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BARGAINING POWER IN MINIMAL WINNING COALITIONS

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Abstract

Riker's "size principle" predicts that only minimal winning coalitions (MWCs) will form in *n*-person zero-sum games that satisfy certain conditions. After summarizing the logic of this principle, a model is proposed in which *n* players can be ordered from most to least weighty. No player has more than half the weight, so only coalitions of at least two players can be MWCs (with more than half the total weight). Two different kinds of MWCs are distinguished:

- those in which every member is "critical" (member-MWCs); and
- member-MWCs that have the smallest weight (weight-MWCs).

A member is *critical* when its defection causes an MWC to become losing.

A listing of the possible categories of member-MWCs and weight-MWCs indicates that their numbers rapidly increase with the number of players (2, 6, 20, and 103 for n = 3, 4, 5, and 6 players). Two quantitative measures of bargaining power in member-MWCs and weight-MWCs, and one of the "essentiality" of members in weight-MWCs, relate weight to bargaining power in the different cases, showing that less weighty players may, on occasion, be more powerful than more weighty players. In fact, the weightiest player is never the common—and hence the indispensable—member of all member-MWCs or of all weight MWCs unless its weight is equal to that of all the other players combined (i.e., half the total weight).

The latter finding, and the quantitative measures for weight-MWCs, suggest conditions under which large size may be more of a liability than an asset. Possible empirical manifestations of the inverse relationship between weight and power in parliamentary coalitions and international politics are discussed.

When Is Size a Liability?

Bargaining Power in Minimal Winning Coalitions

1. Introduction

Although the power of actors would seem to be related to their size, the nature of this relationship is by no means clear. Consider the two best-known measures of power in *simple games*—in which coalitions of players are either winning or losing—proposed by Shapley and Shubik (1954) and Banzhaf (1965). While these measures differ in the power that they assign to different players, they do agree that larger players (e.g., those with more votes in a weighted voting game) never have less power than smaller players.

Both these measures count how often a member of a winning coalition is *critical*—that is, how often this member's defection from such a coalition would cause it to be losing.[1] In some coalitions only one member may be critical, whereas in others all members may be critical.

To illustrate, consider a simple voting body of four members, one having 2 votes and the other three having 1 vote each, or (2,1,1,1). If the decision rule is a simple majority of 3 out of 5 votes, then the 2-vote member is the *only* critical player in the coalition (2,1,1), because the defection of either one of the two 1-vote members would not make this coalition losing.[2]

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[1]The Shapley-Shubik index assumes that all permutations of members are equally likely, whereas the Banzhaf index assumes that all combinations are equally likely. Detailed comparisons of these and other indices are made in Brams (1975), Holler (1982), Lambert (1988), Lucas (1992), and Straffin (1993), among other places, which include both hypothetical examples and real-life applications of the indices.

[2]We do not distinguish here which of the three 1-vote members that the two 1's shown in the coalition (2,1,1) represents. They could be anyone of three different pairs, but which one is immaterial to our argument that the defection of either one of the two 1's is not critical in this coalition. In computing the voting power of each player in this example,

But a question that immediately arises is the following: why would the two 1-vote players join such a coalition? Because neither player alone can threaten to disrupt it, each would seem to have little if any bargaining power, which presumably affects the payoffs that the players receive in a winning coalition.

By contrast, the two coalitions, (2,1) and (1,1,1), *are* subject to disruption by every one of its members. These coalitions, therefore, would seem to have a greater likelihood of forming in the first place, though admittedly the 2-vote player might prefer (2,1,1), in which it would be the sole critical member.

A generation ago, Riker (1962) developed a game-theoretic model to analyze conditions under which minimal-winning coalitions (MWCs) like (2,1) and (1,1,1) would form. After summarizing Riker's model, we shall develop a model that takes MWCs as a starting point and relates the bargaining power of players to their size.[3]

In our model, we assume that there are at least three players, and no player has more than half the weight. To enjoy the benefits of being members of MWCs (with more than half the weight), a player must therefore join with at least one other player.

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however, we will take into account that there are three different cases of (2,1,1), depending on which pair of 1's is included in (2,1,1) (see note 3).

[3]To illustrate the difference between restricting winning coalitions to MWCs and not doing so, we compute the Banzhaf index for the example in the text. With the restriction to MWCs, this index assumes that the three different cases of (2,1), and the one case of (1,1,1), are equally likely. Then the 2-vote member is critical in three of the four cases, and each 1-vote member in two of the cases, giving a power ratio of 3:2, or 50% more power to the 2-vote player. Without the restriction (as is usually assumed), the Banzhaf index counts, in addition, the three cases of (2,1,1), giving a new total of seven cases, with the 2-vote member critical in six cases and each 1-vote member critical in only the original two, giving a power ratio of 6:2 (3:1), or 300% more power to the 2-vote member. (The Shapley-Shubik index gives the same 3:1 ratio in this example, but this equality is coincidental—in general, the two indices give different, though often similar, results.)

The specification of MWCs is of central importance in our study. We distinguish two different kinds of MWCs:

- those in which every member is critical (member-MWCs); and
- member-MWCs that have the smallest weight (weight-MWCs).

We shall show, among other things, that with one exception the weightiest player is never the common member of all member-MWCs or of all weight-MWCs, suggesting that being the largest player may be a liability in bargaining.

These results do not depend on the actual numerical weights of players but only on an ordering of player weights from largest to smallest. This ordinality assumption makes tractable the exhaustive analysis of MWCs when there are as many as five or six players, enabling us to draw some quantitative conclusions about bargaining power and the essentiality of members in MWCs.[4] We briefly indicate the possible applicability of the analysis to assessing the power of players in legislative and international politics.

2. The Size Principle

A *coalition* is a subset of players.[5] Its incentive to form is to become winning, which enables it to enforce its selection of an outcome on other

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[4]In situations involving only three players, Caplow (1968) postulated control over other actors as a goal, from which he developed a theory about what coalitions will form in triads for different distributions of strength (weight) among the three actors. Riker (1962, pp. 132-146) refined Caplow's (1956, 1959) original analysis and Gamson's (1961) later analysis and extended it to four and five players; see also Willis (1962). The model we develop in section 3 simplifies Riker's analysis by not making explicit assumptions about weight distributions for singles, pairs, triples, etc.; rather, it makes such distributional assumptions implicit in the possible MWCs that can form.

[5]The discussion in this section is adapted from Brams (1975, pp. 200-202, 213-222) and Brams (1985, pp. 184-187).

players. Each player, then, faces, the problem of what other players to join with in a coalition that can win, which will depend on what payoffs it might receive in that prospective winning coalition.

We assume a player's payoffs are related to its power in the coalition, which will in turn depend on how crucial it is in the formation or breakup of the coalition. This raises the interesting question: is a player powerful because of its position in a coalition, or does a coalition form because players are powerful in it (i.e., can enforce an agreement as well as obtain something of value)? In politics, it seems, the power of players is inextricably linked with the coalitions of which they are members, and any scheme that attempts to disentangle the players from the coalitions they constitute necessarily simplifies reality.

For analytic purposes, one may either start with players and ask what coalitions will form, or start with coalitions and ask how their value will be apportioned among the players as a function of their bargaining power. We start by assuming that only MWCs form and then analyze the bargaining power of their members, based on how often they are in MWCs or common members in the intersection of MWCs.

We focus initially on winning coalitions—and later on MWCs—because there is not much else in politics to which one can attach value. True, many political actors claim to be interested in maximizing their power, but aside from the measures of voting power alluded to in section 1, there have been no generally accepted definitions of this concept, not to mention models that justify these definitions.

Although models have been developed for measuring the concentration of power in a political system (Brams, 1968, 1969, and references cited therein), the meaning of power outside the domain of voting situations is quite murky. Indeed, declaring power to be an "imprecise notion," Riker (1962, p. 22) boldly replaces it with the notion of winning:

> What the rational political man wants, I believe, is to win, a much more specific and specifiable motive than the desire for power . . . . The man who wants to win also wants to make other people do things they would not otherwise do, he wants to exploit each situation to his advantage, and he wants to succeed in a given situation.

Because winning helps actors achieve these things in elections, wars, and other arenas of political conflict, it is not surprising that they place a high value on this goal. Moreover, as a concept for realizing a whole panoply of ends, the notion of winning offers the political theorist a generalized goal that subsumes particular goals that are often difficult to know or specify.

Any statement about a postulated goal is necessarily based on the *perceptions* of actors. Ordinarily, this causes no problem in political situations like elections, where the method of counting votes and the decision rule for selecting a winner are known and accepted by the contestants—at least in situations where information is assumed to be complete. But it should be noted that although elections usually distinguish unambigously the winner from the losers, the winner may not be perceived as the true victor, as when a candidate does better than expected in a presidential primary and is declared the nominal winner, in spite of receiving fewer votes than an opponent.

Besides positing the goal of winning, Riker (1962) makes several assumptions in his game-theoretic model. Because Riker's book is out of print and seems not to be read much today, these assumptions are worth reviewing here:

1. *Rationality.* Players are rational: they will choose the alternative that leads to their most-preferred outcome—namely winning.[6]

2. *Zero-sum.* Decisions have a winner-take-all character—what one coalition wins the other coalition loses—so the sum of payoffs to all players is zero. In other words, the model embraces only situations of pure and unrelieved conflict where all value accrues to the winner; cooperation among the participants that redounds to the mutual benefit of all is excluded.

3. *Complete and perfect information.* Players are fully informed about the state of affairs at the beginning of the game and about the moves of all other players throughout the game.

4. *Allowance for side payments.* Players can communicate with each other and bargain about the distribution of payoffs in a winning coalition, whose value is divided among its members. (*Side payments* are simply individual payments that players can transfer to each other in dividing up the value.)

5. *Positive value.* Only winning coalitions have positive value.

6. *Positive payoffs.* All members of a winning coalition receive positive payoffs. This assumption, of course, provides an incentive for players to join a winning coalition, but it says nothing about the size of these payoffs, which we shall consider later in analyzing bargaining power in MWCs.

7. *Control over membership.* Members of a winning coalition have the ability to admit or eject members from it.

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[6]Strictly speaking, it is not necessary to postulate winning as the most-preferred outcome. Rather "rationality" may be defined in terms of the choice of the *most-valued* outcome, where the value associated with winning coalitions and the payoffs to their members are stipulated by assumptions 5 and 6.

Assumptions 5, 6, and 7—besides the assumption that the goal of players is to form winning coalitions—are what Riker calls "sociological assumptions," as distinguished from the four "mathematical assumptions" (1, 2, 3, and 4) standard in *n*-person cooperative game theory. The sociological assumptions specify more precisely the goal of winning and thereby enable Riker to derive the size of winning coalitions that is optimal and therefore likely to occur (Riker and Ordeshook, 1973, pp. 179-180; for further details, see Riker, 1962, pp. 40-46, 247-278).

Given these assumptions, Riker shows that there are no circumstances wherein an incentive exists for coalitions of greater than minimal winning size to form. On the other hand, the fact that there is a positive value associated with winning coalitions (assumption 5), each of whose members receives positive payoffs from winning (assumption 6), is a sufficient incentive for such coalitions to form. The incentive for winning coalitions to form, but not to be of greater than minimal winning size, means that the realization of the goal of winning takes form in the creation only of MWCs; Riker call this conclusion the *size principle*.

His reasoning (without the mathematical details) is as follows: Given that only winning coalitions have positive value (assumption 5), the zero-sum assumption (assumption 2) implies that losing coalitions must have complementary negative value. Since a losing coalition has no positive value to distribute among its members, it would form only as a pretender to eventual winning status. Indeed, the possibility that a losing coalition could eventually become winning provides a strong incentive for a winning coalition to pare off superfluous members (permitted by assumption 7) before they and other disaffected members—to whom it cannot offer sufficient payoffs—defect.

If the excess members are not ejected, the winning coalition becomes vulnerable to offers from a losing coalition that could promise enough defectors greater rewards in a prospective MWC so as actually to constitute such a coalition. Because the MWC has complete and perfect information (by assumption 3), its members know exactly when they have enough members in the coalition to win.

Several things should be noted about Riker's derivation of the size principle. First, winning is not itself an explicit goal: rationality (assumption 1) motivates players to obtain the benefits of being in a winning coalition, as stipulated in assumptions 5 and 6. Second, the size principle is a statement about an outcome—the size of winning coalitions—and not about the process of coalition formation, though Riker (1962, pp. 124-148) provides some analysis of the dynamics of coalition building. Finally, since any win is the sole determinant of value (assumption 5), the ejection of superfluous members from a winning coalition, according to assumption 7, means that the same total amount of value can be divided among the fewer members of an MWC.

Thus, each of the members of an MWC can derive more profit from it than it can from a larger winning coalition. But does one need the formalism of a mathematical model to grasp that larger winning coalitions would therefore be expected to reduce their size? While commonsensical, however, this simple explanation does not offer limiting conditions on the veracity of the size principle, whereas the assumptions of a model do.

As an explanation of why the size principle should hold, however, Riker's argument has logical force only. To connect Riker's model with reality, consider how its assumptions can be interpreted as conditions that limit the operation of the size principle when its theoretical concepts are operationally defined and it is posited as an empirical law. Riker (1962, p. 47) offers this translation of the size principle:

> In social situations similar to *n*-person games with side-payments, participants create coalitions just as large as they believe will ensure winning and no larger.

The introduction of the beliefs or perceptions of actors about a subjectively estimated minimum simply acknowledges the real-world fact that players do not have complete and perfect information about the environment in which they act and about the actions of other players. Consequently, players are inclined to form coalitions that are larger than MWCs as a cushion again uncertainty, and oversized coalitions thus become a quite rational response in an uncertain world.

In this manner, complete and perfect information serves as a limiting condition on the truth of the size principle: in its absence, coalitions will *not* tend toward minimal winning size. Similarly, the other assumptions of the model also restrict the applicability of the size principle in the real world, but Riker singles out the effect of information for special attention probably because it is the concept most easily interpreted, if not operationalized, of those embodied in his game-theoretic model.

The enlargement of coalitions to more than minimal winning size due to the effects of incomplete or imperfect information is what Riker calls the *information effect.* Since this effect is endemic in an uncertain world, there naturally exist many examples of nonminimal winning coalitions, even in essentially zero-sum situations. For Riker these examples represent situations in which coalition leaders miscalculated the capabilities, or misread the intentions, of opponents—not because there were irrational but rather because they lacked accurate and reliable information on which to act.

The evidence in support of Riker's empirical statement of the size principle is mixed, but we shall not assess it here.[7] Instead, we shall focus on the relationship, when only MWCs form, between the size of players and their bargaining power.

3. Two Different Kinds of MWCs

Assume players 1, 2, . . ., n have weights $w_1, w_2, \ldots, w_n$, and $\sum_{i=1}^{n} w_i =$ W. Assume $w_1 \geq w_2 \geq \ldots \geq w_n > 0$, and $w_1 \leq W/2$. A coalition is *winning* if and only if the sum of the weights of its members is more than W/2, so only coalitions of two or more players can be winning.

We define two different notions of an MWC, one more restrictive than the other. A winning coalition is a *member-MWC* if the defection of any one of its members causes it to be losing, making all its members critical. A winning coalition is a *weight-MWC* if it is a member-MWC *and* there is no other member-MWC whose weight is never greater than the weight of the first and is sometimes less than the first's weight (based on the ordering of weights, not the numerical w_i's, given above). Thus, weight-MWCs are a subset of member-MWCs.

To illustrate these two different kinds of MWCs, assume $n = 3$. The member-MWCs fall into two mutually exclusive categories, depending on the w_i's, which are shown in Table 1. A *category* is a set of member-MWCs

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[7]It is reviewed in, among other places, Riker and Ordeshook (1973, pp. 187-201); Brams (1975, pp. 222-234); Budge and Keman (1990, pp. 10-19); and Laver and Schofield (1990, pp. 9-10, 36-40).

Table 1 about here

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that is *feasible:* there exist w_i's for each member *i* that yield exactly these MWCs.

Feasible weights are given for each of the two categories in the $n = 3$ case. Thus, weights (2,1,1) are feasible for players {1,2,3} in category 1: the defection of either of the two players from member-MWCs {1,2} or {1,3}—each with a total weight of 3—is critical: it would leave the resulting coalitions with weight that does not exceed 4/2 = 2.

The underscored coalition, {1,3}, is the unique weight-MWC, because there is no other member-MWC in this category whose weight is always less than or equal to the weight of {1,3}. (In general, {1,2} will have more weight, and certainly never less, than {1,3}.[8]) By comparison, {2,3} is the unique weight-MWC for category 2.

That weight-MWCs need not be unique is illustrated by the $n = 4$ case (Table 1), in which there are six mutually exclusive categories. Consider category 1, in which there are three member-MWCs, {1,2}, {1,3,4}, and {2,3,4}. Because {2,3,4} may have less weight and never has greater weight than {1,3,4}, {1,3,4} is not a weight-MWC. By contrast, {1,2} and {2,3,4} are weight-MWCs, because there exists no other member-MWC whose weight is always less than or equal to the weight of {1,2}, or to the weight of {2,3,4}.

It is important to emphasize that the weight-MWCs are based on the weight *orderings*, not the actual weights. To illustrate the difference

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[8]Although {1,3} has exactly the same total weight of 3 as {1,2} for the feasible weights given in Table 1, this would not be the case if $w_2 > w_3$.

between the orderings and the weights, consider a different set of feasible weights than that shown—namely, (2,2,1,1)—in Table 1 for category 1 when $n = 4$. If the feasible weights were (3,3,1,1), the total weight of the member-MWC that is not a weight-MWC, {1,3,4}, would be 5, which is less than that of weight-MWC {1,2}, which has a total weight of 6.

This example demonstrates that some weight-MWCs may not have the smallest numerical weights among all member-MWCs. However, there must be a weight-MWC among the member-MWCs that has minimum weight.

How does one construct the categories of member-MWCs and know that they exhaust all possibilities? We illustrate the process for $n = 4$, which will give one insight into how to construct a general algorithm for any finite n (not done here):[9]

1. Assume first that the coalition of the two largest players, {1,2}, is an MWC. Suppose {2,3,4}, in addition to {1,2}, is an MWC. Then there are three possibilities:

(i) {1,3,4} also is an MWC, which precludes {1,3} and {1,4};
(ii) {1,3}, but not {1,4}, also is an MWC, which precludes {1,3,4};
(iii) both {1,3} and {1,4} also are MWCs, which precludes {1,3,4}.

These three possibilities yield categories 1, 2, and 3, respectively.

2. Continuing with {1,2} as an MWC, suppose that {2,3,4} is winning but not an MWC. Then {2,3} must be an MWC, which precludes {1,4}. On the other hand, if {2,3,4} is *not* winning, then its weight is W/2 (which is

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[9]The algorithm we developed to generate all categories for this and the $n = 5$ and $n = 6$ cases relies on so-called Hasse diagrams. Technical details of the algorithm will be presented in a separate paper.

also the weight of player 1), and in this case {1,4} is an MWC. In addition, {1,3} is an MWC if either {2,3} or {1,4} is. These two possibilities yield categories 4 and 5, respectively.

3. Assume {1,2} is not an MWC. Then {1,2,3} must be an MWC, and so necessarily must all smaller triples, including the smallest triple, {2,3,4}. This gives category 6; the feasible weights of the four players must all be equal in this category, because otherwise {1,2} would be an MWC.

Because the grand coalition, {1,2,3,4}, is not an MWC—no member's defection from it is ever critical—the six categories of member-MWCs given in Table 1 are exhaustive.

Intersections of the weight-MWCs for each of the six categories for $n = 4$ are shown in Table 1.[10] Notice that player 1 is not a common member, except in one category in each case (category 1 when $n = 3$ and category 5 when $n = 4$).[11] That this is a general phenomenon for all n is shown by the

Theorem. *For all n, player 1 is never a common member of either all member-MWCs or of all weight-MWCs in any category unless* $w_1 = W/2$.

Proof. Let $\{\bar{1}\}$ be the complement of {1}. If $w_1 < W/2$, then $\{\bar{1}\}$, or some proper subset of $\{\bar{1}\}$, must be a member-MWC. But $\{\bar{1}\}$ or a proper subset does not contain player 1, so player 1 cannot be a common member of all member-MWCs in any category. Because either $\{\bar{1}\}$ or some proper

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[10]Luce and Rogow (1956) call such intersections "locations of power" in an analysis of presidential-congressional coalition structures; a description and empirical assessment of their model is given in Brams (1975, pp. 202-213).

[11]These are the categories in which player 1's weight is equal to that of all the other players' weights combined—that is, half the total weight.

subset must always be a weight-MWC, player 1 also cannot be a common member of weight-MWCs in any category. However, if $w_1 = W/2$, player 1—together with each and every one of the other players—will be in a member-MWC; hence, player 1 will be common to all member-MWCs and all weight-MWCs. □

The 20 mutually exclusive categories shown in Table 2 for the $n = 5$

Table 2 about here

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case were considerably more tedious to generate than for $n = 4$.[12] As in the $n = 3$ and $n = 4$ cases, intersections of the weight-MWCs for all categories are shown for $n = 5$, some of which contain no common members (indicated by ∅). Although we do not give feasible numerical weights in the latter case, weights that are consistent with the ordinal rankings can always be found.

4. Bargaining Power in MWCs

In this section, we will use the results for $n = 3$, 4, and 5 to calculate two quantitative measures of bargaining power and one of the "essentiality" of members in weight-MWCs. One concept of bargaining power is based on member-MWCs and the other on weight-MWCs. The concept of essentiality, which also can be viewed as a measure of bargaining power, is based on the intersections of the weight-MWCs.

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[12]There turn out to be 103 categories when $n = 6$, but this list is too lengthy to include. Consequently, results for this case will not be discussed in the subsequent analysis.

Define *member* bargaining power (M_i) to be the proportion of member-MWCs—summed across all categories—that player *i* is in for each case.[13] The results are the following for the three cases, where the maximum value is underscored:

$n = 3$: $\underline{M_1 = 4/5}$; $M_2 = 3/5$; $M_3 = 3/5$.
$n = 4$: $\underline{M_1 = 15/20}$; $M_2 = 12/20$; $M_3 = 12/20$; $M_4 = 9/20$.
$n = 5$: $\underline{M_1 = 81/113}$; $M_2 = 63/113$; $M_3 = 66/113$; $M_4 = 55/113$; $M_5 = 49/113$.

Thus, the weightiest player is always in the most member-MWCs, but M_i does not decrease monotonically when $n = 5$: player 3 is in more member-MWCs (66) than player 2 (63).

Now define *weight* bargaining power (W_i) to be the proportion of weight-MWCs—summed across all categories—that player *i* is in for each case. The results are the following for the three cases, where the maximum value(s) are underscored:

$n = 3$: $W_1 = 1/2$; $W_2 = 1/2$; $\underline{W_3 = 2/2 = 1}$.
$n = 4$: $W_1 = 4/9$; $\underline{W_2 = 6/9}$; $\underline{W_3 = 6/9}$; $\underline{W_4 = 6/9}$.
$n = 5$: $W_1 = 22/41$; $W_2 = 23/41$; $W_3 = 21/41$; $W_4 = 23/41$; $\underline{W_5 = 25/41}$.

Thus, the weightiest player is *never* in the most weight-MWCs; instead, the least weighty player is, who in one case ($n = 4$) is tied with two other players. Also, when $n = 5$, W_i does not increase monotonically: player 3 is in fewer weight-MWCs (21) than players 1 and 2 (22 and 23, respectively).

Finally, define *essentiality* (E_i) to be the proportion of intersections of weight-MWCs—summed across all categories—that player *i* is in for each

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[13]One could normalize these proportions so that they sum to 1, which would better reflect the *relative* bargaining power of the different players.

case. The results are the following for the three cases, where the maximum value(s) are underscored:

$n = 3$: $E_1 = 1/2$; $E_2 = 1/2$; $\underline{E_3 = 2/2 = 1}$.
$n = 4$: $E_1 = 1/6$; $\underline{E_2 = 3/6}$; $\underline{E_3 = 3/6}$; $\underline{E_4 = 3/6}$.
$n = 5$: $E_1 = 1/20$; $E_2 = 5/20$; $E_3 = 4/20$; $E_4 = 6/20$; $\underline{E_5 = 7/20}$.

These results mirror those of W_i, with the least weighty player being the most essential player (or tied for this position) in all three cases.

When $n = 5$, player 5 is a common player in more than one-third of all categories (7/20); at least one player is common in 14 of the 20 categories (all except the 6 categories with ∅ as their intersections in Table 2). Thus, in 70% of the categories, some player or players are *essential*: a weight-MWC cannot form without them. Having, in effect, a veto gives these players great leverage, especially when they are the sole common members, which is true in 8 of the 14 categories in which at least one player is essential (57%).

Perhaps most striking is that player 1 is essential in only one category in each case, as guaranteed by the Theorem. Thus, insofar as only member-MWCs or, more stringently, weight-MWCs form, the weightiest player, except when it has exactly half the total weight, is inessential.

Only the M_i index gives the weightiest player the edge, and not generally by a great margin. While the weightiest player is in the most member-MWCs overall, however, it is important to remember that M_i and the two other power indices give only summary results that are not necessarily descriptive of a specific situation.[14]

The particular category in which the weights of the different players place them is, in fact, determinative of the players' bargaining power in any specific situation. Thus, when $n = 5$ and the player weights put them in, say, category 6 (see Table 2), it is players 2 and 3 that are the most powerful, because they are members of more member-MWCs (i.e., 3) than players 1 (2), 4 (1), or 5 (1).

In fact, player 3 would seem to have an advantage over player 2, because player 3 is the unique common member of the two weight-MWCs, {1,3} and {2,3,5}, of category 6. In this situation, therefore, it is the player of only middling size who is indispensable when only weight-MWCs form.

5. Empirical Considerations

The fact that player 1 becomes increasingly dispensable in both member-MWCs and weight-MWCs as n increases, and less weighty players become more powerful by two of the three measures, suggests that (large) size may be a growing liability as the number of players increases beyond five. But, as Riker (1962) emphasized, the size principle may not always be operative, especially when there is incomplete or imperfect information in zero-sum games.

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[14]Unless one assumes that these categories are equally likely, the proportions that these indices give cannot be interpreted as probabilities that the different players will be in member-MWCs, weight-MWCs, or their intersections. Because the equiprobability assumption seems unwarranted, we focus less on the numerical values of the indices than on trends, such as that the most weighty player becomes virtually inessential as n increases, to the benefit of less weighty players. Riker (1962, p. 162) also showed, using a different model (see note 4), that smaller players frequently have "uniquely advantageous positions" in the cases considered here.

Other assumptions underlying the size principle may be violated. Consider the 123 coalition governments formed in twelve European parliamentary democracies between 1945 and 1987, in which no single party held more than 50% of the seats but the coalition governments did have a parliamentary majority.[15] In many of these cases, coalitions were not free-forming but instead were constrained by a left-right policy dimension (Laver and Budge, 1992). Additionally, the payoffs to the parties—measured in terms of cabinet posts or preferred policies—were generally not zero-sum but usually went to the coalition members who had the strongest preferences for these posts and policies, giving some benefit to everybody (Laver and Schofield, 1990, pp. 40-41).[16]

Thus, it is not surprising that in 46 of the 123 governments (37%), the governing coalitions that formed were larger than minimal winning. As for payoffs to the parties, they tended to be proportional; in fact, the best predictor of the numbers and prestige of the ministerial posts that a party received was its size in the legislature, with the post of prime minister being awarded to the largest party 80% of the time (Budge and Keman, 1990, pp. 89-131).

But other norms besides that of proportionality—including one based on the kind of bargaining power proposed here—do operate in some systems, especially those in highly competitive political environments in which cabinets tend to be unstable (Laver and Schofield, 1990, pp. 175-186).

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[15]In addition to these 123 majority governments, there were 73 minority governments (37% of the total) that did not control a majority of seats in parliament (Laver and Schofield, 1990, Table 4.2, p. 70).

[16]Even noncoalition members, on occasion, were paid off for their acquiescence, as happened in Italy for many years before the fall of the Christian Democratic party in 1993.

These cases may provide a better test of the model, especially its prediction that smaller parties may have greater bargaining power than larger ones when only MWCs form.

International politics is another arena in which an inverse relationship between power and weight has sometimes been observed. Small states, like Sweden during World War II (Fox, 1959), North Vietnam in its war with the United States from 1965 to 1973, Panama in its negotiations with the United States over the Panama Canal treaty in the 1970s (Habeeb, 1988), and Finland in its relations with Russia and the Soviet Union since 1945 (Vital, 1971), have exercised influence over outcomes far out of proportion to their size.[17]

It is less clear in these cases than the legislative cases, however, what larger *n*-person coalitional game was being played that helped to give a boost to the smaller players. The Arab-Israeli conflict is perhaps a better illustration of several countries' (including some outside the Middle East) striving to put together MWCs not just to fight wars but to gain economic, political, and other advantages. Israel, while not small by some standards, nevertheless seems to have enjoyed bargaining power disproportionate to its size in coalitional games that have been played in the region, including the Persian Gulf war in 1991.

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[17]A particularly striking theoretical explanation of the paradoxical relationship between small size and large power—deduced from a noncooperative game-theoretic model—is the "paradox of the chair's position" (Farquharson, 1969, pp. 50-51), but it depends on the players' having certain preferences (Brams, Felsenthal, and Maoz, 1986, 1988). Applications of this model to international politics are discussed in, among other places, Zagare (1979), Brams (1990, pp. 252-258), and Maoz (1990, pp. 219-250).

Table 1

Member-MWCs and Weight-MWCs (Underscored)

***n* = 3**

Category	*Feasible Weights*	*Intersection of weight-MWCs*
1. {1,2}, {1,3}	(2,1,1)	{1,3}
2. {1,2}, {1,3}, {2,3}	(1,1,1)	{2,3}

***n* = 4**

Category	*Feasible Weights*	*Intersection of weight-MWCs*
1. {1,2}, {1,3,4}, {2,3,4}	(2,2,1,1)	{2}
2. {1,2}, {1,3}, {2,3,4}	(3,2,2,1)	{3}
3. {1,2}, {1,3}, {1,4}, {2,3,4}	(2,1,1,1)	{4}
4. {1,2}, {1,3}, {2,3}	(2,2,2,1)	{2,3}
5. {1,2}, {1,3}, {1,4}	(3,1,1,1)	{1,4}
6. {1,2,3}, {1,2,4}, {1,3,4}, {2,3,4}	(1,1,1,1)	{2,3,4}

Table 2

Member-MWCs and Weight-MWCs (Underscored): $n = 5$

Category	*Intersection of weight-MWCs*
1. {1,2}, {1,3,4}, {1,3,5}, {1,4,5}, {2,3,4,5}	∅
2. {1,2}, {1,3,4}, {1,3,5}, {2,3,4}, {2,3,5}	{2}
3. {1,2}, {1,3,4}, {1,3,5}, {1,4,5}, {2,3,4}	∅
4. {1,2}, {1,3,4}, {1,3,5}, {1,4,5}, {2,3,4}, {2,3,5}	∅
5. {1,2}, {1,3,4}, {1,3,5}, {1,4,5}, {2,3,4}, {2,3,5}, {2,4,5}	{2}
6. {1,2}, {1,3}, {2,3,4}, {2,3,5}	{3}
7. {1,2}, {1,3}, {1,4,5}, {2,3,4,5}	∅
8. {1,2}, {1,3}, {1,4,5}, {2,3,4}	∅
9. {1,2}, {1,3}, {1,4,5}, {2,3,4}, {2,3,5}	∅
10. {1,2}, {1,3}, {1,4}, {2,3,4,5}	{4}
11. {1,2}, {1,3}, {1,4}, {2,3,4}	{4}
12. {1,2}, {1,3}, {1,4}, {1,5}, {2,3,4,5}	{5}
13. {1,2}, {1,3}, {1,4}, {1,5}	{1,5}
14. {1,2}, {1,3}, {2,3}	{2,3}
15. {1,2,3}, {1,2,4}, {1,2,5}, {1,3,4}, {1,3,5},{1,4,5}, {2,3,4,5}	{4,5}
16. {1,2,3}, {1,2,4}, {1,2,5}, {1,3,4}, {1,3,5},{1,4,5}, {2,3,4}	{4}
17. {1,2,3}, {1,2,4}, {1,2,5}, {1,3,4}, {1,3,5}, {2,3,4}, {2,3,5}	{2,3,5}
18. {1,2,3}, {1,2,4}, {1,2,5}, {1,3,4}, {1,3,5},{1,4,5}, {2,3,4} {2,3,5}	{5}
19. {1,2,3}, {1,2,4}, {1,2,5}, {1,3,4}, {1,3,5},{1,4,5}, {2,3,4} {2,3,5}, {2,4,5}	{2,4,5}
20. {1,2,3}, {1,2,4}, {1,2,5}, {1,3,4}, {1,3,5}, {1,4,5}, {2,3,4}, {2,3,5}, {2,4,5}, {3,4,5}	{3,4,5}

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