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FAIR DIVISION AND POLITICS

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Fair Division and Politics

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Abstract. The origins of fair division—and, in particular, “divide-and-choose”—are traced back to the Bible, ancient Greece, and the work of the 17th-century English political theorist, James Harrington. Its modern manifestations in the Law of the Sea, and “filter-and-choose” in the American legislative arena, are discussed, with particular attention given to how information available to the players may affect their choices.

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Fair Division and Politics

In the Hebrew Bible, the issue of fairness is raised in some of the best-known narratives. Cain's raging jealousy and eventual murder of Abel is provoked by what he considered unfair treatment by God, who "paid heed" to Abel's offering but ignored his (Gen. 4:4).¹ Jacob, after doing seven years of service in return for Laban's beautiful daughter, Rachel, was told that his sacrifice was not sufficient and that he must instead marry Laban's older and plainer daughter, Leah, unless he did seven more years' service, which he regarded as not only the breaking of a contract but also blatantly unfair. Fairness triumphed, however, when King Solomon proposed to divide a baby, claimed by two mothers, in two. When the true mother protested and offered the baby to the other mother (whose baby had died), the truth about the baby's maternity became apparent, and "all Israel . . . stood in awe of the king; for they saw that he possessed divine wisdom to execute justice" (1 Kings 3:28).

Solomon's proposed solution is the first explicit mention of fair division that we know of in recorded history. But it is, of course, no solution at all: Solomon had no intention of dividing the baby in two. Instead, his purpose was to set up a game between the two women, described in Brams (1980, 1990), that would distinguish the mother from the impostor.²

The solutions of the fair-division procedures we shall discuss are for two players.³ They do not depend on the use of outside parties, such as mediators or arbitrators, and they guarantee a certain "minimal" outcome to each player who uses the recommended strategy, regardless of what the other player does. As we shall show, however, sometimes this minimal outcome can be improved upon through strategic maneuvers that, like Solomon's, capitalize on information that might be privileged, or at least not common knowledge.

Such strategizing, because it inevitably requires departing from the script of a fair-division procedure, is risky: it forces a player to give up the security of a guaranteed

minimal outcome. Solomon's deception is no exception. If he had failed to outwit the impostor with his announcement, and she had protested too, Solomon might well have had to carry out his edict to divide the baby, which would have pleased nobody except, perhaps, the impostor.

Divide-and-Choose

As a first step toward specifying criteria that a fair-division procedure should satisfy, consider the well-known procedure for dividing a cake between two people, whom we will call Bob and Carol: Bob cuts the cake, and Carol chooses one of the two pieces. We assume that the cake is *divisible*, so it can be cut at any point without destroying its value. Yet it need not be *homogeneous*, or the same throughout; rather, we suppose it to be *heterogeneous*, with Bob's and Carol's liking different parts that cannot be separated (e.g., as in a layer cake).

"One divides, the other chooses" is a *procedure* or *algorithm*, which gives the rules for a division, but it is not a *solution*, which also entails a description of how to apply these rules to achieve some stated level of satisfaction. The solution to this problem, however, is well known: Bob divides the cake into two pieces, between which he is indifferent; and Carol chooses what she considers to be the larger piece.⁴ It is easy to see that this solution possesses two properties:

1. Bob can guarantee himself what he perceives to be at least $1/2$ the cake (or $1/2$ the value of the cake; see note 4), and Carol can guarantee herself what she perceives to be at least $1/2$ the cake. A fair-division procedure with this property is said to be *proportional*.⁵
2. Neither Bob nor Carol thinks that the other player received a larger piece of cake than he or she did. A fair-division procedure with this property is said to be *envy-free*.

Envy-freeness and proportionality are equivalent when there are only two players—that is, the existence of one property implies the existence of the other. Thus, if

each player receives what he or she thinks is at least $1/2$ the cake (proportionality), neither thinks the other player received more (envy-freeness). Conversely, if neither player envies the other player (envy-freeness), each must think he or she received at least $1/2$ the cake (proportionality).

There is no such equivalence when there are three or more players. For example, if each of three players thinks he or she received at least $1/3$ of the cake, it still may be the case that one player thinks another received a larger piece (say, $1/2$), so proportionality does not imply envy-freeness. On the other hand, if none of the players envies another, each must believe that he or she received at least $1/3$ of the cake.⁶ In general, envy-freeness is the stronger notion of fairness: whenever it exists, so does proportionality, but not vice-versa. However, because one cuts, the other chooses is a two-person procedure that is proportional, it is also envy-free.

The origins of *divide-and-choose*, as we will call it, go back to antiquity. The first mention we have discovered is in Hesiod's *Theogony*, written some 2,800 years ago. The Greek gods, Prometheus and Zeus, had to divide a portion of meat. Prometheus began by placing the meat into two piles, and Zeus selected one.⁷

Divide-and-choose has important applications today. For example, the 1982 Convention of the Law of the Sea, which went into effect on November 16, 1994, with 159 signatories (including the United States), specifies that whenever a developed country wants to mine a portion of the seabed, that country must propose a division of the portion into two tracts. An international mining company called the Enterprise, funded by the developed countries but representing the interests of the developing countries through the International Seabed Authority, plays the role of the other party, choosing the tract it prefers; the developed country receives the other tract. In this manner, parts of the seabed are preserved for commercial development by the developing countries, which—in the absence of Enterprise—could not otherwise afford to mine the seabed. (Mining, however, probably will not begin in earnest for another decade.)

For divide-and-choose and most other fair-division procedures, the crucial assumption is that individual preferences are *additive*: if A is larger than B, and C is larger than D, and there is no overlap between pieces A and C, then A together with C is larger than B together with D. Although more realistic under the size interpretation than the value interpretation (see note 4), additivity is still sensible in many “value” situations, such as seabed mining.

Filter-and-Choose and Applications

The first application of divide-and-choose to politics of which we are aware was proposed by the English political theorist, James Harrington (1611-77), in his book, *The Commonwealth of Oceana* (1656).⁸ Dubbed “Harrington’s law” by Goodwin (1992, p. 94), it was offered in the context of Harrington’s analysis of a utopian polity, Oceana, intended as a model for England (Blitzer, 1960, p. 32; Pocock, 1992, p. xvii).

According to Harrington, the Senate of Knights of Oceana would, after debate, propose legislation, and the House of Deputies would, without debate, vote on it, putting the Senate in the position of Bob and the House in the position of Carol.⁹ Thereby, Harrington argued, the vested interests of neither chamber could become dominant; it was a central tenet of his theory of egalitarian justice that also provided for the regular rotation of officials in office through annual elections to triennial offices, whereby one-third of the representatives would be elected each year (Goodwin, 1992, pp. 131, 197-198; Smith, 1914, p. 47).

Harrington was adamant that the House in Oceana (1,050 members), which would be more than three times larger than the Senate (300 members), should not debate legislation, even to the point of making this the only crime punishable by death in the commonwealth. According to Blitzer (1960, p. 241), Harrington viewed the writing of laws by the more aristocratic Senate as

the most important part of the governmental process . . . a task for experts . . .
[requiring] native intelligence, knowledge of political theory and practice, and

above all calm and dispassionate discussion.

Distinguishing between dividing and choosing, it is the Senate that has the indispensable function of debating, or dividing. [It] should necessarily be performed by the wisest and most virtuous members of the community, meeting together in a body small enough to permit calm and fruitful discussion (Blitzer, 1960, p. 241).

By contrast, the House is

made up simply of representatives of the people. . . . An individual member is not expected to be particularly intelligent; rather, his duty is simply to reflect the wishes of his constituents (Blitzer, 1960, pp. 241-242).

Thus, Harrington proposed a bicameral legislature, similar in some respects to what the United States, 49 states (only Nebraska has a unicameral legislature), and most other democratic countries have today. However, his republic “rested upon a relation of equality between persons [the two houses] who were unequal in their capacities” (Pocock, 1977, p. 66).

In the U.S. Congress, there is no distinction between the divider and the chooser. Although revenue legislation must be initiated in the House, each body can debate and pass its own version of a bill. If this happens, differences between the two versions are reconciled in a so-called conference committee, comprising members from both houses.

Once a compromise bill is agreed to by the conference committee, it cannot be amended when it goes back to the House and the Senate for action. A deal has been struck—that is, a division has been made, with some provisions of each bill included and some excluded in the conference committee compromise. Unlike in Oceana, however, the compromise bill can be debated in each body of Congress before a vote is taken.

Because the vote is either up or down, Harrington’s law survives in emasculated form in Congress, with the conference committee the divider and each house a chooser.

A bill passes if both houses assent to the compromise bill and the president signs this bill into law, or a presidential veto is overridden by 2/3 majorities in both houses.

A crucial difference between divide-and-choose in cake cutting and its application in the legislative process is that the divider does not get one piece and the chooser another. Instead, they both *share* the same piece—the bill that is eventually passed, or the status quo if it is not.¹⁰

Divide-and-choose in the legislative arena might be better characterized as *filter-and-choose*. Under the latter procedure, imagine that, instead of a single cake, a filterer must choose a subset of different pastries from a larger set—by filtering out what he or she considers the best—in such a way that the chooser will also prefer this portion. Too much of the set will give the filterer indigestion, whereas too little will leave him or her hungry, or otherwise unsatisfied by the selection, and likewise for the chooser.

But instead of the chooser's selecting between the subset and its complement (i.e., the other pastries), the chooser must select between the subset favored by the filterer and no pastries. This choice is analogous to passing new legislation (getting the subset, constructed by the filterer) or voting it down (keeping the status quo). Congresswoman Lynn Martin poignantly described the agony of this choice when she explained why she would support the 1986 tax-reform bill that emerged from a conference committee:

I found, as worried as I am about what this bill does, I am even more worried worried about the current code. The choice today is not between this bill and a perfect bill; the choice today is between this bill and the death of tax reform.¹¹

As with the divider under divide-and-choose, the filterer under filter-and-choose—that is, the player who selects the subset—can ensure a proportional and envy-free division. This person does so by selecting a subset that has the same value as receiving no pastries (which may require dividing up one of the pastries to create equality), but in practice, as will be shown in the next section, the filterer can do still better.

Besides conference committees, another example of filter-and-choose in Congress is the “closed rule,” which precludes amendments to bills on the floor of the House. This rule is applied to bills that might otherwise be strewn with amendments, such as appropriation and revenue bills.

A House committee that reports out a bill under this rule plays the role of the filterer, with the entire House the chooser. Since the choice is between this bill and no bill, one may properly think of the committee members as sifting through provisions that, they hope, will garner the approval of the entire House.

In the Senate, by contrast, no such restriction on amendments from the floor can be imposed. In fact, quite the opposite: a single senator can conduct a filibuster that precludes a bill from coming up for a vote, thereby preventing the Senate from even making a choice. However, a filibuster can be broken by a cloture vote, whose passage requires at least 3/5 of the Senate, so individual senators do not have an absolute right to block a bill.¹²

The power of a conference committee, and a House committee reporting out a bill under a closed rule, would appear to be greater than that of a regular committee, which might see its bill drastically altered by amendments from the floor. Remember, however, that if the (unamendable) bills emerging from the former committees fail to gain majority support on the floor, nothing can be done to save them, whereas a slight change in an (amendable) bill can sometimes make the difference between its passage and its failure.

In sum, the lack of control that a regular committee has over the contents of a bill, once it reaches the floor, must be weighed against the fact that an unamendable bill—emerging from a conference committee or under a closed rule in the House—cannot be fine-tuned, so to speak, in a way that might make it acceptable to a majority unwilling to support the unamended bill. On balance, however, having the ability to dictate a choice, without amendment, probably gives a committee greater power than not having this ability.

In the case of conference committees, to be sure, having the exclusive power to filter, and then propose, is somewhat illusory. These committees, after all, are not exactly free-wheeling entities; they must reach agreement within the strictures set by their respective houses, based on the bills each house passed.

By comparison, a House committee reporting out a bill under a closed rule has, ostensibly, more power, because its members—not one of the two houses—wrote the bill. But this fact may not prevent the House as a whole from balking at its committee's recommendation, especially if it is based on a closely divided committee vote, rendering the committee's power far from absolute.

There is still another incarnation of filter-and-choose in Congress. In impeachment proceedings of federal officials, including the president, the House assumes the role of the filterer: it is the body that draws up charges of impeachment, which require a $2/3$ majority to enact. If an official is impeached, which is equivalent to an indictment in a criminal court, the Senate then serves as the jury and can convict the official by a $3/4$ vote, which makes it effectively the chooser in filter-and-choose.

We have traced the proposed use of divide-and-choose in governmental institutions back to the 17th century. But, as we indicated, divide-and-choose was known in ancient Greece more than two millennia before the appearance of Harrington's book. Empirical manifestations of its use in the Law of the Sea treaty, and in Congress today as filter-and-choose, indicate that it is not just played out in cake-cutting exercises on dining room tables.

We next show that the divider may exploit divide-and-choose if he or she possesses information about the chooser's preference. This vulnerability of the procedure, however, does not undermine its properties of proportionality and envy-freeness—or those of its variant, filter-and-choose.

The Role of Information

To exploit divide-and-choose, Bob must know Carol's preferences with respect to two things—(1) different portions of the cake, and (2) the value of obtaining a larger piece for herself compared with the value of denying him a larger piece. If Carol cares only about what *she* receives, Bob can, instead of dividing the cake equally for himself, cut it so as to give himself as large a piece as possible while still making Carol almost indifferent. We say “almost,” because Bob needs to cut the cake so that Carol slightly prefers the piece he does *not* want and would, therefore, choose for herself. Thereby Bob will end up with a larger piece for himself than if he simply created two pieces he considered equal.

On the other hand, if Bob thinks that Carol might see through his exploitative strategy and prefer to spite him for choosing it, he faces a dilemma. Equal division by Bob guarantees him a piece he considers to be $1/2$ —it does not matter what Carol chooses. By comparison, creating near equality for Carol could give Bob more than $1/2$, but at the risk of not even getting $1/2$ if Carol chooses his (Bob's) larger piece out of spite. (To be sure, Carol would hurt herself doing so—in terms of the amount of cake she receives—but this may be a good long-run strategy if this kind of situation is likely to recur and Carol wants to deter future exploitation by Bob.) Thus, even knowing Carol's preferences for the cake does not ensure Bob of a piece he considers to be strictly larger than $1/2$, given that Carol may resent being exploited and instead prefer to spite Bob rather than get a (slightly) larger piece for herself.

Note, by the way, that even to know she is being exploited, Carol must know Bob's preferences (i.e., whether Bob's cut reflects his true preferences or not). Only then can she act out of spite if she wants to do so.¹³ Thus, even the two-person problem is not without some interesting twists, depending on the information available to the players.

Whether Bob as divider is being conservative or exploitative, he makes a 50-50 division—but in two different ways: (1) if he is being conservative, he makes the

division exactly 50-50 in his own estimation; (2) if he is being exploitative, he makes the division almost 50-50 in Carol's estimation. Again, we say "almost" in case (2), because the divider wishes the chooser to select the divider's less preferred pile. Hence, the divider will endeavor to partition the items so that the chooser will slightly prefer one pile to the other, with the pile that the chooser does *not* select containing as large a fraction of the cake—in the eyes of the divider—as he or she can arrange.

Neither of these divisions will be "equitable" in the sense of trying to equalize the fraction of the cake that each perceives he or she is receiving. Thus, although each will receive at least half the cake—in their own estimation—and thus will not want to trade with the other, Bob will think he received only 50% of the cake while Carol will think she received, say, 80% in case (1), whereas there will be a reversal of roles in case (2). Correcting this aspect of the one-sidedness of divide-and-choose is an issue that is addressed elsewhere (Young, 1994; Brams and Taylor, 1996).¹⁴

To sum up for divide-and-choose, dividers are disadvantaged unless they know the chooser's preferences, in which case they can capitalize on this knowledge. With or without this knowledge, dividers create exact or approximate indifference, but for different players: exact indifference for themselves, if they are in the dark about the chooser's preferences; approximate indifference for the chooser, if they know his or her preferences.¹⁵

In the filter-and-choose examples from Congress we gave in the previous section, it is reasonable to suppose that members of committees, before reporting out a bill that can be voted only up or down by one or both houses, think carefully about what is likely to be acceptable to each house. Conference committee members, especially, do not want to propose a bill that will likely go down to defeat in one or the other house. In fact, they work assiduously to balance provisions passed by each house so that the compromise bill will be acceptable to the memberships of both houses.

It is not just committees, though, that play the role of filterer in the federal government. Presidents initiate most major legislation, and they must be wary of proposing bills that are “dead on arrival.” Normally they consult with their own party leaders in Congress, and sometimes opposition leaders, to draft bills that have a good chance of gaining majority support. But, of course, presidential initiatives are always subject to revision in congressional committees and—once the bills are reported out of committee—by each house, so presidents may have their roles as filterers greatly attenuated.

When it is the opposition party in Congress that initiates legislation, there is a reversal of roles. Now it is the administration that must respond by trying to accommodate itself to, or to contest, a proposal, which may well involve putting forward a counterproposal. In fact, there may be several bills that are introduced that reflect the various interests of different members of Congress.

Who, if anybody, is decisive in the filtering process is often unclear until one bill prevails in the end. Even final passage seldom shows up one player as critical, because most legislation is the product of many deals and compromises.

The distinction between filterer and chooser would appear to be more clear-cut when a bill is passed by both houses of Congress, and the president must decide whether or not to sign it into law. Because the president cannot amend it, he or she would seem very much in the role of chooser.

But this distinction is more apparent than real. Presidents influence the content of bills not only by marshaling their own forces in Congress to help shape the legislation but also by the *threat* of a veto should the bill either not contain provisions they deem essential or contain unacceptable features. In turn, Congress may respond as a filterer by trying to accommodate the president and writing in these provisions (if the veto threat is taken seriously), or by defying the president by trying to ensure that the bill is “veto-proof”—it will have the 2/3 support in both houses needed to override a presidential veto.

To review, the president, congressional committees, and Congress are both filterers and choosers at various times. Consequently, the issue is almost never one in which a filterer exploits knowledge about a chooser's preference, or a chooser decides whether to spite the filterer for this advantage. Rather, a president, Congress, and congressional committees move in and out as occupants of these roles: when a president proposes legislation (or floats a "trial balloon" to get an initial reaction before committing himself to it); when congressional committees write a bill; when a conference committee reconciles different versions of a bill after initial passage by both houses; when the compromise bill is sent back to each house; when, after final passage by both houses, the president signs the bill into law or vetoes it; and when Congress attempts to override a presidential veto.

This process is extended when new players are involved, such as the nine members of the Supreme Court if the constitutionality of the legislation is challenged. All 50 states become players in the ratification of constitutional amendments, which requires acceptance by 3/4 of the states once 2/3 majorities of both houses have proposed an amendment. And, finally, there may be strategic considerations quite removed from the legislative process, such as a president's vetoing a bill to enhance his reelection chances, even though he would prefer the bill to the status quo.

Theory Meets Practice

Although certain congressional and constitutional procedures smack of filter-and-choose, it hardly ever occurs in the pristine form that Harrington proposed it for Oceana. True, a president or Congress—or the Supreme Court or the states—may be presented with a bill or a constitutional amendment that they must either accept or reject, but the process by which this bill or amendment was nurtured and shaped may well include their earlier intrusion. Thus, the separation of the roles of filterer and chooser is never airtight in the U.S. federal system.

The lack of role separation, and the involvement of many players, are not necessarily detrimental to making better collective choices. The blurring of roles means that there is no dominant filterer, or gatekeeper, and the multiplicity of players means that several different points of view get expressed.

Players who anticipate and respond to each other at different stages are probably more conducive to producing balanced (though perhaps weaker) legislation. The process may also make for closer outcomes, especially if the players possess relatively complete information about each other's preferences (Riker, 1982; Brams and Fishburn 1995; Fishburn and Brams, 1996). Then the filterer will defer to the chooser, but only insofar as necessary, to gain this player's acquiescence and not provoke a spiteful response.

The fact that much important and controversial legislation often passes Congress by only the slimmest of margins is *prima facie* evidence that (1) there is reasonably good information about preferences in these situations and (2) the filterer takes this information into account in crafting bills. Although filterers may be advantaged in such situations, the process seems counterbalanced by the frequent reversals of role that the Constitution and legislative rules (e.g., that provide for conference committees) mandate, as we previously illustrated.

Harrington prohibited debate by the House so that its members would, as choosers, be handed cogent, well-conceived alternatives from the Senate that they could not adulterate. By acting only on the alternatives handed to them by the more informed and erudite Senate, Harrington had the quaint belief that House members—untainted by their ignorance and lack of intelligence—would make better decisions. Thereby Harrington could preserve democratic choice in the House, but one enlightened by the prior choices of the “rich and wellborn” (to use Alexander Hamilton's apt phrase) in the Senate.¹⁶

This attitude may strike one as anachronistic if not woefully elitist. In fact, however, Harrington's proposals were more democratic—in the populist sense of giving citizens some final, if indirect, say through their elected representatives in the House—

than those advocated by most of his contemporaries. What he sought, at precisely the time of revolutionary change in England when old historical forms had been destroyed, was “a chance to construct new forms immune from the contingencies of history” (Pocock, 1992, p. xvii).¹⁷

Harrington believed, in his own unabashed account, that “if any man in England could show what a commonwealth was, it was myself” (Pocock, 1992, p. x). This may not be far from the truth; what Pocock (1992, p. xvi) most credits Harrington with is “a historical intelligence capable of synthesizing schemes of social change which set the *de facto* disorderliness of the English Civil Wars in a context of long-range and even universal historical processes.”¹⁸

Like Harrington, the American founding fathers also proposed a bicameral legislature. The functions of the two houses they proposed, however, were quite similar, which would make them better able to check each other as well as the president.

As these institutions have evolved, and new ones like political parties and congressional committees have formed, the roles of filterer and chooser have become increasingly mixed and diffuse, as we have seen. To some the process has become incoherent, even chaotic, compared with Harrington’s uncontaminated roles of Senate (divider) and House (chooser).

But with the spread of information (and misinformation) beyond the proverbial smoke-filled rooms, none of these institutions seems unduly advantaged today. If one institution benefits from being the filterer at one stage, other institutions assume its place later, so no dominant filterer or chooser is discernible.

Notes

¹All Bible translations are from *The Torah: The Five Books of Moses* (1967) and *The Prophets* (1978).

²Formal treatments of this game are given in Glazer and Ma (1989) and Yang (1992).

³Solutions for more than two players that satisfy different properties, including proportionality and envy-freeness (see next section), are analyzed in Young (1994) and Brams and Taylor (1996).

⁴We will use expressions like “piece she considers largest” and “piece she values most” interchangeably, although the intuitions behind these expressions are quite different. The “size interpretation” is more useful in thinking about objects like cake, whereas the “value interpretation” is more useful in thinking about objects in which size is not the measure of worth. Note that to say that each player received at least $1/2$ the value requires only an ordinal comparison—each player believes his or her piece is at least as large or valuable as the other player’s—but not a specification of how much larger or more valuable.

⁵If there are n players, *proportionality* means that each thinks he or she received at least $1/n$ of the cake.

⁶That is, if Bob, Carol, and, say, Ted divide the cake among themselves, and Bob thinks he received less than $1/3$, then he must think that Carol and Ted are sharing more than $2/3$. Thus, he will think that at least one of them (Carol, for instance) received more than $1/3$, so he will envy her. It follows that when Bob (and the other players) are not envious, then each must believe that he or she received at least $1/3$.

⁷Lowry (1987, pp. 126-131) discusses at some length the problems that plagued this particular transaction and also reports on a corruption of divide-and-choose (given in one of Aesop's fables) that has given rise to the phrase, "lion's share:"

It seems that a lion, a fox, and an ass participated in a joint hunt. On request, the ass divides the kill into three equal shares and invites the others to choose.

Enraged, the lion eats the ass, then asks the fox to make the division. The fox piles all the kill into one great heap except for one tiny morsel. Delighted at this division, the lion asks, "Who has taught you, my very excellent fellow, the art of division?" to which the fox replies, "I learnt it from the ass, by witnessing his fate" (Lowry, 1987, p. 130).

⁸Immanuel Kant and Jean-Jacques Rousseau also proposed this idea (Goodwin, 1992, p. 94), though somewhat later and probably independently.

⁹The analogy is not exact, because when the Senate makes a particular proposal, its members—or at least a majority of its members—are presumably not indifferent to its passage, as is the cutter indifferent between which of the two pieces of cake that the chooser selects. Moreover, the Senate, in making its proposal, may have some information on what will be acceptable to the House, which is a matter we take up in the next section. Nonetheless, Harrington used the "homey little simile" (Smith, 1914, p. 51) of cake-cutting to justify his proposal.

¹⁰In the parlance of economics, the bill is a *public good*, from which each player can benefit without detracting from the benefit of the other player (Olson, 1965; Hardin, 1982). The "good" for some players, however, may be a "bad" for others.

¹¹Quoted in the *New York Times*, September 26, 1986, p. D17.

¹²Once debate is cut off, it may seem inconsistent that only a simple majority of senators is required for passage of the bill. The cloture rule's justification—that a larger majority should be required to halt the expression of "free speech" than pass a bill—is dubious if

those members who vote for cloture are precisely those members who support the bill. When this is the case, one vote is tantamount to the other, so the 3/5 cloture requirement simply poses a greater hurdle for passage, vitiating its freedom-of-expression justification.

¹³Of course, Carol may prefer to act out of generosity, even knowing that Bob has been deliberately exploitative in making his cut. By the same token, Bob may prefer to cut the cake to as to give Carol what he thinks she will like. We by no means rule out such benevolent preferences but find malevolence more interesting to analyze, because it raises questions about how to protect against it. In games in which the players know each other's preferences, and spite is not a factor, the advantage of the divider is discussed in van Damme (1991, pp. 133-136) and Young (1994, pp. 137-145), which also include a discussion of alternative procedures to neutralize this advantage.

¹⁴An early attempt to deal with this and related problems of divide-and-choose is given in Singer (1962); a more formal later attempt is given in Crawford (1977).

¹⁵Generally speaking, a divider will have some, if not complete, knowledge of a chooser's preferences, enabling him or her to exploit the chooser by his division (unless the chooser is spiteful). Young (1994, pp. 136-145) describes several schemes that neutralize the divider's advantage, some of which implement such classic game-theoretic bargaining solutions as those of Nash (1950), Raiffa (1953), and Rubinstein (1982); Moulin (1984) gives a scheme for implementing the Kalai-Smorodinsky (1975) solution, which is also analyzed in Van Damme (1991, pp. 156-159). One rather theoretically attractive scheme is that of Crawford (1979), in which two players bid to be the divider, which has been generalized to n players (Demange, 1984). But this scheme, like others, relies on using the fallback position of equal division of all items if the players are not able to agree on a mutually better division, which seems to us an unrealistic threat (Brams and Taylor, 1996).

¹⁶To be sure, Harrington wanted it to be an “aristocracy of excellence” (Pocock, 1977, p. 66), not just an aristocracy based on its members’ place in society. But it is unclear how the capacities of this aristocracy would be recognized, selected, and accorded their proper standing. Although the possession of property (land, goods, or money) rather than birth qualified one as fit to serve in the Senate, this qualification imposed a substantial barrier on those not receiving an inheritance. They, as Harrington stressed, would have to acquire property through their own efforts (Pocock, 1977, p. 67).

¹⁷In the years just prior to the publication of *The Commonwealth of Oceana* in 1656, Charles I was executed (1649) and Oliver Cromwell became England’s lord protector (1653). After Cromwell’s death in 1658, a Republican commonwealth took control before the Stuart monarchy was returned to power in 1660, when Charles II reclaimed his father’s throne.

¹⁸Views differ greatly, however, on Harrington’s importance as a political theorist; see Downs (1977, pp. 13-15). But “in the sphere of practical politics,” Cohen (1994, p. 124) asserts, “Harrington was ultimately more influential . . . than Hobbes—or, for that matter, Vauban, Leibniz, Graunt, or Petty—since his doctrines were implemented in the following century, notably in the form of government adopted in the American Constitution.” Cohen (1994, pp. 124-137) shows how Harrington, a great admirer of the biology of William Harvey, attempted to construct a “political anatomy,” analogous to Harvey’s anatomy and Harvey’s analysis of circulation in the animal body.

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