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A MODEL OF CYCLICAL  
INTERNATIONAL CAPITAL MOVEMENTS

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Abstract This paper introduces fiscal increasing returns to domestic capital into a standard optimizing model of international portfolio selection. The resulting system displays surprising dynamic behavior. Endogenous cycles of any length may occur around the steady state equilibrium, with both capital flows and asset prices displaying regular fluctuations. Initial conditions and expectations both matter in selecting the cycle on which the economy converges. Hence, "animal spirits" may determine the course of asset prices and capital flows.

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## I. Introduction

International capital flows are notoriously cyclical. For example, in the last decade and half capital inflows to Latin America have experienced a boom in the late 1970s, a sharp reversal and net outflows during much of the 1980s, and now a new boom at the start of the 1990s.<sup>1</sup> Domestic investment rates, asset prices and real exchange rates have fluctuated accordingly. One way to interpret this phenomenon is to view flows as reacting to exogenous changes in economic fundamentals. This view, of course, underlies much of existing theory. A different approach might view these fluctuations as part of an endogenous cycle, perhaps not unlike the typical business cycle. This paper provides a simple model that explores and formalizes the latter explanation.

Key to the model is the notion that there is an externality associated with the amount of capital an investor holds in an economy: the larger the number of agents who invest in a given economy, the larger the **ex-post** rate of return will be for each. There are many ways of formalizing this externality;<sup>2</sup> the focus here is on a fiscal externality.

There is a difference in the tax status of capital agents held in the domestic economy and abroad. The latter cannot be taxed (in practice taxation is unenforceable), while the former can. Suppose the tax rate on capital held domestically is set by the local

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<sup>1</sup>For a survey of capital movements in and out of developing countries, see World Bank (1992). For a detailed treatment of the recent boom in capital flows to Latin America, see Calvo, Leiderman and Reinhart (1992).

<sup>2</sup>Dornbusch (1991), Laban (1991) and Rodrik stress the connection between capital flows and the sustainability of a given trade or macroeconomic policy, with larger flows enhancing sustainability and hence increasing the ex-post return. Eaton (1987), on the other hand, stresses the fiscal externality discussed here.

fiscal authorities to ensure that a constant amount of spending is financed<sup>3</sup> --hence, the equilibrium tax rate is inversely related to the size of the tax base. In this setup, domestic capital can enjoy what Blanchard and Summers (1987) label "fiscal increasing returns."<sup>4</sup>

By embedding such fiscal returns in a standard optimizing model --with convex costs associated with movements of resources in and out of the domestic economy-- we obtain two main results. First, there is a continuum of cycles around the steady state equilibrium. Along each of these, the stock of domestic capital and its shadow price display endogenous fluctuations. Such cycles are self-sustaining and need not converge to a stationary position.

Second, the dynamics generated by the model leave ample room for expectations to determine outcomes. That is because this economy displays an unusual kind of multiplicity: there are many equilibrium regular cycles rather than many steady states. As usual, multiplicity involves some degree of indeterminacy. From any initial domestic capital stock there is a subset of all possible cycles to which the system can jump. Which cycle is chosen will depend on "animal spirits". Hence, the system need not only display regular cycles, but may also display sharp fluctuations in response to changes in expectations.

From a technical point of view, the model is an example of a Hamiltonian system of non-linear differential equations. Non-linear systems involving multiple equilibria typically have very

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<sup>3</sup>Assume also that taxes on domestic output are the only source of revenue for the government. Adding other taxes changes nothing as long as part of the exogenous spending must be still financed through the tax on domestic output.

<sup>4</sup>In their view, these are "for macroeconomics, the most important type of increasing returns."

complicated dynamics that cannot be solved for explicitly.<sup>5</sup> By contrast, Hamiltonian systems are a rare case where dynamics have a simple diagrammatic representation. The only other recent economic application of Hamiltonian theory is Matsuyama (1991).

The paper is organized as follows. Section II describes the basic components of the model. Section III develops the associated dynamics, and shows the possibility of cycles as well as the effects of exogenous changes in expectations. Finally, Section IV provides some conclusions.

## II. The Model

Consider an economy with one consumption good and one asset: domestic capital, denoted by  $k$ .<sup>6</sup> Total output is the sum of two components: an exogenous flow  $y$  and a variable flow  $ak$ , where  $a > 0$  is the constant marginal product. The opportunity cost faced by agents is the yield on foreign capital (the world real rate of interest), denoted by  $r$ . We also assume  $a > r$ .<sup>7</sup>

The crucial feature of the model is that domestic output is taxed at the rate  $\tau$ , while the return on foreign capital is always beyond the reach of the taxman. The tax rate  $\tau$  is endogenous but is regarded as given by the individual. It should also be interpreted

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<sup>5</sup>For a general discussion, see Hirsch and Smale (1974), pp. 290-292 and Guckenheimer and Holmes (1983), pp. 180-184.

<sup>6</sup>Here the label "domestic" refers to the location of the capital stock and not to the nationality or place of residence of those who own it. In fact, whether the agents whose behavior is studied in this paper are "foreigners" or "nationals" makes little difference throughout.

<sup>7</sup>This is what one would expect (heuristically) if the home country is "underdeveloped," so that the capital-labor ratio is low and the marginal product of capital is therefore high relative to the world interest rate.

broadly to include the inflation tax or any other means by which the government obtains revenue at the expense of capital held domestically.

In what follows, I focus on the standard portfolio decision faced by agents. If  $z$  denotes net withdrawals from the investment in the domestic economy, the problem of the representative agent is

$$\max \int_0^{\infty} z_t \exp(-rt) dt \quad (1)$$

subject to the budget constraint

$$\dot{k}_t = (y + ak_t)(1 - \tau_t) + g - z_t - z_t i\left[\frac{z_t}{k_t + \frac{y}{a}}\right] \quad (2)$$

where  $g$  are fixed government transfers. The initial capital stock is fixed at  $k_0$ .

It is helpful to define

$$w_t = \left(k_t + \frac{y}{a}\right) \quad (3)$$

This is a variable I shall refer to (somewhat loosely) as the stock of domestic wealth.

Moving resources in and out of the domestic economy is assumed to be costly. The function  $i[az/(ak+y)] = i(z/w)$  captures the cost associated with a given flow. We assume  $i(0) = 0$ ,  $i'(\cdot) > 0$  and  $2i'(\cdot) + (z/w)i''(\cdot) > 0$ . That is to say, the cost is an increasing and convex function of the size of the flow  $z$  relative to (before-tax)

domestic wealth.<sup>8</sup> These costs can be interpreted in two ways. One is as "shoe-leather" or "adjustment" costs associated with moving resources in and out of an economy. The other is as the resources devoted to evading the capital controls imposed by authorities who want to prevent resources from being removed from the domestic economy.<sup>9</sup>

Before solving the maximization problem it is helpful to specify how the equilibrium level of taxes is determined. Assume that the government must finance the exogenous level of transfers  $g$  by using only taxes on domestic output. The government budget constraint is

$$\tau[y+ak] - \tau aw = g \quad (4)$$

Assume further that  $y \geq g$ . This assumption simply ensures that domestic output is larger than government spending (and that therefore  $\tau \leq 1$ ), even as the domestic capital stock becomes very small. Notice that, in equilibrium, all tax revenue is returned to the representative domestic agents via transfers.

Agents maximize

$$J_t = z_t + e_t \left[ (y+ak_t)(1-\tau_t) + g - z_t - z_t i \left( \frac{z_t}{w_t} \right) \right] \quad (5)$$

where  $e$  is the costate variable associated with  $k$ , which can be interpreted as the shadow price of domestic capital. Notice that

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<sup>8</sup>This formulation is analogous to the textbook presentation of adjustment costs in investment. See, for instance, Blanchard and Fischer (1989).

<sup>9</sup>Such controls are extremely common in developing countries. See Kiguel and O'Connell (1992).<sup>5</sup>

maximization by the individual treats  $g$ ,  $\tau$  and  $w$  as given.<sup>10</sup>  
 Necessary and sufficient conditions for a maximum are

$$i' \left( \frac{z_t}{w_t} \right) \left( \frac{z_t}{w_t} \right) + i \left( \frac{z_t}{w_t} \right) - \frac{1}{e_t} - 1 \quad \left( \text{from } \frac{\partial J_t}{\partial z_t} = 0 \right) \quad (6a)$$

$$\frac{\dot{e}_t}{e_t} = r - a(1 - \tau_t) \quad \left( \text{from } \dot{e}_t - r e_t - \frac{\partial J_t}{\partial k_t} \right) \quad (6b)$$

plus the usual transversality condition on  $k$ :

$$\lim_{t \rightarrow \infty} [k_t e_t \exp(-rt)] = 0 \quad (7)$$

Substituting in (6b) the equilibrium  $\tau$  from (4) we have

$$\frac{\dot{e}}{e} = r - a \left[ 1 - \frac{g}{aw} \right] \quad (8)$$

Equation (8) has an obvious interpretation: the after-tax marginal return on domestic capital plus the capital gain from holding the domestic asset must equal the international rate of interest. Notice from (4) and (8) that the after-tax return associated with domestic capital is increasing in  $k$  -- hence, domestic capital exhibits "fiscal increasing returns".<sup>11</sup>

Equation (6a), plus the assumptions we have made on  $i(\cdot)$ , imply a relationship

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<sup>10</sup>This, of course, follows from the fact that the representative individual cannot affect aggregate variables. In equilibrium, aggregate and individual variables are the same.

<sup>11</sup>If the production function were not linear, these increasing returns might give rise to multiple equilibria. That case is examined in Velasco (1991).

$$z = \psi(e)w, \quad \psi'(\cdot) < 0, \quad \psi(1) = 0, \quad \psi(e > 1) < 0, \quad \psi(0 < e < 1) > 0. \quad (9)$$

From (9) we see that resources will be moved in or out of the economy as long as the relative price is different from one. When  $e < 1$  resources will be withdrawn, and viceversa.

Combining (2), (4) and (9) we obtain

$$\frac{\dot{w}}{w} = a - \phi(e) \quad (10)$$

where we have defined

$$\phi(e) = \psi(e) [1 + i[\psi(e)]] \quad (11)$$

Accordingly, the flow  $\phi(e)w$  can be interpreted as the total amount removed from the domestic economy --as a result of a net withdrawal and the resource cost associated with that withdrawal-- at each instant in time. Notice also that

$$\phi'(e) = \psi'(e) \left[ i' \left( \frac{z}{w} \right) \left( \frac{z}{w} \right) + i \left( \frac{z}{w} \right) + 1 \right] = \frac{\psi'(e)}{e} < 0 \quad (12)$$

Equations (9) and (10) represent a system of two differential equations in two variables --the stock of domestic wealth and its shadow price. Steady state conditions are

$$r = a(1-\tau) \quad \text{and} \quad a = \phi(e) \quad (13)$$

Hence, the after-tax return on domestic capital naturally equals the world interest rate. Resource movements  $z$  must be positive as the yield on domestic investment is withdrawn from the domestic economy. Hence, the relative price  $e$  takes on a value between zero and one.

### III. Dynamics

One more step is necessary before we can solve explicitly for the dynamics of the system. Define  $s=\log(e)$  and  $x=\log(w)$ . Then, using this transformation on (9) and (10) yields

$$\dot{s} = r-a+g[\exp(-x)] \quad (14a)$$

$$\dot{x} = a-\phi[\exp(s)] \quad (14b)$$

System (14) is the one we will work with. It has the convenient feature that  $\dot{x}=F(s)$  and  $\dot{s}=G(x)$  --that is, the rate of change of one variable with respect to time depends on the other variable only.

The phase diagram corresponding to (14) appears in Figure 1. The arrows of motion suggest the system is not saddle path stable, as is often the case with perfect foresight models. In fact, as can be easily checked, the system has imaginary roots with zero real parts, a feature which gives rise to unusual dynamic behavior.

We can characterize the full global dynamics of the system.<sup>12</sup> Define the following function H:

$$H = \Phi[\exp(s)] - as + (r-a)x - g[\exp(-x)] \quad (15)$$

where

$$\Phi[\exp(s)] = \int \phi[\exp(s)] ds$$

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<sup>12</sup>Characterizing the global dynamics involves characterizing the behavior of the system on the whole of the Cartesian plane, no matter how far from steady state. By contrast, the usual analysis of non-linear system is confined to local dynamics, typically by working with linear approximations around a steady state. For a detailed treatment of this distinction, see Guckenheimer and Holmes (1983).

H has the property that  $\partial H/\partial s = \dot{x}$  and  $\partial H/\partial x = -\dot{s}$ . Hence, it must be the case that H is constant through time, for  $dH/dt = (\partial H/\partial x)(\dot{x}) + (\partial H/\partial s)(\dot{s}) = (\dot{s})(\dot{x}) - (\dot{x})(\dot{s}) = 0$ . Therefore, any solution to (15) must lie on a level curve of H; or, alternatively, the full phase diagram of (14) is the contour map of H. Such a system is a Hamiltonian dynamical system.<sup>13</sup>

This dynamic structure has two main implications. The first can be summarized in the following proposition:

**Proposition 1:** There exists a continuum of cycles around the steady state equilibrium.

Notice first that the steady state is a global maximum of  $H$ <sup>14</sup>, and there is therefore no path leading to it. The only feasible paths correspond to each level curve of H. Thus, there exist possible cycles of any length!<sup>15</sup> While the system cannot converge to  $k_1$ , it can oscillate arbitrarily close to it. Notice if  $\log(w)$  and  $\log(e)$  oscillate,  $b$  will normally oscillate as well. Hence, the system can display regular cycles in the stocks of domestic and foreign capital held and in the relative price between the two.<sup>16</sup>

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<sup>13</sup>The function H comes from Hamiltonian mechanics. The constancy of H along its level curves is interpreted as the "conservation of energy"; see Hirsch and Smale (1974), pp. 291-292. See also Matsuyama (1991) for an application.

<sup>14</sup>That is because, by construction, first derivatives evaluated at the steady state equal zero and here it is also the case that

$$\frac{\partial^2 H}{\partial x^2} = -g \exp(-x) < 0 \quad \text{and} \quad \frac{\partial^2 H}{\partial s^2} = \phi'[\exp(s)] < 0$$

<sup>15</sup>Of which only one is shown in Figure 1.

<sup>16</sup>For other recent macro models that display endogenous fluctuations, see Drazen (1989) and Diamond and Fudenberg (1989). In both of those cases the cycles arise from a local bifurcation, unlike the situation here. A useful survey of some of the endogenous fluctuations literature is found in Boldrin and Woodford

The second main result is that the role of expectations and of initial conditions in selecting the path followed by the economy can be precisely defined. Notice that while in this case the steady state is unique, there is indeterminacy in the sense that the fundamentals alone cannot determine which one of the many cycles the economy will land on. If initial conditions alone determine which path is feasible, then, in Krugman's (1991) terminology, "history" determines the outcome. By contrast, if more than one path is feasible from given initial conditions, then "expectations" become a factor in determining the outcome.

We must therefore characterize the range of outcomes that is feasible from given initial conditions. Notice first that in this dynamic system  $s$  is a jump variable and  $x$  is a state variable. If the system is to "jump" in response to an exogenous shock to fundamentals or expectations, it must do so vertically keeping  $x$  constant.

Consider the example in Figure 2. From initial conditions at point A the system can jump to any cycle except to those contained in (but not including) the orbit that is tangent to the vertical line at point B.<sup>17</sup> This example can be generalized to any point along the horizontal axis. The result can be summarized in:

**Proposition 2:** To any initial condition  $x_0$  corresponds a subset of all possible cycles around the steady state. From that  $x_0$  the system can jump to any cycle except for those in the subset. If  $x_0$  corresponds to the steady state level, then the system can jump to each and every cycle and to the steady state itself.

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(1990).

<sup>17</sup>That is to say, the system cannot jump to any cycle within the shaded region in Figure 2.

Hence, from any starting position there is an infinite number of possible cycles the system can land on. Notice that only expectations determine which one of the cycles reachable from given initial conditions will be chosen. Moreover, if animal spirits are volatile and shifts in expectations change the course of the system, the stocks of foreign and domestic capital and their relative price will be volatile as well.<sup>18</sup>

Notice that while such "animal spirits" may be arbitrary, there is nothing irrational about them. Expectations of future returns are always governed by fundamentals, for these expectations become self-fulfilling. In this sense, the indeterminacy observed here is very different from that present in bubbles. Notice also that a government can attempt to coordinate expectations on a given equilibrium. Since such expectations are self-fulfilling, an announcement should be enough.

#### IV. Conclusions

This paper formalizes one mechanism --a fiscal externality-- that can give rise to increasing returns to domestic capital. Such an externality affects the decision faced by agents choosing whether to hold capital in the domestic economy or abroad.

The model shows that this externality has important implications for the dynamic behavior of asset prices and capital flows. The system displays a continuum of possible cycles around the unique steady state. Furthermore, changes in expectations can move the system across equilibrium cycles. If expectations are volatile, so will be asset prices and capital flows.

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<sup>18</sup>These shocks to expectations would have to be fully unanticipated, however. Otherwise, anticipations of a future change would affect the current behavior of the system.

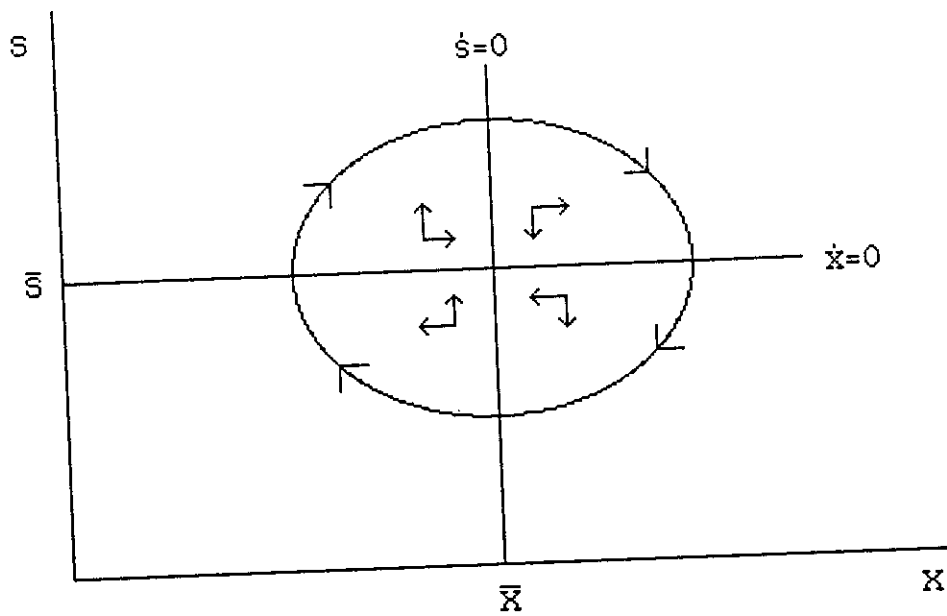


Figure 1

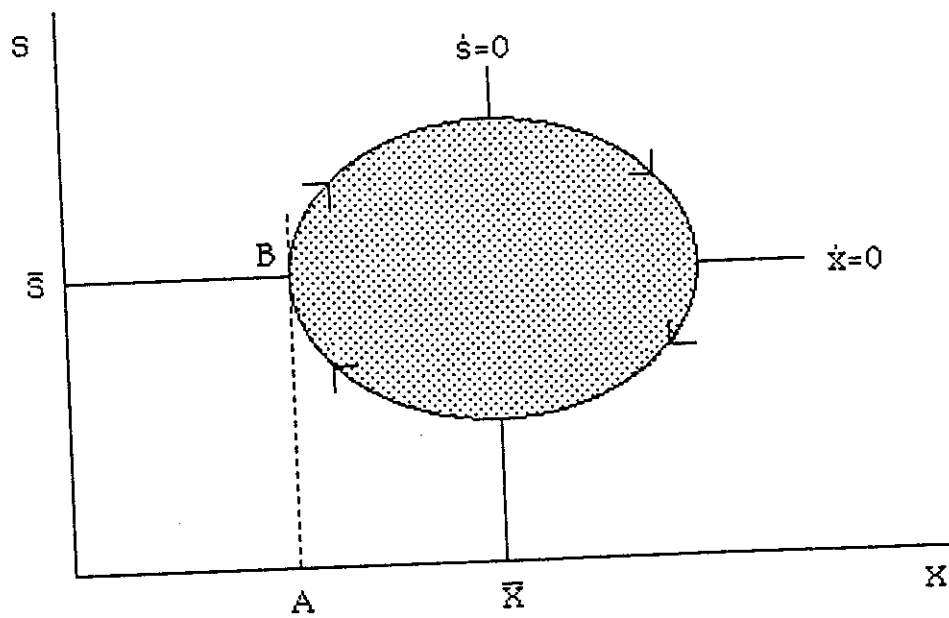


Figure 2

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