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ABSTRACT

In the past, seasonal variation in macroeconomic time series data has often been treated as a purely "statistical" problem. The view prevailed that a proper understanding of seasonality would enable researchers to eliminate its effects more efficiently and thereby facilitate the analysis of "business cycle" frequencies.

An alternative view, advocated by Ramsey (1992), treats the underlying dynamic model as a high frequency oscillator with relatively stable coefficients. Such a model may incorporate "parameter drift" in that the model parameters can be allowed to change slowly over time. The benefits of such an approach include improved interpretation and understanding of the relationships which give rise to the model, as well as insight as to possible causes of observed changes to the system over time.

Ramsey (1992) demonstrated that the time paths of economic variables can be accurately characterized using a simple harmonic oscillator model. The present paper investigates the forecasting properties of this class of model with respect to time series for U.S. durable and non-durable goods production.

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INTRODUCTION

A number of recent studies have shown a new appreciation for the importance of seasonal variation in the analysis of macro-economic and financial time series data. In the past "seasonal variation" has tended to be treated as a purely "statistical" problem. The view prevailed that a proper understanding of seasonality would enable researchers to eliminate its effects more efficiently and thereby facilitate the analysis of "business cycle frequencies". Two approaches have emerged. One is represented by Tiao and Grupe(1980), Osborn (1991) who have modelled seasonal factors as dynamic processes; more precisely seasonal variation has been introduced by specifying that the parameters of a standard ARMA model vary seasonally. Such models are characterized by "slowly varying variable values with fast varying parameters". Appropriate choices for the model specification for such processes can produce periodicity of any desired frequency. However, such models have their own dynamic characteristics and will in general differ substantially from more traditional time series models. Franses (1991), for example, has analyzed the effect of the specification error introduced when a multiple differencing approach to estimation of a time series is used when the data are in fact generated by a seasonally varying parameter model. In another important paper, Boswijk and Franses developed tests for seasonal variation in the mean growth rate, or trend term.

In these models high frequency oscillations are viewed as the result of the interaction between slowly moving variables and rapidly varying parameters. The specification of such models poses not only difficulties for estimation, but also for the interpretation of the implicit dynamics of the model. The former difficulty stems from the fact that all models are estimated by "averaging" in that rapidly varying "errors" are averaged over slowly varying values for the variables. The Osborn approach has been fairly successful from the estimation perspective because the rapidly varying coefficients have been seasonal and can therefore be averaged over many repetitions of the seasonal cycle.

An alternative view of the problem is provided by Ramsey (1992). In this view, the underlying dynamic model is high frequency, seasonal for example, with relatively stable coefficients relative to the rate of oscillation of the dynamic model. Such a model can also incorporate "parameter drift" in that the model parameters can be allowed to change slowly over time. However, for such changes to yield a moderately simple model of oscillations that is interpretable and estimable, it is important that the varying parameters change sufficiently slowly. The changes should in fact be adiabatic; that is, one should be able to model the data for "short periods" as if the parameters were constants. If the parameters vary at the same rate, or even faster than the variables of the system, one has created a very complex dynamical system that is both difficult to analyze and to estimate efficiently. These problems stem from the interaction between the fast varying parameters and the equally fast varying variables. The literature on seasonally varying parameters has in part managed to avoid the worst of these difficulties by capitalizing on the cyclic component of the variation.

There is another aspect that separates the basic viewpoint espoused in this paper and in Ramsey (1992) from that contained in the "rapidly varying parameter" approach. The fundamental insight that underlies the former approach is that economic data, like data in almost all other fields of study, involves dynamical processes that conform to the basic principles of dynamic interaction. Displacements lead to reactive acceleration, an effect on the second derivative, that is usually damped and the dampening is a function of the first derivative. In some cases the forcing terms may be sinusoidal. Short term dynamical systems that are open, as is the case in most economic situations, are subject to parameter drift in that the variations of variables outside the system being modelled will affect the behavior of the model over intermediate time scales. If the variation of the "external variables" is of the same time scale and order of magnitude as the variation within the system being modelled, then the model has to be expanded to include the other rapidly varying variables. To do otherwise, is to guarantee failure in one's modelling strategy.

Model types such as those proposed in Ramsey (1992) when compared to sets of seasonal dummies, provide important insights into some neglected aspects of forecasting. Imagine that, as is true for economic production indices, data are characterized by a twelve period cycle. Ignoring parameter drift for the moment, it is immediately apparent that perfect forecasts can be obtained by estimating the values of the twelve seasonal dummies and projecting their values into the future. In terms of mean square error and similar criteria, such a forecast cannot be bettered. One might then question the benefit of a phenonenologically motivated model of dynamics, such as that presented in Ramsey (1992). The benefit lies in the interpretation and understanding provided by the phenomenologically guided approach. A further benefit is that when the system is changing over time, the latter approach provides insight and guidance to the analysis of the possible causes of the change. For example, in the seasonal dummies approach, if the parameters are shifting over time, there is little one can do but record the shift. But in the alternative approach, precise statements about the nature of the change in the dynamics can be made; for example, that the effect of dampening has lessened, or that the effect of a particular forcing term frequency has increased. With this information, the analyst can begin to investigate further and explore the theoretical reasons for the change.

In Ramsey (1992) it was demonstrated that the time paths of economic variables can be accurately characterized using a simple harmonic oscillator model. In that paper it was shown that a single class of models containing an oscillatory component that is forced by a specific set of frequencies can be used to describe the short term time path of indices of production for durable and non-durable goods. Moreover, it was demonstrated that, up to parameter drift, this class of models holds over the entire recorded history of the two series. Subsequent work showed that the same qualitative results were obtained for OECD definitions of consumer goods production for the U.S., U. K., Germany, France, and Italy.

The present paper explores the forecasting properties of the linearized version of this model. The inclusion of a time varying constant to account for non-stationarity is the only structural modification made.

The Ramsey (1992) paper revealed evidence of parameter drift over the entire sample period, although the rate of drift was relatively slow in most cases and the variance estimates for all the coefficients were remarkably stable over long intervals of time. This paper presents sixty-four month forecasts of growth rates and of the corresponding index levels for both series. Two sets of forecasts are made, one set estimates the coefficients over a prior period to the forecast and then treats the estimated coefficient values as constants for the forecast. The second set uses a window estimation procedure to forecast the parameter values themselves.

Both sets of forecasts seem to be reasonable in terms of capturing the overall dynamic quality of the series and in predicting the turning points, that is the peaks and troughs in the actual growth rates. The forecast index levels track the actual levels well for a while, but then diverge, although the phase of the oscillations is maintained. This divergence between the forecast levels and the actual indicate that the drift in the parameters that prevailed during the estimation period differs from that which prevailed during the latter half of the forecast period.

MODEL SELECTION

The principal variable of interest in the previous paper by Ramsey (1992) was the growth rate, that is the relative first difference of the production index; details are in Ramsey (1992).

The model specification that was obtained in Ramsey (1992) is:

$$\ddot{u}_t + \alpha \dot{u}_{t-1} + [\sum_{i=1}^{4} \beta_i \hat{u}_{it}] = \sum_{i=1}^{4} [a_i \cos(\omega_i t) + b_i \sin(\omega_i t)] \qquad (1)$$

where:

$$\begin{aligned} \hat{u}_{it} &= [\gamma_{1t}u^+, \gamma_{1t}u^-, \gamma_{2t}u^+, \gamma_{2t}u^-] \\ \gamma_{1t} &= 1;\ \textit{if mth} = \textit{Sep-Feb};\ 0\ \textit{Otherwise}; \\ \gamma_{2t} &= 1;\ \textit{if mth} = \textit{Mar-Aug};\ 0\ \textit{Otherwise}; \\ u^+ &= pos(u_t) \\ u^- &= neg(u_t) \end{aligned}$$

The essential idea that is captured by this formulation is that the change in acceleration, that is, the change in the value of the second derivative depends on both the sign of the growth rate and on the period of the year in which the effect occurs. This specification allows the acceleration reaction to differ depending on whether growth rates are positive or negative, so that if they differ, then one can conclude that the process is time irreversible, even in the absence of the linear dampening term that itself produces time irreversibility. The right hand side of equation (1) is the sinusoidal forcing term.

Although this nonlinear differential equation seems to apply to both data sets, a closed form analytical solution to the equation is not available. A simple linearized version that is used in this paper and was analyzed theoretically in Ramsey (1992) is:

$$\ddot{u}_t + \alpha \dot{u}_{t-1} + \beta u_t = \sum_{i=1}^{4} [a_i \cos(\omega_i t) + b_i \sin(\omega_i t)] \tag{2}$$

This equation is easily solved using Laplace transforms, see for example Strang (1986), Ramsey (1992). The specific definitions for the growth rate and for the time derivatives are:

$$\begin{aligned} u_t &= \frac{x_t - x - t - 1}{x_{t-1}} \\ Du_t &= \frac{u_{t+1} - u_{t-1}}{2} \\ D^2 u_t &= \frac{Du_{t+1} - Du_{t-1}}{2} \end{aligned}$$

By substituting these definitions into equation(2) we obtain the reduced form equation in terms of the growth rates themselves:

$$\begin{aligned} u_t = \; & \eta + [2(1-\alpha) - 4\beta] u_{t-2} - (1-2\alpha) u_{t-4} \\ & + 4\sum_{i=1}^{4} [a_i \cos(\omega_i (t-2)) + b_i \sin(\omega_i (t-2))] \end{aligned} \tag{3}$$

This reduced form equation provided the basis for the forecasts for both durable and nondurable goods that are to be studied in this paper. One further modification in this paper is the addition of a constant term to the forcing term to account for the non-stationarity of the actual growth rates. Thus, the total number of parameters to be estimated is 10 or 11 depending on whether the constant term is present or not.

Forecasts for the index levels were calculated by:

$$X_{t_0+k} = X_{t_0} \prod_{t=1}^{k} (1+\hat{u}_t) \tag{4}$$

where $\hat{u}$ is the forecasted growth rate.

Two types of forecasts were generated. First, the parameters were simply estimated over a 120 month period. These estimates were treated as fixed and forecasts were generated accordingly. Secondly, the parameters were estimated using a window technique whereby a sixty month window was estimated for a given fixed time slice. This was repeated 180 times. Each time slice dropped the first observation and added the next. By these means we obtained estimates of the drift path for each parameter. These paths were then smoothed and used to extrapolate the drift over the forecast period. Forecasts were then generated using parameters with drift.

Some summary statistics are presented to provide various measures of the accuracy of the forecasts. The sum of squared differences between the forecasted growth rates and actual growth rates, divided by the number of observations, were calculated for the first 32 months, separately for the second 32 months, and for the whole forecast period. These are labeled as MSQE 1, MSQE 2, and MSQE 3, respectively in Tables 1A and 1B. The corresponding variances of the actual growth rates during the forecast period are presented. We also calculated the ratio of the mean squared errors to the observed variances of the growth rates to obtain an units free measure of the quality of the forecast.

An alternative measure of accuracy which we present are the turning point ratios. These calculate the percentage of relative highs (peaks) or relative lows (troughs) occurring in the actual data that were matched by corresponding peaks or troughs in the forecast series. These ratios were calculated separately with respect to the relative minima and maxima, as well as for all turning points. The results are presented in Table 2.

DATA

Two data series were used. The data are the non-seasonally adjusted U.S. production indices for consumer durables and consumer nondurables, further details are in Ramsey (1992). The data are monthly and run from January 1919 to February 1993. Three observations were lost at the beginning and two at the end of the series because of the calculation of the derivatives of the growth rates.

The Empirical Results

Figures 1 and 2 show the raw production indices for durable goods and for nondurable goods as well as a plot of the growth rate for each index in the inserted subplot. The nonstationarity of both data series is clear, both in terms of the recorded indices and in terms of the growth rates.

The nonstationarity of the data and the corresponding idea of a slow shift in the underlying dynamics is illustrated in Figures 3A, 3B, 4A, and 4B. The first two figures enable the reader to compare the phase space diagrams for the durable goods index between two separated time periods. The phase space plots show the variation in the variable pair (D^2u_t, Du_t), where D^2u_t and Du_t are the second and first derivatives respectively of u_t, the growth rate. A careful examination of these plots shows that between the two periods indicated that there has been a substantial shift in the dynamics of the process generating the data. Similar conclusions have to be drawn from an inspection of Figures 4A and 4B for the phase space plots of the nondurable goods index.

As we indicated in a previous section and in Ramsey (1992), the presence of a changing dynamical process, or the presence of nonstationarity, leads to substantial difficulties for forecasting. Forecasting is typically "time local" and implicitly assumes some form of stationarity, or at least regularity in the series under examination. Reexpressing the matter, any forecast must assume that the mechanism generating the data and as analyzed by the forecaster remains applicable throughout the forecast period. This is not to say that change cannot be handled, but that the change must be modelled as an integral part to the forecast.

Typically, forecasts have been local in the following senses. Forecasts are usually local in that the forecast horizon is short, a few quarters, or a year or two at most. Further, the forecast usually presumes that the coefficient values that were estimated from the most recent data available will continue to hold during the forecast period; that is, the estimated coefficients are treated as constants notwithstanding their long term observed changes. This procedure ignores potentially relevant information from the more distant past. For example, one might incorporate into one's forecast bounds on the outcomes that stem from the behavior of the model over the entire period of observation; such forecasts would incorporate what we call "global constraints", see Ramsey (1992).

The results to be presented below go part way towards addressing this issue. We will compare the results obtained when the model coefficients are "forecast" as constants as is the usual case to the situation in which the coefficients are themselves forecast, even if by crude extrapolative methods. To some extent "global constraints" are imposed as recommended in Ramsey (1992) in that the basic model that has been discovered to hold, at least approximately, for the entire recorded history of the indices will be presumed to continue to apply. What is not assumed in the second version of the analysis is that the constituent coefficients are constant. Indeed, since all the evidence indicates that the coefficients are continuously, albeit slowly, changing over time, that information is incorporated in the modelling of the data.

The basic procedure followed in each case was to estimate the model summarized in equation(3). The forecasts were obtained by inserting into the model the values of the estimated coefficients and the values of the previously forecast growth rates, after the initial conditions used to begin the forecast process; that is, the first four forecasts used at least one value of the observed growth rate, but that thereafter all forecast values depended entirely on the prior forecasts. There was no updating of the coefficients and of the growth rates using information beyond the last observation of the estimation period.

Two sets of variables were forecast. The first is the growth rate and the second is the actual level of the index that was obtained by "integrating back" from the definition of the growth rate. This marks an important improvement over the results quoted in Ramsey (1992), wherein the fits were computed with respect to the values of the dependent variable, D^2u_t. In this paper, we use the expression in equation(3) to obtain by transformation the forecasts for the growth rate, u_t, and by the expression in equation(4) the forecast values for the index itself. These transformations place a considerable potential burden on the accuracy of the estimation process, since the transformation from D^2u_t to u_t involves two integrations and that from u_t to the index one more integration.

For the durable goods index three sets of results were obtained. First the linearized version of the model summarized in equation(2) was fitted and used to forecast the next sixty four months. This model assumes stationarity and consequently does not incorporate a constant term in the regression. The second version incorporates a constant term in the regression of D^2u_t on Du_t and u_t as well as the sinusoidal forcing terms. This inclusion allows for a positive average growth rate in addition to the cyclical variation in the data. The third procedure allowed for the variation in the values of the coefficients themselves. This procedure was accomplished by fitting windowed regressions, each of sample size 60, for a sequence of 180 "time slices" to the observed data; each "time slice" was obtained from the previous slice by dropping the first observation and adding to the last. The 180 by 11 matrix of coefficient values so obtained was smoothed by a spline smoothing procedure and the next sixty-four coefficient forecasts were obtained from the smoothed values by linear extrapolation of the splined smooth. Thus, the input to the forecast for the growth rates includes a 64 by 11 matrix of predicted coefficient values. The windowed estimation period is from August 1967 to July 1987.

Figures 5A and 5B compare the results from forecasting the index without and with a constant term in the regression of equation(3). The inclusion of the constant term allows for a steady state growth rate in the index; the constant is clearly needed to forecast levels. A comparison of Figures 5A and 5B indicates that the nonstationary model, that with the non-zero growth rate, indicates very clearly that in late 1991 there was a substantial drop in the average level of the index, even though the cyclic pattern was in large part maintained. The potential policy implications are both obvious and important.

Table 1 summarizes all the results from the calculation of mean square error comparisons for both indices. Table 2 summarizes the results for the "turning point" comparisons. A graphical representation of the mean squared error comparisons for both indices is in Figure(13). All the comparisons are for the forecasts of the growth rates, u_t. The comparisons are made for the first and second halves of the forecast period as well as for the whole period. This provides some idea of the potential for degradation of forecast success over the forecast horizon and facilitates the analysis of the source of any differences found in forecast accuracy.

Table 1A

DURABLE MEAN SQUARED ERRORS

	NO CONSTANT NO DRIFT	CONSTANT NO DRIFT	CONSTANT DRIFT	SEASONAL DUMMIES
MSQE 1	.00020625	.00020004	.00044085	.0001707
MSQE 2	.00024431	.00025058	.00041233	.0002614
MSQE 3	.00022393	.00022465	.00042497	.0002142
VAR 1	.00062290	.00062290	.00062290	.0006229
VAR 2	.00065732	.00065732	.00065732	.0006573
VAR 3	.00063806	.00063806	.00063806	.0006380
RATIO 1	.3311279	.3211473	.7077435	.2740000
RATIO 2	.3717678	.3812222	.6272968	.3259000
RATIO 3	.3509523	.3520891	.6660429	.3357000

Table 1B

NONDURABLE MEAN SQUARED ERRORS

	NO CONSTANT NO DRIFT	CONSTANT NO DRIFT	CONSTANT DRIFT	SEASONAL DUMMIES
MSQE 1		.00030043	.00061382	.00006889
MSQE 2		.00051697	.00066423	.00010413
MSQE 3		.00031192	.00064379	.00008591
VAR 1		.00075203	.00075203	.00075203
VAR 2		.00073807	.00073801	.00073801
VAR 3		.00072045	.00072045	.00072045
RATIO 1		.3995097	.8162173	.09160741
RATIO 2		.7005075	.9000282	.14111111
RATIO 3		.4329586	.8936026	.11870000

The mean squared error results shown in Table 1 deserve some considerable comment. The mean squared error ratio is the ratio of the observed mean squared error to the observed variance of the actual data. This provides a units free measurement of the degree of approximation that is provided by the model. The ratio depends not only on how the fit changes over the forecast period, but also on the possible change in the variance of the data themselves. Obviously, this consideration is most important when the forecast data are clearly nonstationary. For the durable index, the variance of the data increase slightly over the second half of the forecast period. The mean square error ratio increased during the second period, but not substantially so. At first sight, this modest increase in the growth rate mean square error is seemingly inconsistent with the clearly substantial degradation in the forecastability of the levels. However, this result is an important example of the magnification of the effects of small changes in the growth rate on the change in the levels. The average mean growth rate declined after mid 1990. While this will not necessarily have a noticeable effect on the overall cyclic fit of the model to the growth rates, it has a substantial effect on the fit of the levels.

This result provides a striking example of the benefits of the forecasting approach that we are advocating in this paper. Time slice plots of the coefficients over this period indicate very clearly that there was a dramatic change in May of 1990, whereas there was no noticeable difference in the levels until the beginning of 1991. Over the sixty months that cover this period, the constant term that represents the instantaneous growth rate, the coefficient on the forcing term, and the coefficient on the 2.4 month cycle all decreased in magnitude, whereas the coefficients on the dampening term, the two cycle per year, and the four cycles per year all increased; the coefficient on the three cycles per year declined and then rose steeply. In May of 1990, the slopes increased dramatically. The effect of this change on the model coefficients, as opposed to the cyclic terms, that is, the effect on the constant term, alpha and beta, was to reduce the growth rate and the degree of variation of the growth rate. Dampening increased, the response to a shock decreased, and the mean growth rate declined. The effect of the cyclic coefficients was to increase the weight on the three and four cycles and to decrease the weight on the other two.

Returning to the main development, the application of an allowance for drift in the coefficients during the period prior to the forecast period, that is, from August 1987 to November 1992, is shown in Figures 6-8. The windowed estimation period is from August 1967 to July 1987. Figure 6 shows the forecast variation in alpha and beta; noticeable, but not striking. The effect of these small changes is demonstrated on the growth rates in Figure 7 and on the levels in Figure 8. In comparing the results with those in Figures 5A and 5B, one observes little difference with respect to the growth rates, but a noticeable difference with the levels. The forecasts without allowance for the change in the coefficients tends to under forecast the levels and the allowance for historically observed drift tends to over estimate the levels up to the dramatic decline beginning in early 1991. The actual degree of change in the coefficients lies between these two extremes.

The relative mean square error values cited in Table 1 seem to be very bad. However, this use of the mean square error concept is not as unambiguous as one naively suppose. Mean square error differences make the most sense when the errors are nearly orthogonal to the axis of the forecast line, or the axis line of the values of the variable being forecast. As the axis line becomes increasingly more steep, the justification for minimizing error sums of squares parallel to the "y axis" makes less and less sense; what is preferable in this case is minimization orthogonal to the local axis of the curve being fitted, see, for example, Malinvaud(1966, pp.7-12). The mean squared error results that are reported in Table 1 suffer dramatically from this problem. If one examines the plots of the actual and forecast growth rates shown in Figure 7, it is clear that notwithstanding the observed large values for the mean squared errors, the "fits" are remarkably good form a visual perspective. There are two problems. The first is that the forecast growth rates underestimate certain months. The second problem is that one of the observed major peaks in the cycle is off phase by one month; this gives rise in the next month to a very large contribution to the mean squared error in that a peak is being matched to a trough.

Table 2

TURNING POINT RATIOS

	DURABLES NO CONSTANT NO DRIFT	DURABLES CONSTANT NO DRIFT	DURABLES CONSTANT DRIFT	NONDURABLES CONSTANT DRIFT
MIN	.6111	.6111	.7222	.6666
MAX	.5555	.5555	.5555	.5555
ALL	.5833	.5833	.6388	.5833

Table 2 presents the extent to which the forecast matches the turning points in the data. The ratio shown in Table 2 is the proportion of forecast peaks or troughs that match an actual peak or tough. An examination of Figure 7 indicates that except for a one month phase shift in the peak month the matches are in fact remarkable. One can with some justification claim that missed peaks, or toughs, are due to random highly transitory shocks to the actual data; see, for example, the period in early 1989.

Figures 9-14 provide similar information with respect to nondurable goods index as that presented for the durable goods index in the Figures already discussed. The same comments apply. However, there are some qualitative differences. Allowing for parameter drift improves the quality of fit for the forecast of the growth rates; the timing and seasonal pattern are correct, but there is one component of the year in which the forecast seriously underestimates the data. The reader will recall that these forecasts have all been calculated with respect to the linearized version of the model discussed in Ramsey(1992), the addition of the nonlinear terms would allow one to capture more of the fine detail in the series. Comparing Figures 10 and 14, one sees that the allowance for drift improves the performance of the forecasts quite significantly,even though the early period is substantially under forecast in both cases.

Figure 13 illustrates clearly the effect of the variation in phase on the size of the mean squared error; a few mistimed observations contribute overwhelmingly to the total mean squared error. It is interesting that with but two exceptions, the extreme values for the squared errors occur at different points for durable and nondurable indices.

The last task is to compare the results obtained above on the basis of a phenomenologically useful equation to the fitting of twelve seasonal dummies. Given that we dealing with a dynamic mechanism with a period of twelve months, then for the inter cycle fitting of the data, no model can beat the fit of twelve seasonal dummies. It is interesting to note that the nature of the major errors of forecast based on the twelve dummies are the same as for the oscillator model; see, for example, the errors of forecast for early 1989 shown in Figure 15 and compare those errors to the forecast and actual growth rates shown in Figure 7. Correspondingly, Figure 16 can be usefully compared to Figure 5B. Similar comments apply to the results for the nondurable index forecasts. As a final comment, a comparison between Figure 19 showing the squared errors for the fit of the twelve dummies and Figure 13 that shows the same calculations for the model forecasts, indicates that the outliers are at different points in time and that with one exception in nondurable goods the orders of magnitude are the same.

SUMMARY AND CONCLUSIONS

The work reported in Ramsey (1992) demonstrated that the model described in equation (1) provided, up to parameter drift, a stable relationship between D^2u_t, Du_t, u_t, and a set of four forcing frequencies for both durable and nondurable goods production indices. These results have subsequently been confirmed by the application of the model to an OECD classification of consumer goods production for the U.S. and four European countries.

This paper extends those results by examining the forecasting implications of the model for the growth rates and for the levels of the indices for U.S. durable and nondurable goods production.

The previous paper indicated that careful attention would have to be paid to the obvious elements of nonstationarity that are present in the data. The effect of nonstationarity is potentially far greater in this paper because we are essentially forecasting the second and third integrals of the variable that was forecast in the previous paper; that is, the effects of nonstationarity are often far less in the context of analyzing the time paths of derivatives than is the case when analyzing levels.

Equations (3) and (4) provided the basic forecasting equations for the growth rate and for the levels. Equation (3) contains a constant term to represent the small, but nonnegative, average growth rate that prevails in the context of the seasonal variation in the growth rates over the year.

Four sets of comparative estimates were made for the durable goods index and three for the nondurable goods index. The four alternatives were to forecast the growth rates using no constant, no drift in the coefficients; a constant, but no drift in the coefficients, a constant with drift allowed in the coefficients, and the use of twelve dummies.

The forecasts were all based entirely for the whole forecast period on historical data. There was no updating of the time series and no updating of the coefficients using information beyond the estimation period.

For the forecast period examined,that is, from August 1987 to November 1992, based on coefficient estimates from data for the period July 1977 to July 1987 for the "no drift" case, or for the period July 1967 to July 1987 for the "drift" case the following conclusions can be drawn. We stress that much of the detail in the analysis of the results depends in large part on the periods of time chosen. However, it is fortuitous that the periods chosen were most revealing for the difficulties involved in this type of analysis, even though the results seem at first sight to be modest in their forecast accuracy.

It is clear that the periods chosen for the estimation and for the forecast differed in terms of their dynamics. Interestingly, in no case did the estimation of the drift component of the "constant" term indicate a significant drift in that coefficient. Consequently, the striking change in the levels beginning in 1991 is due almost entirely to the effects of the terms in the dynamical part of the equation; in short, on the alpha and beta coefficients. This experience and the comparison between the drift and non drift results indicates that the estimation period could not forecast the whole of the forecast period, since the drift of the coefficients past the estimation period was essentially different, beginning in early 1991. The constant term, no drift, model under forecasts, while the drift forecast tends to over forecast.

Notwithstanding the apparent lack of performance compared to the dummy estimates for the growth rates, we indicated that much useful information about the change in the dynamics of the system was obtained from the phenomenologically driven model. Our results form the basis for an informed investigation of the causes for the downturn in the durable goods index of production. The first questions are what caused the increase in the dampening and why should the beta coefficient decrease at the beginning of 1991?

A close examination of the turning point analysis indicates that the facts seem to be more in favor of the model than at first sight seems to be the case. Overall, the growth rate predictions seem to track the data very closely. We recognize that we are using the linearized model and from Ramsey (1992) know that means that the fine scale variation has been ignored.

The main difficulty in forecasting the levels, in particular, is that we have neither model nor understanding of the time path of the coefficients. Notwithstanding this caveat, our understanding of the system up to parameter drift is, we would hazard, much improved. However, with these results we are now in a position to investigate the "business cycle" questions that are implicitly posed by our empirical findings with respect to the time path of the coefficients.

The initial presumption that the indices could be modeled by a dynamic model with slowly varying coefficients seems to have been supported.

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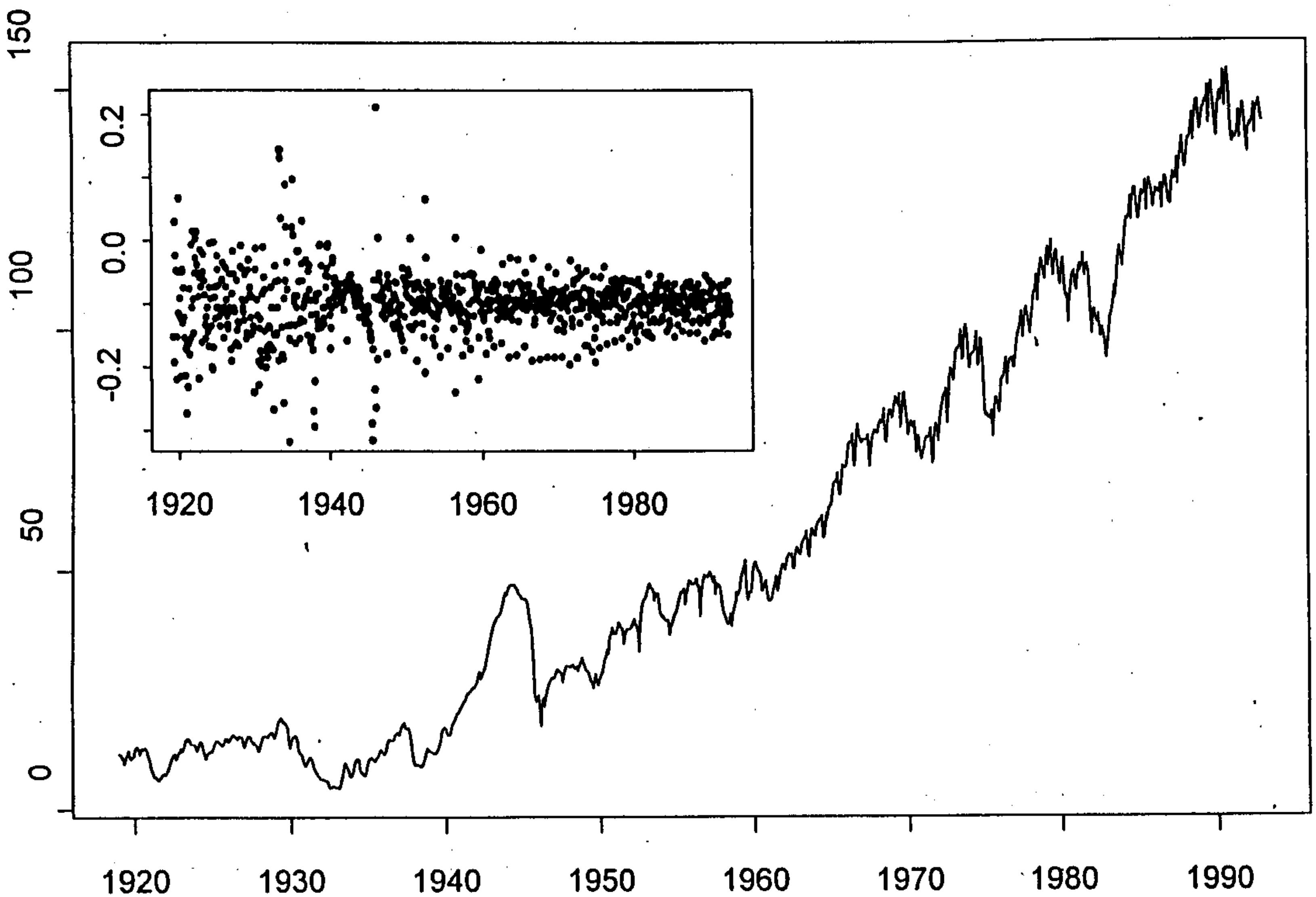
Figure 1

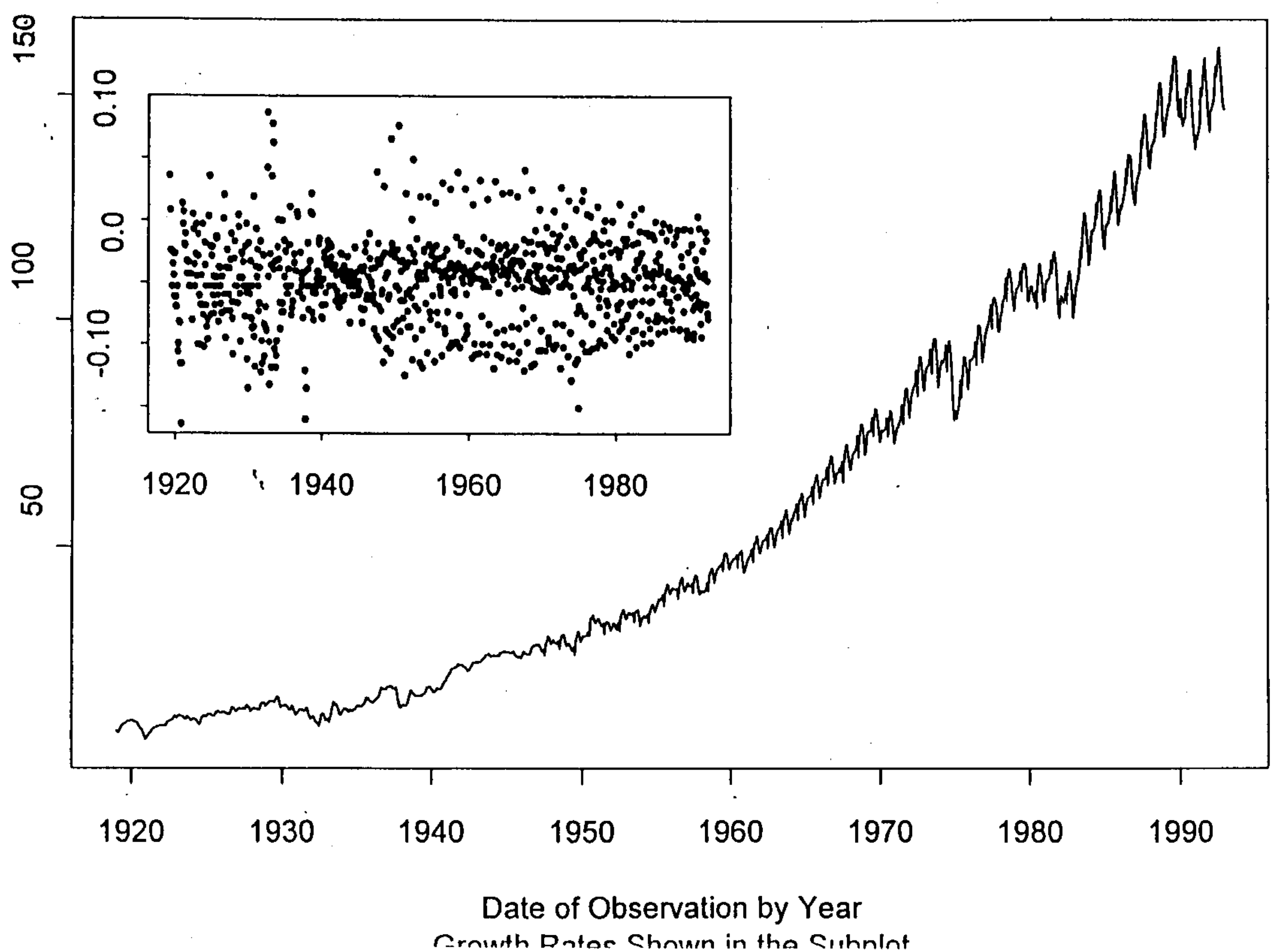
TIME SERIES FOR THE DURABLE GOODS INDEX:1919 to 1992



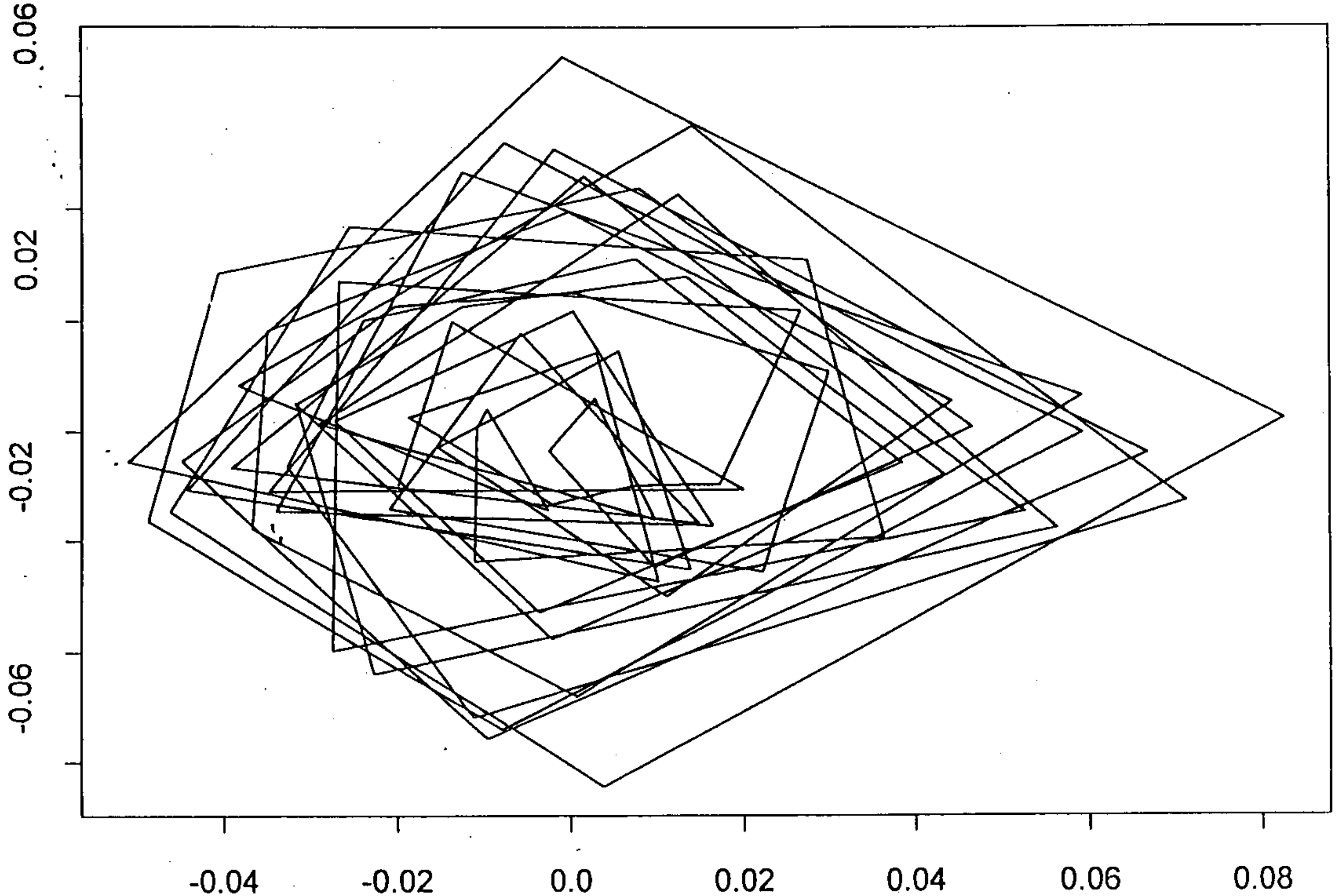
Date of Observation by Year
Growth Rates Shown in the Subplot

Figure 2

TIME SERIES: NONDURABLE GOODS INDEX:1919 to 1992



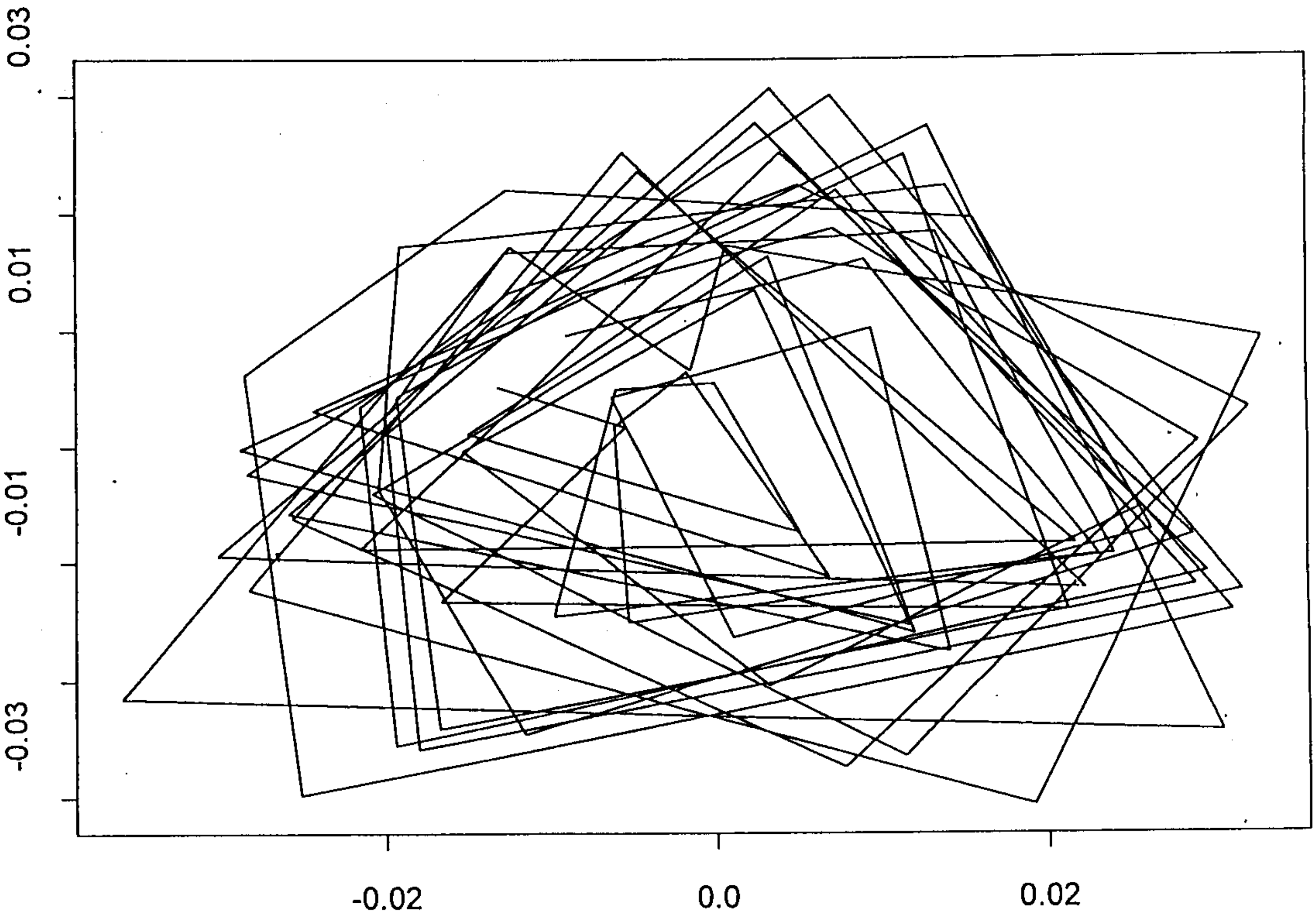
PHASE SPACE DIAGRAM: DURABLE GROWTH RATE



FIRST DERIVATIVE: DURABLE GROWTH
April,1969 to April, 1976

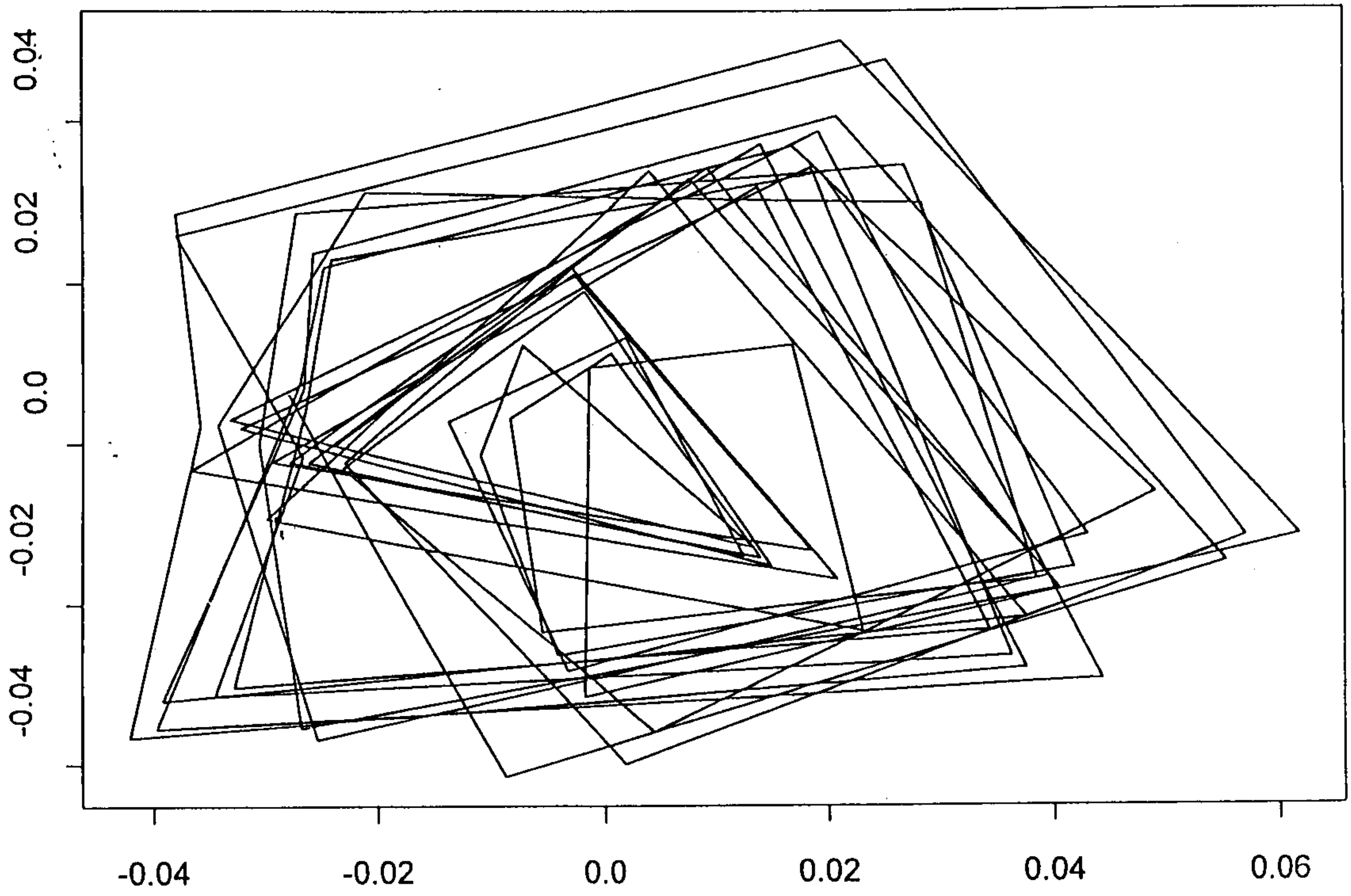
Figure 3B

PHASE SPACE DIAGRAM: DURABLE GROWTH RATE



FIRST DERIVATIVE: DURABLE GROWTH
Dec. 1985 to Nov. 1992

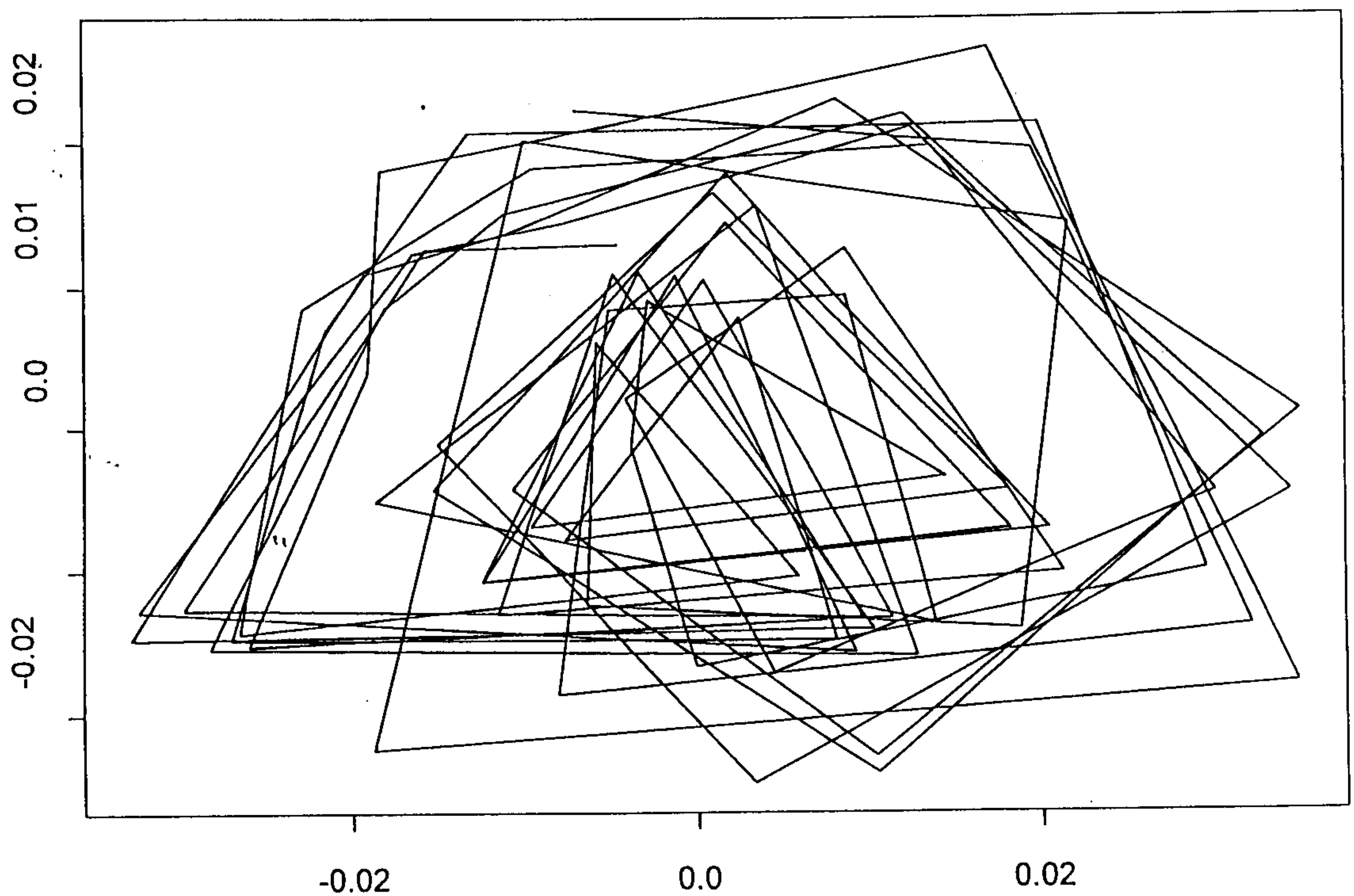
PHASE SPACE DIAGRAM: NONDURABLE GROWTH RATE



FIRST DERIVATIVE: NONDURABLE GROWTH
April,1969 to April,1976

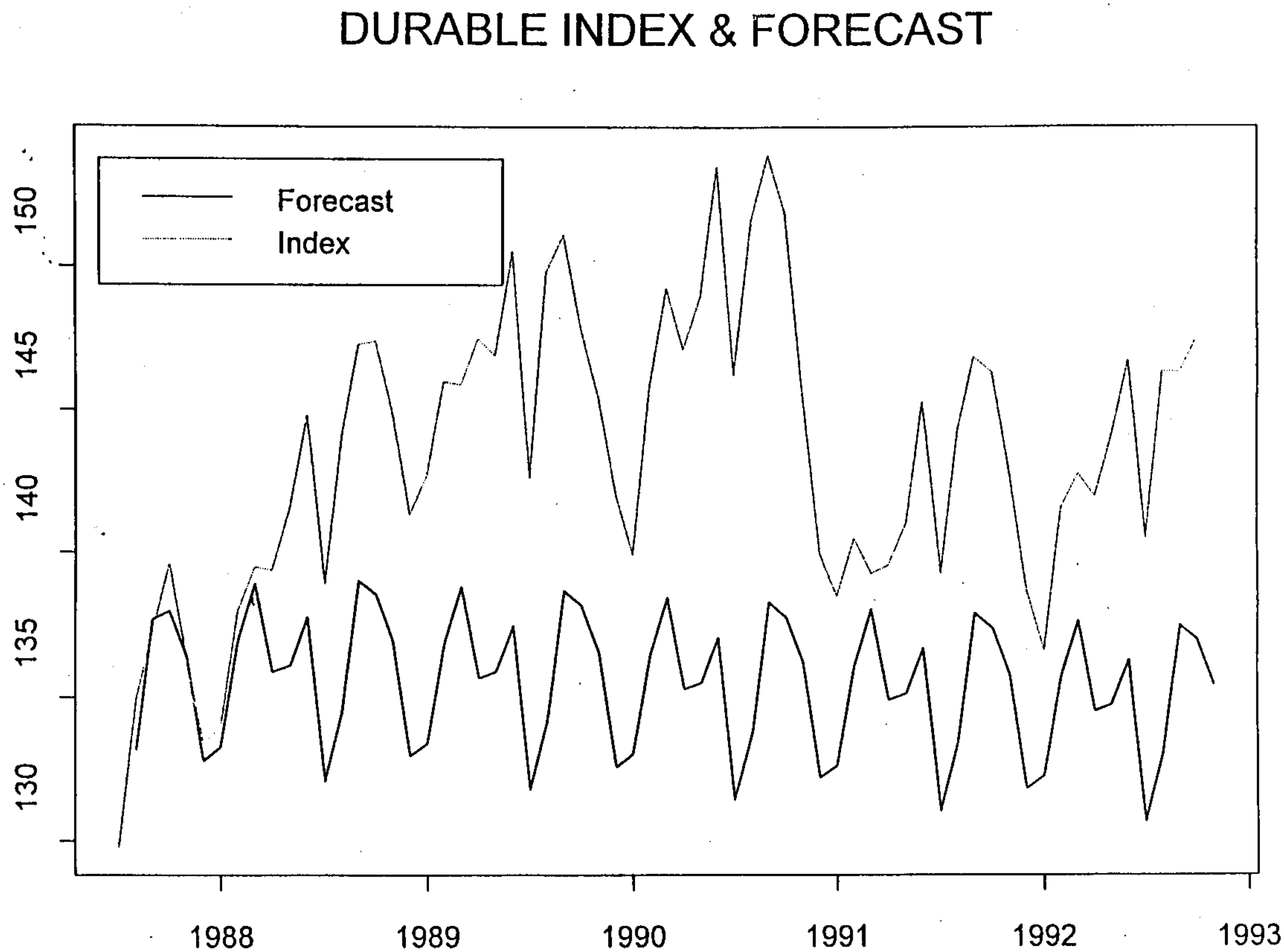
Figure 4B

PHASE SPACE DIAGRAM: NONDURABLE GROWTH RATE



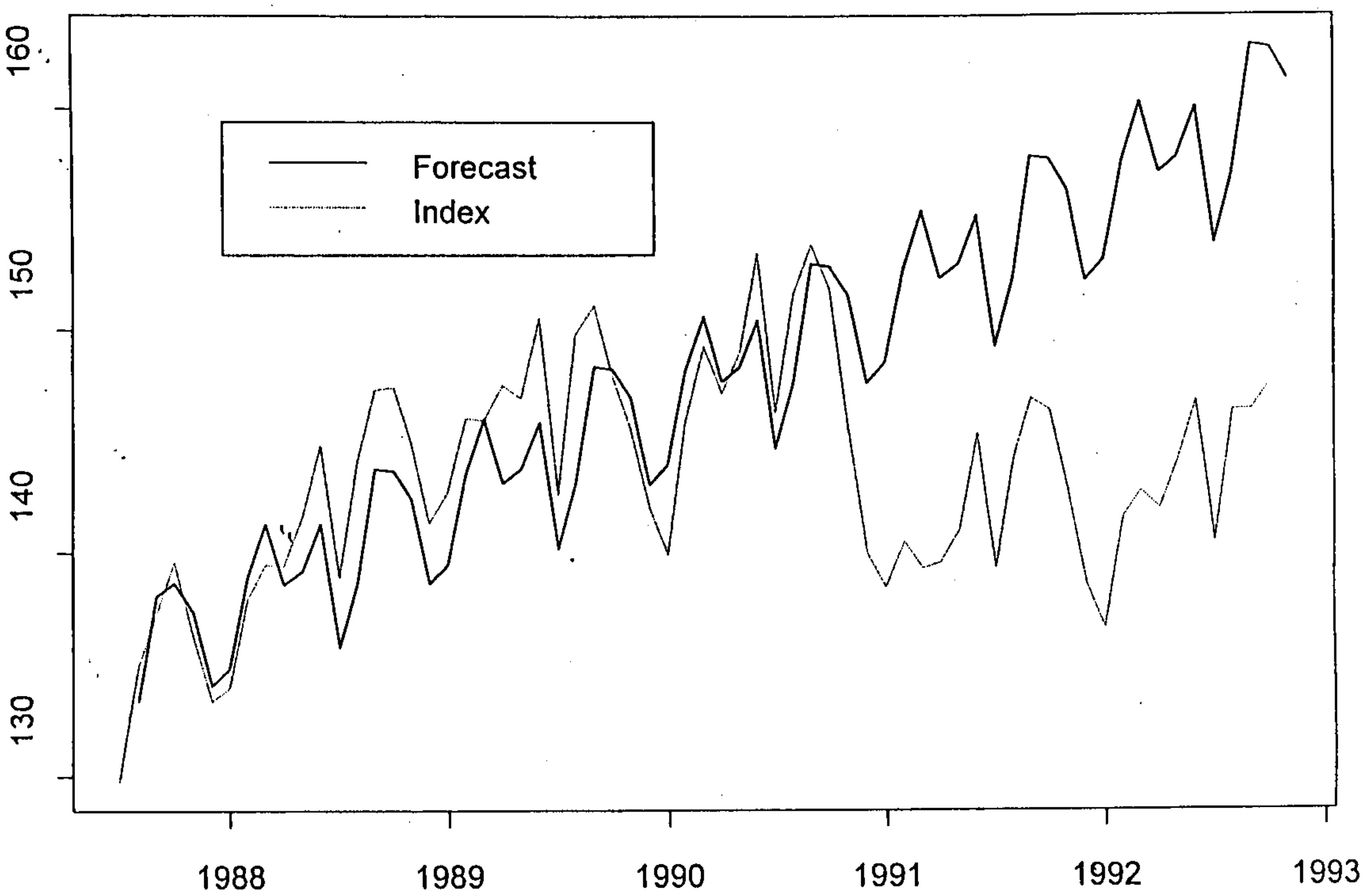
FIRST DERIVATIVE: NONDURABLE GROWTH
Dec. 1985 to Nov. 1992

Figure 5A



No Constant; no drift allowed.

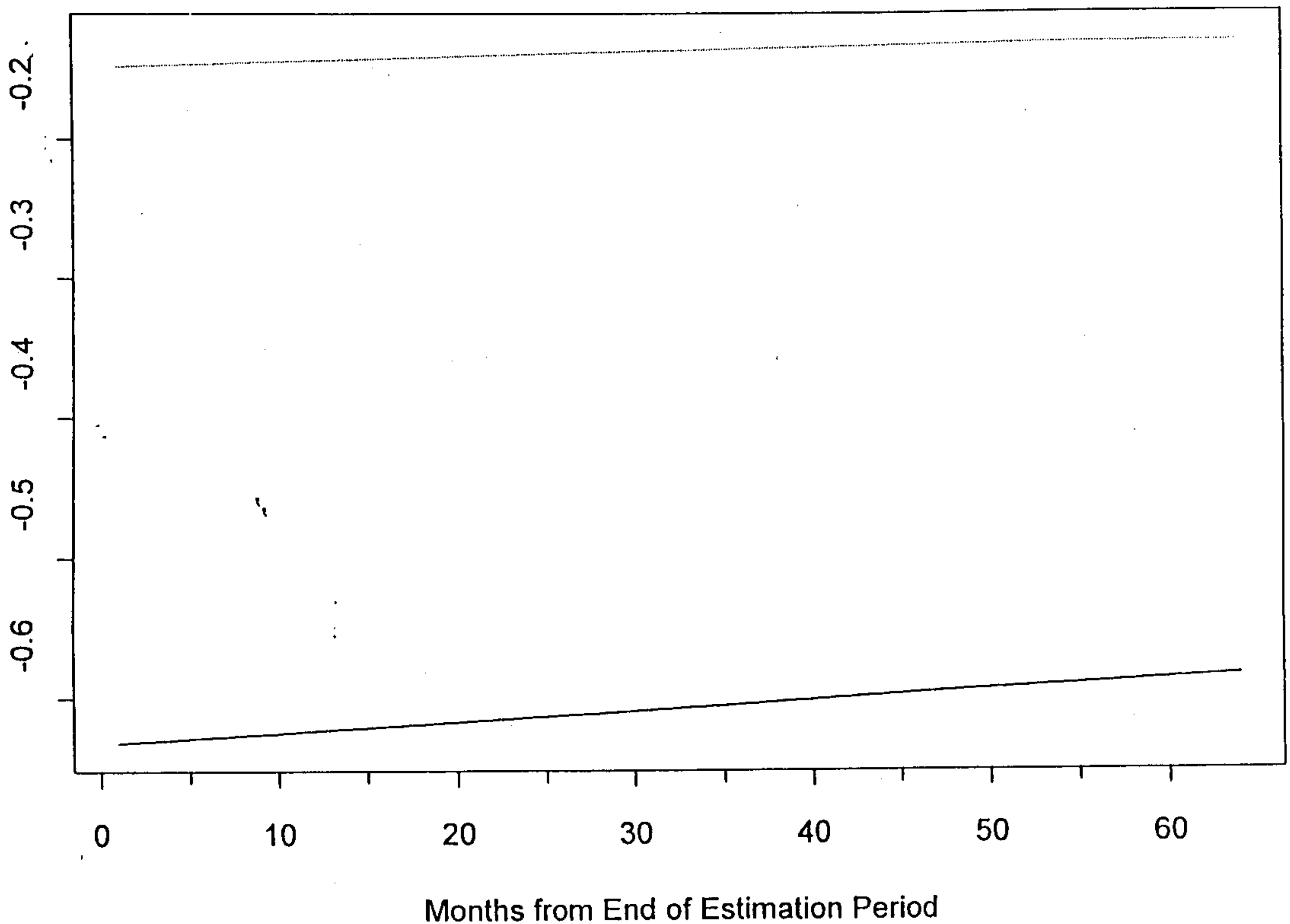
DURABLE INDEX & FORECAST



Constant term included; no drift allowed.

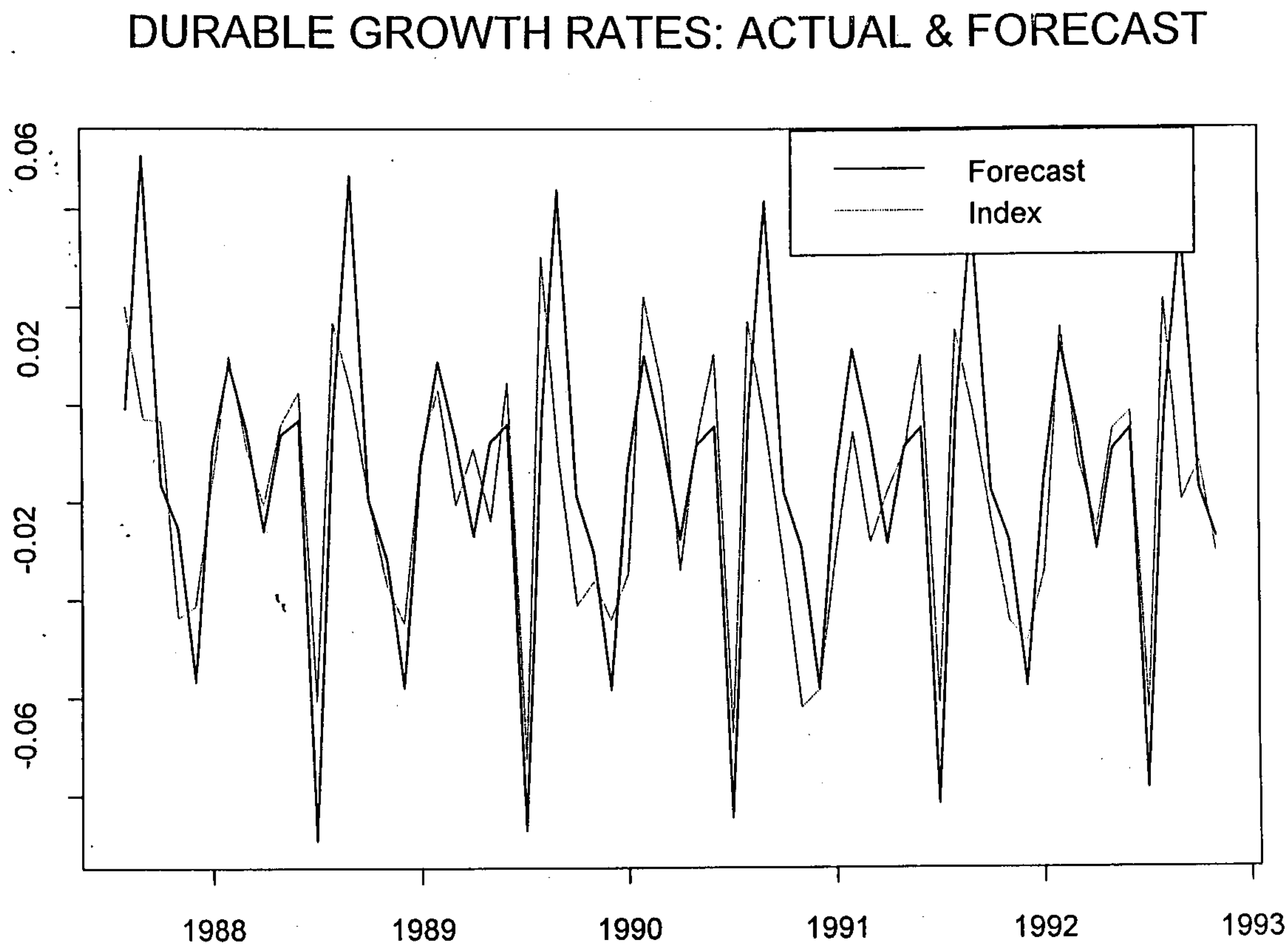
Figure 6

Prediction Plots for the Coefficients Alpha & Beta
Durable Goods Index



Predictions based on a linear extrpolation of a smoothed spline.

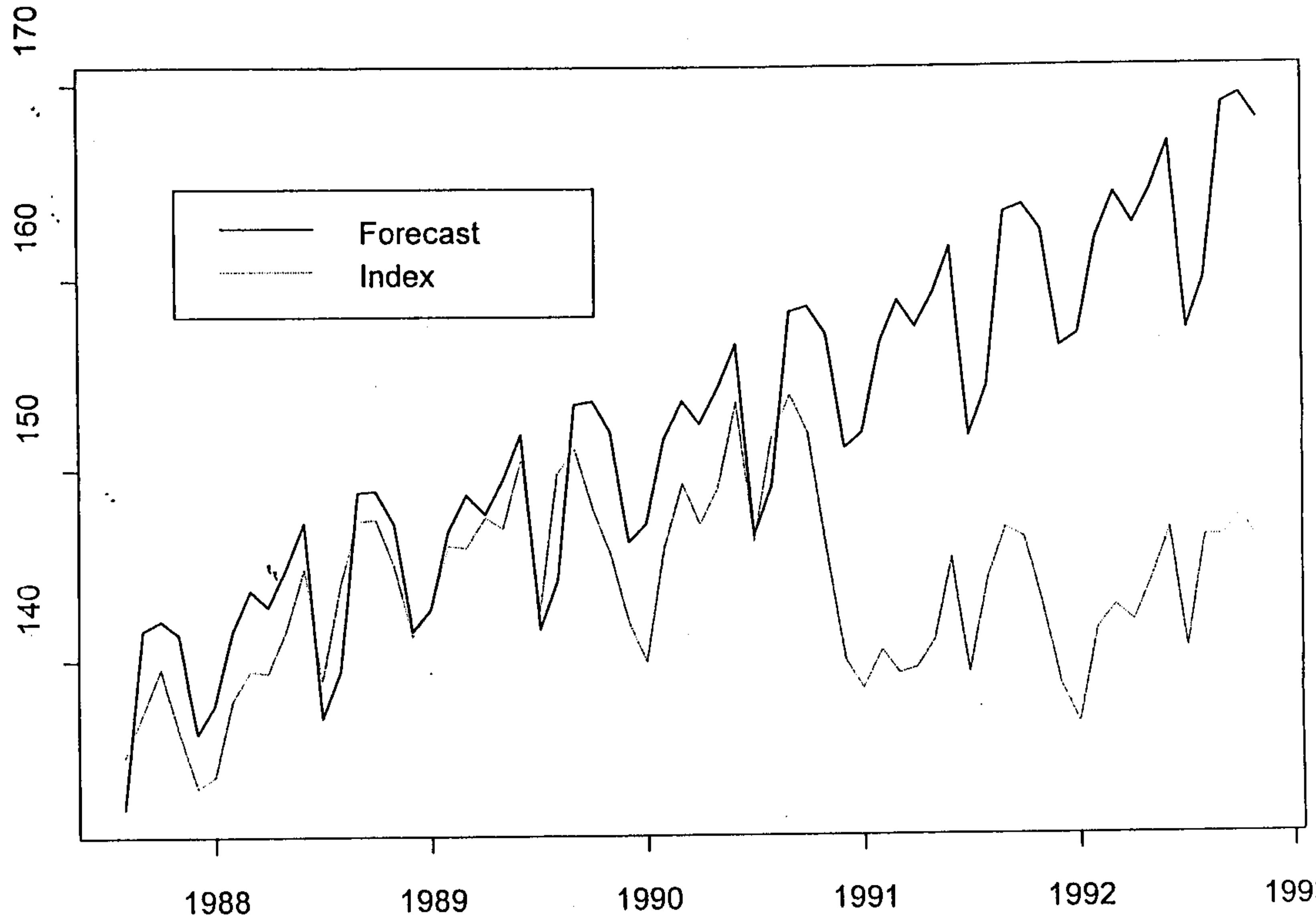
Figure 7



Constant included; DRIFT in coefficients.

Figure 8

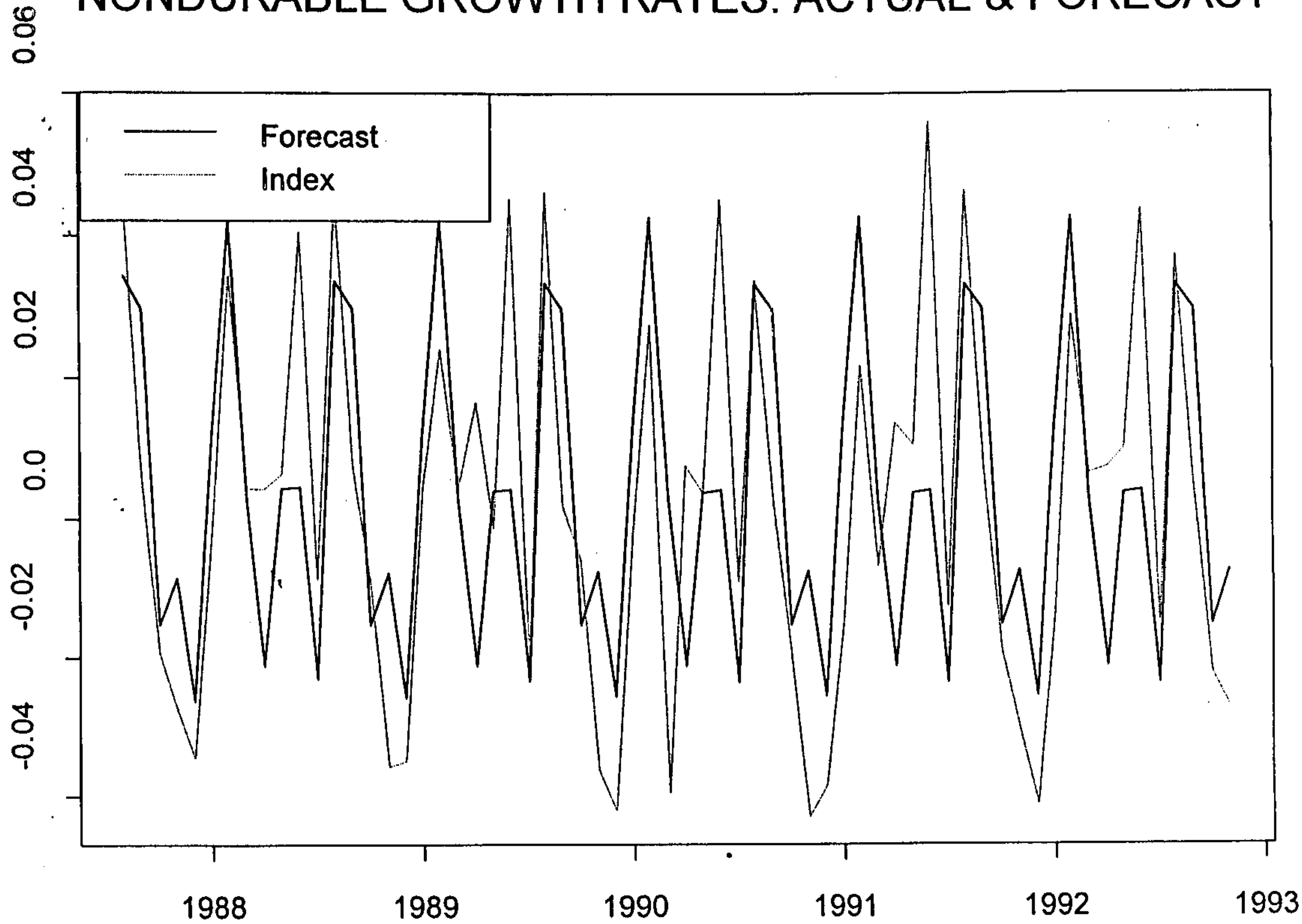
DURABLE PROD. INDEX: ACTUAL & FORECAST



Constant included; DRIFT in coefficients.

Figure 9

NONDURABLE GROWTH RATES: ACTUAL & FORECAST



Date of Observtion
Constant included; no drift allowed.

Figure 10

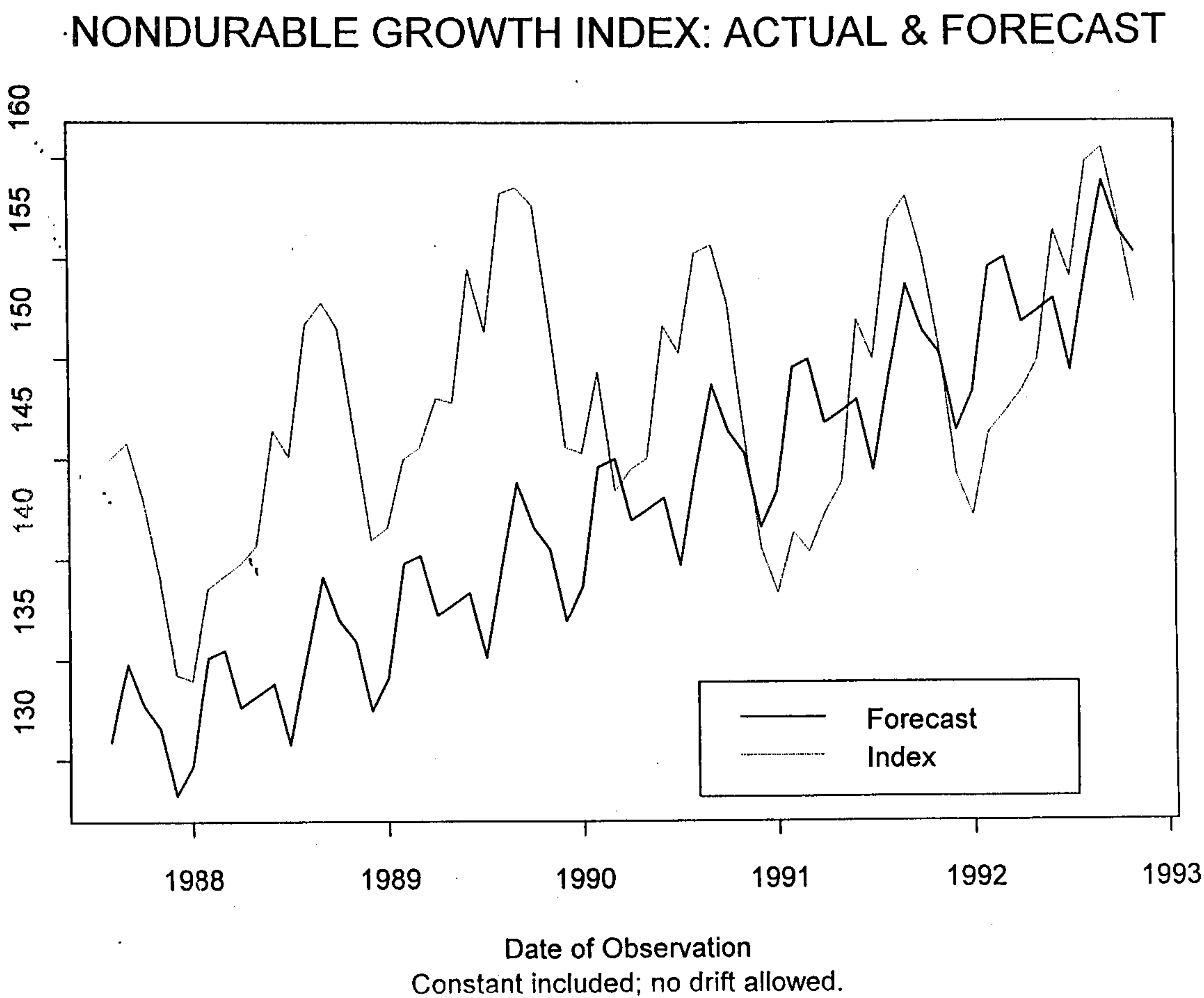

NONDURABLE GROWTH INDEX: ACTUAL & FORECAST
160
155
150
145
140
135
130
1988
1989
1990
1991
1992
1993
Forecast
Index
Date of Observation
Constant included; no drift allowed.

Figure 11

Prediction Plots for the Coefficients Alpha & Beta
Nondurable Goods Index

Figure 12

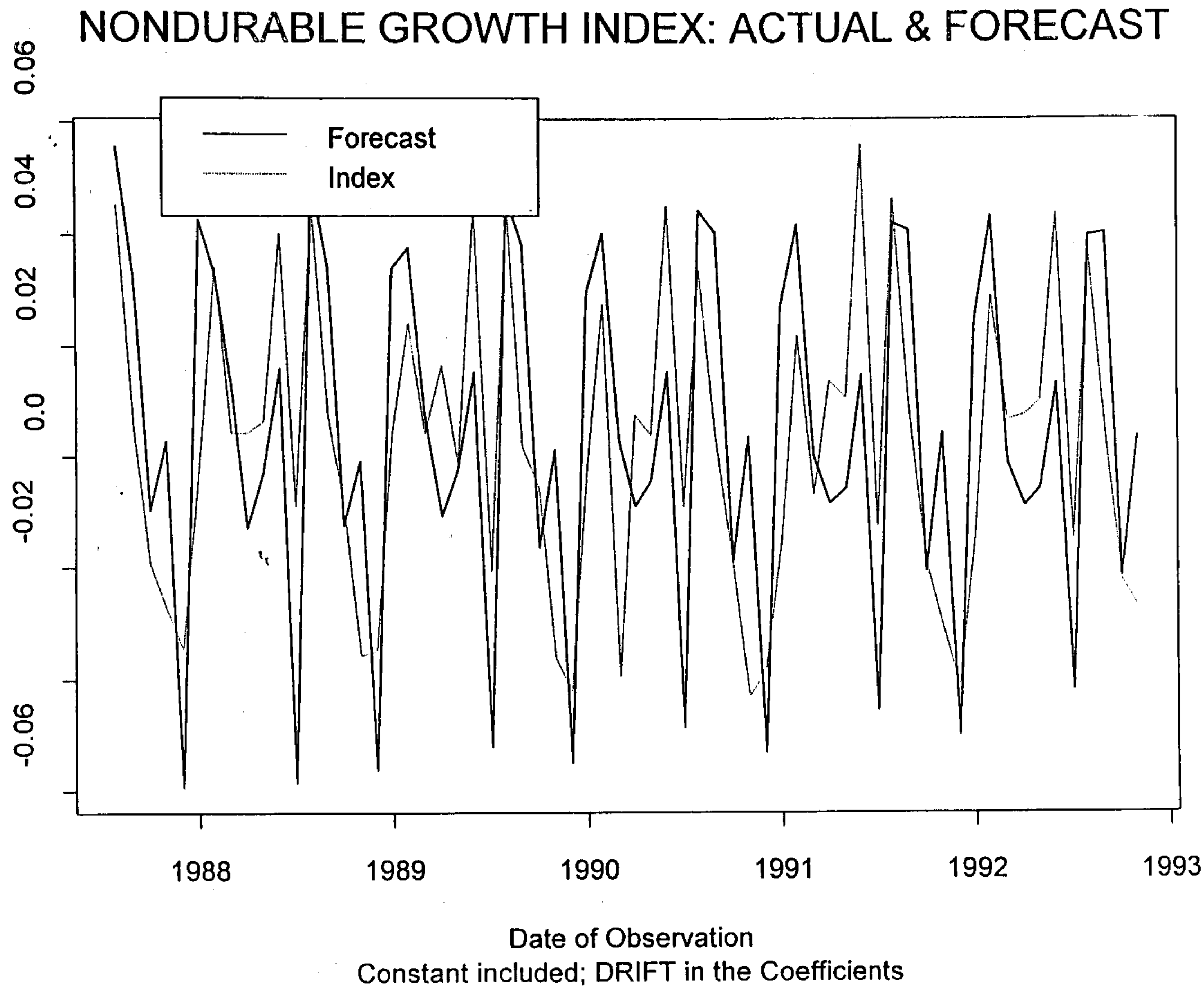
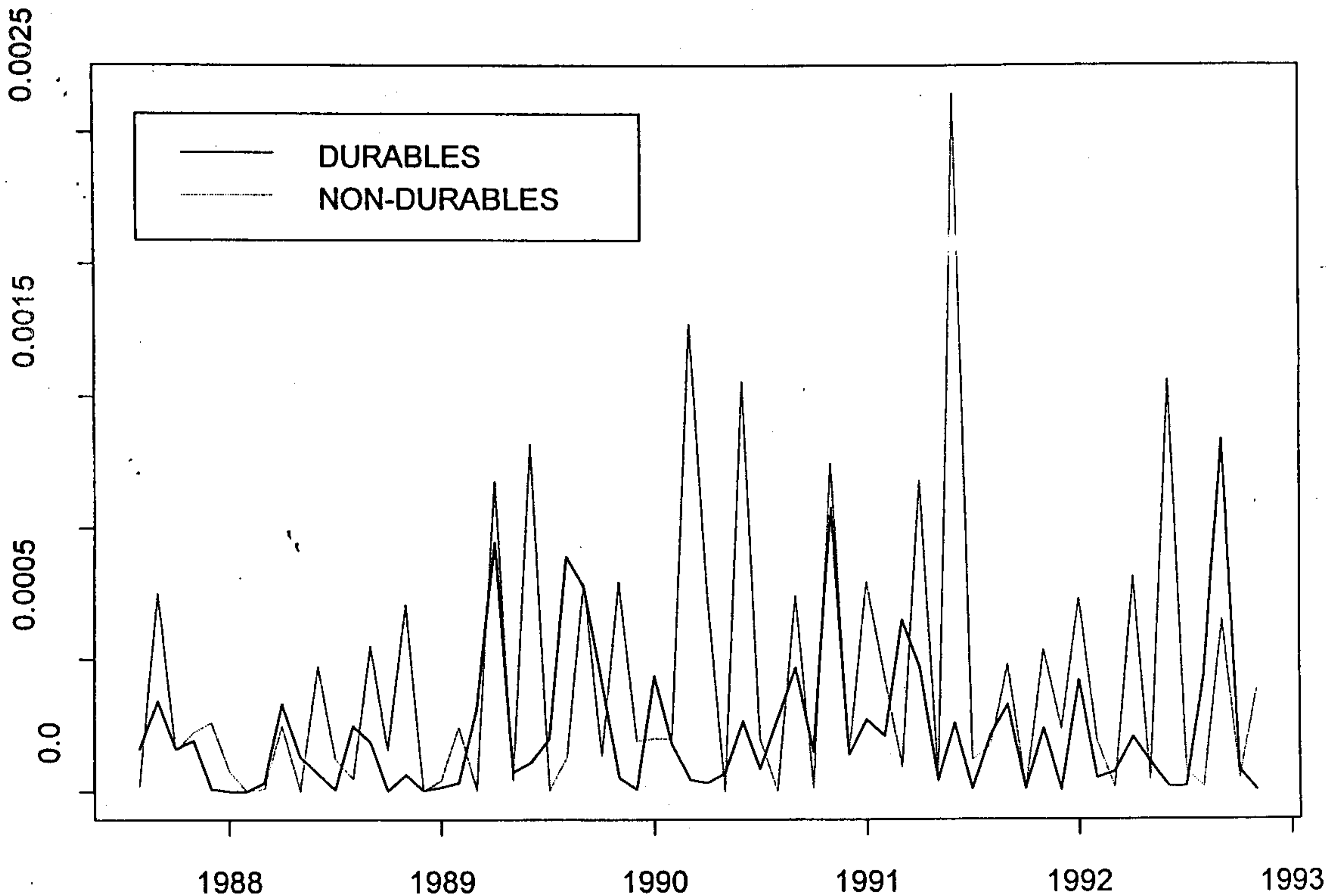

NONDURABLE GROWTH INDEX: ACTUAL & FORECAST
Forecast
Index
0.06
0.04
0.02
0.0
-0.02
-0.06
1988
1989
1990
1991
1992
1993
Date of Observation
Constant included; DRIFT in the Coefficients

Figure 13

FORECAST SQUARED ERRORS



Constant coefficients; no drift

Figure 14

NONDURABLE INDEX: ACTUAL & FORECAST

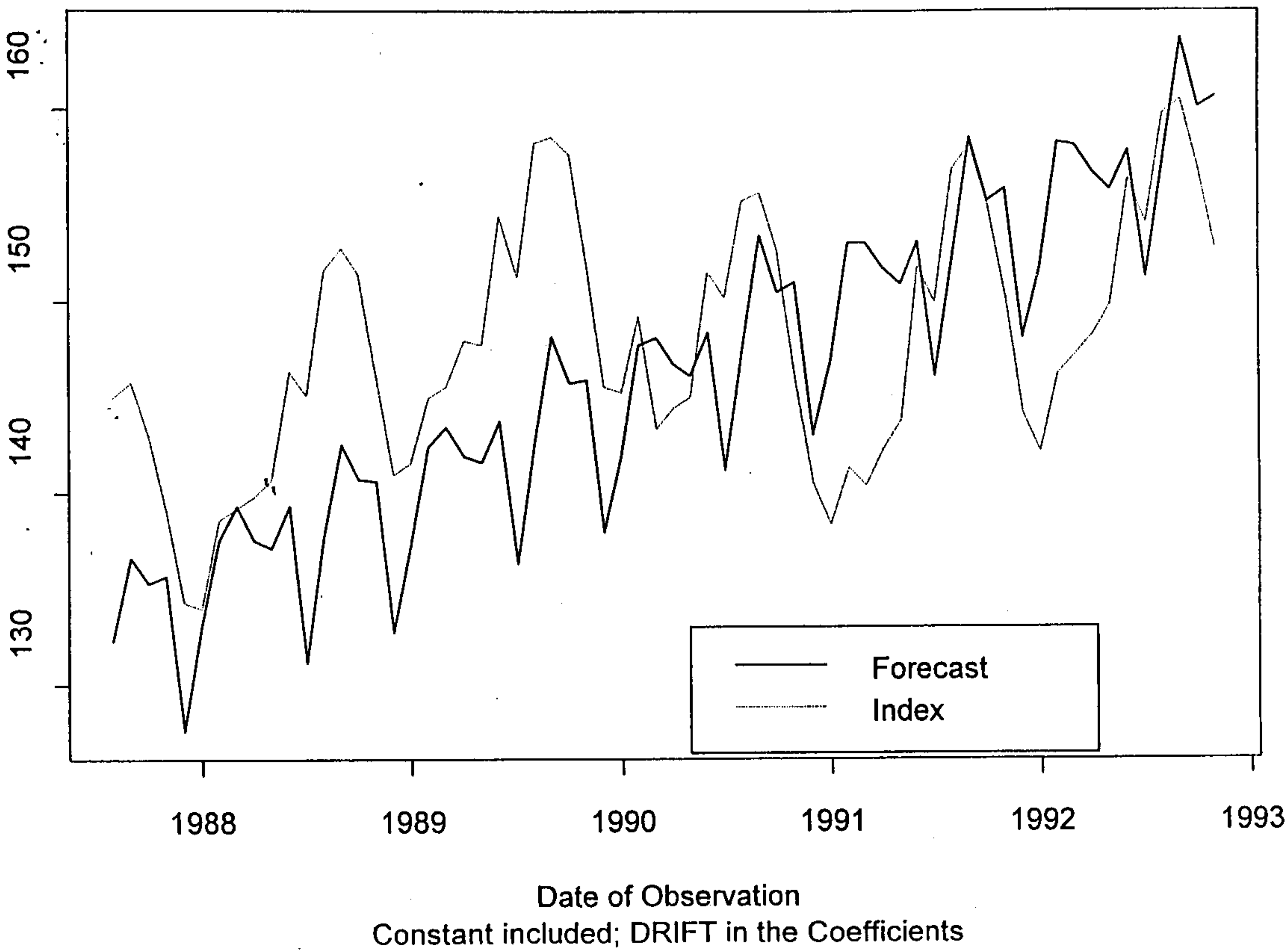
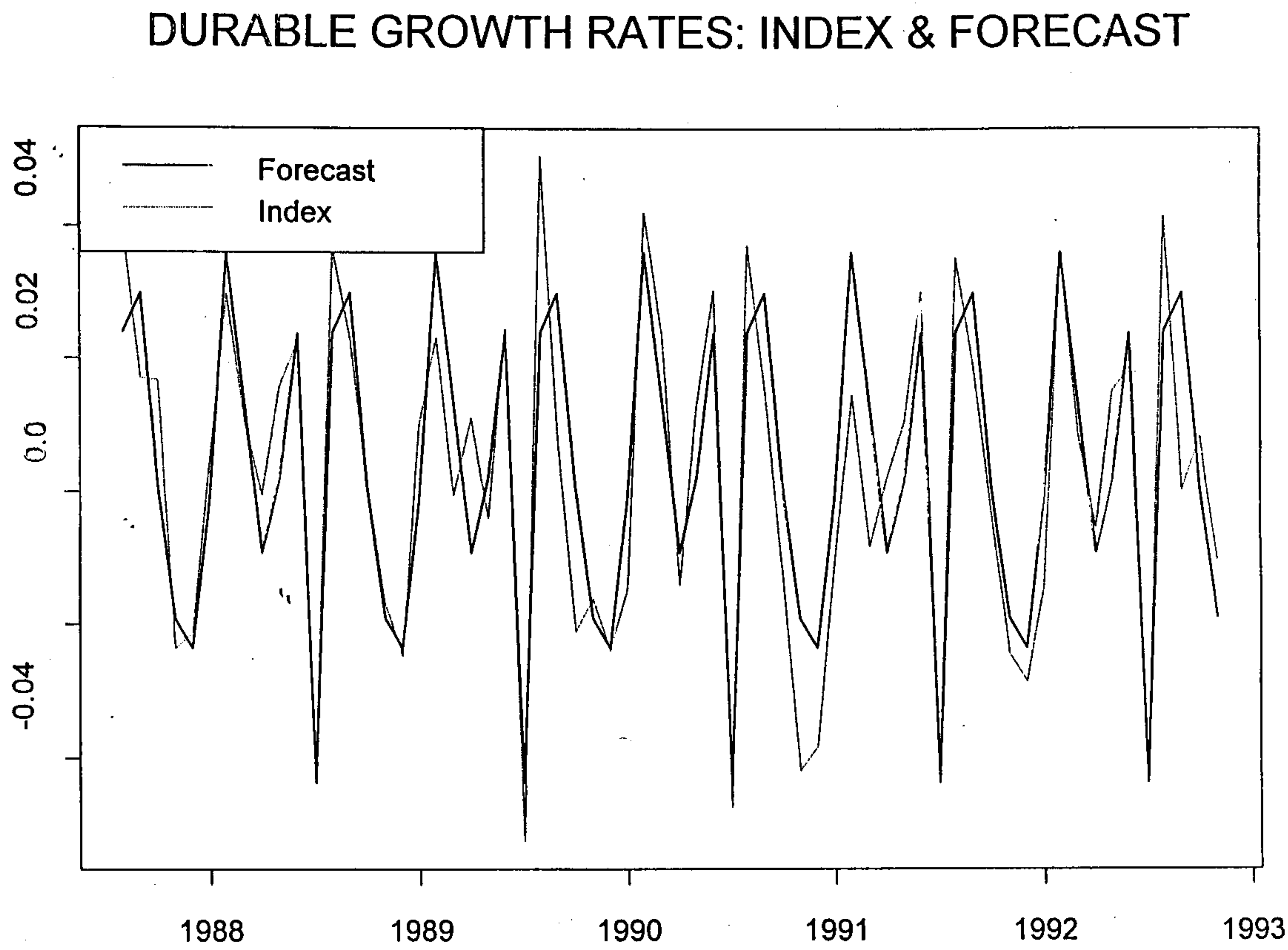
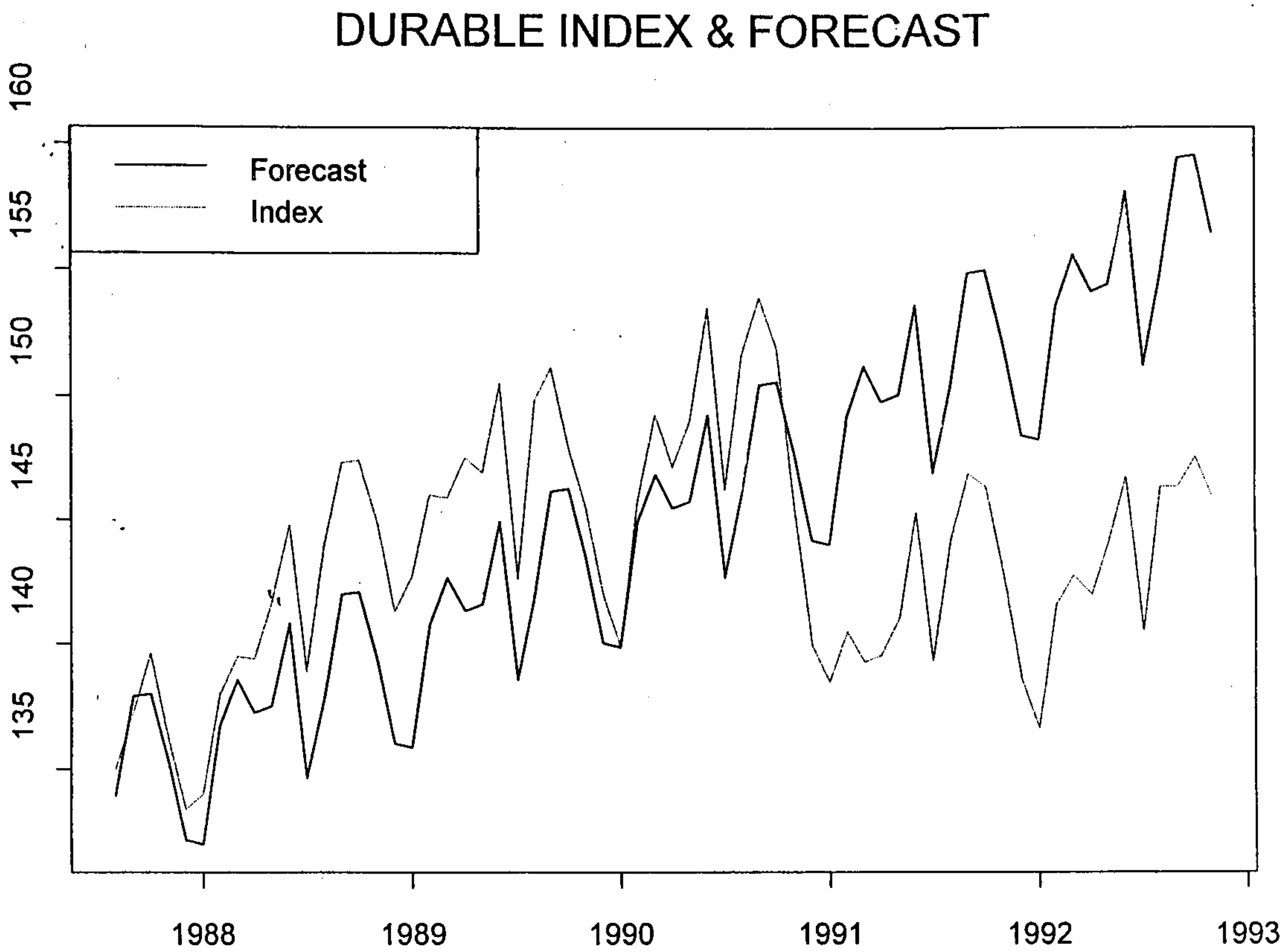


Figure 15



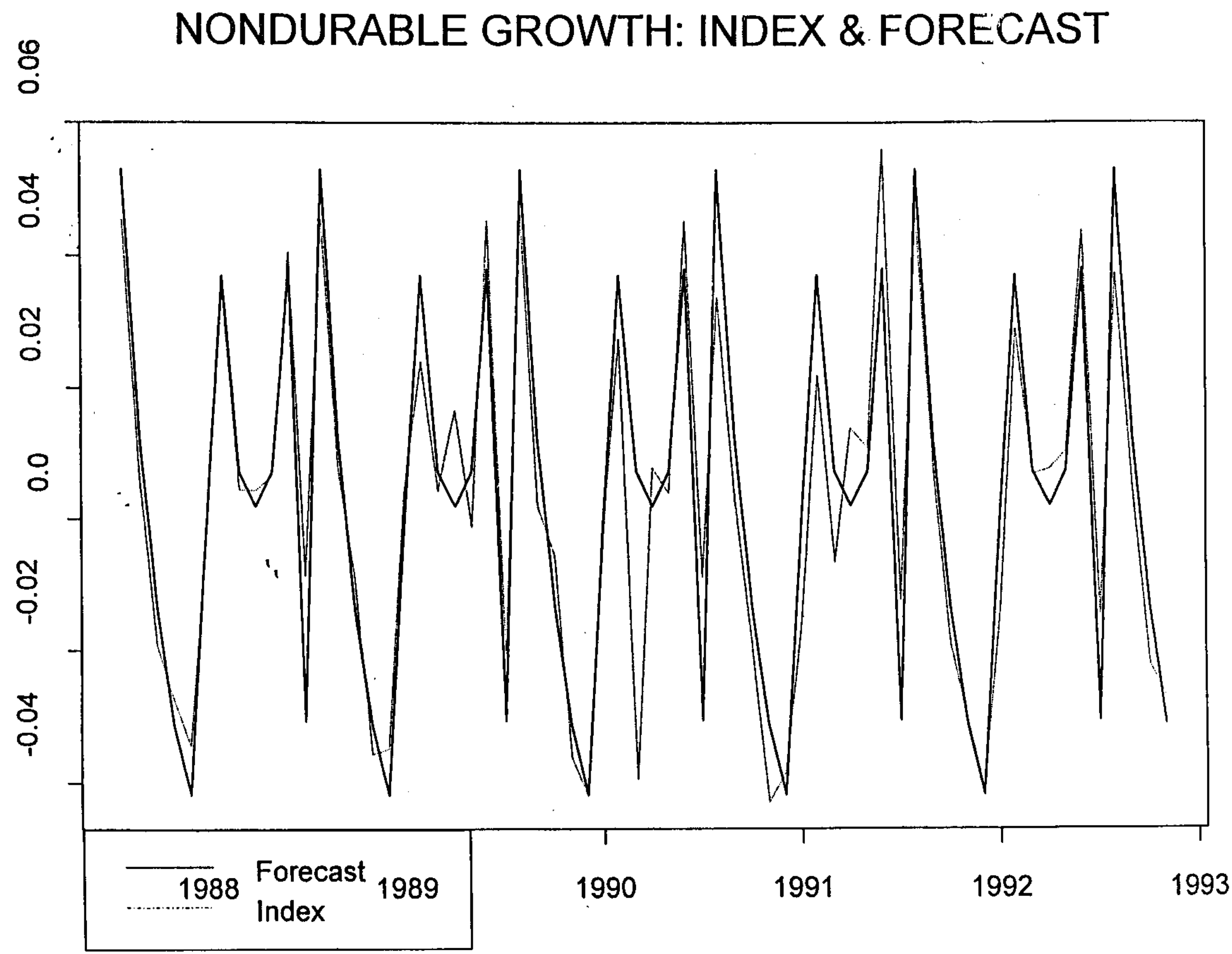
Forecast on seasonal dummies only

Figure 16



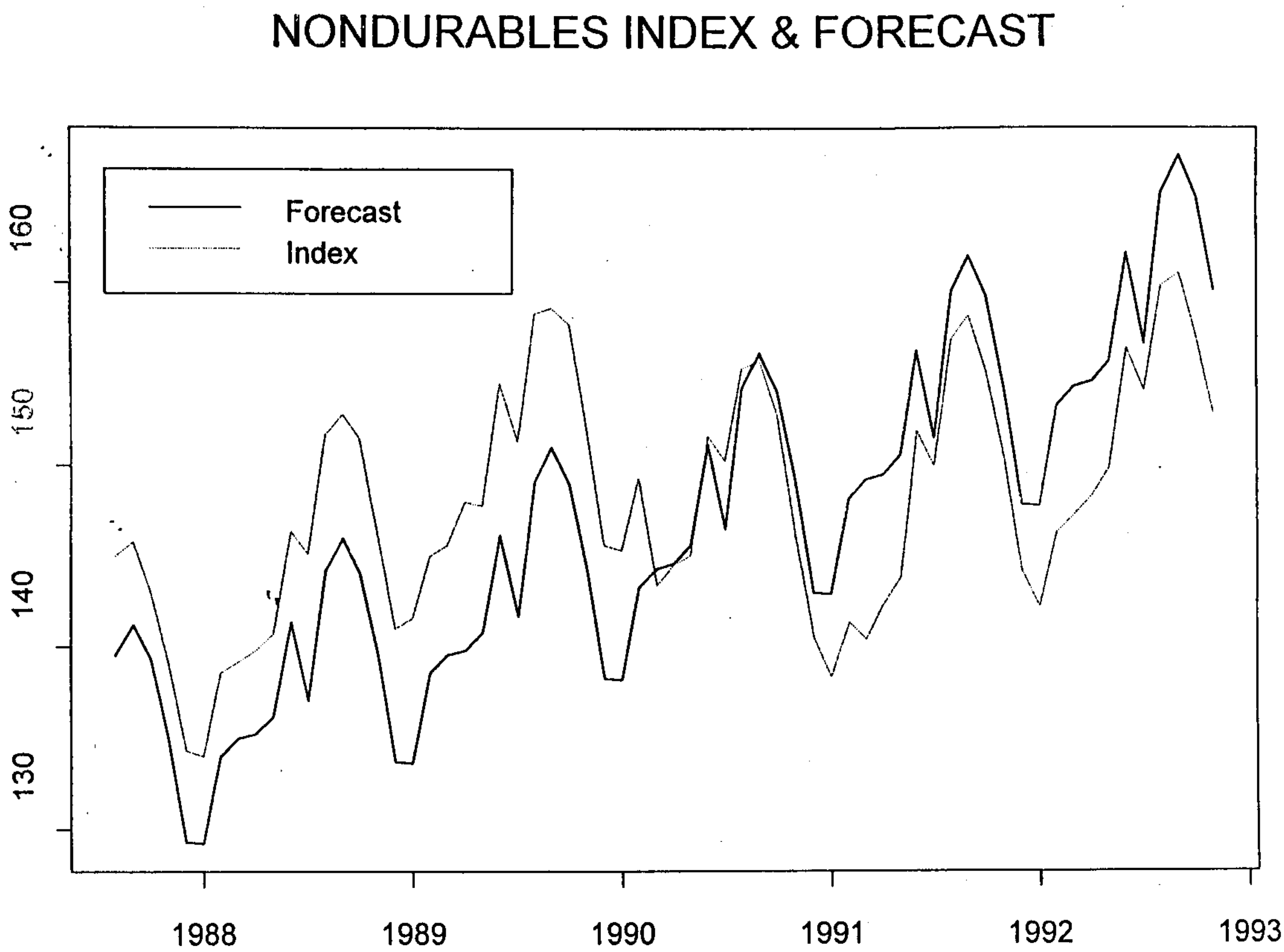
Forecast on seasonal dummies only

Figure 17



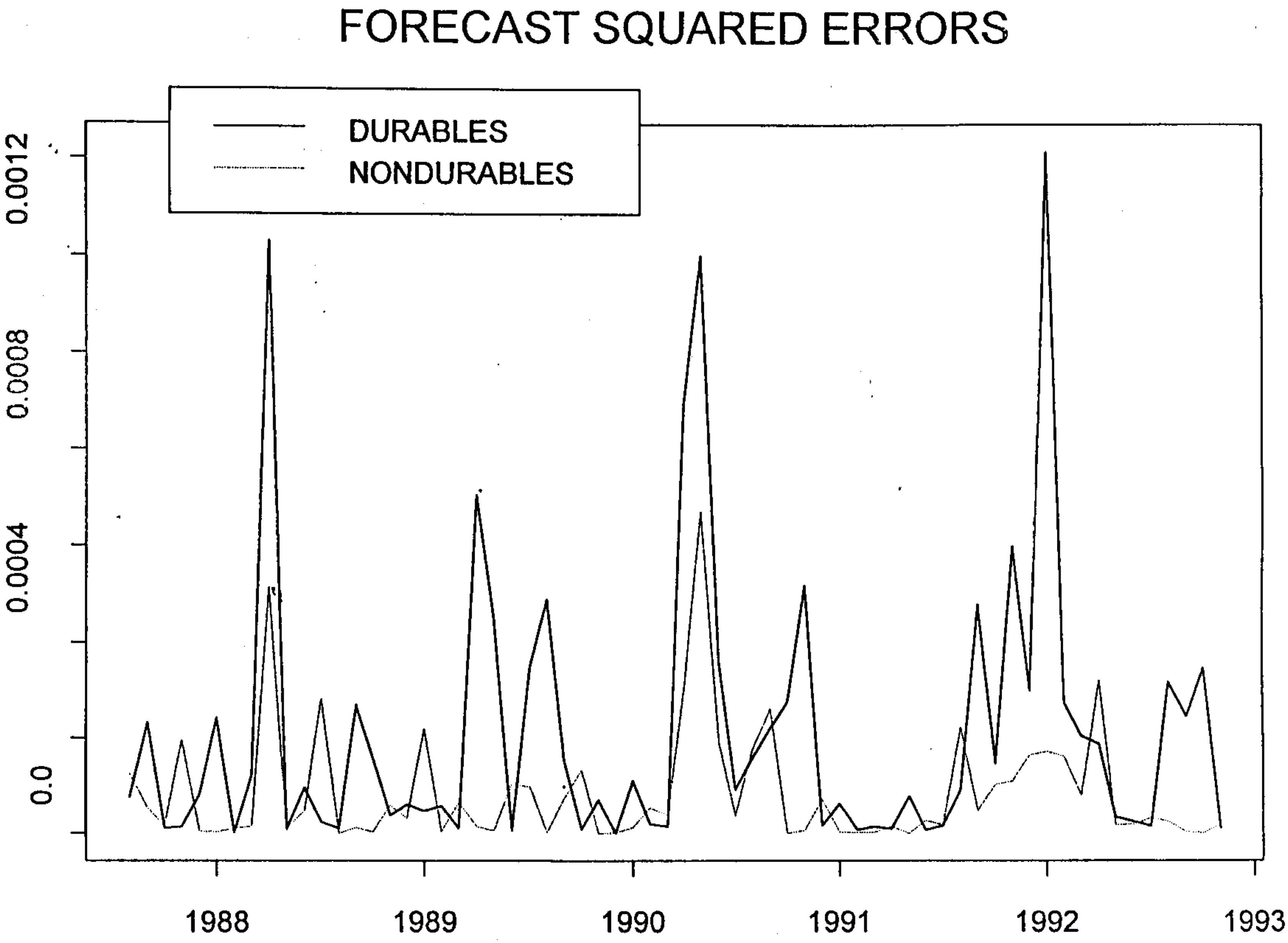
Forecast on seasonal dummies only

Figure 18



Forecast on seasonal dummies only

Figure 19



Forecast on seasonal dummies only