrix},$$

$$j_1 = f_1 + h_1 = \{c(u_2)-c(u_1)\} - 2(u_2-u_1)c'(u_1), \text{ and}$$

$$j_2 = f_2 + h_2 = \{c(u_2)-c(u_1)\} - 2(u_2-u_1)c'(u_2) < 0.$$

Since we know that $|R|$ is positive, the signs of $|J_1|$ and $|J_2|$ depend on j_1. When j_1 is positive, the effects of λ on u_1^E and u_2^E are not clear. However, when j_1 is negative, $|J_1|$ and $|J_2|$ are both positive. That is, the government subsidies have an unambiguous positive impact on the quality of both products.

When the government subsidizes both products simultaneously, the effects of the subsidies on the low quality product and the high quality product compete, so that the results are ambiguous.

Comparing the three policies discussed above, we find that $h_1 > j_1 > f_1$. That is, when u_2 is given, the effect of γ is the greatest (λ is the next, and δ has the smallest effect) in improving product 1. On the other hand, since $f_2 < 0$, j_2 is always less than h_2. Thus, when u_1 is given, if $h_2 > 0$, γ is the most efficient policy in improving product 2, while, if $h_2 < 0$, λ is the most efficient. If the interactions between the firms are considered, when $h_2 > 0$, γ unambiguously enhances the quality of product 2 the most. However, even if $h_2 < 0$, λ is not guaranteed to be the most efficient policy because the effect from the improvement of product 1 is less.

V. CONCLUDING REMARKS

In this paper, we have analyzed a two-stage non-cooperative game where duopolists decide sequentially upon quality and prices, and have derived the conditions under which a unique interior solution for optimal qualities exists. We have also found that government subsidization is unambiguously quality improving when only the production of the low quality good is subsidized.

The conditions we have derived are based on the assumption that the consumers' quality valuation is uniformly distributed. However, if the distribution is more peaked around its center, like the normal distribution, the conditions may be less stringent because the firms are less likely to attempt to differentiate their products since they are now most concerned with the taste of the median consumer. Thus interior solutions will be more easily obtained.

APPENDIX

1. **Derivation of Reaction Functions:** We look for the best quality choice of firm 1 for each given u_2. We know that Π_1^E increases at a decreasing rate as u_1 decreases from u_2. The maximum point of Π_1^E, u_1^*, is obtained from

$$\left.\frac{\partial \Pi_1^E}{\partial u_1}\right|_{\text{at } u_1^*} = \frac{X}{9(u_2-u_1^*)^2}\Big[(2a-b)(u_2-u_1^*)+c(u_2)-c(u_1^*)-2(u_2-u_1^*)c'(u_1^*)\Big] = 0,$$

where $c'(u_1) = dc(u)/du$ at u_1. Here, we assume that Π_k^E is defined even at the points out of the range of available qualities. Now we will check whether u_1^* is greater than $\underline{u}$ or not: (i) If $\partial\Pi_1^E/\partial u_1$ at $\underline{u}$ is negative, i.e.,

$$m(\underline{u},u_2) \le b-2a+2c'(\underline{u}), \qquad \cdots\cdots\cdots \text{(A-1)}$$

firm 1 will choose $\underline{u}$. (ii) If $\partial\Pi_1^E/\partial u_1$ at $\underline{u}$ is positive, i.e.,

$$m(\underline{u},u_2) > b-2a+2c'(\underline{u}), \qquad \cdots\cdots\cdots \text{(A-2)}$$

firm will choose u_1^*.

Since $c(u)$ is assumed to be convex, $m(\underline{u},u_2)$ is increasing in u_2, so that condition (A-1) holds for $u_1 < u_2 \le u_2^m$. Thus, firm 1's best quality choice is $\underline{u}$. For $u_2^m < u_2 < \bar{u}$, condition (A-2) holds, so that firm 1 chooses u_1^*. Therefore, the reaction function of firm 1 is

$$R_1(u_1;u_2) = \begin{cases} u_1-\underline{u} = 0 & \text{when } u_1<u_2\le u_2^m \\ (2a-b)(u_2-u_1)+c(u_2)-c(u_1)-2(u_2-u_1)c'(u_1)=0 & \text{when } u_2^m<u_2<\bar{u} \end{cases}$$

Similarly, the maximum point of Π_2^E is obtained from

$$\left.\frac{\partial\Pi_2^E}{\partial u_2}\right|_{\text{at } u_2^*} = \frac{Y}{9(u_2^*-u_1)^2}\left[(2b-a)(u_2^*-u_1)+c(u_2^*)-c(u_1)-2(u_2^*-u_1)c'(u_2^*)\right] = 0,$$

where $c'(u_2) = dc(u)/du$ at u_2. We now check whether u_2^* is greater or less than $\bar{u}$: (i) If $\partial\Pi_2^E/\partial u_2$ at $\bar{u}$ is positive, i.e.,

$$m(u_1,\bar{u}) \ge a-2b+2c'(\bar{u}), \qquad \cdots\cdots\cdots \text{(A-3)}$$

firm 2 will choose $\bar{u}$. (ii) If $\partial\Pi_2^E/\partial u_2$ at $\bar{u}$ is negative, i.e.,

$$m(u_1,\bar{u}) < a-2b+2c'(\bar{u}), \qquad \cdots\cdots\cdots \text{(A-4)}$$

firm 2 will take u_2^*.

Since $m(u_1,\bar{u})$ is increasing in u_1, from conditions (A-3) and (A-4), we find the reaction function of firm 2 as

$$R_2(u_2;u_1) = \begin{cases} u_2-\bar{u} = 0 & \text{when } u_1^m\le u_1<u_2 \\ (2b-a)(u_2-u_1)+c(u_2)-c(u_1)-2(u_2-u_1)c'(u_2)=0 & \text{when } \underline{u}<u_1<u_1^m \end{cases}$$

2. **Proof of Proposition:** From each reaction function, we have

$$R_{12} = \partial R_1/\partial u_2 = -m(u_1,u_2)+c'(u_2),$$

$$R_{21} = \partial R_2/\partial u_1 = m(u_1,u_2)-c'(u_1),$$

$$R_{11} = \partial R_1/\partial u_1 = m(u_1,u_2)-c'(u_1)-2(u_2-u_1)c''(u_1)$$
$$= -R_{12}+c'(u_2)-c'(u_1)-2(u_2-u_1)c''(u_1), \text{ and}$$

$$R_{22} = \partial R_2/\partial u_2 = -m(u_1,u_2)+c'(u_2)-2(u_2-u_1)c''(u_2)$$
$$= -R_{21}+c'(u_2)-c'(u_1)-2(u_2-u_1)c''(u_2).$$

Now, from the second order conditions and the convexity of c(u), we find

$$R_{11} < 0,\ R_{22} < 0,\ R_{12} > 0, \text{ and } R_{21} > 0.$$

Since

$$\left.\frac{du_2}{du_1}\right|_{R1=0} = -\frac{R_{11}}{R_{12}}, \text{ and } \left.\frac{du_2}{du_1}\right|_{R2=0} = -\frac{R_{21}}{R_{22}},$$

the slope of each reaction curve is positive. Furthermore,

$$(-R_{11})-R_{12} = -\{c'(u_2)-c'(u_1)\} + 2(u_2-u_1)c''(u_1), \text{ and}$$

$$(-R_{22})-R_{21} = -\{c'(u_2)-c'(u_1)\} + 2(u_2-u_1)c''(u_2).$$

Therefore, under conditions (8) and (9), the slope of firm 1's reaction curve is greater than one, and the slope of firm 2's reaction curve is less than one.

From Figure 2, the value of u_2 at point A is the one which satisfies $m(u_1^m,u_2) = b-2a+2c'(u_1^m)$, while $m(u_1^m,\bar{u}) = a-2b+2c'(\bar{u})$ is satisfied at point B. Since the function $m(u_1^m,u_2)$ is increasing in u_2, when $b-2a+2c'(u_1^m) > a-2b+2c'(\bar{u})$, i.e.,

$$\frac{3}{2}(b-a) > c'(\bar{u})-c'(u_1^m),$$

the value of u_2 at point A is greater than $\bar{u}$. Analogously, the value of u_1 at point C is the one which satisfies $m(u_1,u_2^m) = a-2b+2c'(u_2^m)$, while $m(\underline{u},u_2^m) = b-2a+2c'(\underline{u})$ at point D. Since the function $m(u_1,u_2^m)$ is increasing in u_1, when $a-2b+2c'(u_2^m) < b-2a+2c'(\underline{u})$, i.e.,

$$\frac{3}{2}(b-a) > c'(u_2^m)-c'(\underline{u}),$$

the value of u_1 at point C is less than $\underline{u}$. Therefore, from Figure 2, we can see that, under conditions (10) and (11), we find that at least one intersection of the reaction curves exists where $\underline{u} < u_1 < u_1^m$ and $u_2^m < u_2 < \bar{u}$. Also, conditions (8) and (9) guarantee its uniqueness. Moreover, the matrix

$$R = \begin{pmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{pmatrix}$$

is negative definite everywhere in the range of $\underline{u} < u_1 < u_1^m$ and $u_2^m < u_2 < \bar{u}$. Therefore, the equilibrium is globally stable.

3. **Case of Concave Cost Curve:** If $c(u)$ is concave, the reaction functions are

$$R_1(u_1;u_2) = \begin{cases} (2a-b)(u_2-u_1)+c(u_2)-c(u_1)-2(u_2-u_1)c'(u_1)=0 & \text{when } u_1<u_2<u_2^m \\ u_1-\underline{u} = 0 & \text{when } u_2^m \leq u_2<\bar{u} \end{cases}$$

$$R_2(u_2;u_1) = \begin{cases} (2b-a)(u_2-u_1)+c(u_2)-c(u_1)-2(u_2-u_1)c'(u_2)=0 & \text{when } u_1^m<u_1<u_2 \\ u_2-\bar{u} = 0 & \text{when } \underline{u}<u_1 \leq u_1^m \end{cases}$$

In order for the reaction curves to intersect each other in the range of $u_1^m \leq u_1 < u_2$, $u_1 < u_2 \leq u_2^m$, $(2a-b)(u_2-u_1)+c(u_2)-c(u_1)-2(u_2-u_1)c'(u_1)$ must be equal to $(2b-a)(u_2-u_1)+c(u_2)-c(u_1)-2(u_2-u_1)c'(u_2)$; that is,

$$c'(u_2)-c'(u_1) = \frac{3}{2}(b-a).$$

Since this condition can not be satisfied under the concavity of $c(u_k)$, no interior solution can be obtained.

Also, when $u_1^m < u_1 < u_2$ and $u_1 < u_2 < u_2^m$, the slope of each reaction curve is

$$\left.\frac{du_2}{du_1}\right|_{R1=0} = -\frac{R_{11}}{R_{12}} < 0, \text{ and } \left.\frac{du_2}{du_1}\right|_{R2=0} = -\frac{R_{21}}{R_{22}} < 0.$$

Since, by the second order conditions and the concavity of $c(u)$, we find

$$R_{11} < 0,\ R_{22} < 0,\ R_{12} < 0, \text{ and } R_{21} < 0,$$

the slope of $R_1(u_1;u_2) = 0$ is less than -1, and the slope of $R_2(u_2;u_1) = 0$ is greater than -1. (i) When $u_1^m > \underline{u}$ and $u_2^m < \bar{u}$, i.e., $b-2a+2c'(\underline{u}) \geq m(\underline{u},\bar{u}) \geq a-2b+2c'(\bar{u})$, the equilibrium qualities are $(\underline{u}, \bar{u})$, as we see in Figure A-1. In the area of $u_1^m < u_1 < u_2$ and $u_1 < u_2 < u_2^m$, the reaction curves do not intersect each other because the slope of $R_1(u_1;u_2) = 0$ is less than the slope of $R_2(u_2;u_1) = 0$. (ii) When $u_1^m > \underline{u}$ and $u_2^m \geq \bar{u}$, i.e., $m(\underline{u},\bar{u}) \geq \min\{a-2b+2c'(\bar{u}), b-2a+2c'(\underline{u})\}$, and the u_2 value at point A is less than that at point B, i.e., $c'(\bar{u})-c'(u_1^m) \leq 3(b-a)/2$, the equilibrium is $(u_1^E,\bar{u})$, as in Figure A-2. (iii) When $u_1^m \leq \underline{u}$ and $u_2^m < \bar{u}$, i.e., $m(\underline{u},\bar{u}) \leq \min\{a-2b+2c'(\bar{u}), b-2a+2c'(\underline{u})\}$, and the u_1 value at point C is greater than that at point D, i.e., $c'(u_2^m)-c'(\underline{u}) \geq 3(b-a)/2$, the equilibrium is $(\underline{u},u_2^E)$, as in Figure A-3. (iv) The case of $u_1^m \leq \underline{u}$ and $u_2^m \geq \bar{u}$, i.e., $b-2a+2c'(\underline{u}) < m(\underline{u},\bar{u}) < a-2b+2c'(\bar{u})$, is not feasible because $b-2a+2c'(\underline{u}) > a-2b+2c'(\bar{u})$.

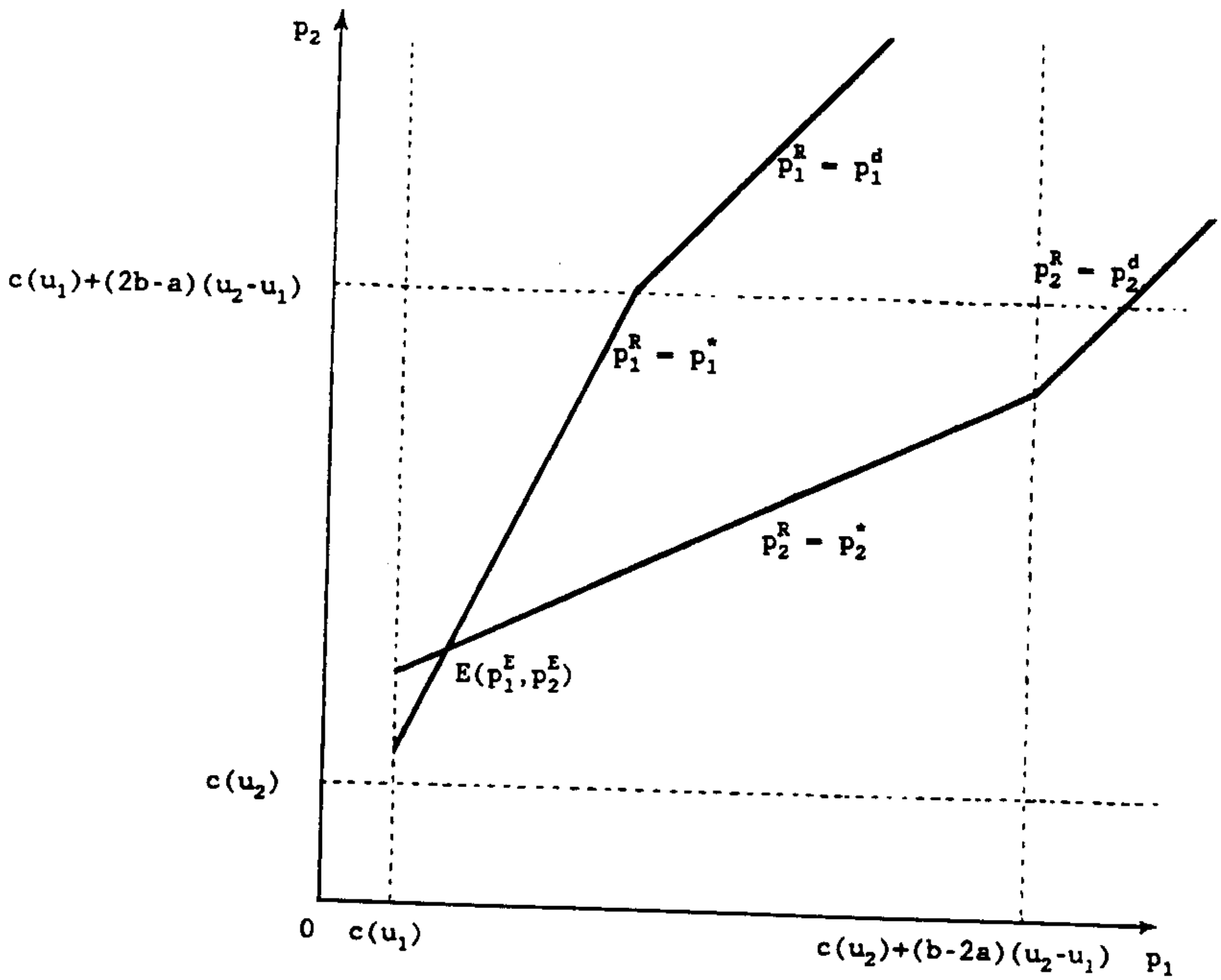

Figure 1: Price Equilibrium

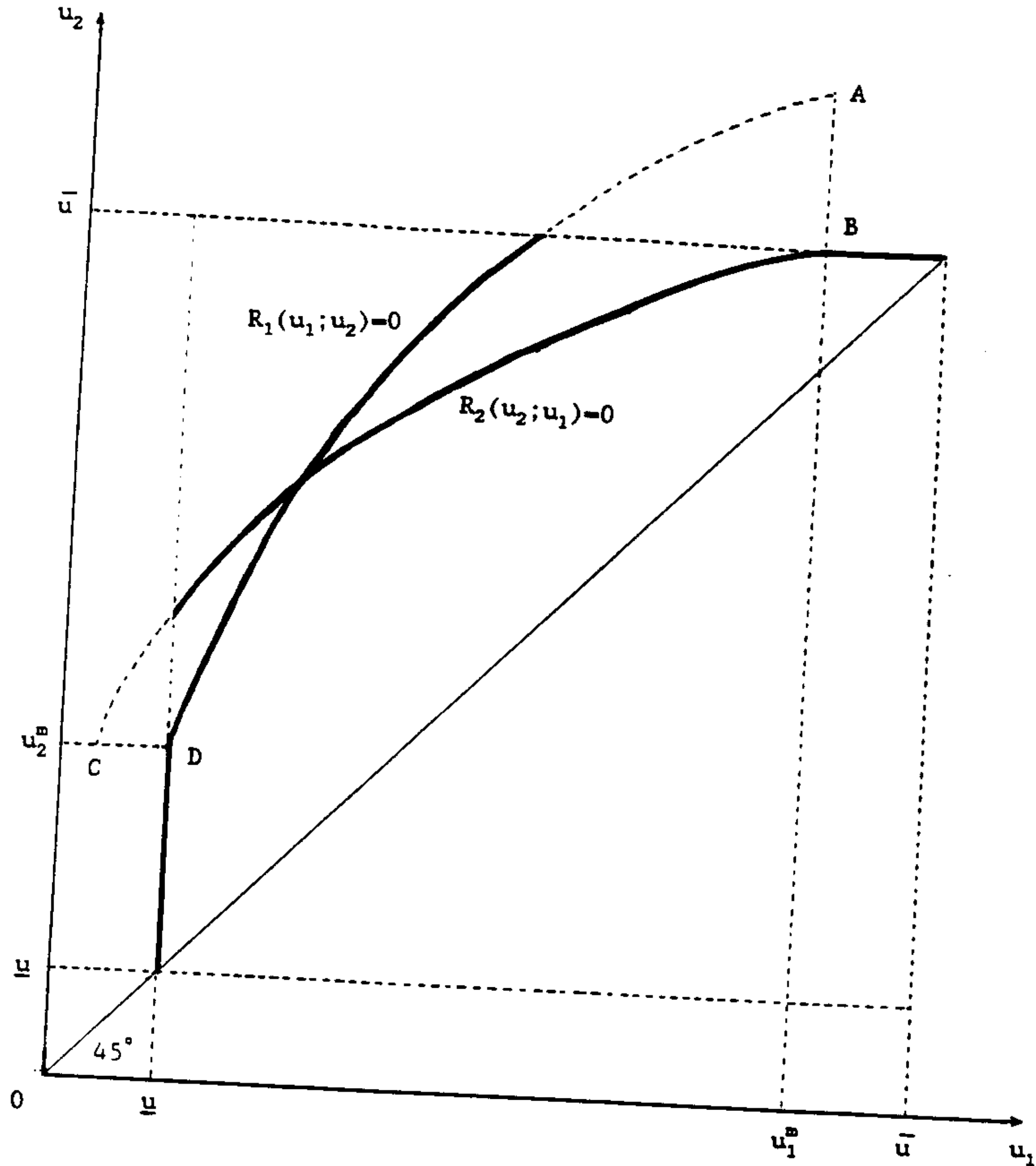

Figure 2: Quality Equilibrium

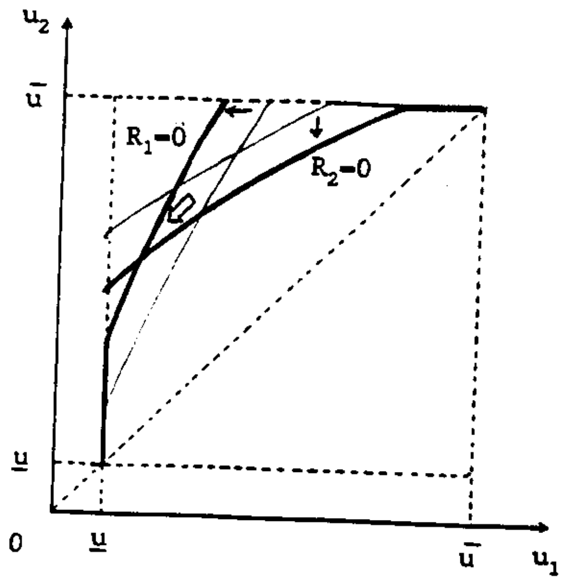

Figure 3: Movement of Quality Equilibrium by a Government Subsidy for the High Quality Good (when $h_2 \geq 0$)

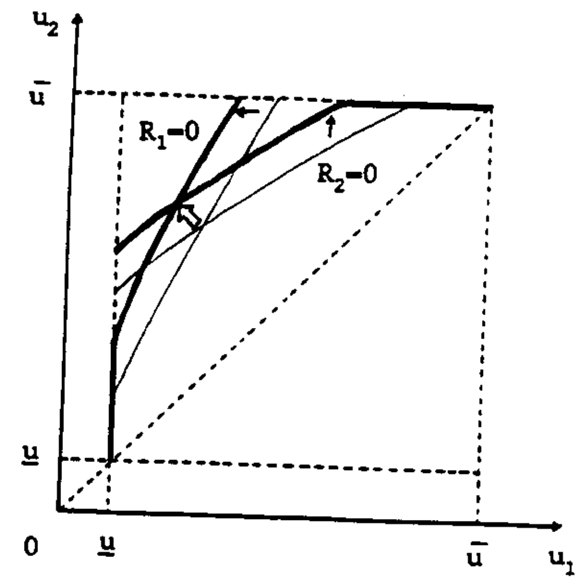

Figure 4: Movement of Quality Equilibrium by a Government Subsidy for the High Quality Good (when $h_2 < 0$)

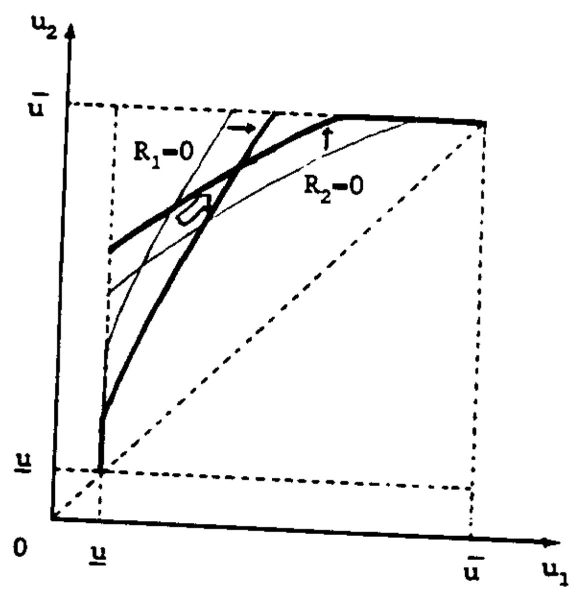

Figure 5: Movement of Quality Equilibrium by a Government Subsidy for the Low Quality Product

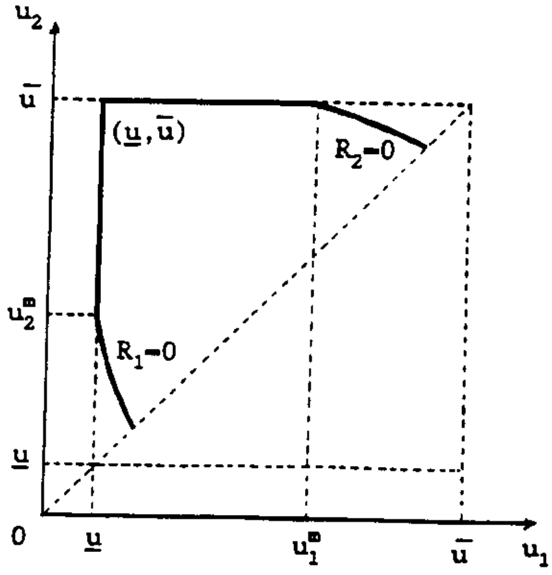

Figure A-1

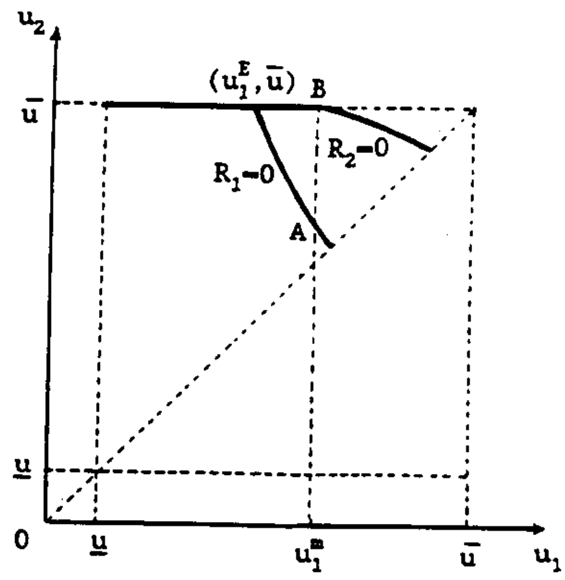

Figure A-2

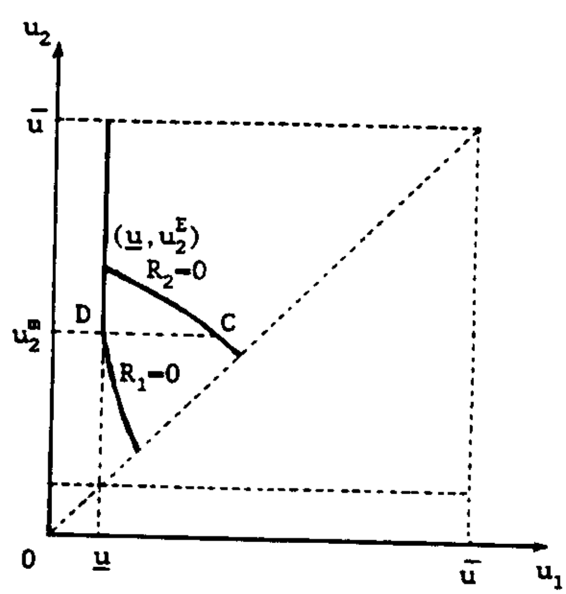

Figure A-3

REFERENCES

Anderson, S.P., de Palma, A and Thisse, J.-F.(1989) "Spatial Price Policies Reconsidered," *The Journal of Industrial Economics*, 38, 1-18.

Archibald, G.C. and Rosenbluth, G.(1975) "The New Theory of Consumer Demand and Monopolistic Competition," *Quarterly Journal of Economics*, 80, 569-90.

Bonanno, G.(1987) "Location Choice, Product Proliferation and Entry Deterrence," *Review of Economic Studies*, 54, 37-45.

Capozza, D.R. and Van Order, R.(1978) "A Generalized Model of Spatial Competition," *American Economic Review*, 68, 896-908.

Capozza, D.R. and Van Order, R.(1980) "Unique Equilibria, Pure Profits, and Efficiency in Location Models," *American Economic Review*, 70, 1046-53.

Cooper, T.(1989) "Indirect Competition with Spatial Product Differentiation," *The Journal of Industrial Economics*, 37, 241-57.

d'Aspremont, C., Jaskold Gabszewicz, J. and Thisse, J.F.(1979) "On Hotelling's Stability in Competition," *Econometrica*, 47, 1045-50.

d'Aspremont, C., Jaskold Gabszewicz, J. and Thisse, J.F.(1983) "Product Differences and Prices," *Economics Letters*, 11, 19-23.

de Palma, A., Ginsburgh, V., Papageorgiou, Y.Y. and Thisse, J.F.(1985) "The Principle of Minimum Differentiation Holds Under Sufficient Heterogeneity," *Econometrica*, 53, 767-81.

de Palma, A., Ginsburgh, V. and Thisse, J.-F.(1987) "On Existence of Location Equilibria in the 3-Firm Hotelling Problem," *The Journal of Industrial Economics*, 36, 245-52.

Dixit, A. and Stiglitz, J.(1977) "Monopolistic Competition and Optimum Product Diversity," *American Economic Review*, 67, 297-308.

Eaton, B.C. and Lipsey, R.G.(1975) "The Principle of Minimum Differentiation Reconsidered: Some New Development in the Theory of Spatial Competition," *Review of Economic Studies*, 42, 27-49.

Forbes, K.F.(1988) "Product Differentiation, Oligopolistic Interdependence, Market share, and Profits," *Economics Letters*, 26, 381-386.

Hotelling, H.(1929) "Stability in Competition," *Economic Journal*, 39, 41-7.

Jaskold Gabszewicz, J., Shaked, A., Sutton, J. and Thisse, J.F.(1981) "International Trade in Differentiated Products," *International Economic Review*, 22, 527-34.

Jaskold Gabszewicz, J., Shaked, A., Sutton, J. and Thisse, J.F.(1986) "Segmenting the Market: The Monopolist's Optimal Product Mix", *Journal of Economic Theory*, 39, 273-89.

Jaskold Gabszewicz, J. and Thisse, J.F.(1979) "Price Competition, Quality and Income Disparities," *Journal of Economic Theory*, 20, 340-59.

Jaskold Gabszewicz, J. and Thisse, J.F.(1980) "Entry (and Exit) in a Differentiated Industry," *Journal of Economic Theory*, 22, 327-38.

Jaskold Gabszewics, J. and Thisse, J.F.(1982) "Product differentiation with Income Disparities: An Illustrative Model," *The Journal of Industrial Economics*, 31, 115-29.

Jaskold Gabszewics, J. and Thisse, J.F.(1986) "Spatial Competition and the Location of Firms," in Jaskold Gabszewicz, Thisse, Fujita M. and Schweizer U., *Location Theory*, Hardwood, Chur (Switzerland).

Judd, K.L.(1985) "Credible Spatial Preemption," Rand Journal of Economics, 16, 153-66.

Kohlberg, E. and Novshek, W.(1982) "Equilibrium in a Simple Price-Location Model," *Economics Letters*, 9, 7-15.

Lancaster, K.(1974) "Socially Optimal Product Differentiation", *American Economic Review*, 65, 567-85.

Lerner, A.P. and Singer, H.W.(1937) "Some Notes on Duopoly and Spatial Competition," *Journal of Political Economy*, 45, 145-86.

Martinez-Giralt, X.(1989) "On Brand Proliferation with Vertical Differentiation," *Economics Letters*, 30, 279-86.

Martinez-Giralt, X. and Neven, D.J.(1988) "Can Price Competition Dominate Market Segmentation?," *The Journal of Industrial Economics*, 36, 431-42.

Neven, D.(1985) "Two Stage (Perfect) Equilibrium in Hotelling's Model," *Journal of Industrial Economics*, 33, 317-25.

Neven, D.(1986) "On Hotelling's Competition with Non-Uniform customer Distribution," *Economics Letters*, 21, 121-26.

Novshek, W.(1980) "Equilibrium in Simple Spatial (or Differentiated Product) Models," *Journal of Economic Theory*, 22, 313-26.

Ordover, J.A. and Willig, R.D.(1981) "An Economic Definition of Predation: Pricing and Product Innovation," *Yale Law Journal*, 91, 8-53.

Osborne, M.J. and Pitchik, C.(1987) "Equilibrium in Hotelling's Model of Spatial Competition," *Econometrica*, 55, 911-22.

Perloff, J. and Salop, S.C.(1985) "Equilibrium with Product Differentiation," *Review of Economic Studies*, 52, 107-20.

Phlips, L. and Thisse, J.F.(1982) "Spatial Competition and the Theory of Differentiated Markets: An Introduction," *The Journal of Industrial Economics*, 31, 1-9.

Prescott, E.C. and Visscher, M.(1977) "Sequential Location among Firms with Forsight," *Bell Journal of Economics*, 8, 378-93.

Salop, S.C.(1979) "Monopolistic Competition with Outside Goods," *The Bell Journal of Economics*, 10, 141-56.

Schmalensee, R.(1982) "Product Differentiation Advantage of Pioneering Brands," *American Economic Review*, 72, 349-65.

Selten, R.(1975) "Re-examination of the Perfectness Concept for Equilibrium Points in Extensive Games," *International Journal of Game Theory*, 4, 22-55.

Shaked, A. (1982) "Existence and Computation of Mixed Strategy Nash Equilibrium for 3-Firms Location Problem," *The Journal of Industrial Economics*, 31, 93-6

Shaked, A. and Sutton, J.(1982) "Relaxing Price Competition Through Product Differentiation," *Review of Economic Studies*, 49, 3-14.

Shaked, A. and Sutton, J.(1983) "Natural Oligopolies," *Econometrica*, 51, 1469-84.

Shaked, A. and Sutton, J.(1984) "Natural Oligopolies and International Trade," in Kierzkowski(ed.) *Monopolistic Competition and International Trade*, Oxford University Press, Oxford.

Shaked, A. and Sutton, J.(1987) "Product Differentiation and Industrial Structure," *Journal of Industrial Economics*, 36, 131-46.

Smithies, A.(1940) "Optimal Location in Spatial Competition," *Journal of Political Economy*, 49, 423-9.

Spence, M.(1976) "Product Selection, Fixed Costs, and Monopolistic Competition," *Review of Economic Studies*, 43, 217-35.

Sutton, J.(1986) "Vertical Product Differentiation: Some Basic Themes," *American Economic Association(papers and proceedings)*, 76, 393-8.

Waterson, M.(1989) "Models of Product Differentiation," *Bulletin of Economic Research*, 41, 1-27.