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by **Jess Benhabib**
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Roger E. A. Farmer

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C.V. STARR CENTER FOR APPLIED ECONOMICS

NEW YORK UNIVERSITY
FACULTY OF ARTS AND SCIENCE
DEPARTMENT OF ECONOMICS
WASHINGTON SQUARE
NEW YORK, NY 10003-6687

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Abstract

In this paper we take as given that market economies are characterized by a set of stylized responses to increases in the stock of money. Innovations to the stock of money lead to increased output and reductions in short term interest rates in the short run and only in the long run do nominal prices respond. These features of the monetary transmission mechanism have been discussed at least since David Hume. Most authors have attributed the real effects of money in the short run either to mistaken expectations or to non-market clearing or both. In this paper we argue that neither of these channels is needed to explain the facts. We show that a competitive market clearing model in which money enters the production function is fully capable of mimicking the broad features of the data. Our argument relies on an explanation of "price stickiness" that exploits a multiplicity of equilibria in a rational expectations model.

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"...in every kingdom, into which money begins to flow in greater abundance than formerly, everything begins to take a new face: labour and industry gain life; the merchant becomes more enterprising, the manufacturer more diligent and skilful, and even the farmer follows his plough with greater alacrity and attention."

David Hume *Of Money*

(1) Introduction

In simple equilibrium business cycle models that are amended to include money – it is difficult to set things up in a way that causes simulated time series from the model to mimic the time series generated by actual data. The problem is that in most equilibrium models there is too much price flexibility. Money shocks feed immediately into prices and these models display not only long run neutrality of money, but also short run neutrality. In the data this is not what we observe. Instead, money shocks cause real output responses in the short run and only after a considerable period of time do prices adjust to insulate real quantities from nominal disturbances.[1]

There are two popular views of why equilibrium models fail. One view holds that markets or expectations or both are typically in disequilibrium. According to this view some suitably amended version of the IS-LM model will accurately describe the world if only a clever theorist can solve the tricky problem of explaining why prices do not clear markets in the short run. According to the second view, the correlations that we observe in the data are examples of reverse causation; output causes money rather than the other way around and hence there is no puzzle to be explained.

In this paper we argue that there *is* a puzzle for equilibrium business cycle theory but that this puzzle can be resolved within a market clearing model in which agents have rational expectations. We argue that the world in which we live is a world in which the assumption of rational expectations is insufficient to pin down a particular equilibrium. In fact there are infinitely many beliefs that are consistent with rational expectations and market clearing. We argue that agents in the real world have resolved this multiplicity by

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[1] The folk evidence for a transmission mechanism with these features extends as least as far as David Hume's essay "Of Money" from which our opening quote is taken. Formal analysis of macroeconomic time series using vector autoregressions points in the same direction. For a discussion of the monetary transmission mechanism based on the evidence from vector autoregressions see the recent article by Bernanke and Gertler (1995) in the *Journal of Economic Perspectives*.

coordinating on a particular equilibrium and that this equilibrium has the property that prices are predetermined one period in advance.

The argument that indeterminacy can be used to explain the observed behavior of prices has been made before, in the context of the overlapping generations model (Farmer and Woodford (1984), Farmer (1991), (1992)), in an economy with productive externalities (Matheny (1992), Lee (1993), Beaudry and Devereux (1993)) and in an economy in which money itself has external effects (Benhabib and Farmer (1991)); the fact that monetary models might display indeterminate equilibria has been known at least since the work of Brock (1974).[2] There have however been few attempts to investigate the *empirical* plausibility of indeterminacy arising from the productive or utility producing role of money[3] and for this reason most macroeconomists have tended to dismiss the idea that indeterminacy of equilibrium can explain the monetary transmission mechanism. In this paper we hope to make a case for the multiple equilibrium approach to "price stickiness" by showing that a suitably calibrated model can fit many of the facts without imposing strong or unusual assumptions on preferences and technologies. In section (4) we give a very simple calibration that is fairly robust to small perturbations in the parameters. Nevertheless our model remains quite simple and abstracts from growth, capital accumulation, technical progress, labor and goods market imperfections, capacity utilization and labor hoarding, and many issues concerning complex government policies. As such, the model is simply intended to suggest the empirical plausibility of a monetary transmission mechanism, distinct from other mechanisms that rely on increasing returns and externalities, that gives rise to indeterminacy and that can explain at least some of the broad stylized facts associated with the dynamics of interest rates, prices, real balances, and output.

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[2] Other authors who have studied this issue include Calvo (1979), Benhabib and Bull (1983), Obstfelt and Rogoff (1983), Gray (1984) and Woodford (1987). More recent explorations of the existence of indeterminacy in monetary models were undertaken by Matsuyama (1990) and Woodford (1994) who showed that existence of indeterminate equilibria in the cash-in-advance model of Lucas and Stokey (1983).

[3] One such attempt is given by Benhabib and Farmer (1991), who rely on aggregate monetary externalities; another empirical approach is taken by Beaudry and Devereux (1993), who use Benhabib and Farmer's (1995) model that produces indeterminacies in a non-monetary model of increasing returns to scale, but augment it to incorporate money. The route to indeterminacy in the Beaudry Devereux paper is quite different from the channel that we explore in this paper since nowhere do we rely on increasing returns to scale. We conjecture that the Beaudry Devereux model will likely display indeterminacy in *two dimensions* (rather than one) for parameterizations of the exchange technology similar to the one that we explore in this paper.

(2) The Mechanics of an Example

This section introduces our main idea by means of an example. Although the example is relatively simple it contains all of the elements that we believe to be important in understanding why nominal disturbances have real short run effects on output. The main idea that we will expand on below is that, in simple monetary economies, equilibria can be represented as those solutions to a difference equation that remain bounded. Sometimes the difference equation that characterizes equilibria has a unique bounded solution, sometimes it does not. We argue that models that display multiple bounded solutions capture many of the features of the monetary transmission mechanism that are otherwise difficult to understand.

We begin by deriving the difference equation that characterizes equilibrium. We suppose that a single commodity is produced from a fixed factor and marketed using the services of real balances. Let the technology be given by the function:

$$(1) \qquad Y_t = F\left(\frac{M_t}{P_t}\right) \equiv \left(\frac{M_t}{P_t}\right)^a,$$

where M is nominal money, P is the price of commodities in terms of money, the ratio M/P represents real balances, and Y is output. We suppose that the firm is owned by a representative consumer who solves the problem:

$$(2) \qquad \text{Max } U = \sum_{t=0}^{\infty}\left(\frac{1}{1+\rho}\right)^t \frac{C_t^{1-r}}{1-r},$$

subject to the constraints:

$$(3) \qquad \frac{\overline{M}_{t+1}}{P_t} + \frac{B_{t+1}}{P_t} = \frac{\overline{M}_t}{P_t} + \frac{B_t}{P_t}(1+i_{t-1}) + Y_t - C_t + \frac{T_t}{P_t}$$

$$(4) \qquad \text{Lim}_{s\to\infty} Q_t^s\left(\frac{M_s + B_s}{P_s}\right) \geq 0, \quad Q_t^s = \frac{P_s}{P_t}\prod_{v=1}^{s}\frac{1}{(1+i_v)}$$

with B_0 and M_0 given. The consumers choose the sequences of C_t, B_{t+1}, and $\overline{M}_{t+1}$, $t > 0$. The money supply in period t reflects the agent's choice $\overline{M}_t$ plus the transfer that they subsequently receive: $M_t = \overline{M}_t + T_t$. The constraint (3) is a budget constraint that allows the consumer to save by holding money, M, or government bonds, B, that we assume to be in zero net supply. The term T represents a lump sum nominal transfer from the government that we include to allow for the disbursement of the seignorage revenues from money creation. The reason for introducing bonds into the model is to define the interest rate in equilibrium from the first order condition for holding bonds. The inequality (4) prevents the consumer from running Ponzi schemes in which consumption is financed by continually borrowing from the future.

We suppose that the government expands the money supply at a fixed rate each period and we define the money growth rule in equation (5).

$$(5) \qquad M_t = \mu M_{t-1}.$$

We further assume that all output is consumed:

$$(6) \qquad Y_t = C_t.$$

Given the assumption that the household maximizes utility by choosing sequences of money and bonds and given further the assumption that all output is consumed by the representative consumer we characterize equilibria of this economy by substituting the market clearing equations into the first order conditions of the household. There are two such equations; equation (7) that describes the optimal choice of money, and equation (8) that describes the choice of bonds:

$$(7) \qquad \frac{1}{P_t}(C_t)^{-r} = \frac{1}{1+\rho}\frac{1}{P_{t+1}}(C_{t+1})^{-r}\left(1 + F_m\left(\frac{M_{t+1}}{P_{t+1}}\right)\right).$$

$$(8) \qquad \frac{1}{P_t}(C_t)^{-r} = \frac{1}{1+\rho}\frac{1}{P_{t+1}}(C_{t+1})^{-r}(1 + i_t).$$

One also requires that the transversality condition:

$$\text{Lim}_{t\to\infty}\left(\frac{1}{(1+\rho)\mu}\right)^t \frac{M_{t+1}}{P_{t+1}}(C_{t+1})^{-\rho} = 0$$

should hold. It is the transversality condition that imposes the requirement that sequences of real balances should remain bounded in equilibrium.

Equation (7) contains the key to understanding the equilibria of the model. Notice that consumption is equal to output which, from equation (1), can be written as a function of real balances. Similarly, the ratio of the price level at consecutive dates can be written as a function of real balances at consecutive dates by substituting from the money growth rule, (5). Making these substitutions one arrives at the equation:

(9) $$m_t F(m_t)^{-r} = \left(\frac{1}{1+\rho}\right)\frac{1}{\mu} m_{t+1} F(m_{t+1})^{-r}\left(1 + F_m(m_{t+1})\right)$$

which is a single difference equation in real balances, M/P. In the remainder of the paper we refer to this variable with a lower case m. An equilibrium for our economy is fully described by a bounded sequence of real balances that satisfies equation (9) at all points in time.

(3) The Properties of Equilibria

In this section we will describe the types of equilibria that can occur in this model paying particular attention to parameter configurations for which there are multiple equilibria. We begin by rewriting the difference equation (9) in the form:

(10) $$G(m_t) = \frac{1}{(1+\rho)\mu} G(m_{t+1}) X(m_{t+1})$$

where the functions G and X are defined below:

(11) $$G(m) = mU_c(F(m)),$$

(12) $$X(m) = 1 + F_m(m).$$

The function G is the marginal utility of consumption (expressed as a function of real balances) multiplied by real balances. The function X is the gross return to holding money; this return follows from the fact that money is useful in organizing transactions in the subsequent period. In the remaining part of this section we will establish two facts. First we will show that the difference equation (10) has a unique monetary steady state. Second, we will show that this difference equation is stable if the function G is decreasing in real balances. Since a stable difference equation is one that remains bounded for a set of initial conditions in the neighborhood of the steady state, we will also be able to show that when G is decreasing, this model will display multiple perfect foresight equilibria.

We begin with the existence of a monetary steady state. This follows from (10) which, expressed at the steady state, implies that:

(13) $$F_m(m) = (1+\rho)\mu - 1 = i.$$

The second equality comes from arranging equations (7) and (8) which together imply that the rate of interest must equal the marginal product of money in production. For the example in which F is the power function "m^a" equation (13) has a unique steady state given by the expression

(14) $\quad \overline{m} = \left((1+\rho)\mu - 1\right)^{1/a}.$

Clearly, for more general functions the steady state will also be unique and interior given suitable boundary conditions on the function F(m).

Now we will investigate the behavior of non-stationary equilibria in the neighborhood of the steady state $\overline{m}$. Let the parameters ε_G and ε_X be defined by the expressions:

(15) $$\varepsilon_G = \left.\frac{d\log G(m)}{d\log m}\right|_{m=\overline{m}}, \quad \varepsilon_X = \left.\frac{d\log X(m)}{d\log m}\right|_{m=\overline{m}}.$$

These parameters represent the elasticities of the functions G and X evaluated at $\overline{m}$. Linearizing equation (10) around $\overline{m}$ leads to the approximate equation:

(16) $$\tilde{m}_{t+1} = \nu\tilde{m}_t, \qquad \nu \equiv \frac{\varepsilon_G}{\varepsilon_G + \varepsilon_X},$$

where $\tilde{m}_t$ is the deviation of the logarithm of real balances from its steady state. For the example of the power production function, ε_X is given by the expression:

(17) $$\varepsilon_X = \left(\frac{(1+\rho)\mu - 1}{(1+\rho)\mu}\right)(a-1),$$

and ε_G by:

(18) $$\varepsilon_G = 1 - ra.$$

Since the parameter "a" is less than one, ε_X will be a number between 0 and minus 1 for any reasonable values of the parameter r and the money growth factor μ. It follows that a sufficient condition for the slope of the difference equation ν to be a positive number between zero and one is that ε_G is negative[4]. In the following section we show that when ε_G is negative it will not be possible for agents in this economy to uniquely forecast the path of the economy for any given value of fundamentals; in this example fundamentals means technology, endowments, the money supply and the structure of the money growth

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[4] Note that an alternative route to indeterminacy is to allow the parameter "a" to be greater than one, that is through increasing returns. Since this route has been explored in other papers, we rule out increasing returns in this case by postulating that a < 1.

rule. Any initial price, and any corresponding self-fulfilling beliefs about the path of future prices, will be consistent with equilibrium. The condition $\varepsilon_G < 0$ translates, for this example, into the condition:

(19) $ar > 1.$

The parameter r measures the willingness of the consumer to accept time varying consumption streams. A large value of r represents extreme displeasure at time varying consumption. The value of "a" represents the relative importance of money in production. If the product of these parameters is large, the example will display a stable steady state.

The intuition for the indeterminacy result can be explained with the help of equations (9) and (19). In a model with a unique equilibrium the steady state of equation (9) is unstable if one expresses m_{t+1} as a function of m_t. In a model of this kind an increase in nominal money balances, holding constant the current price level and holding constant expectations of future prices, would increase the demand for money. One possible way to restore equilibrium would be to adjust future prices downward to increase the return on money and induce agents to willingly accept the higher real balances. If the expected fall in future prices is self-fulfilling, however, then on this path money balances must increase in the subsequent period, requiring further deflation in the future. A path of this kind cannot be an equilibrium since it would result in an explosive trajectory for real balances that is ruled out by the transversality condition. The only equilibrium, in the case when the steady state of (9) is unstable, is one in which there is a jump in the initial price level which offsets the increase in real balances: the left-hand side of (9) does not increase.

In the model that we study in this paper, it is possible for the steady state of (9) to be stable. In our model money is productive and an increase in money balances causes output to rise, either directly, or indirectly through an effect on labor demand that we discuss in section (5). If the output effect of money is sufficiently strong, contrary to the standard case, an increase in the nominal quantity of money (holding fixed the current price), may *decrease* the left side of (9) leading to an excess demand for money and an excess supply of goods. In this case it may be possible to restore equilibrium through a self-fulfilling expectation that the *future* price will increase thereby reducing the current return on money in order to eliminate its excess demand. In an equilibrium of this kind inflation will cause real balances to shrink in the subsequent period. This process will continue until money balances gradually return to their initial stationary level. In this case the non-stationary equilibrium path is compatible with the transversality condition since at all points, real balances remain bounded. There are also many other paths that are valid equilibria. For example, a path in which money has a small initial effect on prices that is not sufficient to eliminate the initial excess supply of money. There is in fact an infinite multiplicity of equilibria that can be parameterized by the way in which initial prices respond to the injection of money.

The critical elements that are necessary to generate an excess supply rather than an excess demand for money in response to a monetary injection are the output effect of money, represented by "a" in equation (19), and the curvature of utility of consumption, represented by the parameter r. In our simple example indeterminacy occurs whenever ar >1.[5] In the following section we elaborate on the arguments of this section and we illustrate how the existence of a stable steady state to equation (9) is associated with the possibility of multiple equilibria and, in particular, with an equilibrium in which money has real short run effects on output.

(4) Introducing Variable Labor

This section introduces variable labor supply. Our reason for complicating the model in this way is that time series evidence from US data suggests that the parameter "a" is small, of the order of 1%. If "a" is of the order of 1%; the parameter r would have to be greater than 100 if the model were to display indeterminacy. Although there is some evidence from the asset pricing literature that r may be large; a value of 100 or more is well beyond the bounds that most studies have found. Logarithmic utility implies, for example, that r is equal to one. For agents with time separable Von-Neumann Morgenstern preferences over consumption streams the coefficient r measures both aversion to temporal fluctuations *and* aversion to risk. Many authors claim that a high value of r is inconsistent with observations of the kind of risks against which most families are willing to insure.

The evidence that the parameter "a" is small comes from the assumption that money is a competitive factor that is paid its marginal product. This assumption implies that we should observe the marginal product of money equated to its marginal cost. Since the opportunity cost of holding money is equal to the rate of interest the equality between marginal cost and marginal product implies that:

$$(20) \qquad F_m(m) = i, \quad \text{or} \quad \frac{mF_m(m)}{F(m)} = \frac{im}{Y}.$$

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[5] It can also be easily shown that in a model where money enters the utility rather than the production function, the critical element for generating indeterminacy is the cross partial $U_{cm}(c,m)$: multiple equilibria are possible if an increase in money reduces the marginal utility of consumption. A condition for indeterminacy very similar to that given by (19) can also be derived for this case.

The left side of the second equality is the elasticity of money in the production function. The right side is the rate of interest divided by the velocity of circulation; a term that we call the *share* of money as a productive factor in GDP. If we take the concept of money to be M1, an upper bound for the share of money in U.S. data is 2%. 1% would be a more reasonable figure. The data on the share of money in US data is given in figure 1.

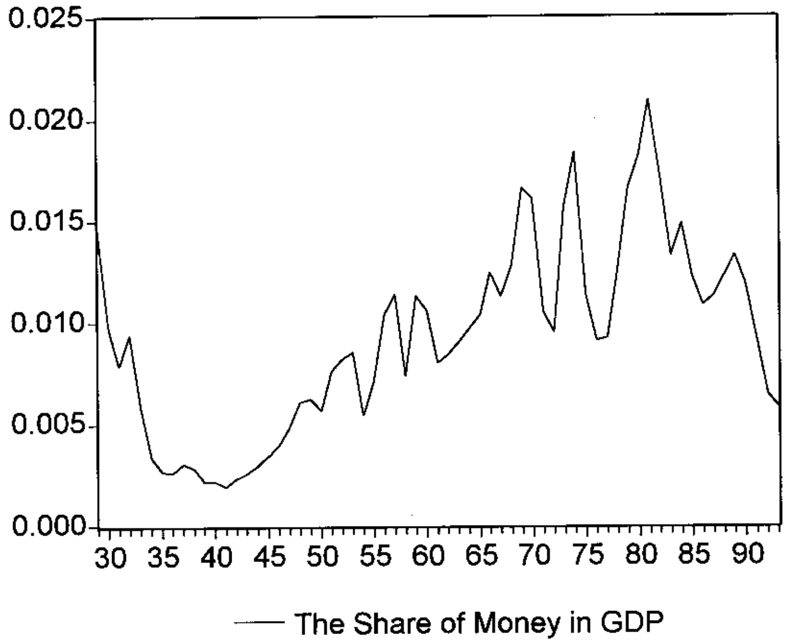

Figure 1

The fact that the share of money is small implies that money cannot be *directly* important in affecting GDP. However, if labor is variable then it *is* possible that money has strong second order effects.

Suppose that labor is variable and that the period utility function is equal to U(C,1−L) where L is labor supplied to the market and 1−L is leisure. Suppose that output is produced using labor and real balances using the production function Y=F(L,m). In this case, competition in the labor market will give rise to an equation of the form:

$$\frac{U_{1-L}(C,1-L)}{U_C(C,1-L)} = F_L(L,m). \tag{21}$$

Since, in our economy, all output is consumed one may replace C by F(L,m). Equation (21) will then lead to a relationship between labor and real balances that we denote with the notation:

$$L = h(m). \tag{22}$$

By totally differentiating (21) one arrives at equation (23); an expression for the elasticity of the function h(m).

$$\text{(23)} \qquad \frac{\partial h}{m}\frac{m}{h(m)} \equiv \varepsilon_h(m) = \frac{\alpha_{Lm} + (\delta_{CC} - \delta_{LC})\alpha_m}{(\delta_{LC} - \delta_{CC})\alpha_L - (\delta_{LL} - \delta_{CL}) - \alpha_{LL}}.$$

where α_L is the elasticity of F(L,m) with respect to L and α_m is the elasticity with respect to m. The terms δ_{xy} and α_{xy} refer to the *elasticities* of *marginal utilities* and *marginal products* with respect to x and y. For example,

$$\text{(24)} \qquad \alpha_{Lm} \equiv \frac{F_{Lm}(L.m)}{F_L(L,m)}m; \ \delta_{CL} = \frac{U_{c,1-L}}{U_c}L; \ \delta_{CC} = \frac{U_{cc}}{U_c}c.$$

where F_{Lm} is the cross partial derivative of F_L with respect to m etc. The important point of this analysis is to notice that the facts that we cited about the importance of money in production are facts that enable us to place a value on the direct effect of money, that is, on α_m since it is α_m that is equated to money's share. The importance of money may well derive instead from the fact that it is highly complementary with other factors. This means, in terms of equation (23), that α_{Lm} may be large even if α_m is small.

Figures 2 and 3 present time series plots of the first differences of employment, real GDP and money in annual U.S. data. An OLS regression of the log. difference of employment on real balances yields a regression coefficient of 0.35. We interpret this as evidence that the elasticity ε_h is relatively large. If one takes seriously the idea that output is produced using employment and money then one might estimate the production function directly. The least squares regression of output on real balances and employment (in first differences) leads to a coefficient on money of 0.24 with a standard error of 0.04. However, when the regression is instrumented to allow for simultaneity bias using lagged variables of differenced variables as instruments the significance of money disappears. We interpret this data as evidence that the direct effect of money in production is small – but that money is useful precisely because it is complementary with other factors.

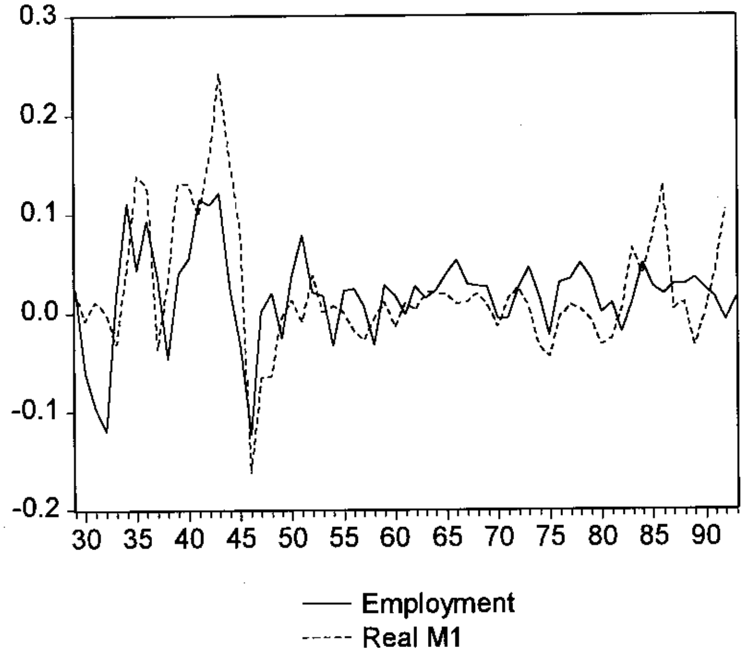

Figure 2

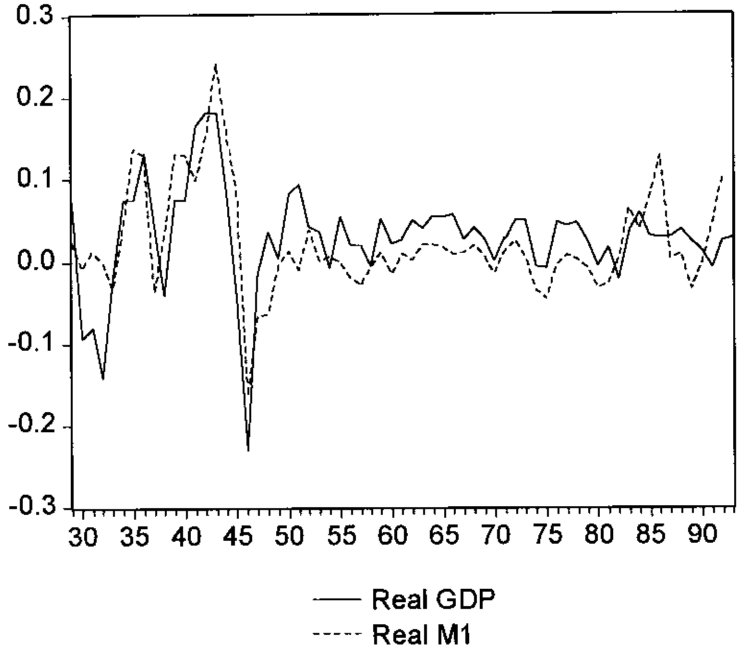

Figure 3

In the following section we present a parametric example of an economy that can mimic the correlation that we observe between employment and money.

(5) An Example of A Model with Variable Labor

In this section we give an example of a utility function and a production function that can be calibrated to fit the facts that we have cited so far and which admits indeterminate equilibria for relatively modest values of risk aversion. Let U be given by the function[6]:

$$(25) \qquad U = \frac{C^{1-r}}{1-r} - \frac{L^{1+\gamma}}{1+\gamma}$$

The parameter γ can be interpreted as the inverse labor supply elasticity for this economy. Let the production function be given by:

$$(26) \qquad Y = A\left((1-\varphi)m^{\theta} + \varphi L^{\alpha\theta}\right)^{1/\theta}; \quad 0 < \varphi < 1..$$

This production function combines money and raw output in a CES format to produce the market good Y. Raw output is produced with labor alone because we have chosen at this stage to abstract from capital: we have in mind a Cobb- Douglas production function $L^{\alpha}K^{1-\alpha}$ where we have set K = 1. The parameter θ measures the degree to which labor and money are complements in production. For very large and negative values of θ the isoquants between money and labor become close to the Leontief case, which basically represents a cash-in advance specification for the firm[7]. It is precisely the case of a large negative value of θ that one needs to generate a large positive value of the elasticity of the function ε_h. For this example the function h(m) is found by applying the inverse function theorem to the first order condition:

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[6] We are aware that in models that account for economic growth this functional form is inconsistent with the existence of stationary hours for the representative consumer when r is different from unity. However, the data on hours per person (full and part time equivalent employees deflated by population) indicates that hours per person have grown on average at .06% per year in annual data since 1929. Our view is that a serious attempt to match growth statistics *and* high frequency statistics of data needs to move beyond the representative agent assumption which we use in this paper for purely illustrative purposes.

[7] Note that a cash-in-advance specification in the absence of multiple equilibria and indeterminacy fails to generate the output, price and interest rate responses to a monetary injection that are observed in the data, as discussed by Christiano and Eichenbaum (1992). Christiano and Eichenbaum rely on rigidities in the allocation of money between financial assets and consumption goods prior to the realization of the monetary injection to generate liquidity effects. For another study assessing the empirical properties of monetary models see King and Watson (1995).

(27) $$C^{r}L^{\gamma} = A\left((1-\varphi)m^{\theta} + \varphi L^{\alpha\theta}\right)^{\frac{1-\theta}{\theta}} \alpha\varphi L^{\alpha\theta-1},$$

where C is given by the market clearing condition:

(28) $$C = Y = A\left((1-\varphi)m^{\theta} + \varphi L^{\alpha\theta}\right)^{\frac{1}{\theta}}.$$

The elasticity of the function ε_h is given by the formula:

(29) $$\varepsilon_h = \frac{\delta_m(1-r) - \delta_m\theta}{1+\gamma-\delta_L(1-r)-\alpha\theta\delta_m}.$$

Table 1 calibrates the parameters of this economy in a way that is consistent with the features of the data that we have cited so far. We have chosen θ equal to -238 to match the correlation of real money and

Table 1: Parameter	*Magnitude*	*Evidence*
δ_m	0.01	Money's Share of GDP
r	6	Risk aversion (1 is log preferences)
$\delta_L = \alpha(1-\delta_m)$	0.7	Labor's share of Income
γ	1	Inverse labor supply elasticity
θ	-238	Makes ε_h match observed correlation of 0.35

employment in de-trended data. The value of r of 6 is high but not outrageously so given the range considered "reasonable" in the asset pricing data and the inverse labor elasticity, set at 1, is consistent with a range of cross-section and time series studies.[8] The parameters α and δ_m are obtained from observations of the share of labor and money in GDP.

In the example with variable labor, equation (10) still describes equilibria; but the functions G and X are defined differently. The amended definitions are given in equations (30) and (31).

[8] One possible drawback of a large value of r is that it may give rise to larger variability in the real wages than we see in the data. Even when γ is zero, we can see from the first order condition for labor given by (27) that income effects due to flucuations in consumption will not result in a flat labor supply curve, and that labor demand fluctuations will move the wage rate. We are indebted to Craig Burnside for this observation.

(30) $$G(m) \equiv mU_c\left(F(h(m),m),1-h(m)\right),$$

(31) $$X(m) \equiv 1+F_m(h(m),m).$$

The interpretation of G and X is the same as in the previous model. G is equal to real balances multiplied by the marginal utility of consumption; X is the gross return from holding money. These functions are now more complicated because there are two channels by which money can affect marginal utility. First, money may *directly* affect output. This is the channel that we argued above is likely to be small since the share of money in GDP would be equated in a competitive economy to the direct elasticity of money in production. Second, there is an *indirect* effect of money on output since money and labor are highly complementary inputs in production. In fact, our calibration in which we have set θ equal to -238 makes the model close to one which imposes cash-in-advance on firms in their labor hiring decisions.

Since the difference equation that describes equilibria is the same in this model, as in the simpler model with constant labor, the analysis of equilibria is also the same. There will exist multiple equilibrium paths when ε_G is negative. For our calibrated example, the elasticities of the functions G and X are given by the expressions:

(32) $$\varepsilon_G = 1-r(\delta_L\varepsilon_h+\delta_m)$$

(33) $$\varepsilon_X = (\theta-1)(1-\delta_m-\delta_L\varepsilon_h)\frac{i}{(1+i)}$$

where i is the interest rate which is equal in the steady state to the marginal product of money in production. Now equation (16) in section (3) also applies to our model with variable labor, so that the critical parameter for indeterminacy is still given by $\nu \equiv \varepsilon_G/(\varepsilon_G+\varepsilon_X)$. We chose an interest rate of 5%

Table 2

Parameter	*Magnitude*
ε_G	-0.53
ε_X	-14.08
ν	0.036

in our calibrations[9]. Given the values for the other parameters reported in table 1 our calibrated economy leads to a model with an indeterminate steady state and values of ε_G, ε_X and v given in table 2.

In addition to the information that we used to calibrate the model, our parameterization has implications for other features of the data, some of which are more reasonable than others. For example, our model gives rise to a simple demand for money function that can be derived from equations (20) and (26). From the agent's perspective the expected interest rate at time t will be given by $i_t = E_t(F_2(h(m_{t+1}), m_{t+1})$ so that $\hat{i}_t = (1-\theta)(\hat{Y}_{t+1} - \hat{m}_{t+1})$ where hatted variables are in log differences. The implied money demand function, in linearized form, is then given by:

$$(34) \qquad \hat{m}_t = \hat{Y}_t - \frac{1}{(1-\theta)v}\hat{i}_t$$

The interest elasticity of the demand for money implied by (34) is -0.115 for the calibrations given above.

Although our model leads to an indeterminate steady state, the degree of persistence in real balances is low. In table 2 we report a value of v of 0.0 36 whereas the correlation in the data of the difference of the logarithm of real balances with itself lagged is closer to 0.6. This correlation in the data should however be interpreted with caution since velocity in the data is non-stationary. A more complex specification of the role of money in technology, and of technological progress that allows for trends in velocity, are possible modifications of the model that may allow a closer match to these empirical regularities.

(6) Multiple Equilibria and Rational Expectations

In this section and the section that follows we elaborate on our earlier comments that, in an economy with multiple equilibria, one should treat beliefs as fundamentals. To make the discussion more relevant we will treat the case of a stochastic economy in which the only shocks are to the money growth rate. For the purposes of this exercise we replace equation (5) with the amended policy rule:

$$(5') \qquad M_t = \mu_t M_{t-1},$$

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[9] Given our CES technology we have $i = F_m = A^{\theta}(1-\varphi)\left(\frac{Y}{m}\right)^{1-\theta}$. From $\alpha_m = \frac{F_m m}{Y}$ we obtain $i = A(1-\varphi)^{1/\theta}\alpha_m^{(\theta-1)/\theta}$.

The composite free parameter A(1-φ) allows us to uncouple the interest rate from the share parameter α. Note that for our calibrations which are close to a cash-in-advance specification (-φ is small), this composite parameter must be close to zero because α_m is small.

where the monetary growth rate μ_t is white noise. For this economy it is relatively simple to show that an equilibrium can be described as a stochastic process for the state variable m that obeys the expectational difference equation:

(35) $$G(m_t) = E_t\left[\frac{1}{(1+\rho)}\frac{1}{\mu_{t+1}}G(m_{t+1})X(_{t+1})\right].$$

Equation (35) incorporates two assumptions. The first is the assumption that the demand and the supply of money are equal and the second is that agents know the probability distribution of future uncertainty; the rational expectations assumption. To keep the algebra manageable we will linearize equation (35) around the steady state to give:

(36) $$\nu(\tilde{M}_t - \tilde{P}_t) = E_t[(\tilde{M}_{t+1} - \tilde{P}_{t+1})]$$

where $\tilde{M}_t$ and $\tilde{P}_t$ are deviations of the logarithms of the nominal money stock and the price level from their growth paths. This equation really should be broken into two parts. Letting a superscript E denote the *subjective* expectation of a variable and using the superscript D to denote demand we can express the family's *demand* for real balances a s follows:

(37) $$\tilde{M}_t^D - \tilde{P}_t = \frac{1}{\nu}\tilde{M}_{t+1}^E - \frac{1}{\nu}\tilde{P}_{t+1}^E .$$

Our notation reads as follows. A tilde over a variable continues to refer to the deviation from the equilibrium steady state. This steady state is one in which real balances are constant but in which nominal money and price are growing along a balanced growth path. In the following discussion all variables that we discuss are deviations from this balanced path.

The notation $\tilde{M}_t^D$ represents the agent's demand for nominal balances at date t. The notation $\tilde{M}_{t+1}^E$ refers to the agents *subjective* belief about the mean of the *supply* of money at date t+1 and $\tilde{P}_{t+1}^E$ is his subjective belief about the future value of the price. The right hand side of equation (37) combines the agent's forecasts about future prices and about the real value of his wealth and instructs him how to behave at date t for given beliefs about the behavior of future prices and about the future stock of money. An *equilibrium* requires the assumption that:

(38) $$\tilde{M}_t^D = \tilde{M}_t$$

where $\widetilde{M}_t$ refers to the *supply* of money at date t. To arrive at equation (36) as a characterization of equilibrium one requires both equation (38) *and* the rational expectations assumption:

$$(39) \qquad \widetilde{M}^E_{t+1} = E_t\left[\widetilde{M}_{t+1}\right], \qquad \widetilde{P}^E_{t+1} = E_t\left[\widetilde{P}_{t+1}\right],$$

where the expectations operator is taken with respect to the *true* distribution of the variables $\widetilde{M}_{t+1}$ and $\widetilde{P}_{t+1}$.

How does the agent arrive at a forecast of future prices and future money stocks so that he may behave rationally in response to new information at date t? In standard rational expectations models this is straightforward. Since there is a unique rational expectations equilibrium there is only one behavior possible that is consistent with market clearing and rational expectations at all future dates. But in a world of multiple rational expectations equilibria this question is central since it is expectations of the future that will determine prices today and hence the response of the economy to nominal shocks. In section 8 we show how the parameterization of beliefs with different forecast functions can lead to economies in which output and prices respond in very different ways to nominal shocks.

(7) Selecting an Equilibrium – Beliefs as Fundamentals

In this section we give two alternative rules that agents might use to forecast future prices and we show how the alternative rules lead to economies with very different stochastic properties. In economy 1 agents forecast future prices using the function[10]:

$$(40) \qquad \widetilde{P}^E_{t+1} = v^2\left(\widetilde{P}_{t-1} - \widetilde{M}_{t-1}\right),$$

For reasons that will become apparent we call economy 1 the price responsive economy.

[10] In (40) agents forecast $\widetilde{P}^E_{t+1}$ using $\widetilde{P}_{t-1}$, $\widetilde{M}_{t-1}$ and in (41) using $\widetilde{P}_{t-1}$, $\widetilde{M}_{t-1}$ and $\widetilde{M}_t$. It is straightforward to allow forecasts to be conditioned on $\widetilde{P}_t$ as well. The forecast rules below are alternatives to (40) and (41), and do not alter any of the results that follow in the paper:

$$(40)' \qquad \widetilde{P}^E_{t+1} = \lambda v^2\left(\widetilde{P}_{t-1} - \widetilde{M}_{t-1}\right) + (1-\lambda)v(\widetilde{P}_t - \widetilde{M}_t),$$

$$(41)' \qquad \widetilde{P}^E_{t+1} = \lambda v^2\left(\widetilde{P}_{t-1} - \widetilde{M}_{t-1}\right) + (1-\lambda)v(\widetilde{P}_t - \widetilde{M}_t) - \lambda v \widetilde{M}_t,$$

where $0 < \lambda < 1$.

In economy 2 (we refer to this economy as the quantity responsive economy) agents use the forecast rule:

$$(41)\qquad \widetilde{P}^{E}_{t+1} = \nu^2\left(\widetilde{P}_{t-1} - \widetilde{M}_{t-1}\right) - \nu\widetilde{M}_t .$$

It is straightforward to check that both of these forecast rules induce rational expectations equilibria since in each case, the actual behavior of real balances will be given by a difference equation that satisfies the expectational equation (36). In the following two subsections we will derive the difference equation that characterizes the actual behavior of real balances and we will analyze the effects of monetary policy in each of the two economies.

The Price Responsive Economy

In the price responsive economy agents forecast deviations of prices from their growth path using equation (40). The forecast of the price level is obtained by (i) figuring out the stationary level of real balances (ii) figuring out the money stock next period (iii) calculating the price level associated with the stationary money stock and (iv) figuring out the deviation from this path arising due to random fluctuations in the money growth rate. Agents using the rule given in equation (40) would react to monetary shocks by adjusting their demand for money. This demand for money would be calculated by plugging the forecast rule (equation (40)) into the right side of equation (37) and setting demand equal to supply. This procedure leads to the equation:

$$(42)\qquad \widetilde{M}^{D}_t - \widetilde{P}_t = \frac{1}{\nu}\left[E_t\widetilde{M}_{t+1} - \nu^2\left(\widetilde{P}_{t-1} - \widetilde{M}_{t-1}\right)\right] = \widetilde{M}_t - \widetilde{P}_t ,$$

where the first equality uses the forecast function (40) to derive the demand for money and the second equality sets demand equal to supply. Since expectations are rational, the conditional expected deviation of money from its growth path is identically zero: that is $E_t\left[\widetilde{M}_{t+1}\right] = 0$. Rearranging terms it follows that if agents use the forecast rule (40) the actual behavior of real balances (and hence the actual path of prices) can be found from the equation:

$$(43)\qquad \left(\widetilde{M}_t - \widetilde{P}_t\right) = \nu\left(\widetilde{M}_{t-1} - \widetilde{P}_{t-1}\right).$$

Equation (43) determines how prices evolve in this economy if agents use equation (40) to forecast future prices. Notice that (43) satisfies the expectational equation (36) and the price forecast rule, equation (40), is therefore rational in the sense that if agents use this rule then their subjective expectations will equal the actual expectation of future prices. Since ν is a fraction between zero and 1, in this economy the deviations of real balances from their steady state will converge to zero. It follows that if agents forecast prices with the rule given in equation (40) that real balances will converge to their steady state. But if real balances are equal to their steady state value then any nominal shocks to this economy must affect prices one for one. It is for this reason that we refer to this economy as a *price responsive* economy since it is an economy in which nominal shocks are instantly transmitted to prices and real quantities are perfectly insulated from real disturbances.

The Quantity Responsive Economy

In the quantity responsive economy agents forecast deviations of future prices from their steady state path using equation (41) rather than equation (40). We will show below that the difference of (41) from (40) is that if agents forecast the future using equation (41) they will allow current nominal disturbances to affect quantities rather than prices.

In the quantity responsive economy the equation that determines current real balances (and therefore current prices) is found by equating the demand and supply of money. The demand for money is found by substituting (41) into the right side of (37) and equating demand to supply:

$$(44) \qquad \tilde{M}_t^D - \tilde{P}_t = \frac{1}{v}\left[E_t\tilde{M}_{t+1} - v^2\left(\tilde{P}_{t-1} - \tilde{M}_{t-1}\right) + v\tilde{M}_t\right] = \tilde{M}_t - \tilde{P}_t .$$

Once again one can solve this equation to find how current real balances are determined:

$$(45) \qquad \left(\tilde{M}_t - \tilde{P}_t\right) = v\left(\tilde{M}_{t-1} - \tilde{P}_{t-1}\right) + \tilde{M}_t .$$

We call this the *quantity responsive* economy because it follows from equation (45) that the price in period t is independent of the monetary shock. $\tilde{M}_t$ cancels from the left and right sides of equation (45) to give an expression that determines the deviation of price from its steady state path solely as a function of the past realizations of real balances. In the quantity responsive economy, nominal money shocks act as a driving force to real balances causing asymptotically declining real effects.

From the discussion above we can derive the effects of a monetary injection on output, prices and interest rates in the case of the *quantity responsive* economy. The initial increase in real balances raises output, both directly through the production function and through its effects on employment. Prices will rise slowly and real balances will decline over time, causing output to return back to its steady state. The initial increase in output, followed by a gradual decline, implies that consumption will initially increase and then return to its steady state. Since consumption returns asymptotically to its steady state, the *ratios* of consumption in adjacent periods (the variable C_{t+1}/C_t) will drop initially but then increase slowly, eventually approaching unity. From equation (7) the real interest factor in this economy is given by:

$$(46) \qquad \frac{P_{t+1}}{P_t}(1+i_t)=\frac{1}{1+\rho}\left(\frac{C_{t+1}}{C_t}\right)^{r}.$$

The fact that the ratios C_{t+1}/C_t tend back to unity implies then that the real rate of interest initially declines and then increases back to its steady state value. One can show that the nominal interest rate $i=F_m(h(m),m)$ also declines initially if ε_X is negative, but climbs back to its steady state as real balances fall back to their stationary value. Such behavior of the interest rates is also consistent with the empirical observations of Bernanke and Gertler (1996).

(8) Conclusions

The idea that general equilibrium models can generate indeterminate equilibria has been understood for some time although it is only recently that such models have been calibrated to fit existing data. There are two criticisms that are frequently leveled at economic models with indeterminacy. The first is that the degree of increasing returns required to generate indeterminacy is implausible. The second is that models with a multiplicity of equilibria cannot be used to make concrete predictions. In this paper we have tried to address both of these points.

By studying a monetary model in which money is highly complementary with other inputs we have shown that indeterminacy can arise in plausibly parameterized economies even when the technology satisfies constant returns to scale. We have also argued that models of multiple equilibria are not devoid of predictive content. In fact, each of the equilibria that might arise has a very different concrete prediction for the behavior of data.[11] Provided one imposes the discipline that agents form expectations in a stable way, the existence of indeterminacy should provide no more of a problem for econometricians than the assumption that utility functions are stable over time. In recent literature, a number of authors have exploited the idea that equilibria may be indeterminate to generate explanations of business cycles might that are driven and propagated by "animal spirits".[12] In this paper we have argued that equilibrium models in which there may be an indeterminate set of equilibria may also be used to provide a positive explanation of the monetary transmission mechanism.

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[11] For an elaboration this point see the paper by Farmer and Guo (1995) and the discussion by Aiyagari (1995).

[12] See the collection of papers on this issue in the *Journal of Economic Theory*, Vol. 63 no. 1, 1994.

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