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WITH INVOLUNTARY UNEMPLOYMENT

by

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Real Business Cycles with Involuntary Unemployment

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Abstract

We develop and analyze a real business cycle model in which both goods and labor markets are characterized by imperfect competition. In equilibrium, unemployment emerges as the result of the market power exercised by *insiders* at the firm level. We show that a calibrated version of the model is capable of generating both a procyclical labor supply and a countercyclical unemployment rate, in a way qualitatively consistent with the U.S. data.

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1 Introduction

Stochastic dynamic general equilibrium models have become in recent years the central paradigm for the analysis and understanding of macroeconomic fluctuations. Though early applications were generally restricted to model economies for which technology shocks were the only source of fluctuations, and where built-in classical assumptions guaranteed the optimality of equilibrium allocations (thus leaving no room for any meaningful welfare or policy analysis), the flexibility of that paradigm has been illustrated in a number of recent papers which have developed models of economies characterized by the presence of non-classical features[1] and/or alternative sources of fluctuations[2]. As a result, and in contrast with the earlier literature, many of the recent models yield equilibrium allocations that are inefficient, and thus provide a rationale for corrective policies.

Despite the previous effort to enrich the basic framework in order to improve its empirical relevance and performance, most existing models–classical and non-classical–embed the assumption of continuously clearing, perfectly competitive labor market, thus effectively ruling out the possibility of involuntary unemployment. That feature flies in the face of actual market economies' experience, which–to a degree that varies both across countries and historical periods–are characterized by significant levels of unemployment, as well as large and persistent fluctuations in that variable. From the viewpoint of many societies, the magnitude of the unemployment problem, its social repercussions, and the central role it plays in the policy debate make the notion of "macroeconomics without unemployment" seems almost a contradiction in terms.[3]

[1]Examples include models with productive externalities (e.g., Baxter and King (1991)), imperfect competition (e.g., Chaterjee and Cooper (1993), Rotemberg and Woodford (1992)), policy distortions (Greenwood and Huffman (1991),Galí (1994)), and cash-in-advance constraints (e.g., Cooley and Hansen (1989)), among other non-classical assumptions.

[2]Including shocks to government spending (e.g., Christiano and Eichenbaum (1992)), distortionary tax shocks (e.g., Braun (1994)), and even sunspots (e.g., Farmer and Guo (1994), Galí (1994)).

[3]Even though unemployment is often viewed by economists and commentators as a European disease, its cyclical variations have historically played a major role in American

On the other hand, the traditional macroeconomic literature on unemployment[4], though rich and full of insights, has been largely restricted to static or partial equilibrium models, thus falling short of adopting the explicitly dynamic, optimizing, general equilibrium framework that has proved so useful in modelling other aspects of economic fluctuations. Unemployment models in that tradition include models with unions (e.g., Oswald (1985)), efficiency wages (e.g., Shapiro and Stiglitz (1984)), as well as models with insider-outsiders (e.g., Lindbeck and Snower (1988)), among others. A common theme of those models is the absence of a perfectly competitive labor market, with wages being instead set (by firms, workers, or both) at some level above their market clearing level, thus leading to involuntary unemployment, i.e. the inability of (some) individual workers to sell as much labor services as they wish to supply, given the prevailing wages and other labor conditions. In those models unemployment is thus a consequence of a *market failure* (manifested in wages which fail to equate the opportunity cost of work).[5]

In the present paper we try to bridge part of the gap between recent business cycle modelling strategies and traditional models of unemployment with imperfect competition in labor markets. Specifically, we develop and analyze a dynamic general equilibrium model that is consistent (at least qualitatively) with a number of stylized facts regarding the cyclical behavior of the labor market, in addition to other features of their business cycles. In particular, we want to explain (a) employment is highly procyclical and almost as variable as GNP, (b) the labor force is mildly procyclical and substantially less volatile than GNP and employment, and (c) the unemployment rate is highly countercyclical and roughly half as variable as GNP. The three previous observations are illustrated in Figures 1, 2, and 3 (respectively) and Table 1. The dashed line in each figure corresponds to the HP-filtered time series for (log) US GNP. The solid lines plot, respectively, the HP filtered logs of measures of employment, the labor force, and unemployment. A de-

politics and election outcomes. In fact, Americans' concerns about unemployment seem to show fluctuations as large as unemployment itself. Over the period 1981-1992, the percentage of respondents to the Gallup Poll who ranked unemployment as the most important problem facing the United States fluctuated between 3 % (1990) to 53 % (1983). The difference in unemployment rates between those two years was of about 2 percentage points.

[4]See, e.g., Layard et al. (1991) for a review of some of that literature, and Bean (1994) for a recent survey on its empirical applications to the European case.

[5]See also Silvestre (1993) for a discussion of the connection between unemployment and market power in goods and labor markets.

scription of the data and sources can be found in section 5.4 . Though (a) is well known and frequently reported, (b) and (c) have generally been ignored by the recent business cycle literature.

In order to focus on the mechanisms that bring about unemployment and to compare our results with those of other researchers we choose to depart as little as possible from the standard RBC paradigm. In particular, we assume that technology shocks are the only source of fluctuations and that the economy is populated by a continuum of identical infinite-lived consumers. The latter assumption (combined with the absence of labor indivisibilities) implies that in our model unemployment takes the form of rationing at the *intensive* margin, i.e. it involves constraints on the number of hours each person works (as opposed to the number of people who work). More specifically, we define involuntary unemployment in our model economy as the difference between (a) hours of work that a perfectly competitive agent wishes to supply *given* the actual law of motion for wages and interest rates, and (b) actual employed hours.[6] In our model the existence of a gap between (a) and (b) is a consequence of the exercise of market power by *insiders* (i.e., incumbent workers insulated from competition by the existence of large labor turnover costs) who manage to bring wages above their market clearing levels. The existence of cyclical variations in that gap follows from the cyclical variations in workers' degree of market power that result from our assumptions on technology and market structure. We want to stress, however, that neither the notion of involuntary unemployment nor the methodology to compute the equilibrium process for the unemployment rate introduced in this paper hinge on the specific mechanism that is assumed to generate unemployment in our model, and could be easily applied to any other dynamic models with non-clearing labor markets.

In contrast with much of the literature on wage setting by unions or insiders–which typically treats capital as either a fixed or irreversible factor–our model sticks to the real business cycle tradition of assuming the existence

[6]Though below we present some evidence on the quantitative importance and strong cyclical behavior of unemployment at the intensive margin in the U.S. economy, it is clear that the bulk of unemployment in most market economies takes the form of some people not working at all, as opposed to every person not working as much as he would want to. Here we are not trying to give a literal interpretation to the type of unemployment generated by the model, but rather view it as a shortcut– associated with the use of a representative consumer framework–to the modelling of "total unemployment" (i.e., unemployment at both the intensive and extensive margins).

of a competitive capital rental market which allows firms to adjust their capital stock levels at any time. In that context, and given the assumed homogeneity properties of technology, the presence of imperfect competition in the goods market is a *necessary* condition for workers' at the firm level to have any market power and thus for involuntary unemployment to exist. Furthermore, under our (standard) assumptions on preferences and technology, we show that the existence of *fluctuations* in the unemployment rate requires cyclical variations in the degree of market power–reflected in countercyclical markups and resulting (in our model) from entry and exit of firms. [7]

We approximate the equilibrium of our model economy in a neighborhood of the steady state using the log-linearization technique described in Campbell (1994). Once we determine the equilibrium law of motion for quantities and prices, we proceed to construct the corresponding equilibrium process for the unemployment rate. Given our definition, that involves solving a partial equilibrium dynamic optimization problem of a perfectly competitive consumer-worker who chooses his optimal labor supply while taking as given the law of motion for factor prices that characterizes the (imperfectly competitive) equilibrium. Interestingly, the Campbell solution method can also be used to solve for the optimal labor supply rule of the competitive agent in terms of the economy-wide state variables. The law of motion for equilibrium unemployment is then easily obtained by combining the competitive labor supply decision rule and the equilibrium law of motion for employment.

Before we introduce our formal model we briefly discuss some of the differences between our approach and the related work of other authors. First, we want to stress that the concept of unemployment found in the indivisible labor economies analyzed by Rogerson (1988) and Hansen (1985) differs from the one adopted in this paper. In their models unemployment of a (randomly selected) fraction of the labor force emerges in an optimal allocation in the presence of labor indivisibilities (i.e., when each consumer can either work a fixed number of hours or not work at all). That optimal allocation can be decentralized by introducing labor contracts that involve a lottery that determines whether a given individual will have to work or not and by allowing each worker to choose the probability associated with that lottery. Even if we take this (unrealistic?) market arrangement as given, there is no sense in which unemployment in the Hansen-Rogerson economies

[7] Rotember and Woodford (1991) provide some empirical evidence for the presence of such countercyclical markups.

is involuntary, for all consumers freely enter such contracts at the prevailing market conditions, and they do not face any "quantity" constraints in their choice of a probability of working.[8]

Our concept of unemployment also differs from that found in search models. In the latter a *technological constraint*, usually in the form of a matching function[9] or a time cost for job reallocation[10], prevents those who lose or quit their job from finding employment immediately somewhere else. In that context, unemployment is associated with time allocated to search activities. In some search models (e.g., Greenwood et al.) unemployment can be viewed as voluntary, in the sense that it is consistent with a (privately) optimal decision by workers to quit their current jobs (given the wage and opportunity cost of remaining in the job). In other search models (e.g., Mortensen (1990)), unemployment results from exogenous separations, that occur independently of the wage. In contrast, our model focuses on insiders' market power and the resulting wage distortions as a source of unemployment.

In a spirit much closer to the present paper, Danthine and Donaldson (1990,1992) have examined the implications of introducing a variety of non-Walrasian labor market features in an otherwise standard RBC model. Such features include efficiency wages (Danthine and Donaldson (1990)[11], as well as risk-sharing arrangements, minimum wages and unemployment subsidies (Danthine and Donaldson (1992)). As is well known, some of those arrangements can lead to wages above their competitive market-clearing level and, thus, generate involuntary unemployment or rationing, as in the present paper. Yet, and in contrast with the framework developed below, the previous authors impose an inelastic labor supply, and thus assume away the need to solve for the optimal labor supply rule of a competitive consumer-worker: in their model unemployment is simply given by the difference between the quantity of labor demanded and an *exogenous* aggregate labor endowment. In contrast with our model their structure cannot account for the cyclical fluctuations in the labor force and/or in the desired workweek, a feature of the data present in most market economies.[12]

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[8]Furthermore, given the full consumption insurance implied by their specification, the unemployed are (ex-post) better off since they consume the same amount of physical good, and enjoy more leisure.

[9]See, e.g., Mortensen (1990) and Pissarides (1990).

[10]See, e.g., Lucas and Prescott (1974), Jovanovic (1987), and Greenwood et al. (1994).

[11]See also MacLeod et al. (1994).

[12]See, e.g., Elmeskov and Pichelmann (1993).

The plan of the paper is as follows. Section 2 presents the model. Section 3 defines an equilibrium. Section 4 discusses the problem faced by a perfectly competitive agent and defines unemployment. Section 5 describes the solution method and the statistical properties of a number of calibrated versions of the model. Section 6 concludes and points to some possible extensions.

2 The Model

The production side of the economy consists of two sectors which turn out, respectively, a single final good and a continuum of intermediate inputs. Next we describe the market structure and technology in some detail.

2.1 Final Good

The final good is produced by a perfectly competitive representative firm. That firm has access to a constant returns technology that transforms intermediate inputs into the final good, and which is represented by the CES production function

$$Y = \left(\int_0^1 X(z)^{\frac{\epsilon-1}{\epsilon}} dz \right)^{\frac{\epsilon}{\epsilon-1}} \tag{1}$$

where Y denotes the output of the final good, $X(z)$ is the quantity of input $z \in [0,1]$, and $\epsilon > 1$ is the elasticity of substitution among intermediate inputs. The representative firm maximizes its profits at each point in time

$$\pi_y \equiv \max \; Y - \int_0^1 p(z) \; X(z) \; dz$$

subject to (1), where $p(z)$ denotes the price of input z in terms of the final good (which is taken as the numéraire). The first order conditions for the problem above take the form of a set of demand functions for intermediate inputs

$$X(z) = \left(\frac{p(z)}{P} \right)^{-\epsilon} \left(\frac{I}{P} \right), \qquad z \in [0,1] \tag{2}$$

where $I \equiv \int_0^1 p(z) \; X(z) \; dz$, and $P \equiv \left(\int_0^1 p(z)^{1-\epsilon} \right)^{\frac{1}{1-\epsilon}}$. It is easily checked that for an equilibrium with positive production to exist it must be the case that $P = 1$, which in turn implies $Y = I$, and $\pi_y = 0$.

2.2 Intermediate goods

The intermediate sector is made up of a continuum of industries represented by the unit interval. Each industry, indexed by $z \in [0,1]$, consists of a finite number of firms producing a homogeneous intermediate good that is sold to the final goods sector. A typical firm (say, firm j) in a given industry has access to a production function

$$X_{jt} = \exp(\phi_t)\ (K_{jt} - \upsilon\ \gamma^t\ )^\alpha\ (\gamma^t\ L_{jt})^{1-\alpha} \tag{3}$$

X_{jt}, K_{jt} and L_{jt} denote, respectively, the level of output and the quantities of capital and labor services employed by firm j in period t.[13] Each firm has an overhead capital requirement $\upsilon\ \gamma^t$, which grows at an exogenous gross rate γ, which also corresponds to the rate of growth of labor-augmenting technology.[14] $\{\phi_t\}$ denotes the stochastic component of technology which follow the first-order autoregressive process

$$\phi_t = \rho\ \phi_{t-1} + e_t \tag{4}$$

with $|\rho| < 1$, $Ee_te_{t-j} = 0$, for $j \neq 0$, and $Ee_t^2 = s^2$, $t = 0, 1, 2, ...$ Parameters υ and γ, as well as the realizations of $\{\phi_t\}$ are assumed to be common to all industries and firms.

Each intermediate firm rents labor and capital services from consumers, taking the wage W_j and the rental cost of capital q as given. Thus, the (static) profit maximization problem faced by an intermediate firm each period can be formalized as follows:

$$\pi_{jt} \equiv \max\ p_{jt}\ X_{jt} - W_{jt}\ L_{jt} - q_t\ K_{jt}$$

subject to (3) and the inverse demand schedule (derived from (2))

$$p_t = \left[\frac{(m_t - 1)\ \overline{X}_{-jt} + X_{jt}}{Y_t}\right]^{-\frac{1}{\epsilon}} \tag{5}$$

where m denotes the number of active firms in the industry, $\overline{X}_{-j}$ is the average output for the $(m-1)$ firms in the industry (other than firm j) . We assume Cournot competition at the industry level, with each firm taking

[13]In order to lighten the notation and given the symmetry across industries embedded in the model we omit the industry index z whenever there is no risk of confusion.

[14]That assumption is needed for a balanced growth path with positive growth to exist.

aggregate demand Y_t, the quantities produced by other firms in the industry $\overline{X}_{-jt}$, and the number of firms in the industry m as given.

The associated first order conditions equate each factor's marginal revenue product to its rental cost, and are given by

$$p_t \left(1 - \frac{1}{\xi_{jt}}\right)\left(\frac{\partial X_{jt}}{\partial K_{jt}}\right) = q_t \tag{6}$$

$$p_t \left(1 - \frac{1}{\xi_{jt}}\right)\left(\frac{\partial X_{jt}}{\partial L_{jt}}\right) = W_{jt} \tag{7}$$

where $\xi_j = \epsilon \left(1 + \frac{(m-1)\,\overline{X}_{-j}}{X_j}\right)$ is the price-elasticity associated with firm j's demand schedule.

Letting $w_{jt} \equiv \frac{W_{jt}}{\gamma^t}$, the quantity of labor employed by the firm (L_{jt}) is (implicitly) determined by

$$J(L_{jt}, w_{jt}, \theta_{jt}) = 0 \tag{8}$$

where $\theta_{jt} \equiv [\phi_t, q_t, \overline{x}_{-jt}, m_t, y_t]$, with $\overline{x}_{-jt} \equiv \frac{\overline{X}_{-jt}}{\gamma^t}$ and $y_t \equiv \frac{Y_t}{\gamma^t}$, and where $J : \Re^7 \rightarrow \Re$ is a continuously differentiable function formally derived in the appendix. θ_{jt} is a vector of variables taken as given by each individual firm when maximizing profits (and which will also be taken parametrically by its workers in the wage-setting process, as we will see below). We note that (8) already embeds the firm's optimal choice of capital, as well as its price-setting decision.

We focus on a symmetric equilibrium in which all firms in all industries produce the same quantities and employ the same levels of inputs. In that case we have $p_t = 1$, $\overline{X}_{-jt} = X_{jt}, \xi_{jt} = \epsilon\, m_t$, and $K_{jt} = \frac{K_t}{m_t}$, for $j = 1, 2, ...m$, where K_t denotes the aggregate capital stock.

Conditions (6) and (7) can be used to derive an expression for an individual firm's profits in a symmetric equilibrium:

$$\pi_{jt} = X_{jt} \left[\frac{1}{\epsilon\, m_t} - \alpha \left(1 - \frac{1}{\epsilon\, m_t}\right)\left(\frac{m_t\, v}{k_t - m_t\, v}\right)\right] \tag{9}$$

where $k_t \equiv \frac{K_t}{\gamma^t}$.

Under the assumption of free entry and zero profits we can solve for the number of firms as a function of the aggregate capital stock

$$m(k_t) = \left(\frac{1-\alpha}{2\alpha\epsilon}\right)\left(\sqrt{1+\Psi\left(\frac{k_t}{\upsilon}\right)}-1\right) \tag{10}$$

where $\Psi \equiv \frac{4\alpha\epsilon}{(1-\alpha)^2}$. The previous result, combined with the symmetric equilibrium condition $y_t = m_t x_{jt}$, (where $x_{jt} \equiv \frac{X_{jt}}{\gamma^t}$ and $y_t \equiv \frac{Y_t}{\gamma^t}$) allows us to derive the following reduced-form aggregate production function

$$y_t = \exp(\phi_t)\ \varphi(k_t)^\alpha\ L_t^{1-\alpha} \tag{11}$$

where $\varphi(k_t) \equiv k_t - m(k_t)\upsilon$ and $L_t \equiv m(k_t)\ L_{jt}$.

Furthermore, using (10) we can derive an expression for the wage-elasticity of a firm's labor demand $\eta_{jt} \equiv \frac{\left(\frac{\partial J_{jt}}{\partial w_{jt}}\right)}{\left(\frac{\partial J_{jt}}{\partial L_{jt}}\right)} \frac{w_{jt}}{L_{jt}}$ evaluated at the symmetric equilibrium, as a function of the aggregate capital stock

$$\eta(k_t) = \alpha + (1-\alpha)\left(\frac{(\epsilon\ m(k_t)-1)\ \epsilon\ m(k_t)}{2\ \epsilon\ m(k_t)-(1+\epsilon)}\right) \tag{12}$$

Henceforth we assume that $\eta > 1$ and $\eta'(k) > 0$. Sufficient conditions that guarantee that the previous inequalities hold (at least in a neighborhood of the steady state) are very weak and will generally hold for any reasonable set of parameter values.[15] In that case the wage elasticity of labor demand η is positively related to the price elasticity of the demand for the intermediate good $\xi = \epsilon\ m(k_t)$, which in turn is increasing in the aggregate capital stock. The intuition for that result is as follows: in the face of an idiosyncratic wage increase, the firm's optimal response involves (a) an increase in the capital/labor ratio and (b) a reduction in output (with the consequent price increase). Both (a) and (b) work in the direction of reducing employment. The size of the downward adjustment in employment will depend on the optimal reduction in output, and the latter will be greater the higher is the price-elasticity of demand.[16]In the limiting case of perfect competition

[15]It is easy to check that $\eta' > 0$ will hold if $m > \frac{(1+\epsilon)+\sqrt{\epsilon^2-1}}{2\ \epsilon}$. Using 10 we can see that such an inequality will be satisfied as long as the steady state capital stock is above a certain threshold. That condition was easily satisfied in all the simulations reported below.

[16]The existence of a connection between goods markets competitiveness and the the degree of workers' market power is a standard result in the wage bargaining literature (see, e.g., chapter 2 in Layard et al. (1991)).

in the goods market (as implied, say, by $\varepsilon \to \infty$), the individual firm's labor demand schedule becomes perfectly flat (i.e., $\eta \to \infty$) and, as a result, workers become *de facto* wage takers.[17]

2.3 People

We assume the existence of a continuum of agents (consumer-workers) uniformly distributed over the unit interval and indexed by $i \in [0,1]$. Individual preferences are represented by the utility function

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(\log c_{it} - \chi \, \sigma \, n_{it}^{\frac{1}{\sigma}} \right) \tag{13}$$

where c_{it} is (normalized) consumption, n_{it} is hours of work, $\beta \in (0,1)$ is the discount factor, and $\sigma \in (0,1)$ determines the curvature of labor disutility (and thus the wage elasticity of labor supply, which is given by $\frac{\sigma}{1-\sigma}$).

Agent i's dynamic budget constraint is given by

$$\gamma \; k_{i,t+1} = R_t \; k_{it} + w_{jt} \; n_{it} - c_{it} \tag{14}$$

where $R_t \equiv ((1-\delta)+q_t)$, δ is the depreciation rate, and w_{jt} is the wage paid by the firm (j, say) where consumer i works.

Let us now turn to the wage setting and employment decisions. The assumed structure aims at capturing in a stylized manner some of the labor market rigidities that may be at the root of any market power enjoyed by workers. At the beginning of period t, firm j enters a labor contract with a set of workers of measure $\tau_{jt} \in [0,1]$. Labor contracts last one period. Contracts entitle (a) workers to set the wage rate unilaterally (after observing the technology shock), and (b) the firm the right to choose its desired level of employment (i.e., the number of man-hours), given that wage.[18] Early termination of the contract by the firm is effectively ruled out by the existence of (sufficiently large) firing costs. Furthermore, we assume that either de iuris–because of closed shop like restrictions–or de facto –because of insiders'

[17] In other words, in that case the wage is pinned down by the factor price frontier, given the rental cost of capital. That result follows from technology's homogeneity of degree one assumption combined with the absence of a "predetermined" input.

[18] The assumed wage setting structure corresponds to models of "monopoly union with right-to-manage". As discussed in McDonald and Solow (1981), those contracts are known to be inefficient in general (i.e., both workers and firms could be better off if they could bargain over both wages and employment).

threat not to cooperate or to harass any additional hires–the firm is prevented from hiring any additional workers ("outsiders") once the shock has been observed, and before the beginning of the following period. Thus, each firm's insiders effectively hold the monopoly on the supply of labor services to that firm. In particular, and because of the labor turnover costs suggested above, they cannot be underbid by external workers. Accordingly, the labor demand schedule faced by firm j's insiders in period t is given by

$$J(\tau_{jt}\ n_{it}, w_{jt}, \theta_{jt}) = 0 \tag{15}$$

As will become clear below, the symmetry embedded in our model implies that workers in the same firm effectively face an identical problem. They also have an obvious incentive to exploit to the full extent their market power in the wage setting process. That leads them to jointly determine the wage, consumption and savings consistent with the maximization of (13) subject to the dynamic budget constraint (14) and the labor demand schedule (15), while taking as given the equilibrium process for economy and industry-wide variables in θ_{jt}.

The optimality conditions for that problem are given by

$$w_{jt} = \lambda_{jt}\ c_{it}\ n_{it}^{\frac{1-\sigma}{\sigma}} \tag{16}$$

$$\beta\gamma^{-1}\ E_t\ \left\{\left(\frac{c_{it}}{c_{i,t+1}}\right)\ R_{t+1}\right\} = 1 \tag{17}$$

$$\lim_{T\to\infty} E_t\ \beta^T\ \left(\frac{k_{iT}}{c_{iT}}\right) = 0 \tag{18}$$

for $t = 0, 1, 2, 3...$ where $\lambda_{jt} \equiv \frac{\eta_{jt}}{\eta_{jt}-1}$ is the wedge between the real wage and the marginal relation of substitution between consumption and work hours driven by market power in labor markets, and which can be interpreted as a *wage markup*. In a symmetric equilibrium with $\tau_{jt} = \frac{1}{m_t}$ for all j, it follows from (12) that the wage markup will be common to all firms and given by $\lambda(k_t) \equiv \frac{\eta(k_t)}{\eta(k_t)-1}$.

3 Equilibrium

We define a (symmetric) equilibrium of our model economy as a stochastic sequence $\{k_t, y_t, c_t,\ n_t, w_t, R_t\}_{t=0}^{\infty}$, satisfying

$$\gamma\ k_{t+1} = (1-\delta)\ k_t + y_t - c_t \tag{19}$$

$$y_t = \exp(\phi_t)\ \varphi(k_t)^{\alpha}\ {n_t}^{1-\alpha} \tag{20}$$

$$\beta\gamma^{-1}\ E_t\ \left\{\left(\frac{c_t}{c_{t+1}}\right)\ R_{t+1}\right\} = 1 \tag{21}$$

$$w_t = \lambda(k_t)\ \chi\ c_t\ n_t^{\frac{1-\sigma}{\sigma}} \tag{22}$$

$$w_t = \left(\frac{1}{\mu(k_t)}\right)(1-\alpha\ )\exp(\phi_t)\ \left(\frac{\varphi(k_t)}{n_t}\right)^{\alpha} \tag{23}$$

$$R_t = (1-\delta) + \left(\frac{1}{\mu(k_t)}\right)\alpha\ \exp(\phi_t)\ \left(\frac{\varphi(k_t)}{n_t}\right)^{-(1-\alpha)} \tag{24}$$

$$\lim_{T\to\infty} E_t\ \beta^T\ \left(\frac{k_T}{c_T}\right) = 0 \tag{25}$$

together with (4), and where $\mu(k_t) \equiv \frac{\epsilon\ m(k_t)}{\epsilon\ m(k_t)-1}$ is the price markup. Under our assumptions, $\mu' < 0$ and $\lambda' < 0$, reflecting the fact that as the aggregate capital stock accumulates, entry of new firms leads to a reduction in equilibrium markups, for both prices and wages.

Given the recursive structure of the model, the equilibrium process $\{k_t, y_t, c_t, n_t, w_t, R_t\}_{t=0}^{\infty}$ can be in principle represented as a first order difference equation for the vector of state variables, i.e. $[k_t, \phi_t]' = f([k_{t-1}, \phi_{t-1}]')$, together with a set of equilibrium conditions linking each aggregate variable with the contemporaneous values of the two state variables, i.e., $y_t = y(k_t, \phi_t)$, $c_t = c(k_t, \phi_t)$, $n_t = n(k_t, \phi_t)$, $w_t = w(k_t, \phi_t)$, and $R_t = R(k_t, \phi_t)$. Unfortunately, the nonlinear nature of equilibrium conditions (19)-(25) does not allow us to obtain an exact solution for the equilibrium functions but, as we describe below, we can approximate their behavior in a neighborhood of a steady state by means of the log-linearization method of Campbell (1994).

4 Unemployment

We define *unemployment* in a given period as the difference between (a) the quantity of labor services that the representative consumer-worker would wish to sell *if he took the equilibrium law of motion for wages and interest rates generated by the imperfectly competitive economy as given*, and (b) the actual quantity of labor services employed.

Given the law of motion for employment implied by (19)-(25), determining the equilibrium process for unemployment requires solving the partial equilibrium dynamic optimization problem faced by a hypothetical perfectly competitive consumer-worker, whose decisions have a negligible impact on the economy. Formally, this involves maximizing

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(\log c_t^* - \chi \, \sigma \, n_t^{*\frac{1}{\sigma}} \right)$$

subject to

$$\gamma \, k_{t+1}^* = R_t \, k_t^* + w_t \, n_t^* - c_t^* \tag{26}$$

and the equilibrium laws of motion

$$[k_t, \phi_t]' = f([k_{t-1}, \phi_{t-1}])$$

$$w_t = w(k_t, \phi_t)$$

$$R_t = R(k_t, \phi_t)$$

where we let variables with a '$*$' superscript denote the (normalized) choice variables of the competitive consumer-worker. The optimality conditions associated with that problem are given by

$$\beta \gamma^{-1} \, E_t \left\{ \left(\frac{c_t^*}{c_{t+1}^*} \right) \, R_{t+1} \right\} = 1 \tag{27}$$

$$w_t = \chi \;\; c_t^* \; n_t^{*\frac{1-\sigma}{\sigma}} \tag{28}$$

and the transversality condition $\lim_{T \to \infty} E_t \, \beta^T \, \left(\frac{k_T}{c_T} \right) = 0$.

The optimal labor supply choice for the wage-taking consumer can be written as a function of the two aggregate state variables k_t and ϕ_t and the individual state variable k_t^*

$$n_t^* = n^*(k_t^*, k_t, \phi_t)$$

Given our definition, the *unemployment rate* u is determined by the following function of (k_t, ϕ_t):

$$u_t = u(k_t, \phi_t) \equiv \log \left(\frac{n^*(k_t, k_t, \phi_t)}{n(k_t, \phi_t)} \right) \tag{29}$$

where the symmetry assumption $k_t^* = k_t$ is imposed so that n_t^* can be interpreted as the labor supply choice of the representative agent if, behaving as a price-taker, he did not face any rationing (i.e., he could sell as much labor as he wished at the current wage) with his individual decision having a negligible aggregate effect. Unemployment thus defined can be interpreted as *involuntary* in the following sense: in the absence of labor market rigidities restricting the hiring of outsiders, an *individual* worker would choose to work longer hours, for a wage no greater than the current wage, and any firm would be willing to hire his services.

5 Approximate Equilibrium Dynamics

In order to solve for the laws of motion describing the economy's equilibrium behavior, we apply the method of undetermined coefficients to a log-linear approximation of (19)-(25) around the associated perfect foresight steady state.[19] Interestingly, the same method can be applied to derive a log-linear approximation to the competitive labor supply policy rule $n^*(k_t^*, k_t, \phi_t)$, a result which can then be used to approximate the equilibrium unemployment function $u(k_t, \phi_t)$.

5.1 Steady State

Setting $\phi_t = 0$, all t, and dropping all time subscripts (and the expectation operator) in (19)-(24) we obtain a system of equations implicitly determining the (perfect foresight) steady state vector $\{k, y, c, n, w, R\}$:

$$R = \frac{\gamma}{\beta} \tag{30}$$

$$c = \left(\frac{r + \delta}{\alpha} \right) \mu(k) \ \varphi(k) \ - (\gamma - 1 + \delta) \ k \tag{31}$$

[19]See Campbell (1994) for an exposition of that solution method.