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Unequal Societies

by

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Unequal Societies:

Income Distribution and the Social Contract

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Abstract

I propose a theory of inequality and the social contract which explains how countries with similar preferences and technologies, and equally democratic institutions, can nonetheless sustain such different systems of social insurance, fiscal redistribution and education finance as those of the United States and Western Europe. With imperfect credit and insurance markets some redistributive policies have a positive effect on ex–ante welfare, and this implies a political support which *decreases with inequality*, at least over some range. Conversely, with capital market imperfections lower redistribution translates into more persistent inequality. Hence the potential for *multiple steady-states,* with mutually reinforcing high inequality and low redistribution, or vice–versa.

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JEL classification: D31, E62, P16, O41, I22.

Introduction

The social contract varies considerably across nations. Some have low tax rates, others a steeply progressive fiscal system. Many countries have made the financing of education and health insurance the responsibility of the state; some, notably the United States, have left them in large part to families, local communities and employers. The extent of implicit redistribution through labor market policies or the mix of public goods also shows persistent differences. Can these societal choices be explained without appealing to exogenous differences in tastes, technologies, or political systems?

Adding to the puzzle is the fact that redistribution is often correlated with income inequality in just the opposite way than predicted by standard politico–economic theory: among industrial democracies the more unequal ones tend to redistribute less, not more. The archetypal case is that of the United States versus Western Europe, but the observation holds within the latter group as well; thus Scandinavian countries are both the most equal and the most redistributive. In the developing world a similar contrast is found in the incidence of public education and health services, which is much more progressive in East Asia than in Latin America (e.g., South Korea versus Brazil). Turning finally to time trends, it is rather striking that the welfare state is being cut back in most industrial democracies at the same time that an unprecedented rise in inequality is occurring.

The purpose of this paper is to develop a joint theory of inequality and the social contract which can help resolve these puzzles. In the process, it also serves to reconcile certain empirical findings of the recent literature on political economy and growth. Several authors such as Persson and Tabellini (1994) or Alesina and Rodrik (1994) have documented a negative relationship between initial disparities of income or wealth and subsequent aggregate growth. The proposed explanation is that greater inequality translates into a poorer median voter relative to the country's mean income, as in Meltzer and Richards (1981). This leads to increased pressure for redistributive policies, which in turn reduce incentives for the accumulation of physical and human capital. The data, however, do not support this explanation. Perotti (1994), (1996) and most of the other studies reviewed in Bénabou (1996c) find no relationship between inequality and the share of transfers or government expenditures in GDP. Among advanced countries the effect is actually negative, as suggested by the above examples (Rodriguez (1997)).[1] As to the effect of transfers on growth, most studies yield estimates which are in fact significantly positive.

The point of departure for this paper is a rather different view of both the role of the state and the workings of the political process. When capital and insurance markets are imperfect, a variety of policies which redistribute wealth from richer to poorer agents can have a *positive* net effect on aggregate output, growth, or more generally ex–ante welfare. Examples considered here will include social insurance through progressive taxes and transfers, state funding of public education, and residential integration. Net efficiency

[1] Using panel data for 20 OECD countries and controlling for national income, population and the age distribution, Rodriguez (1997) finds that pre-tax inequality has a significantly negative effect on every major category of social transfers as a fraction of GDP, as well as on the capital tax rate. Henriet and Rochet (1997) show, for a cross–section of 18 OECD countries, that public health coverage is more extensive where other forms of redistribution are also higher.

gains lead to very different political economy consequences from those of standard models: popular support for such redistributive policies *decreases with inequality*, at least over some range. Intuitively, efficient redistributions meet with a wide consensus in a fairly homogenous society but face strong opposition in an unequal one. Conversely, if agents engage in any type of investment, capital market imperfections imply that lower redistribution translates into more persistent inequality. The combination of these two mechanisms creates the potential for *multiple steady-states:* mutually reinforcing high inequality and low redistribution, or low inequality and high redistribution. Temporary shocks to the distribution of income or the political system can then have permanent effects.

I formalize these ideas in a stochastic growth model with incomplete asset markets and heterogeneous agents who vote over redistributive policies, whether fiscal or educational. In the short run, redistribution is shown to be U–shaped with respect to inequality; in the long run they are negatively correlated across steady–states. There are two important ingredients in the analysis. The first one is that redistribution enhance ex–ante welfare, at least up to a point. I thus examine policies which reduce the variance and possibly increase the mean of family income, by providing insurance against idiosyncratic shocks and relaxing credit constraints. The second one is a simple extension of the standard voting model, reflecting the fact that some groups have more influence in the political process than others. I present extensive evidence that the propensities to vote, give political contributions, work on campaigns and participate in most forms of political activity rise with income and education. In the model the pivotal agent is richer than the median, but he need not be richer than the mean. It should be emphasized that I do not appeal to variations in political rights or participation to explain countries' different societal choices: this parameter is kept fixed across steady-states. In the comparative statics analysis I vary the efficiency costs and benefits of redistribution on the one hand (via the elasticity of labor supply and the degree of risk aversion), the political system on the other, so as to identify their respective contributions to the results. In particular, there exists a *critical level* for the (normalized) gain in ex–ante welfare from a redistributive policy, such that: (i) below this threshold, no allocation of political influence can sustain more than a single social contract; (ii) above this threshold, multiple steady-states occur provided the political weight of the rich is neither too large nor too small.

When two "unequal societies" arise from common fundamentals they cannot be Pareto–ranked, although the more redistributive one has lower inequality and greater social mobility. As to macroeconomic performance, the tradeoff between tax distortions and liquidity–constraint effects allows for two interesting scenarios. One, termed "growth–enhancing redistributions", can help account for the positive coefficients of transfers in growth regressions, as well as the contributions of education and land policies in East–Asian and Latin American countries to their respective developments (or lack thereof). The other, termed "eurosclerosis," explains how European voters can choose to sacrifice more employment and growth to social insurance than their American counterparts, even though both populations have the same basic preferences. Another notable prediction of the model is that, depending on their source, exogenous shocks to income inequality will bring about sharply different evolutions of the social contract. Thus, an increased variability of sectoral shocks will lead to an expansion of the social safety net, while a surge in immigration may prompt large–scale cutbacks.

The model also has several interesting methodological features. The first is analytical tractability. Individual transitions are linear, reflecting the absence of non-convexities; yet one obtains multiplicity. The distribution of wealth remains log-normal, leading to easily interpretable closed–form solutions. Second, whereas the previous literature has focused on proportional taxation, I show how to incorporate progressivity. Third, I formalize political influence in an intuitive and tractable manner. These modelling devices could be useful in other settings.

There are three strands of recent literature to which the paper is related. The first one emphasizes the political economy of redistribution (Bertola (1993), Perotti (1993), Saint-Paul and Verdier (1993), Alesina and Rodrik (1994), Persson and Tabellini (1994), (1996), Benhabib and Rustichini (1996)). The second one is concerned with the financing and accumulation of human capital (Loury (1981), Glomm and Ravikumar (1992), Galor and Zeira (1993), Bénabou (1996a), Durlauf (1996a), Fernandez and Rogerson (1994), Gradstein and Justman (1997)). The third one stresses the wealth and incentive constraints which bear on entrepreneurial investment (Banerjee and Newman (1991), (1993), Aghion and Bolton (1997), Piketty (1997)). Most directly related are the models in Bénabou (1996a), (1996c), upon which I build, and the paper by Saint-Paul (1994), which identifies another politico-economic mechanism with properties related to those obtained here. Saint-Paul points out that increases in inequality whose adverse impact is concentrated in the lower tail of the income distribution may be accompanied by a rise in median income relative to the mean. The middle class, which remains politically decisive, will then reduce its transfers to the poor. If exit from poverty requires some kind of investment, capital market imperfections may then lead to multiple steady-states: a large underclass which persists due to low redistribution, or one which is kept small by significant transfers. Piketty (1995) provides another explanation for international differences in redistribution, similar to a collective form of the bandit problem. Because individual experimentation is costly, agents never fully learn the extent to which income is affected by effort rather than predetermined by social origins. The citizens of otherwise identical countries may then end up with different distributions of beliefs concerning social mobility, which translate into different perceived tradeoffs between the insurance and incentive effects of redistribution.

The paper is organized as follows. Section I explains the main ideas using the simplest possible setup, which treats the aggregate impact of redistribution as exogenous. Section II presents the actual economic model, which combines incomplete asset markets and progressive taxation. Section III describes the political system, then shows how a non–monotonic relationship between inequality and redistribution arises in the short run. Section IV analyzes the steady–state of an endowment economy. The extent of social insurance chosen by voters increases with income uncertainty but may decrease with the persistence of endowment shocks. Section V solves the full model with accumulation. The range of economic and political "fundamentals" which allow multiple steady–states to arise is characterized, and the rates of social mobility and aggregate growth under alternative regimes are compared. Section VI recasts the model so as to explain differences in countries' systems of education finance, which represent the most striking example of divergence in the social contract. Section VII discusses other applications such as altruism, residential integration and the mix of public goods. Section VIII concludes. All proofs are gathered in the appendix.

I A Simplified Presentation of the Main Ideas

As a prelude to the the actual model, I present in this section a highly stylized reduced form which offers a direct shortcut to the main intuitions and results.

A *The Standard View*

Let there be a continuum of agents, $i \in [0,1]$, with log-normally distributed endowments: $\ln y^i \sim \mathcal{N}(m, \Delta^2)$. The log-normal is a good approximation of empirical income distributions, leads to tractable results and allows for an unambiguous definition of inequality, as increases in Δ^2 shift the Lorenz curve outward. This variance also measures the distance between median and per capita income: $m = \ln y - \Delta^2/2$, where $y \equiv E[y^i]$. Suppose now that agents are faced with the choice between two stylized policies:

($\mathcal{P}$) *laissez-faire:* each consumes his own endowment, $c^i = y^i$, for all i.

($\widehat{\mathcal{P}}$) *complete redistribution:* resources are pooled, and everyone consumes $c^i = y$.

Sharing is restricted to be either zero or one for simplicity, but nothing changes with a menu of tax rates $\tau \in [0,1]$. How many people are in favor of redistribution, in this benchmark case where its has no aggregate impact? Clearly, all those with endowment below the mean , i.e. a proportion

$$p = \Phi\left(\frac{\Delta^2/2}{\Delta}\right) = \Phi\left(\frac{\Delta}{2}\right) \tag{1}$$

where $\Phi(\cdot)$ is the c.d.f. of a standard normal. Because the income distribution is right–skewed the median is below the mean, so $p > 1/2$. A strict majority rule would thus predict that redistribution should always take place. In reality the poor vote with lower probability than the rich and, to some extent, money buys political influence. Therefore the relevant threshold for redistribution to occur may not be $p^* = 50\% = \Phi(0)$, but $p^* = \Phi(\lambda)$, $\lambda > 0$. For instance if the π poorest agents never vote, $\Phi(\lambda) = (1+\pi)/2$.[2]

What is important and robust in (1) is therefore not the level effect, $p > 1/2$, but the comparative statics, $\partial p/\partial\Delta > 0$: in a more unequal society there is greater political support for redistribution. For *any* degree of bias λ in the voting system, positive or negative, the likelihood that redistribution takes place increases with inequality –or more specifically, skewness. This result is only reinforced under the standard assumption that redistribution entails some deadweight loss (endogenized later on), reducing available

[2] Section III will formalize in more detail –as well as present extensive evidence on– the influence of income and human wealth on most forms of political activity.

resources from y to ye^{-B}, $B > 0$. Given the choice between laissez-faire and sharing this reduced pie, the extent of political support for the *inefficient redistribution* is:

$$p = \Phi\left(\frac{-B + \Delta^2/2}{\Delta}\right) = \Phi\left(-\frac{B}{\Delta} + \frac{\Delta}{2}\right). \tag{2}$$

Note that now $p \gtrless 1/2$ but $\partial p/\partial\Delta$ is even more positive than before. As inequality increases, so does the likelihood that a policy which reduces aggregate income gets implemented. This, in essence, is the mechanism by which inequality reduces growth in models such as Alesina and Rodrik (1994), Persson and Tabellini (1994) or some cases of Bertola (1993) and Perotti (1993).[3] The idea that distributional conflict hampers economic performance appears to be supported by the evidence: a number of studies have confirmed Persson and Tabellini's and Alesina and Rodrik's findings of a negative effect of inequality on growth.[4] This correlation, however, does not seem to arise through increased redistribution. Perotti (1994) (1996), Lindert (1996) and Keefer and Knack (1995) show that there is no relationship between the income share of the middle class (which corresponds to the median voter) and any tax rate or share of government transfers in GDP. Clarke (1992) finds no correlation between any measure of inequality and government consumption. As casual empiricism suggests, more unequal countries do not redistribute more. Among advanced nations, they actually redistribute less (Rodriguez (1997)). Furthermore, the coefficients on transfers in growth regressions are most often significantly *positive:* see among others Lindert (1996), Sala-i-Martin (1996), Devarajan, Swaroop and Zou (1993), and especially Perotti (1994) (1996), who controls for the endogeneity of redistribution.

B *Efficiency Gains and Redistribution: the Static Case*

In a world of incomplete insurance and loan markets, some policies with redistributive features can have a positive effect on total welfare and even output (as the evidence on transfers and growth may suggest). Ex–ante welfare gains, in turn, imply a political support that varies with inequality in a radically different way from the traditional one. Indeed, suppose that by redistributing resources agents achieve increased efficiency, so that each gets to consume ye^B. For now I continue to take $B > 0$ as exogenous, but later on I shall derive it from a variety of channels: insurance, altruism, or credit constraints on the accumulation of human and physical capital. The fraction of people who support an *efficient redistribution* is:

$$p = \Phi\left(\frac{B + \Delta^2/2}{\Delta}\right) = \Phi\left(\frac{B}{\Delta} + \frac{\Delta}{2}\right). \tag{3}$$

It is of course always higher than (1) and (2), but the important point is the one illustrated on **Figure 1.**

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[3] Naturally, this simple reduced form fails to capture the richness of the original models.

[4] See Bénabou (1996c) for a survey of empirical studies on inequality and growth. As with nearly all growth regressions, one should bear in mind that this finding is robust to a variety of, but not to all, changes in specification. In any case, this correlation is not essential to our results, which primarily involve inequality and redistribution. Thus, Section V.B identifies parameters such that inequality and growth are positively, or negatively, correlated across steady–states.

Proposition 1 *When a redistributive policy generates gains in ex–ante efficiency, political support for it initially declines with inequality. In the present framework, the relationship is U-shaped.*

The intuition is simple: when dispersion is relatively small compared to the average gain, there is near unanimous support for the policy. As inequality rises, the proportion of those who stand to lose from the redistribution increases. At high enough levels of inequality, however, the standard skewness effect eventually dominates: there are so many poor that they impose redistribution no matter what its aggregate impact. There is thus *no monotonic relationship* between income inequality or the relative position of the median agent (both measured here by Δ^2) and the likelihood of redistribution.[5]

To relate the extent of popular support for a policy to actual outcomes one needs to specify the mechanism through which preferences are aggregated. I shall continue to assume that redistribution occurs if p exceeds a threshold $p^* \equiv \Phi(\lambda)$, where λ reflects the degree to which financial or human wealth contributes to political influence. More generally, the probability of implementation could be some increasing function of p. **Figure 1** shows that *only if* redistribution's aggregate impact B is positive can the policy be abandoned as the result of greater inequality –no matter how biased the political system might be. Conversely, a positive λ is needed for the political outcome to reflect the drop in popular support. The full model will confirm the joint importance of efficiency gains and wealth bias in shaping a declining relationship between inequality and redistribution. The arguments seen here for a zero–one policy choice will then apply to marginal changes in the tax rate, i.e. to every electoral contest between τ and $\tau + d\tau$.

Finally, consider the dynamic implications of Proposition 1. A society which starts with enough wealth disparity to find itself below p^* on the U-shaped curve of Figure 1 will not implement the redistributive policy; as a result, high inequality will persist into the next period or generation. Conversely, low inequality creates wide political support for efficient policies which prevent disparities from growing. The dynamic feedback operates whenever some form of investment is credit–constrained, so that current resources affect future earnings. Thus the same type of market imperfections which can give rise to a (partly) decreasing relationship between inequality and transfers also provide the second ingredient required for multiple steady-states. These dynamics are represented on **Figure 2** (which will be formally derived later on), for a continuous rate of redistribution $\tau \in [0, 1]$. The U-shaped curve $\tau = T(\Delta)$ is similar to that of **Figure 1**, while the declining $\Delta = D(\tau)$ locus arises from the accumulation mechanism.

The main part of the paper, to which I now turn, will show that all the intuitions obtained in this section carry over to a fully specified intertemporal model of individual behavior and collective choice. In particular, the welfare gains and losses from redistribution will be endogenous.

[5] The fact that inequality affects political support for redistribution in opposite ways, depending on the sign of its aggregate impact, is essentially *independent* of distributional assumptions. For any symmetric distribution $F(x)$ with mean μ, a symmetric, mean–preserving spread leads to a decline in popular support $p = F(\mu + B)$ for all $B > 0$, and an increase for all $B < 0$. In the case of log-normally distributed wealth, a rise in Δ combines this general effect of dispersion with a fall in $m = \mu - \Delta^2/2$ due to increased skewness; hence the U-shape. These properties remain true when B is a function of Δ, as long as it increases less than one for one with Δ. Such will be the case when B is endogenized later on.

II Incomplete Asset Markets and Progressive Taxation

A *Technology, Preferences and the Timing of Decisions*

The economy is populated by overlapping–generations families, $i \in I \equiv [0,1]$. In generation t, adult i combines his (human or physical) capital endowment k_t^i with effort l_t^i to produce output, subject to an i.i.d. productivity shock z_t^i :

$$y_t^i = z_t^i \left(k_t^i\right)^\gamma \left(l_t^i\right)^\delta . \tag{4}$$

Taxes and transfers, specified below, transform this gross income y_t^i into a disposable income $\hat{y}_t^i$ which finances both the adult's consumption, c_t^i, and his investment, e_t^i :

$$\hat{y}_t^i = c_t^i + e_t^i \tag{5}$$

$$k_{t+1}^i = \kappa\, \xi_{t+1}^i \,(k_t^i)^\alpha\, (e_t^i)^\beta \tag{6}$$

Capital depreciates geometrically at the rate $1-\alpha \geq \beta\gamma$, and investment is subject to i.i.d. productivity shocks ξ_t^i. There is no loan market for financing individual investment or educational projects (e.g., children cannot be held responsible for the debts of their parents) and no insurance or securities market where the idiosyncratic risks z_t^i and ξ_{t+1}^i could be diversified away. These are extreme forms of market incompleteness, but all that really matters is that there be some imperfections.[6] Both shocks are assumed to be log–normal with mean one, and initial human capital is log–normally distributed across families: thus $\ln z_t^i \sim \mathcal{N}(-v^2/2, v^2)$, $\ln \xi_t^i \sim \mathcal{N}(-w^2/2, w^2)$ and $\ln k_0^i \sim \mathcal{N}(m_0, \Delta_0^2)$.

Adults have lifetime preferences defined over their own consumption and effort as well as their child's endowment of capital, all of which are random variables. Following Kreps and Porteus (1979), Epstein and Zin (1989) and Weil (1990), these preferences will be defined recursively with respect to the sequence of shocks; see **Figure 3.**

endowed with k_t^i	votes on τ_t	learns z_t^i	chooses l_t^i, produces y_t^i	receives $\hat{y}_t^i$	consumes c_t^i, invests e_t^i	child endowed with k_{t+1}^i
↓	↓	↓	↓	↓	↓	↓

	↓		↓			
	$U_t^i = U(k_t^i, \tau_t)$		$V_t^i = V(k_t^i, z_t^i, \tau_t)$			

Figure 3: Preferences and the Timing of Decisions

[6] Perotti (1994) provides evidence that credit market frictions reduce aggregate investment, especially where the income share of the bottom 40% is low. Additional evidence on asset market incompleteness as a constraint on investment decisions in education and farming is discussed in Bénabou (1996c).

Upon discovering his productivity z_t^i agent i chooses effort, consumption and savings so as to maximize:

$$\ln V_t^i \equiv \max_{l_t^i,\, c_t^i} \left\{(1-\rho)[\ln c_t^i - (l_t^i)^\eta] + \rho \ln\left((E_t[(k_{t+1}^i)^{r'} \mid k_t^i, z_t^i])^{1/r'}\right)\right\}. \tag{7}$$

The disutility of effort is measured by $\eta > 1$, which corresponds to a compensated elasticity of labor supply equal to $1/(\eta - 1)$. The discount factor ρ defines the relative weights of the adult's own felicity and of his bequest motive, while his (relative) risk–aversion with respect to the child's endowment k_{t+1}^i is measured by $1 - r'$. At the beginning of period t, on the other hand, when evaluating and voting over redistributive policies, the agent has not yet learned z_t^i. In other words, he knows his type (k_t^i, z_t^i) *imperfectly.*[7] The resulting uncertainty over his ex–post utility level V_t^i is reflected in his ex–ante preferences,

$$U_t^i \equiv \ln\left(E_t[(V_t^i)^r \mid k_t^i]^{1/r}\right), \tag{8}$$

with a risk–aversion coefficient of $1-r$. When $r = 0$ preferences are time–separable and $1/(1-r)$ coincides with the intertemporal elasticity of substitution, which by (7) is fixed at one (for analytical tractability, as in much of the literature on income distribution dynamics). The usefulness of the recursive specification, however, lies in allowing $1 - r$ to parametrize the insurance value of redistributive policies, and therefore their impact on the benefit side of efficiency –just as the elasticity of labor supply $1/(\eta - 1)$ parametrizes their impact on the cost side, through standard distortions. Finally, it should be noted that while I have assumed overlapping generations with "imperfect" altruism, many of paper's results can also be derived with infinitely lived agents.[8]

B *Fiscal Policy*

In period t, agent i's market income y_t^i is transformed into disposable income $\hat{y}_t^i$ through the following tax-and transfer scheme:

$$\hat{y}_t^i = (y_t^i)^{1-\tau_t} (\tilde{y}_t)^{\tau_t}, \tag{9}$$

where the break–even income level $\tilde{y}_t$ is determined by the government's budget constraint: net transfers must sum to zero or, denoting per capita income by y_t :

$$\int_0^1 (y_t^i)^{1-\tau_t} (\tilde{y}_t)^{\tau_t}\, di = y_t\,. \tag{10}$$

[7] Therein lies the important difference between ξ_t^i and z_t^i, rather than in the fact that one affects human capital and the other production; those roles could be switched. Also, because the policies which I shall consider provide no insurance against ξ_{t+1}^i (only against z_t^i), the value of $1 - r'$ will turn out not to play an important role in the analysis. It could therefore be set (for instance) to zero, to one, or to $1 - r$, which is the more essential risk–aversion parameter defined below.

[8] See footnote 12 in Section III for further discussion. The infinite–horizon version of the recursive preferences presented here is obtained by simply replacing k_{t+1}^i with V_{t+1}^i in (7), with (8) unchanged. In an earlier version of this paper (Bénabou (1996b)) I analyzed the time–separable case ($r = 1$) with endogenous policy, while in Bénabou (1997) I study the general case with exogenous policies, both analytically and quantitatively.

The elasticity τ_t of post-tax income measures the degree of *progressivity* (or regressivity) of fiscal policy.[9] An alternative interpretation is that of wage compression through labor market institutions favorable to workers with relatively low skills. As usual, confiscatory rates $\tau_t > 1$ must be excluded as not incentive-compatible. Nothing in principle prevents a regressive tax, so I shall allow it; restricting τ_t to be non-negative would require dealing with corner solutions but would not change the nature of any result.

Proposition 1 *Given a tax rate τ_t, agents in generation t choose a common labor supply and savings rate:*

$$\begin{aligned} l_t &= \chi \cdot (1-\tau_t)^{1/\eta} \\ e_t^i/\hat{y}_t^i &= \mathfrak{s} \end{aligned}$$

where $\chi^\eta \equiv (\delta/\eta)(1-\rho+\rho\beta)/(1-\rho)$ and $\mathfrak{s} \equiv \rho\beta/(1-\rho+\rho\beta)$.

I will refer from here on to $1/\eta$ as "the" elasticity of labor supply.[10] Because taxes are progressive rather than merely proportional they affect effort, $l_t = l(\tau_t)$, in spite of the unitary intertemporal elasticity of substitution. They would also affect savings, were it not for adults' simple bequest motive. Even with infinitely–lived agents, however, the savings distortion can be fully offset by a balanced–budget combination of consumption taxes and investment subsidies, and moreover this can be shown to be Pareto–optimal (Bénabou (1996b), (1997)). In any case, one distortion is enough to demonstrate how the tradeoff between the costs and benefits of redistribution shapes the range of politico–economic equilibria.

C *Redistribution and Accumulation*

Given Proposition 1, and substituting (9) into (6), capital accumulation simplifies to:

$$(11) \qquad \begin{aligned} \ln k_{t+1}^i &= \ln \xi_{t+1}^i + \beta(1-\tau_t)\ln z_t^i + \ln\kappa + \beta\ln\mathfrak{s} \\ &\quad +(\alpha+\beta\gamma(1-\tau_t))\ln k_t^i + \beta\delta(1-\tau_t)\ln l_t + \beta\tau_t \ln\tilde{y}_t. \end{aligned}$$

Due to the symmetry of agents' effort and savings decisions, wealth and income remain log-normally distributed over time. If $\ln k_t^i \sim \mathcal{N}(m_t, \Delta_t^2)$, the government's budget constraint (10) easily yields the break–even point of the redistributive scheme (see the appendix):

$$(12) \qquad \ln\tilde{y}_t = \gamma m_t + \delta \ln l_t + (2-\tau_t)\gamma^2\Delta_t^2/2 + (1-\tau_t)v^2/2.$$

[9] The marginal tax rate rises with pretax income, to an extent which increases with τ_t. Similarly, agents with the average level of income are made better off ($\tilde{y}_t > y_t$) if and only if $\tau_t > 0$. Measuring (local) progressivity by the elasticity of after-tax to pretax income was initially proposed by Musgrave and Thin (1948). Jakobsson (1976) and Kakwani (1977) later showed this to be the "right" measure of income equalization: the post–tax distribution resulting from one fiscal scheme will Lorentz–dominate that of a second one (for all pre–tax distributions) if and only if its elasticity is everywhere smaller. I have recently become aware of a couple of (static) models where an isoelastic or "constant residual progression" scheme like (9) was used to study insurance or risk–taking: Feldstein (1969), Kanbur, (1979) and Persson (1983).

[10] It is indeed the uncompensated elasticity to the net-of-tax rate $1-\tau_t$ of the fiscal scheme, and varies monotonically with the standard compensated elasticity to proportional variations in the real wage, $1/(\eta-1)$.

From (11), we then obtain two simple difference equations which govern the evolution of the economy:

$$(13) \qquad m_{t+1} = (\alpha+\beta\gamma)\, m_t + \beta\delta \ln l_t + \beta\, \tau_t\, (2-\tau_t)\, (\gamma^2 \Delta_t^2 + v^2)/2 + \ln\left(\kappa \mathfrak{s}^{\beta}\right) - (w^2 + \beta v^2)/2$$

$$(14) \qquad \Delta_{t+1}^2 = (\alpha + \beta\gamma(1-\tau_t))^2\, \Delta_t^2 + \beta^2 (1-\tau_t)^2\, v^2 + w^2$$

The effect of redistribution on the dynamics of inequality is clear: the progressivity rate τ_t determines the *persistence of family wealth,* $\alpha + \beta\gamma(1-\tau_t)$. The impact on the dynamics of aggregate income is more complex, as it involves a tradeoff between labor supply and credit–constraint effects:

Proposition 2 *The distribution of pre–tax income at time t is* $\ln y_t^i \sim \mathcal{N}(\gamma m_t + \delta \ln l_t - v^2/2, \gamma^2 \Delta_t^2 + v^2)$, *where* m_t *and* Δ_t^2 *evolve according to the linear difference equations (13)–(14) and* $l_t = \chi(1-\tau_t)^{1/\eta}$. *The growth rate of per capita income is:*

$$(15) \qquad \ln(y_{t+1}/y_t) = \ln\tilde{\kappa} - (1-\alpha-\beta\gamma)\ln y_t + \delta(\ln l_{t+1} - \alpha \ln l_t) - \mathfrak{L}_v(\tau_t) v^2/2 - \mathfrak{L}_\Delta(\tau_t)\gamma^2\Delta_t^2/2$$

where $\ln\tilde{\kappa} \equiv \gamma(\ln\kappa + \beta\ln\mathfrak{s}) - \gamma(1-\gamma)w^2/2$ *is a constant and*

$$\mathfrak{L}_v(\tau) \equiv \beta\gamma(1-\beta\gamma)(1-\tau)^2 > 0$$
$$\mathfrak{L}_\Delta(\tau) \equiv \alpha + \beta\gamma(1-\tau)^2 - (\alpha+\beta\gamma(1-\tau))^2 > 0.$$

The term in $\ln y_t$ in the growth equation reflects the standard convergence effect; it disappears under constant aggregate returns, namely when $\alpha + \beta\gamma = 1$ or when κ is replaced by an appropriate spillover κ_t (see Section V.B). The term in l_t, capturing the effect of labor supply on accumulation, is also of a "representative agent" nature. The terms in $\mathfrak{L}_v(\tau)$ and $\mathfrak{L}_\Delta(\tau)$, on the other hand, represent growth losses specific to the heterogenous economy with imperfect credit markets. Suppose first that all adults in generation t have the same level of human capital ($\Delta_t^2 = 0$). Everyone then faces the same technology for accumulation, with decreasing returns measured by the concavity $\beta\gamma(1-\beta\gamma)$. The shocks z_t^i generate ex–post income disparities which are only partially offset by redistribution; due to the absence of credit these translate into inefficient variations in investment levels, reducing the growth rate of total output by $\mathfrak{L}_v(\tau_t)(1-\tau_t)^2 v^2/2$. Consider now initial disparities in endowments of human or physical capital, $\gamma^2\Delta_t^2$. When $\alpha = 0$ these have the same effect as income shocks: $\mathfrak{L}_v \equiv \mathfrak{L}_\Delta$. The marginal return to investment is higher for the poor than for the rich, because the former are more severely liquidity–constrained. When $\alpha > 0$, however, preexisting capital stocks k_t^i represent *complementary inputs* which generate a differential productivity of investment expenditures across agents, and thereby reduce the desirability of equalizing resources. Thus $\mathfrak{L}_\Delta(\tau)$ is minimized for $\tau = (1-\alpha-\beta\gamma)/(1-\beta\gamma)$, which decreases with α.

D *Individual Welfare*

I now turn from the evolution of the economy as a whole to individuals' evaluation of alternative fiscal polices. Given the optimal labor supply and savings responses to a tax rate τ_t it is straightforward to compute agent i's disposable income $\hat{y}_t^i$, for any productivity realization z_t^i. Substituting his consumption $c_t^i = (1-\mathfrak{s})\hat{y}_t^i$ and next period wealth $k_{t+1}^i = \kappa\xi_{t+1}^i(k_t^i)^\alpha(\mathfrak{s}\hat{y}_t^i)$ into (7) then yields his ex–post welfare, V_t^i. It is before learning their type, however, that agents must vote over redistribution, so the relevant preferences over τ_t are defined by the ex–ante utility U_t^i, according to (8).

Proposition 3 *Given a rate of fiscal progressivity τ_t, agent i's intertemporal welfare is:*

$$U_t^i = \bar{u}_t + A(\tau_t)(\ln k_t^i - m_t) + C(\tau_t) - (1-\rho+\rho\beta)(1-\tau_t)^2\left(\gamma^2\Delta_t^2 + Bv^2\right)/2 \tag{16}$$

where $\bar{u}_t$ is independent of the policy τ_t and:

$$A(\tau) \equiv \rho\alpha + (1-\rho+\rho\beta)\gamma(1-\tau) \tag{17}$$

$$C(\tau) \equiv (1-\rho)(\delta\ln l(\tau) - l(\tau)^\eta) + \rho\beta\delta\ln l(\tau) \tag{18}$$

$$B \equiv 1 - r + \rho r(1-\beta) \geq 0. \tag{19}$$

The first component of intertemporal utility, $\bar{u}_t$, depends only on the state variables m_t and Δ_t^2, on the economy's parameters, and on the (endogenous but constant) investment rate $\mathfrak{s}$. The second term in (16), which disappears through aggregation, makes clear the *redistributive* effects of tax policy, including its impact on the persistence of social positions, $\alpha + \beta\gamma(1-\tau)$. The last two terms represent the aggregate *welfare cost* and aggregate *welfare benefit* of a progressivity rate τ_t. Thus $C(\tau_t)$, which is maximized for $\tau_t = 0$, reflects the distortions in labor supply entailed by such a policy. Conversely, the term $-(1-\tau_t)^2(\gamma^2\Delta_t^2 + Bv^2)$, which is maximized for $\tau_t = 1$, embodies the efficiency gains which arise from better insurance and the redistribution of resources from low to high marginal–product investments.

To make the role of market incompleteness more explicit I consider again each source of heterogeneity in turn. By (16), idiosyncratic income variability lowers everyone's utility by $(1-\rho+\rho\beta)B(1-\tau)^2v^2/2$. When $\beta = 1$ the coefficient on $(1-\tau)^2v^2/2$ is simply the aversion to risk, $1-r$: with constant returns the ex–ante value of redistribution reduces to the insurance it provides. In general, it also contributes to efficiency through the relaxation of credit constraints. This is best seen with risk–neutral parents who only care only about their offspring: when $r = \rho = 1$ the utility loss is $\beta(1-\beta)(1-\tau)^2v^2/2$, which is the shortfall in expected (and aggregate) bequeathed wealth resulting from imperfectly offset resource shocks and a concave investment technology. Consider now preexisting inequality Δ_t^2, which reduces aggregate welfare by $(1-\rho+\rho\beta)\gamma^2(1-\tau)^2\Delta_t^2/2$. This simple term –or equivalently its decline with redistribution– is actually the sum of two effects, only one of them due to market incompleteness. First, reallocating investment resources towards the poor again increases the growth rate of total wealth, $\ln(k_{t+1}/k_t)$. Second, there is the standard effect of concave (logarithmic) utility functions, whereby average welfare increases when individual consumptions (of c_t^i and k_{t+1}^i) are distributed more equally.[11] Equivalently in this model, it captures the effect of *skewness,* as in Section I: the median agent, whose levels of consumption and bequest are below average, gains when these resources are redistributed progressively. Finally, note that by varying $1-r, \beta, v^2$ and $1/\eta$ the ex–ante efficiency gain measured by $Bv^2/2$, or more generally by $(1-\rho+\rho\beta)(1-\tau)^2 Bv^2/2 - C(\tau)$, can be made arbitrarily large or small relative to preexisting income inequality, $\gamma^2\Delta_t^2$.

III The Political System

I now turn to the determination of the equilibrium policy. Each generation chooses the rate of fiscal progressivity τ_t to which it will be subject, before the individual productivity shocks z_t^i are realized. Agent i's ideal policy would thus be to maximize U_t^i.[12] These individual preferences are aggregated through a political process in which some groups have more influence than others.

A *Preferred Policies*

For expository purposes it will often be useful to start with the case where labor supply is fixed, namely $1/\eta = 0$. The utility function U_t^i is then quadratic in τ_t, and maximized at:

$$\frac{1}{1-\tau_t^i} = \frac{\gamma^2\Delta_t^2 + Bv^2}{\gamma \max\{\ln k_t^i - m_t, 0\}}. \tag{20}$$

Voters below the median desire the maximum feasible tax rate, $\tau_t^i = 1$, which is also the ex–ante efficient one in the absence of distortions. Voters above the median desire a tax rate $\tau_t^i < 1$ which decreases with their initial wealth and increases with the variance of productivity shocks, for both insurance and investment reasons (concavity of preferences and concavity of the technology).

With endogenous labor supply complete redistribution is never chosen, as it would lead to zero effort and output. Agent i's desired policy is given by the first–order condition $\partial U_t^i/\partial\tau = 0$, or:

[11] Formally, one can rewrite $(1-\rho+\rho\beta)\gamma^2(1-\tau)^2 = \rho[\beta\gamma^2(1-\tau)^2 - (\alpha+\beta\gamma(1-\tau))^2] + (1-\rho)\,\gamma^2(1-\tau)^2 + \rho(\alpha+\beta\gamma(1-\tau))^2$ $= \rho\ln(k_{t+1}/\ln k_t) + (1-\rho)\,var_{j\in I}[\ln c_t^j] + \rho var_{j\in I}[\ln k_{t+1}^j] + \mu_t$, where $var_{j\in I}[\cdot]$ denotes a cross–sectional variance, μ_t a quantity independent of τ_t, and I have assumed $v = w = \delta = 0$ for notational simplicity.

[12] Because of the overlapping–generations structure these decisions do not involve any intertemporal strategic considerations. Infinite lives or "perfect" parental altruism, by contrast, would generate a dynamic game where voters try to influence future political outcomes τ_{t+k} by altering the evolution of the wealth distribution Δ_{t+k}^2 through their choice of τ_t (see (14)). This problem is notoriously intractable, so the standard practice is to assume that voters are "myopic", either failing to recognize their influence on future outcomes or –as here– not caring about it due to a limited bequest motive. Notable exceptions are Saint–Paul (1994) and Grossman and Helpman (1996); an alternative approach is to look for solutions numerically, as done by Krusell, Quadrini and Ríos-Rull (1997). In Bénabou (1996b) I assume a different form of political myopia on the part of infinitely–lived agents, and obtain results very similar to those presented here. I first allow an arbitrary initial generation to *specify a constitution,* by choosing among all *constant sequences* $\{\tau_t = \tau\}_{t=0}^{\infty}$ the one to which it would like to commit the economy. I then require *consistency* between the initial income distribution, which determines the choice of τ, and that which results from the policy, through the law of motion (14). In such a steady-state ($\Delta_t^2 = \Delta_\infty^2$) every generation, if given the same choice set as its predecessors, will validate the existing social contract. The *"self–restraint"* implicit in the restriction to time–invariant policies has been used in some papers on dynamic taxation (e.g., Chamley (1985)), but none of them solved for a consistent steady-state. It can be given choice–theoretic foundations based on the work of Cohen and Michel (1991) and Caillaud, Cohen, and Julien (1994).

$$(21) \qquad (1-\tau)(\gamma^2\Delta_t^2 + Bv^2) - \gamma(\ln k_t^i - m_t) - \frac{\delta}{\eta}\left(\frac{\tau}{1-\tau}\right) = 0.$$

This quadratic equation always has a unique solution less than 1, which will be denoted τ_t^i.

Proposition 4 *Each agent's utility U_t^i is strictly concave in the policy τ_t. His preferred tax rate τ_t^i decreases with his wealth k_t^i and increases with Bv^2. Finally, $\left|\tau_t^i\right|$ decreases with the labor supply elasticity $1/\eta$.*[13]

These results are intuitive. Lower personal wealth or greater ex-ante efficiency benefits from redistribution increase an agent's demand for such policies. A more elastic labor supply increases the deadweight loss caused by taxes and transfers, whether progressive or regressive, which cause individuals to distort their labor supply away from the first–best level. Finally, as a prelude to the analysis of political equilibrium, note how (21) embodies the same intuitions as the stylized model of Section I. For any $\tau < 1$ the proportion of agents who would like further redistribution at the margin,

$$(22) \qquad p(\tau, \Delta_t) \equiv \#\left\{i \left| \frac{\partial U_t^i}{\partial \tau} > 0\right.\right\} = \Phi\left(\frac{(1-\tau)Bv^2 - \delta\tau/(\eta(1-\tau))}{\gamma\Delta_t} + (1-\tau)\gamma\Delta_t\right),$$

is U-shaped in Δ_t provided the resulting gain in ex–ante efficiency $(1-\tau)Bv^2$ dominates the distortion $\delta\tau/(\eta(1-\tau))$; otherwise, $p(\tau, \Delta_t)$ is strictly increasing in Δ_t. Also as on **Figure 1**, the variations in popular support for efficient increases in τ take place above the 50% level, so they will influence policy outcomes only under some departure from the pure "one person, one vote" democratic ideal.

B *Wealth and Political Influence*

It is well–known that poor and less educated individuals have a relatively low propensity to register, turn out to vote, and give political contributions. These are among the facts clearly documented in **Table 1**, which presents data from Rosenstone and Hansen (1993); see also Wolfinger and Rosenstone (1980), Edsall (1984), or Conway (1991). For each form of participation in electoral and governmental politics, the *representation ratio* of a given socioeconomic group is the ratio between its share of the population engaged in this activity and its share of the general population. Thus the poorest 16% account for only $.76 \times 16 = 12.2\%$ of the votes and $.25 \times 16 = 4.0\%$ of the number of campaign contributors, while the richest 5% account for $1.27 \times 5\% = 6.4\%$ of the votes and $3.25 \times 5 = 16.3\%$ of contributors. Put differently, the representation ratios are simply the slopes of the (piecewise linear) *Lorenz curve* which describes the concentration, by income or education, of a given form of political influence.

The data in **Table 1** are striking in several respects. The propensity to participate in *every* reported form of political activity rises with income and education. For voting itself the tendency is relatively moderate, whereas for contributing to political campaigns it is drastic. In the latter case the actual bias

[13] Specifically: for $\ln k_t^i \leq m_t + \gamma\Delta_t^2 + Bv^2/\gamma$ the ideal τ_t^i is positive and decreases towards zero as $1/\eta$ rises. For $\ln k_t^i > m_t + \gamma\Delta_t^2 + Bv^2/\gamma$, $\tau_t^i < 0$ and it increases towards zero as $1/\eta$ rises.

is still understated, as the data reflects only the number of contributions and not their amounts, which undoubtedly also rise steeply with income. It is intuitive that the wealthy should be overrepresented in money–intensive channels of political influence: such lobbying is a form of collective investment where liquidity constraints are even more likely to bind than usual. One might have expected poorer, less skilled agents to have a countervailing advantage for attending meetings, working on campaigns, writing Congress, and other such time–intensive activities for which they have a lower opportunity cost. But, remarkably, the pro–wealth (financial and human) bias is here again not only positive, but extremely strong.

I shall not seek to explain the source of these biases, only to model them in a convenient and fruitful manner.[14] Let each agent's opinion be affected by a relative weight, or probability of voting, $\omega^i / \int_0^1 \omega^j \, dj$. If individual preferences are single–peaked and the preferred policy is monotonic in wealth, or more generally if preferences satisfy a sorting condition, a median–voter–type result applies but where the median is computed on an appropriately renormalized population. With a log-normal wealth distribution, the following schemes yields particularly simple results.

Proposition 5 *Suppose that agents* $i \in [0,1]$ *have preferences* $U(k^i, \tau)$ *over some policy variable* $\tau \in \mathbb{R}$*, and that these satisfy the single–crossing condition: for all* $k < k'$ *and* $\tau < \tau'$*, if* $U(k', \tau') > U(k', \tau)$ *then* $U(k, \tau') > U(k, \tau)$.

1) If an agent's political weight depends on his rank in the wealth distribution, $\omega^i = \omega(p^i)$*, the pivotal voter is the one with rank* $p^* = \Phi(\lambda)$ *and (log) wealth* $\ln k^* = m + \lambda\Delta$*, where* $\lambda \lesseqgtr 0$ *is defined by* $\left(\int_0^{\Phi(\lambda)} \omega(p)dp\right) \Big/ \left(\int_0^1 \omega(p)dp\right) = 1/2$.

2) If an agent's political weight depends on the absolute level of his wealth, with $\omega^i = (k^i)^\lambda$ *for some* $\lambda \lesseqgtr 0$*, the pivotal voter has rank* $p^* = \Phi(\lambda\Delta)$ *and (log) wealth* $\ln k^* = m + \lambda\Delta^2$.

Ordinal schemes ensure that each person's weight and the identity of the pivotal voter remain invariant when the distribution of wealth shifts due to growth or when it becomes more unequal.[15] Previous discussions of political rights have generally assumed that influence depends on one's absolute level of wealth. This was motivated by historical examples such as voting franchises restricted to citizens owning a minimum amount of property (Saint-Paul and Verdier (1993), Persson and Tabellini (1994)) or membership in a ruling elite which requires some investment expenditures (Ades and Verdier (1997)). I find it somewhat more plausible that even such cutoff levels should be relative ones, keeping up with aggregate growth and reflecting the competitive nature of bids for political influence. I shall therefore focus on ordinal schemes, but the second part of Proposition 5 shows that absolute income effects are just as easy to capture; the case $\lambda = 1$, for instance, corresponds to a "one dollar, one vote" rule. Moreover, this alternative formulation

[14] Roemer (1995) shows how the presence of a second dimension in the political game (morals, religion) can result in a similar bias, by limiting the size of the coalition which can be mustered in favor of redistribution. It is worth emphasizing again that I shall not appeal to differences in the allocation of political power or influence to explain why the extent of redistribution varies across countries (although the model can readily incorporate this "easier" explanation).

[15] Also, for any ordinal scheme $\omega(\cdot)$ the associated λ is a sufficient statistic: it is as if the bottom $2\Phi(\lambda) - 1$ voters (or the top $1 - 2\Phi(\lambda)$ when $\lambda < 0$) systematically abstained. Note that even though $\lambda > 0$ is the more empirically relevant case, the model will be solved for all $\lambda \gtreqless 0$.

would only reinforce the paper's results, as it implies that the political system becomes more biased towards the wealthy as inequality rises.[16]

In the last column of **Table 1** I interpolated the empirical $\omega(p)$ function to compute the *position of the pivotal agent* p^* for each separate form of political participation, i.e. as if it were the only one that mattered.[17] No data exist which would allow them to be weighted by their relative importance in determining the final political outcome. For voting the wealth bias is moderate, with the pivotal voter at the 56^{th} percentile rather than the usually assumed median. For all other forms of influence it is much stronger, with the pivotal agent always above the 60^{th} percentile, and quite often the 70^{th}. From this evidence one can safely conclude that the decisive political group is located *above the median* in terms of income and human wealth. Depending on the relative efficacy of the different channels of political influence it may even be above the mean, which in the US income distribution falls around the 63^{d} percentile. As we shall see below, the first condition –but *not* the second– will be required for redistribution to decline with inequality.

C *Political Equilibrium, Inequality, and Redistribution*

We are now ready to establish the paper's first main result. By virtue of Propositions 4 and 5, the outcome of the political process is obtained by simply setting $\ln k_t^i - m_t = \lambda\Delta_t$ in the first-order condition $\partial U_t^i/\partial\tau = 0$, or equivalently $p(\tau, \Delta_t) = p^* = \Phi(\lambda)$ in (22). Again, consider first the case where there are no labor supply distortions $(1/\eta = 0)$. For $\lambda > 0$, this yields:

$$\frac{1}{1-\tau_t} = \frac{\gamma^2\Delta_t^2 + Bv^2}{\lambda\gamma\Delta_t}. \tag{23}$$

The equilibrium tax rate is clearly *U–shaped* in Δ_t, and minimized where $\Delta^2 = Bv^2/\gamma^2$. Similarly, in the general case it is the unique solution $T(\Delta_t) < 1$ to the quadratic equation derived from (21):

$$(1-\tau_t)\left(\frac{\gamma^2\Delta_t^2 + Bv^2}{\gamma\Delta_t}\right) - \frac{\delta}{\eta\gamma\Delta_t}\left(\frac{\tau_t}{1-\tau_t}\right) = \lambda. \tag{24}$$

Proposition 6 *The rate of fiscal progressivity $\tau_t = T(\Delta_t)$ chosen in generation t has the following features:*

1) τ_t increases with the ex–ante efficiency gain from redistribution (gross of distortions) Bv^2, and decreases with the political influence of wealth, λ.

2) $|\tau_t|$ decreases with the elasticity of labor supply $1/\eta$.[18]

[16] This is somewhat similar to Ades and Verdier (1996), in that increased inequality pushes a greater fraction of the population into political disenfranchisement (low ω^i, due to low wealth), thereby concentrating power on a smaller "elite".

[17] The weights $\omega(p^i)$ are obtained by rescaling the representation ratios by the population's average propensity to participate in the activity under consideration (given in column 1). If a group of size n^i has a propensity to participate equal to ω^i it accounts for a share $\pi^i = \omega^i n^i/(\sum \omega^j n^j)$ of the political activity, so its representation ratio is $\varsigma^i = \pi^i/\left(n^i/\sum n^j\right)$. Therefore $\omega^i = \varsigma^i\bar{\omega}$, where $\bar{\omega} \equiv \left(\sum \omega^j n^j\right)/\left(\sum n^j\right)$. As to the relationship between voters' income and their political preferences, McCarthy, Poole and Rosenthal (1997) provide substantial evidence that U.S. politics are highly –and increasingly– unidimensional, along the axis of rich/opposed to redistribution versus poor/favoring redistribution.

[18] Thus if $Bv^2 > \lambda^2/4$ then $\tau_t > 0$ and it declines with $1/\eta$. If $Bv^2 < \lambda^2/4$, then $\partial\tau_t/\partial(1/\eta)$ has the sign of $-\tau_t$.

3) τ_t is U-shaped in the initial level of inequality Δ_t, for any $\lambda > 0$ (and increasing for any $\lambda \leq 0$). It starts at the ex–ante efficient rate, given by:

$$\frac{1}{1-T(0)} = \frac{1}{2}\left(1+\sqrt{1+4(\eta/\delta)Bv^2}\right)$$

for $\Delta_t = 0$, declines to a minimum $\underline{\tau}$ at some $\underline{\Delta}$ (with $\underline{\Delta} > 0$ if and only if $\lambda > 0$), then rises back towards one as Δ_t tends to infinity. The larger Bv^2, the wider the range $[0, \underline{\Delta})$ where $\partial\tau_t/\partial\Delta_t < 0$.

The first two results show that the equilibrium tax rate depends on the costs and benefits of redistribution, as well as on the allocation of political influence, in a very intuitive manner. The third result provides an *endogenously derived* analogue to **Figure 1**, with the continuous policy outcome τ now replacing the proportion of people supporting redistribution in a zero-one decision. It also confirms two important claims made earlier about the result that *redistribution may decline with inequality.* First, it is not predicated on the pivotal agent being richer than the mean, or becoming richer relative to the mean. Second, it is more likely to occur the larger the ex–ante welfare gain from redistribution, e.g. the larger Bv^2.[19] Some bias $\lambda > 0$ with respect to pure majority rule is needed as well, because the median agent always wants to push redistribution beyond its range of efficiency. As shown in (22), it is only within that range that political support for a tax cut, $1 - p(\tau, \Delta_t)$, can rise with inequality.

All these results apply equally in an endowment economy ($\beta = 0$) and in the presence of accumulation. In the first case the efficiency gains arise from insurance. In the second they also reflect the reallocation of resources to agents whose marginal product of investment is higher, due to tighter liquidity constraints. In studying which social contracts emerge in the long–run, I shall consider each case in turn.

IV Inequality and Social Insurance in an Endowment Economy

Should we expect a more generous welfare state in countries with greater disparities of income and wealth, as predicted by standard models, or a less generous one, as a comparison between Sweden and the United States would suggest? To study the political economy of pure social insurance, let us focus on an endowment economy ($\beta = 0$).[20] By (6), each dynasty's endowment k_t^i then simply follows a geometric AR(1) process with serial correlation α. The same is true for income $y_t^i = z_t^i(k_t^i)^\gamma(l_t)^\delta$ except that the deterministic term is endogenous, reflecting optimal effort. By Proposition 6, the rate of tax progressivity chosen at time t is U-shaped in Δ_t. In the long run, however, inequality converges to its state–state level $\Delta_\infty^2 \equiv w^2/(1-\alpha^2)$,

[19] With respect to the first claim, note that $\ln k^* - \ln E[k] = m + \lambda\Delta - \Delta^2/2$ is increasing only on $[0, \lambda]$ and positive only on $[0, 2\lambda]$. Neither interval coincides with, nor contains, $[0, \underline{\Delta}]$. The second claim follows from Proposition 6, but even when $Bv^2 = 0$ one sees from (22) that for all $\Delta \in [0, \underline{\Delta}]$ a marginal rise in τ above $T(\Delta)$ increases ex–ante efficiency. These gains arise from lowering the effort distortions due to regressionary taxes, as $Bv^2 = 0$ implies $T(0) = 0$, hence $\tau_t < 0$ on $[0, \underline{\Delta}]$.

[20] The important assumption here is the absence of insurance markets. The incompleteness of the loan market is inessential, and indeed this section's results remain unchanged in the static case where $\rho = 0$. This one–shot model is close to that of Persson (1983), except that he assumes no initial difference between agents ($\Delta_0 = 0$) and no political bias ($\lambda = 0$). He studies the ex–ante optimal policy (similar to $T(0)$ in Proposition 6) and contrasts it with the one which the median voter might want to deviate to ex–post, once all the income shocks have been realized.

and the tax rate therefore to $\tau_\infty \equiv T(\Delta_\infty)$. When $1/\eta = 0$, for instance,

$$\frac{1}{1-\tau_\infty} = \frac{w}{\lambda}\left[\frac{\gamma}{\sqrt{1-\alpha^2}} + (1-(1-\rho)r)\left(\frac{v}{w}\right)^2\left(\frac{\sqrt{1-\alpha^2}}{\gamma}\right)\right]. \tag{25}$$

More generally, it is easily shown that:

Proposition 7 *The steady-state rate of fiscal progressivity τ_∞ increases with agents' degree of risk–aversion $1-r$ and with income uncertainty v^2, but is U-shaped with respect to the variance of endowment realizations w^2 and the persistence of the endowment process, α.*[21] *It decreases with the political influence of wealth λ, and declines in absolute value with the labor supply elasticity $1/\eta$.*

Recalling that steady–state income dispersion is $\gamma^2 w^2/(1-\alpha^2) + v^2$, these results indicate that the relationship between inequality and redistribution is not likely to be monotonic. What matters is not just the *amount* of income inequality, but also *its source.* To the extent that high income disparities in some countries reflect larger uninsurable shocks (or more imperfect insurance markets), higher taxes and transfers should be observed. But if greater inequality is due to more persistent wealth dynamics or to greater ex–ante heterogeneity at the time of the policy decision –correlated for instance with observable ethnic or regional differences– the reverse correlation may be observed.[22] In particular, greater persistence can reduce risk-sharing, even though it implies more volatile lifetime income. The reason is that it also increases the number of agents for whom the value of insurance is more than offset by their vested interest in the status quo.

V History–Dependent Social Contracts

A *Dynamics and Multiple Steady-States*

I now turn to the paper's second main idea, sketched at the end of Section I: if more inequality leads to less redistribution and if pre-tax resources depend on past transfers, multiple equilibria can arise. To demonstrate this point I solve the full model with capital accumulation subject to wealth constraints ($\beta > 0$). The critical difference with the endowment economy is that the wealth distribution is now endogenous, through the effect of fiscal policy on persistence $\alpha + \beta(1-\tau_t)$. The joint evolution of inequality and policy is thus described by the recursive dynamical system:

$$\begin{cases} \tau_t &= T(\Delta_t) \\ \Delta_{t+1} &= \mathfrak{D}(\Delta_t, \tau_t) \end{cases} \tag{26}$$

[21] As in Proposition 6, the declining portion of the U–shaped curve is non–degenerate if and only if $\lambda > 0$, which is the empirically relevant case.

[22] A related result obtains in Persson and Tabellini (1996), where two regions bargain over the degree of risk–sharing in the federal constitution. In both models the underlying assumption is the inability to make taxes and transfers contingent only on unpredictable innovations to individual or regional income, as distinguished from its permanent component. See also Cassamatta, Cremer and Pestiau (1997) for a study of how ex–post political equilibrium constrains the ex–ante design of public health insurance systems.

where $T(\Delta_t)$ is the unique solution less than one to (24), while $\mathfrak{D}(\Delta_t, \tau_t)$ is given by the recursion equation (14). Under a time–invariant policy, in particular, long–run inequality decreases with redistribution:

$$\Delta_\infty^2 = \frac{w^2 + \beta^2(1-\tau)^2 v^2}{1-(\alpha+\beta\gamma(1-\tau))^2} \equiv D^2(\tau). \tag{27}$$

A steady-state equilibrium is an intersection of this downward-sloping locus, $\Delta = D(\tau)$, with the U-shaped curve $\tau = T(\Delta)$ described in Proposition 6; see **Figure 2.** Substituting (27) into (24), this corresponds to a rate of tax progressivity solving the equation:

$$f(\tau) \equiv (1-\tau)\left(\frac{\gamma^2 D(\tau)^2 + Bv^2}{\gamma D(\tau)}\right) - \frac{\delta}{\eta\gamma D(\tau)}\left(\frac{\tau}{1-\tau}\right) = \lambda. \tag{28}$$

The solutions correspond to the zeroes of a polynomial of degree eight, making this a complex problem. Yet, by exploiting geometric intuitions on the shape of f I shall establish a series of propositions which formalize some of the paper's main economic ideas. As illustrated on **Figure 4**, two countries with the same economic and political fundamentals can nonetheless evolve into *different societies*, provided:

(a) the ex–ante welfare *benefits* of redistribution are high enough, relative to the *costs;*

(b) the political *power* of the wealthy lies in some *intermediate* range.

The intuition behind the proofs is the following. Consider the right–hand side of (28) as a function of $1-\tau \in (0, (1-\alpha)/\beta\gamma)$. Since D is monotonic, the first fraction in large brackets is U–shaped in $1-\tau$. After multiplication by $1-\tau$ the product typically becomes N-shaped, so that it will have three intersections with the horizontal λ, for some range of values of λ. The larger B, the more pronounced this N–shape, which is also that of the vertical difference $T(\Delta) - D^{-1}(\Delta)$ on **Figure 2**. Conversely, the last term in (28), reflecting labor supply distortions, tends to make f strictly increasing in $1-\tau$ (at least where $\tau > 0$) and therefore works towards uniqueness.

Theorem 1 *Let $1-\alpha < 2\beta\gamma$. When the normalized efficiency gain $B \equiv 1 - r(1-\rho+\rho\beta)$ is below some critical value $\underline{B}$ (equivalently, when risk–aversion $1-r$ is less than some $1-\underline{r}$), there is a unique, stable, steady–state. When $B > \underline{B}$, on the other hand, there exist $\underline{\lambda}$ and $\bar{\lambda}$ with $0 < \underline{\lambda} < \bar{\lambda}$, such that:*

1) For any λ in $[\underline{\lambda}, \bar{\lambda}]$ there are (at least) two stable steady-states.[23] *Inequality is lower, and social mobility higher, under a more redistributive social contract.*

2) For $\lambda < \underline{\lambda}$ or $\lambda > \bar{\lambda}$ the steady–state is unique.

Where multiple steady-states occur, history matters. Temporary shocks to the distribution of wealth (immigration, educational discrimination, shifts in demand or technology) as well as to the political system (slavery, voting rights restrictions) can permanently move society from one equilibrium to the other, or more generally have long–lasting effects on the economy and the social contract. For instance, even if

[23] Specifically, there are $2 \leq n \leq 4$ stable steady states. If $1/\eta$ is small enough then $n \leq 3$, and if α is also small enough then $n = 2$. See the theorem's proof in the appendix.

the factors behind the recent rise in wage inequality in industrialized countries (e.g., skill-biased technical progress) abate over the next few decades, the current economic polarization of these societies and the cutbacks in the welfare state which are accompanying it are likely to endure much longer.

This history-dependence contrasts sharply with traditional politico-economic models, where countries can deviate at most temporarily from a common steady-state level of inequality and redistribution (given stable "fundamentals"). In particular, fiscal policy operates there as a stabilizing force on the distribution of wealth: more inequality today means more redistribution, hence less inequality tomorrow.[24] Here, on the contrary, there emerges in the long–run a *negative correlation between inequality and redistribution* across societies, as indeed one observes between the United States and Europe, or among advanced countries in general (Rodriguez (1997)). Along the adjustment path there is no monotonic relationship between these two variables, and indeed the correlation is zero in samples which include developing and poor countries (e.g., Perotti (1996)). Multiple steady-states due to a negative impact of inequality on redistribution is the distinguishing feature which the present model shares with Saint-Paul (1994). Consistent with the general argument that this entails redistributions which increase the size of the pie, Saint Paul's model has the property that transfers raise aggregate income.

Also of interest is the predicted *negative correlation between inequality and social mobility,* which is consistent with the results obtained by Erikson and Goldthorpe (1992) for a sample of fifteen developed countries. But what of the conventional wisdom of the United States as an exceptionally mobile society? In fact, most econometric studies of income mobility find either no significant difference, or even somewhat greater mobility in European "welfare states": see Björklund and Jäntti (1997b) for a survey.[25] Indeed, the most extreme forms of social immobility at the lower end, such as urban ghettos or the persistence of welfare dependency across generations, do seem more exacerbated in American society. Things could well be different for the middle class, so a more satisfactory comparison across countries would need to take into account such non–linearities in the mobility process (e.g., Cooper, Durlauf and Johnson (1994)). These remain beyond the scope of the present model and of most existing comparative studies.

Having demonstrated how the sustainability of different societal choices depends on the importance of risk–aversion and credit constraints (summarized in B) as well as on the allocation of power in the political system (λ), I now consider the other parameters. Due to the complexity of the problem their effects on the multiplicity threshold, which is a function $\underline{B}(v/w;\eta v^2)$, are studied under additional assumptions.

[24]In Persson and Tabellini (1994) the degree of inequality, hence also the policies implemented, depends only on the fixed underlying distribution of talent. In Alesina and Rodrik (1994) and Bertola (1993) the deterministic nature of the models allows any distribution of initial endowments to persist indefinitely. Incorporating uninsurable idiosyncratic shocks would normally lead to a unique steady-state distribution. Uniqueness also obtains in Perotti (1993) and Saint-Paul and Verdier (1993). In Saint-Paul and Verdier (1992) greater inequality again results in higher taxes and spending on public education, but this stabilizing effect is more than compensated by a divergence in the incentives of the rich and the poor to invest privately in additional human capital. Hence two possible steady-states: high (low) inequality and public education expenditures, with private accumulation by the rich only (by both classes). This generates the same correlations as movement along the convergence path in Saint-Paul and Verdier (1993): greater inequality is associated with increased redistribution and, up to a point, with higher growth. Both correlations are somewhat problematic in view of the empirical evidence.

[25]For instance, Couch and Dunn (1997) find greater mobility –especially in terms of education– in Germany than in the US, and Björklund and Jäntti (1997a) find similar results for Sweden. Rustichini, Ichino and Checchi (1997), on the other hand, find lower mobility in Italy than in the US.

Proposition 8 *Let* $1/\eta = 0$. *The efficiency threshold for multiplicity* $\underline{B}$ *is a decreasing function of* v/w, *with* $\lim_{v/w \to 0} \underline{B} = +\infty$ *and* $\lim_{v/w \to +\infty} \underline{B} = 0$.

This result also appears on **Figure 4**. The intuition is that income uncertainty interacts with market incompleteness in generating efficiency gains from redistribution, as reflected by the term Bv^2 in (16). For a given B, multiplicity therefore occurs when v^2 is large enough compared to the other source of income dispersion, namely the variance w^2 of the shocks which agents learn prior to choosing policy. The concrete implications are important: depending on their source, changes in the economic environment which have similar short–run effects on income inequality will bring about radically *different evolutions of the social contract.* Thus, an increase in the variability of sectoral shocks (similar to v^2) will lead to an expansion of the welfare state, including public education. Conversely, a surge in immigration which results in a greater heterogeneity of the population (similar to an exogenous rise in Δ_t^2 or w^2) may lead to cutbacks or even a large–scale dismantling. In the first case the policy response will mitigate the shock's impact on long–run inequality, in the second it will aggravate it.

If greater benefits of redistribution increase the likelihood of multiplicity, greater distortions will reduce it. These costs are parametrized by the labor supply elasticity $1/\eta$ (just as B or $1 - r$ parametrizes the gains), but only for positive values of τ. Where effort is distorted by regressionary taxes and transfers, on the contrary, a rise in τ represents an efficiency gain which increases with $1/\eta$; recall that $C(\tau)$ is maximized for $\tau = 0$. When regressive fiscal policy is ruled out, as in Section VI below, or more generally for economies that operate in a region where $\tau > 0$, distortions do rise monotonically with $1/\eta$, and the scope for multiple regimes correspondingly declines.

Proposition 9 *Consider an economy with* $B > \underline{B}(v/w; \eta v^2)$ *and such that for some* λ *in* $[\underline{\lambda}, \bar{\lambda}]$ *there are (at least) two stable steady-states with positive rates of progressivity,* $0 < \tau_1 < \tau_2 < 1$. *Then, for any* $1/\eta' < 1/\eta$, $B > \underline{B}(v/w; \eta' v^2)$ *and there is some range* $[\underline{\lambda}', \bar{\lambda}']$ *for which the economy with parameters* (B, v, w, η') *has (at least) two stable steady-states, with at least one corresponding to a positive rate of progressivity.*

B *Growth Implications of Different Social Contracts*

The steady–states corresponding to two different social contracts are clearly not Pareto–rankable. How do they compare in terms of overall growth and average welfare? Recall from (15) that the effect of fiscal progressivity τ_t on short–run growth (i.e., given y_t and Δ_t) is determined by the tradeoff between tax distortions and credit–constraint effects. This remains true for long–run output, as seen by taking limits in (15) with a constant τ. Moreover, any comparison of long–run levels can easily be translated into a ranking of *long-term growth rates*, through knowledge spillovers or public goods complementing private investment.

For instance, let the constant κ in (6) be replaced by $(\kappa_t)^\omega$, where the human or physical capital aggregate

$$\kappa_t \equiv \left(\int_0^i (k_t^i)^\gamma\right)^{1/\gamma} \tag{29}$$

captures external effects of the economic environment on accumulation, other than those of policy. As κ_t does not enter into the determination of the politico–economic equilibrium (Δ_t and τ_t) all previous results remain unchanged, with κ simply replaced by κ_t wherever it appeared. The presence of the spillover only affects the growth rate along each equilibrium trajectory, transforming for instance finite steady–states (when $\omega < 1 - \alpha - \beta\gamma$) into endogenous growth paths (when $\omega = 1 - \alpha - \beta\gamma$).[26] The following results therefore apply equally to short– and to long–run economic growth.

Proposition 10 *A more redistributive social contract*

1) has higher income growth when $1/\eta = \alpha = 0$ *and* $\beta\gamma < 1$;

2) has lower income growth when $1/\eta > 0$, $\alpha = 0$ *and* $\beta\gamma = 1$.

Since both conditions are compatible with Theorem 1's requirement that $1 - \alpha < 2\beta\gamma$ they allow the comparison of steady–states corresponding to different, self–sustaining values of τ. Two interesting empirical scenarios can be accounted for by the model.

Case 1: "Growth–enhancing redistributions". The fact that both equilibria have (endogenously) the same savings rate makes clear that the superior growth performance under the more redistributive social contract arises from a more efficient allocation of investment expenditures, which relaxes the credit constraints of the poor.[27] Tax distortions, meanwhile, remain relatively small. This scenario is particularly relevant for human capital investment (which is considered in more detail in the next section) and public health expenditures; the contrasted paths followed by East Asian and Latin American countries come to mind. More generally, it offers a potential explanation, in a context of endogenous policy choice, for the fact that regression estimates of the effects of social and educational transfers on growth are usually positive and significant (e.g., the references in Section I.A).

Case 2: "Eurosclerosis and the welfare state". In this converse case, the credit–constraint effect is weak compared to the tax distortions. European countries, it is often argued, have chosen a higher degree of social insurance and compression of inequalities than the United States, at the cost of higher unemployment and slower growth.[28] Whether this is viewed as enlightened policy or dismal "eurosclerosis", it begs the question of why voters on both sides of the Atlantic would choose such different points on the

[26] Because it aggregates individual contributions with the same elasticity of substitution as total output, κ_t is heterogeneity–neutral, in the sense that it does not introduce any additional effects of income distribution on growth. It just makes more permanent those due to imperfect credit markets, by reducing or even eliminating the "convergence" term $-(1-\alpha-\beta)\ln y_t$ from (15). Alternative CES aggregates with elasticities other than $1/(1-\gamma)$ could easily be dealt with, as in Bénabou (1996a).

[27] The equality of investment rates is true in the infinite–horizon version of the model as well. A higher τ then implies a lower private savings rate, but also a higher equilibrium rate of consumption taxation and investment subsidization.

[28] See for example Freeman (1995). It is in recent years that European growth has fallen short of US growth, but unemployment has been higher for nearly two decades.

equity–efficiency, or insurance–growth, tradeoff. In our model Europeans choose more redistribution than Americans not because they are intrinsically more risk–averse, but because in more homogenous societies there is less erosion of the consensus over social insurance mechanisms which, ex–ante, would be valued enough to compensate for lesser growth prospects. At any given level of income–sharing a smaller fraction of the population wants to opt out, so the equilibrium rate of redistribution is higher.

Consider finally aggregate welfare, which here is also that of the median voter. Since multiplicity requires some minimal political bias ($\underline{\lambda} > 0$) it is clear that, in each steady–state, a *marginal* increase in redistribution would raise average welfare; see (16) and (21). This corresponds to the requirement, in the stylized model of Section I, of an aggregate gain from the redistributive policy $\widehat{\mathcal{P}}$ relative to $\mathcal{P}$. Comparing steady–states, on the other hand, involves non-marginal variations in τ. When tax distortions are small enough (such as in Case 1 above) the more redistributive steady–state does have higher total welfare, but in general this need not be the case; see equation (16).

VI Explaining International Differences in Education Finance

A *Alternative Systems of School Funding*

Education finance provides perhaps the most compelling case of a redistributive policy with positive efficiency implications. Loan market imperfections are more likely to affect investment in human than in physical capital, which can serve as collateral. The same is true for decreasing returns. The financing of primary and secondary schools also constitutes a striking example of persistent international differences in the extent of redistribution. Japan and most European countries have state-funded public education, which essentially equalizes expenditures across pupils. The United States, in contrast, relies in large part on local financing; because communities are heavily income-segregated, expenditures reflect parental resources to a large extent, making education a quasi-private good. In Bénabou (1996a) I demonstrate how a move from local to state funding of schools can raise the economy's long-run output level, and even its long-term growth rate. Calibrating a model with local funding to US data, Fernandez and Rogerson (1994) find that such a move to state finance could raise steady-state GDP by about 3%. Whether or not one subscribes to this view, differences in national education systems which persist for over a century represent a puzzle, unless one is willing to appeal to intrinsic differences in tastes, technologies or political rights.[29]

To examine this issue, let k_t^i now specifically represent human wealth. The term $(k_t^i)^\alpha$ in (6) captures the transmission of human capital or ability within the family, while the shocks ξ_{t+1}^i represent the unpredictable component of innate talent. Finally, instead of progressive taxes and transfers I now consider *progressive*

[29] In Glomm and Ravikumar (1992) private finance of education leads to a higher long-run growth rate than public funding because it gives individuals better incentives to accumulate human wealth. Bénabou (1996a) shows that taking into account the randomness in children's ability tends to reverse this ranking, as does the presence of economy-wide spillovers. Gradstein and Justman (1993) study similar issues in a model with labor supply, then examine voters' choice among different funding regimes. They obtain a unique equilibrium. Saint-Paul and Verdier's (1992) model yields multiplicity with respect to the level of public education funding, as explained in footnote 24.

subsidies to educational investment. Thus (9) is replaced by $\hat{y}_t^i = y_t^i$, while in (6) parental savings e_t^i are replaced by the net (after–tax) resources invested in their child's education, namely:

$$\hat{e}_t^i = e_t^i \, (\tilde{y}_t / y_t^i)^{\tau_t}, \tag{30}$$

with $\tilde{y}_t$ still determined by (10). Because agents will still choose a common savings rate the government's budget constraint, which is now

$$\int_0^1 (\hat{e}_t^i - e_t^i) \, di = 0, \tag{31}$$

will again be satisfied. The progressivity rate τ_t is the elasticity of the tax price of education with respect to wealth. Given agents' savings behavior, $e_t^i = \nu_t y_t^i$, it also measures the extent to which education is *publicly and equally* provided: thus $\tau = 0$ corresponds to private finance, while $\tau = 1$ is equivalent to a European–style system where universal public education is funded by a proportional income tax.

Proposition 11 *Given a rate of education finance progressivity τ_t, agents in generation t choose a common labor supply and savings rate: $e_t^i / \hat{y}_t^i = \rho\beta/(1 - \rho + \rho\beta) \equiv \mathfrak{s}$, as before, while:*

$$l_t = \left(\frac{\delta}{\eta}\right)^{1/\eta} \left(\frac{1 - \rho + \rho\beta(1 - \tau_t)}{1 - \rho}\right)^{1/\eta}.$$

The effort distortion is smaller than in the taxation case because the policy bears only on the part of their income which agents reinvest, and not on that which they consume.[30] Up to that difference in $l_t = l(\tau_t)$ the implied dynamics of human wealth are identical to (11)–(15), leading to the following result.

Proposition 12 *Given a rate of education finance progressivity τ_t, agent i's intertemporal welfare is:*

$$U_t^i = \bar{u}_t + A(\tau_t)(\ln k_t^i - m_t) + C(\tau_t) - \rho\beta(1 - \tau_t)^2 \gamma^2 \Delta_t^2 + B(\tau_t) v^2/2 \tag{32}$$

where $\bar{u}_t$ is independent of the policy τ_t and

$$A(\tau) \equiv \rho\alpha + (1 - \rho + \rho\beta(1 - \tau))\gamma \tag{33}$$

$$C(\tau) \equiv (1 - \rho)(\delta \ln l(\tau) - l(\tau)^\eta) + \rho\beta\delta \ln l(\tau) \tag{34}$$

$$B(\tau) \equiv -(1 - \rho + \rho\beta(1 - \tau)^2) + r(1 - \rho + \rho\beta(1 - \tau))^2. \tag{35}$$

This utility function resembles closely the one which arose under fiscal redistribution, with two fairly minor differences. On the cost side, $C(\tau)$ is the same function of effort $l(\tau)$ as before but $l(\tau)$ is now different. In particular, distortions remain bounded, so even though U_t^i is strictly concave in τ_t it may

[30] In the infinite–horizon version of the model private savings decision are also less distorted than with income taxes and transfers. But, once again, voters all agree on a Pareto–improving mix of consumption taxes and investment subsidies which restore everyone's investment rate to $\mathfrak{s}$.

be maximized at $\tau = 1$ for a poor enough agent. The other difference occurs in the benefits term. For $\rho = 1$, $B(\tau) = -\beta(1 - r\beta)(1 - \tau)^2$ as in (16), and with the same economic interpretation in terms of insurance and reallocation of liquidity–constrained investments. But in general $B(\tau)$ is not proportional to $-(1-\tau)^2$ any more, and when $r > 0$ it is not even monotonic in $1 - \tau$. Thus efficiency may require less than full equalization of educational resources ($B'(0) > 0 > B'(1)$), even when abstracting from deadweight losses and differences in family environments.[31]

Given the closely related form of preferences and distributional dynamics, the political equilibrium of the education model is very similar to that of the tax model, both in the short and in the long run. However, the formal analysis is rendered more difficult by the possibility of corner solutions ($\tau_t = 1$) and the different form of $B(\tau)$. In any case, deriving an exact analogue to Theorem 1 would be repetitious. I shall focus instead on a simpler case which yields explicit results for the steady–states and their comparative statics, bringing out the underlying intuitions most clearly.

B *Sustainability of Centralized and Decentralized Education Systems*

From here on, I restrict policy to two options. Under laissez-faire or *decentralized funding,* $\tau = 0$, education expenditures are determined by family or community resources; the two are essentially equivalent when communities are stratified by socioeconomic status. *Public funding* of education corresponds to $\tau = 1$ or more generally to $\tau = \bar{\tau}$, where $0 < \bar{\tau} \leq 1$. Given an initial income distribution of human capital Δ_t this system is adopted if $U_t^i(\bar{\tau}) > U_t^i(0)$ for at least a fraction $p^* \equiv \Phi(\lambda)$ of the population. Setting $\ln k_t^i - m_t = \lambda \Delta_t$ in (32), this means:

$$\lambda < \left((B(\bar{\tau}) - B(0)) \left(\frac{v^2}{2} \right) - (C(0) - C(\bar{\tau})) \right) \frac{1/\rho\beta\bar{\tau}}{\gamma\Delta_t} + \left(\frac{2 - \bar{\tau}}{2} \right) \gamma\Delta_t. \quad (36)$$

Intuitively, the *political influence of wealth* must not be too large compared to the *aggregate welfare benefit* of redistributive education finance (relative to laissez–faire), and given the existing level of inequality. Preexisting inequality raises the hurdle which the policy must overcome, as reflected by the term $1/\gamma\Delta_t$ multiplying the net welfare benefit. This effect tends to make adoption of state finance more difficult where it has not previously been in place (e.g., the United States), because of the greater human capital disparities which result over time from a decentralized system. Conversely, the term in $\gamma\Delta_t$ incorporates the combined effects of *skewness* and credit–constraints, which always generate a higher demand for redistribution. As a result of these two offsetting forces the right-hand side of (36) has the usual U-shape in Δ_t, and is in fact very similar to (3) in the stylized model of Section I. To focus on the long–run, let us now replace Δ_t by the asymptotic variance under public provision (partial or complete), namely $\Delta_\infty = D(\bar{\tau})$ given by (27).

[31] This is due to the fact that adults face a tradeoff between insuring their children's human capital k_{t+1}^i against a low realization of their own productivity z_t^i, and smoothing the intertemporal profile of the two goods they care about, namely c_t^i and k_{t+1}^i. A high τ_t helps with the first objective but hurts with the second, given that c_t^i is not insulated from the shock z_t^i. The result is not specific to the myopic bequest motive adopted in this paper. It also occurs with infinitely–lived agents, given a low enough degree of risk–aversion; see Bénabou (1997).

Redistributive public funding of education is thus a steady-state when:

$$\lambda < \left((B(\bar{\tau}) - B(0))\left(\frac{v^2}{2}\right) - (C(0) - C(\bar{\tau}))\right)\left(\frac{1/\rho\beta\bar{\tau}}{\gamma D(\bar{\tau})}\right) + \left(\frac{2-\bar{\tau}}{2}\right)\gamma D(\bar{\tau}) \equiv \bar{\lambda}. \tag{37}$$

Conversely, private or local financing is a steady-state, with inequality $\Delta_\infty = D(0)$, when:

$$\lambda > \left((B(\bar{\tau}) - B(0))\left(\frac{v^2}{2}\right) - (C(0) - C(\bar{\tau}))\right)\left(\frac{1/\rho\beta\bar{\tau}}{\gamma D(0)}\right) + \left(\frac{2-\bar{\tau}}{2}\right)\gamma D(0) \equiv \underline{\lambda}. \tag{38}$$

The two regimes can coexist if and only if $\underline{\lambda} < \bar{\lambda}$, which occurs when the differential gain

$$B(\bar{\tau}) - B(0) = \rho\beta\bar{\tau}\left[2 - \bar{\tau} - r(2 - 2\rho + \rho\beta(2 - \bar{\tau}))\right] \tag{39}$$

exceeds the differential cost

$$C(0) - C(\bar{\tau}) = \frac{\delta}{\eta}\left[(1 - \rho + \rho\beta)\ln\left(\frac{1 - \rho + \rho\beta}{1 - \rho + \rho\beta(1 - \bar{\tau})}\right) - \rho\beta\bar{\tau}\right] \tag{40}$$

by a sufficient amount, specified below. It is easily seen that the distortion $C(0) - C(\bar{\tau})$ is positive and increasing in $1/\eta$. I will assume that $B(\bar{\tau}) > B(0)$, so that there is an actual gain to be compared to this cost; sufficient conditions include $1 - r \geq 1$, or $\rho(2 + \beta) \geq 1$, or that $\bar{\tau}$ not be too large. Finally, let

$$G(\bar{\tau}) \equiv \rho\beta\bar{\tau}(2 - \bar{\tau})\gamma^2\left(\sqrt{\frac{1 + \beta^2 v^2/w^2}{1 - (\alpha + \beta\gamma)^2}}\right)\left(\sqrt{\frac{1 + \beta^2(1 - \bar{\tau})^2 v^2/w^2}{1 - (\alpha + \beta\gamma(1 - \bar{\tau}))^2}}\right). \tag{41}$$

Theorem 2 *If the gain in ex–ante welfare from progressive public financing of education, relative to private or decentralized financing, is large enough, namely*

$$B(\bar{\tau}) - B(0) \geq (C(0) - C(\bar{\tau}))\left(\frac{2}{v^2}\right) + G(\bar{\tau})\left(\frac{w^2}{v^2}\right) \equiv \underline{B}(v/w, \eta v^2),$$

there exist $0 < \underline{\lambda} < \bar{\lambda}$ *such that:*

1) for $\lambda \in [\underline{\lambda}, \bar{\lambda}]$ *both public and decentralized school funding are stable steady-states. The first regime* ($\tau = \bar{\tau}$) *has lower inequality and greater social mobility than second* ($\tau = 0$).

2) for $\lambda < \underline{\lambda}$ *public funding is the only steady-state, while for* $\lambda > \bar{\lambda}$ *it is decentralized funding.*

These results demonstrate how such fundamentally different systems of school finance as those of the United States and Western European countries can be self–perpetuating, once arisen from historical circumstances.[32] As to which type of education system leads to faster growth, this depends once again on the tradeoff between the positive impact of redistributive education finance on wealth constraints and its

[32] Generalizing Theorem 2 to any policy pair $\{\underline{\tau}, \bar{\tau}\}$ with $\underline{\tau} < \bar{\tau} \leq 1$ is straightforward. Like all the results in the paper, it can also be extended to allow for optimal time allocation by parents between work and child-rearing, and for a production structure where families are linked through complementarities between different types of workers, or through economy-wide spillovers in production or accumulation.

adverse effect on incentives: Proposition 10 applies unchanged. Because the distortions are less severe than with progressive taxes and transfers, however, the likelihood that state intervention enhances growth is now greater.

Theorem 2 also shows how the efficiency threshold for multiplicity varies with the costs and benefits of redistributive school finance, as well as different sources of income inequality: $\underline{B}(v/w, \eta v^2)$ increases with $1/\eta$ and w^2, while it decreases with v^2. Equivalently, there is a lower bound for risk–aversion with the same properties. These results have the same interpretations as Propositions 8–9, and can again be represented on **Figure 4**, with $B \equiv B(\bar{\tau}) - B(0)$ now on the vertical axis. In particular, a more elastic labor supply shifts the $\underline{B}$ locus from the solid to the dotted curve. One difference with the case of a continuous policy variable τ is that for $B < \underline{B}$ there might be no steady–state, as $\bar{\lambda}$ is then less than $\underline{\lambda}$. Equations (37)–(38) show that for $\lambda \notin [\bar{\lambda}, \underline{\lambda}]$ there is a unique (and intuitive) steady–state, but for $\lambda \in [\bar{\lambda}, \underline{\lambda}]$ there is none. The economy can instead be shown to cycle between the two regimes, as in Gradstein and Justman (1997), with whose model this one shares important elements (theirs does not generate multiplicity, however). Actual instances of such cycling are hard to come by, and indeed Theorem 1 indicates that non-existence is an artefact of restricting policy to discrete values.[33]

I now return to the central issue, namely the coexistence of multiple regimes. The *range of political systems* which allow this indeterminacy is illustrated on **Figure 5**.

Proposition 13 *The scope for the political system to generate multiple equilibria increases with the efficiency benefits of redistribution and decreases with their efficiency costs: as* $B \equiv B(\bar{\tau}) - B(0)$ *increases, due to greater risk–aversion or more decreasing returns, both* $\underline{\lambda}$ *and* $\bar{\lambda}$ *rise but the interval* $[\underline{\lambda}, \bar{\lambda}]$ *widens. A higher labor supply elasticity has the opposite effects.*

VII Other Applications

A *Concern for Equity*

Apart from social insurance and capital market imperfections, one of the main reasons for income redistribution is simply that most people dislike living in a society which is too unequal. This may be due to pure altruism or to the fact that inequality generates social tensions, crime, and similar problems which have direct costs. To capture these ideas one can simply augment (7) as follows:

$$\ln \hat{V}_t^i = \ln V_t^i - (\mathcal{A}/2) \left[(1-\rho)\, var_{j \in I}[\ln c_t^j - (l_t^j)^\eta] + \rho\, var_{j \in I}[\ln k_{t+1}^j] \right]. \tag{7'}$$

[33] More specifically, with a continuous τ the analogue of equation (28) for education funding is easily shown to always have at least one stable steady–state. Conversely, an analogue of Theorem 2 can be derived for progressive taxation, with the progressivity rate τ restricted to $\{0, \bar{\tau}\}$. This is interesting because it shows that the non–linear redistributive schemes used in the paper, namely (9) and (30), are not driving the results: for the polar values $\tau = 0$ and $\tau = \bar{\tau} \equiv 1$ (and focusing on the case where $1/\eta = 0$), such a geometric scheme and the standard linear one coincide exactly. The model of Gradstein and Justman (1996) corresponds to the case where $1 - r = 1$ (time–separable, logarithmic utility), the disutility of effort $-l^\eta$ is replaced by $\ln(1-l)$, $\tau \in \{0, 1\}$ (pure public or private system), $\lambda = 0$ (pure democracy) and $v^2 = 0$ (no uncertainty at the time of voting). From Theorem 2 we see that the restrictions $\lambda = 0$ and $v^2 = 0$ are what precludes multiple equilibria.

The coefficient $\mathcal{A}$ represents everyone's aversion to disparities in felicity, measured by the cross-sectional variances of consumption (including leisure) and bequeathed wealth. For a given tax rate this altruistic motive does not affect savings or effort decisions, so the economy's laws of motion remain unchanged and the political equilibrium quite similar. For simplicity, let us focus again on the endowment economy, $\beta = 0$, where the altruism term is just $(\mathcal{A}/2)\left[(1-\rho)(1-\tau)^2(\gamma^2\Delta_t^2 + v^2) + \rho(\alpha^2\Delta_t^2 + w^2)\right]$. Individuals' ideal tax rates τ_t^i and the equilibrium policy τ_t are then the same as before, except that the ubiquitous $\gamma^2\Delta_t^2 + Bv^2$ is replaced by $(1+\mathcal{A})\gamma^2\Delta_t^2 + (B+\mathcal{A})v^2$: inequality aversion is equivalent to a simultaneous increase in risk–aversion and in the concavity of the aggregate welfare function (previously logarithmic). For instance, when $1/\eta = 0$ the steady–state tax rate becomes

$$\frac{1}{1-\tau_\infty} = \frac{w}{\lambda}\left[(1+\mathcal{A})\left(\frac{\gamma}{\sqrt{1-\alpha^2}}\right) + (1+\mathcal{A}-(1-\rho)r)\left(\frac{v}{w}\right)^2\left(\frac{\sqrt{1-\alpha^2}}{\gamma}\right)\right], \qquad (24')$$

instead of (25). Naturally, progressivity increases with $\mathcal{A}$ and decreases with λ. If one observed two countries, the first with low pretax inequality yet extensive redistribution, the other with high inequality yet limited redistribution, one would indeed be tempted to conclude that the citizens of the first country were more altruistic, or their poor better organized politically. In fact it could be that preferences are identical and political institutions equivalent, but that the second country's more unequal distribution reflects a more persistent income process. This could be due to exogenous factors, as with α here, or be endogenous, as in the case of multiple steady–states.

B *The Mix of Public Goods*

Some public services such as the legal system, the protection of property, prisons, etc., benefit citizens largely in proportion to their levels of wealth or investment. Others, such as public infrastructure or education, have more uniformly or even progressively distributed benefits. Deininger and Squire (1995) find in cross–country regressions that public investment affects the growth of income equally for all quintiles, while public schooling expenditures benefit the bottom 40% most, the middle class to a lesser extent, and the rich not at all. Let us therefore think of the government as choosing, at the margin, from among a menu of public goods which are complementary to private capital accumulation: if g_t is spent on a public good with characteristics $(\kappa, \alpha, \beta, \gamma)$, the private sector faces the following technology:

$$k_{t+1}^i = \kappa \cdot \xi_{t+1}^i \cdot (k_t^i)^\alpha\,(e_t^i)^\beta (g_t)^\gamma. \qquad (42)$$

The first type of good corresponds to a high value of $\alpha + \beta$ and a low value of γ; the second type has low $\alpha + \beta$ and high γ. In the simple case where the policy decision consists of choosing a single public good (broadly defined) from such a menu, the problem is analogous to the earlier ones. As a result, countries can sustain different choices without any underlying differences in tastes. In reality, many public goods are provided simultaneously and the debate is over the appropriate mix.[34] Nonetheless, the same intuitions should carry over to this more complicated problem.

C *The Socioeconomic Structure of Cities*

The presence in human capital accumulation of peer effects, role models and other neighborhood interactions implies that residential stratification increases the persistence of income disparities across families (e.g., Bénabou (1993), Durlauf (1996a)). Urban ghettos are but the most extreme example of this phenomenon, which is the subject of a large empirical literature.[35] The degree of socioeconomic segregation (often correlated with ethnic segregation) varies a lot from one city to another. So do the level of political support for, and resources committed to, housing, schooling and infrastructure policies aimed at reducing or limiting this polarization. To see how neighborhood effects can give rise to same multiplicity of politico–economic regimes as fiscal and education policy, consider the general law of motion for human capital studied in Bénabou (1996a):

$$h_{t+1}^i = F(\xi_{t+1}^i, h_t^i, L_t^i, H_t), \tag{43}$$

where L_t^i and H_t are human capital averages capturing respectively local externalities (e.g., peer effects) and economy-wide interactions (e.g., production complementarities, knowledge spillovers, etc.) which affect the accumulation of human capital. Thus L_t^i is computed over the population of family i' s neighbors or peers, while H_t reflects the general population's distribution of characteristics. In a metropolitan area where families have sorted into socioeconomically homogenous communities or "clubs", $L_t^i = h_t^i$ for every i, so $h_{t+1}^i = F(\xi_{t+1}^i, h_t^i, h_t^i, H_t)$. Conversely, under perfect integration each community is a representative sample of the population at large, so everyone shares in the same level of local externality or public good, $L_t^i = L_t$; hence $h_{t+1}^i = F(\xi_{t+1}^i, h_t^i, L_t, H_t)$. Looking at the implied serial correlations for human wealth, the formal similarity with the cases $\tau = 0$ and $\tau = 1$ in the present model is quite apparent, especially when F is Cobb–Douglas. Moreover, I show in Bénabou (1993), (1996a) that the long-run impact of segregation on aggregate surplus is likely to be negative; put in another way, equilibrium segregation tends to be inefficiently high. The two conditions identified here for multiple steady-states are therefore satisfied: there are redistributive policies (broadly speaking, subsidizing integration) which can reduce persistence and increase ex–ante welfare. Yet a high degree of segregation can be self-perpetuating, precisely due to the increased inequality which it induces.[36] Theories which ascribe international differences to a multiplicity of equilibria always face the challenge of identifying the original source of these diverging paths. In the

[34] Formally, $k_{t+1}^i = \int \kappa \cdot \xi_{t+1}^i \cdot (k_t^i)^\alpha (e_t^i)^\beta (g_t(\kappa, \alpha, \beta\gamma))^\gamma \, d\kappa \, d\alpha \, d\beta \, d\gamma$, with $\int g_t(\kappa, \alpha, \beta, \gamma) \, d\kappa \, d\alpha \, d\beta \, d\gamma$ equal to government revenue. Benoît and Osborne (1994) study a related issue in the context of crime, examining what determines a country's chosen mix between the severity of punishment and preventive social expenditures.

[35] Recent references include Borjas (1995), Cooper, Durlauf and Johnson (1994) and Topa (1995). Jencks and Mayer (1990) provide an extensive survey of earlier empirical studies, and Manski (1993) a critical discussion of methodology. Indeed the identification of these social spillovers remains the subject of some controversy; see for instance Oates and Schwab (1992).

[36] An alternative source of multiplicity is the one explored by Durlauf (1996b), where it is only when income disparities are not too large that rich families are willing to share with poorer ones the fixed costs of running a community and its schools.

case of segregation, the historical circumstances which set the United States and a few other countries on a permanently different course do not seem hard to pinpoint.[37]

VIII Conclusion

This paper has proposed a theory of the social contract which explains how countries with similar preferences and technologies, as well as equally democratic political systems, can nonetheless make very different choices with respect to fiscal progressivity, social insurance, and education finance. The answer involves two mechanisms which arise naturally in the absence of complete insurance and credit markets. First, redistributions which would increase ex–ante welfare command less political support in an unequal society than in a more homogenous one. A lower rate of redistribution, in turn, increases inequality of future incomes due to wealth constraints on investment in human or physical capital. This leads to two stable steady–states, the archetypes for which could be the United States and Sweden: one with high inequality yet low redistribution, the other with the reverse configuration. These two societies are not Pareto rankable, and which one has faster income growth depends on the balance between tax distortions to effort and the greater productivity of investment resources reallocated to more severely credit–constrained agents.

These ideas were formalized in a stochastic growth model with missing markets, progressive fiscal or education policy, and a more realistic political system than the standard median voter setup. The resulting distributional dynamics remain simple enough to allow a number of extensions. In Bénabou (1997) I develop and calibrate an infinite–horizon version of the incomplete markets model, then use it to quantify the effects of fiscal and educational redistribution on growth, risk, and welfare. Alternatively, one could enrich the model's representation of the production sector to study the interactions between technological progress, the education system, and the political sphere. Yet another interesting problem is to endogenize the kind of wealth–biased political mechanism used here, where those with more resources command more influence; Bourguignon and Verdier (1997) and Rodriguez (1997) are recent examples of such models. Finally, the original question of why the social contract differs across countries, and whether these choices are sustainable in the long run, remains an important topic for further research.

[37] In addition to the obvious legacy of racial segregation, the post-war period saw a combination of sudden technological change (the spread of the automobile) and federal government policies (subsidies to infrastructure and home ownership) which made extensive suburbanization viable.

Appendix

Proof of Proposition 1 Once agent i knows his productivity z_t^i, hence also his pre– and post–tax incomes $y_t^i = z_t^i (k_t^i)^\gamma (l_t^i)^\delta$ and $\hat{y}_t^i = (y_t^i)^{1-\tau_t}(\tilde{y}_t)$, his decision problem takes the form:

$$\begin{aligned}
\text{(A.1)} \quad \ln V_t^i &= \max_{l,\nu} \left\{(1-\rho)[\ln((1-\nu)\hat{y}_t^i) - l^\eta] + (\rho/r')\ln E_t[(k_{t+1}^i)^{r'}] \,|\, k_{t+1}^i = \kappa\xi_{t+1}^i (k_t^i)^\alpha (\nu\hat{y}_t^i)^\beta\right\} \\
&= \max_\nu \{(1-\rho)\ln(1-\nu) + \rho\beta\ln\nu\} + \max_l \{-(1-\rho)l^\eta + (1-\rho+\rho\beta)(1-\tau_t)\delta\ln l\} \\
&\quad +\rho(\ln\kappa - (1-r')w^2/2) + [\rho\alpha + (1-\rho+\rho\beta)\gamma(1-\tau_t)]\ln k_t^i \\
&\quad +(1-\rho+\rho\beta)\left[(1-\tau_t)\ln z_t^i + \tau_t \ln\tilde{y}_t\right],
\end{aligned}$$

where $\nu_t^i \equiv e_t^i/\hat{y}_t^i$ is the savings rate. Strict concavity in ν and l is easily verified, and the first–order conditions directly yield the stated results. ‖

Proof of Proposition 2 Let us start by computing the redistributive scheme's cutoff level $\tilde{y}_t$. If $\ln k_t^i \sim \mathcal{N}(m_t, \Delta_t^2)$, then (4) implies that aggregate income is:

$$\text{(A.2)} \qquad \ln y_t = \ln E[z_t^i] + \ln E[(k_t^i)^\gamma] + \delta\ln l_t = \gamma m_t + \delta \ln l_t + \gamma^2\Delta_t^2/2.$$

The level of transfers $\tilde{y}_t$ which satisfies the government budget constraint (10) is then given by:

$$\begin{aligned}
\tau_t \ln\tilde{y}_t &= \ln y_t - \delta(1-\tau_t)\ln l_t - \ln E\left[(z_t^i)^{1-\tau_t}\right] - \ln E\left[(k_t^i)^{\gamma(1-\tau_t)}\right] \\
&= \ln y_t - \delta(1-\tau_t)\ln l_t + \left((1-\tau_t) - (1-\tau_t)^2\right)v^2/2 - \left((1-\tau_t)\gamma m_t + (1-\tau_t)^2\gamma^2{\Delta_t}^2/2\right)
\end{aligned}$$

since the z_t^i's and k_t^i's are independent. Thus:

$$\text{(A.3)} \qquad \tau_t\ln\tilde{y}_t = \gamma\tau_t m_t + \delta\tau_t \ln l_t + \tau_t(2-\tau_t)\gamma^2{\Delta_t}^2/2 + \tau_t(1-\tau_t)v^2/2,$$

as claimed in (12). Equation (14) follows from taking variances in (11), while (13) follows from taking averages with $\tau_t \ln \tilde{y}_t$ replaced by (A.3). Finally, combining both laws of motions with (A.2) yields:

$$\begin{aligned}
\ln y_{t+1} &= \delta\ln l_{t+1} + \gamma\left[(\alpha+\beta\gamma)\,m_t + \beta\delta\ln l_t + \beta\,\tau_t\,(2-\tau_t)\,(\gamma^2\Delta_t^2 + v^2)/2 + \ln(\kappa\mathfrak{s}^\beta) - (w^2+\beta v^2)/2\right] \\
&\quad +\gamma^2\left[(\alpha+\beta\gamma(1-\tau_t))^2\,\Delta_t^2 + \beta^2(1-\tau_t)^2\,v^2 + w^2\right]/2 \\
&= \gamma(\ln\kappa + \beta\ln\mathfrak{s} - (1-\gamma)w^2/2) + \delta(\ln l_{t+1} - \alpha\ln l_t) + (\alpha+\beta\gamma)[\gamma m_t + \delta l_t + \gamma^2\Delta_t^2/2] \\
&\quad -\beta\gamma[1-\tau_t(2-\tau_t) - \beta\gamma(1-\tau_t)^2]v^2/2 - [\alpha+\beta\gamma - (\alpha+\beta\gamma(1-\tau_t))^2 - \beta\gamma\tau_t(2-\tau_t)]\gamma^2\Delta_t^2/2 \\
&= \ln\tilde{\kappa} + \delta(\ln l_{t+1} - \alpha\ln l_t) + (\alpha+\beta\gamma)\ln y_t - \beta\gamma(1-\beta\gamma)(1-\tau_t)^2v^2/2 - \mathfrak{L}_\Delta(\tau_t)\gamma^2\Delta_t^2/2,
\end{aligned}$$

hence the result, given the definitions of $\ln\tilde{\kappa}$, $\mathfrak{L}_v(\tau)$ and $\mathfrak{L}_\Delta(\tau)$. ‖

Proof of Proposition 3 Substituting the optimal $l_t^i = l_t$ and $\nu_t^i = \mathfrak{s}$ into (A.1), and denoting $\ln \kappa' \equiv \ln \kappa - (1-r')w^2/2$, yields:

$$\text{(A.4)} \qquad \begin{aligned} \ln V_t^i &= \rho \ln \kappa' + (1-\rho)\ln(1-\mathfrak{s}) + \rho\beta \ln \mathfrak{s} - (1-\rho)l_t^\eta + (1-\rho+\rho\beta)(1-\tau_t)\delta \ln l_t \\ &+ [\rho\alpha + (1-\rho+\rho\beta)\gamma(1-\tau_t)] \ln k_t^i + (1-\rho+\rho\beta)\left[(1-\tau_t)\ln z_t^i + \tau_t \ln \tilde{y}_t\right]. \end{aligned}$$

Thus *conditional* on k_t^i, $\ln V_t^i$ is normally distributed, with variance $(1-\rho+\rho\beta)^2(1-\tau_t)^2 v^2$. This implies:

$$\text{(A.5)} \qquad \ln U_t^i \equiv \frac{1}{r} \ln E\left[\left(V_t^i\right)^r \mid k_t^i\right] = E\left[\ln V_t^i \mid k_t^i\right] + r(1-\rho+\rho\beta)^2(1-\tau_t)^2 v^2.$$

Therefore, to obtain $\ln U_t^i$ one simply needs to replace in (A.4) the term in $\ln z_t^i$ by $-(1-\rho+\rho\beta)(1-\tau_t)\left[1 - r(1-\rho+\rho\beta)(1-\tau_t)\right] v^2/2$. Finally, substituting in the value of $\tau_t \ln \tilde{y}_t$ from (A.3) yields the claimed result, with

$$\text{(A.6)} \quad \bar{u}_t = \ln[(1-\mathfrak{s})^{1-\rho}\mathfrak{s}^{\rho\beta}] + \rho(\ln\kappa - (1-r')w^2/2) + (\rho\alpha + (1-\rho+\rho\beta)\gamma)m_t + (1-\rho+\rho\beta)\gamma^2\Delta_t^2/2. \ \|$$

Proof of Proposition 4 We can rewrite (21) as a second–degree polynomial in $x \equiv 1-\tau$,

$$\text{(A.7)} \qquad P(x) \equiv x^2(\gamma^2\Delta_t^2 + Bv^2) - (\gamma(\ln k_t^i - m_t) - \delta/\eta)x - \delta/\eta = 0,$$

which always has two real roots of opposite sign. Since $\tau_t^i \leq 1$ necessarily, the relevant root is $x_t^i = 1 - \tau_t^i > 0$ and at that point $P'(x_t^i) > 0$. It is easy to compute τ_t^i explicitly, but for comparative statics I shall simply use the implicit function theorem. Since $\partial P/\partial(Bv^2) > 0 > \partial P/\partial \ln k_t^i$ and $P'(x_t^i) > 0$, the theorem implies that $\partial x_t^i/\partial \ln k_t^i > 0 > \partial x_t^i/\partial(Bv^2)$. Similarly, $\partial x_t^i/\partial \Delta_t < 0$. Finally, $-\partial P/\partial(1/\eta) = \delta(1-x)$, so $\partial x_t^i/\partial(1/\eta)$ has the sign of $1 - x_t^i = \tau_t^i$, or equivalently $\partial \tau_t^i/\partial(1/\eta)$ has the sign of $-\tau_t^i$. Finally, $\tau_t^i > 0$ if and only if $x_t^i < 1$, which means $P(1) > 0$, or $\gamma(\ln k_t^i - m_t) < \gamma^2\Delta_t^2 + Bv^2$. $\|$

Proof of Proposition 5 Let us index agents by their log–wealth, $\theta^i \equiv \ln k^i$, and denote its c.d.f. by $F(\theta)$. For any weighting scheme $\omega^i = g(\theta^i)$ the proportion of votes cast by agents with $\theta^i \leq \theta$ (more generally, their total political weight) is $G(\theta)/G(\infty)$ where $G(\theta) \equiv \int_{-\infty}^{\theta} g(z)\, dF(z)$. For an ordinal scheme, $g(z) = \omega(F(z))$, so $G(\theta) = \int_0^{F(\theta)} \omega(p)\, dp$. Given the single-crossing condition satisfied by preferences, the agent with log-wealth θ^* and rank $p^* = F(\theta^*)$ defined by $G(\theta^*)/G(\infty) = 1/2$ is clearly pivotal (see Gans and Smart (1996)). In the log-normal case $F(\theta) = \Phi((\theta - m)/\Delta)$, so if we define $\lambda \equiv \Phi^{-1}(p^*)$ then $\theta^* = m + \lambda\Delta$. This wealth level is the same as if the whole distribution of $\theta = \ln k$ were shifted up by $\lambda\Delta$. Let us now turn to the cardinal scheme $g(\theta) = e^{\lambda\theta}$. Simple derivations show that

$$\text{(A.8)} \qquad G(\theta) \equiv \int_{-\infty}^{\theta} e^{\lambda z}\, dF(z) = e^{\lambda(m+\lambda\Delta^2/2)} \cdot F(\theta - \lambda\Delta^2),$$

hence $G(\theta)/G(\infty) = F(\theta - \lambda\Delta^2)$. The whole distribution is thus shifted by $\lambda\Delta^2$, and so is the solution to $G(\theta^*)/G(\infty) = 1/2$. $\|$

Proof of Proposition 6 The equilibrium tax rate is the one preferred by the agent with $\ln k_t^i - m_t = \lambda\Delta_t$, so claims (1) and (2) follow directly from the properties of τ_t^i established in Proposition 4. To establish the third claim let us rewrite that agent's first-order condition, $\partial U_t^i/\partial\tau = 0$, in terms of $x_t \equiv 1 - \tau_t$. By (A.7), x_t is the unique positive root of the polynomial:

$$Q(x) \equiv x^2(\gamma^2\Delta_t^2 + Bv^2) - (\gamma\lambda\Delta_t - \delta/\eta)x - \delta/\eta = 0. \tag{A.9}$$

Now, $Q'(x_t) > 0$ and $\partial Q/\partial\Delta_t = 2x^2\gamma^2\Delta_t - \gamma\lambda x$, so $\partial x_t/\partial\Delta_t$ has the sign of $\lambda - 2\gamma x_t\Delta_t$. Therefore $\partial\tau_t/\partial\Delta_t > 0$ if and only if $x_t > \lambda/2\gamma\Delta_t$. For $\lambda \leq 0$ this is always true, hence τ_t is strictly increasing in Δ_t. For $\lambda > 0$, on the other hand, the condition is equivalent to:

$$\begin{aligned} {\Delta_t}^2 Q(\lambda/2\gamma\Delta_t) &= (\lambda/2\gamma)^2(\gamma^2\Delta_t^2 + Bv^2) - (\gamma\lambda\Delta_t - \delta/\eta)(\lambda\Delta_t/2\gamma) - (\delta/\eta)\Delta_t^2 < 0 \Leftrightarrow \\ R(\Delta_t) &\equiv -\left(\lambda + 4\delta/\eta\lambda\right)\Delta_t^2 + 2(\delta/\eta\gamma)\Delta_t + \lambda Bv^2/\gamma^2 < 0. \end{aligned}$$

This second–degree polynomial in Δ_t has two real roots of opposite sign. Denoting $\underline{\Delta}$ the positive one (with, clearly, $\partial\underline{\Delta}/\partial(Bv^2) > 0$), we conclude that $\partial\tau_t/\partial\Delta_t > 0$ if and only if $\Delta_t > \underline{\Delta}$. Thus τ_t is indeed U–shaped in Δ_t, and its limiting values at $\Delta_t = 0$ and as $\Delta_t \to \infty$ are readily obtained from (A.9). Finally, recall that $\tau_t^i > 0$ if and only $\gamma(\ln k_t^i - m_t) < \gamma^2\Delta_t^2 + Bv^2$; therefore

$$\tau_t > 0 \Leftrightarrow \gamma^2\Delta_t^2 - \gamma\lambda\Delta_t + Bv^2 > 0. \tag{A.10}$$

When $Bv^2 > \lambda^2/4$ the condition always holds, so $\tau_t > 0$. When $Bv^2 < \lambda^2/4$ there is a range $[\Delta', \acute{\Delta}''] \subset (0, 2\lambda)$ such that $\tau_t < 0$ if and only if Δ_t is in that interval. ‖

Proof of Theorem 1 Let us start with a lemma characterizing stable and unstable steady–states.

Lemma 1 *A stable steady–state is a point τ^* where the function $f(\tau)$ cuts the horizontal λ from above, or equivalently a point (Δ^*, τ^*) where the curve $\Delta = D(\tau)$ cuts the curve $\tau = T(\Delta)$ from above. An unstable steady state corresponds in each case to an intersection from below.*

Proof: The dynamical system (26) reduces to a one–dimensional recursion: $\Delta_{t+1} = \mathfrak{D}(\Delta_t, T(\Delta_t))$. A fixed point $\Delta^* = \mathfrak{D}(\Delta^*, T(\Delta^*))$ is stable if and only if $(d\mathfrak{D}(\Delta_t, T(\Delta_t))/d\Delta_t)_{\Delta=\Delta^*} < 1$, or:

$$\mathfrak{D}_1(\Delta^*, \tau^*) + T'(\Delta^*)\mathfrak{D}_2(\Delta^*, \tau^*) < 1, \tag{A.11}$$

where $\tau^* \equiv T(\Delta^*)$ and a j–subscript denotes a j-th partial derivative. Now, the function $\tau = T(\Delta)$ is implicitly given by the first–order condition (24), or:

$$\psi(\tau, \Delta) \equiv (1 - \tau)\left(\frac{\gamma^2\Delta^2 + Bv^2}{\gamma}\right) - \frac{\delta}{\eta\gamma}\left(\frac{\tau_t}{1 - \tau_t}\right) - \lambda\Delta = 0, \tag{A.12}$$

therefore $T'(\tau) = -(\psi_2/\psi_1)(T(\Delta), \Delta)$ and the stability condition becomes:

$$\text{(A.13)} \qquad \mathfrak{D}_1(\Delta^*, \tau^*) - \left(\frac{\psi_2(\tau^*, \Delta^*)}{\psi_1(\tau^*, \Delta^*)}\right) \mathfrak{D}_2(\Delta^*, \tau^*) < 1.$$

Next, recall from (28) that f is defined by: $f(\tau) - \lambda = \psi(\tau, D(\tau))/D(\tau)$, so that $f' < 0$ if and only if $\psi_1 + (\psi_2 - \psi/D)D' < 0$. Finally, $D(\tau)$ is defined by (27) as the (unique) fixed point solution to $D(\tau) = \mathfrak{D}(D(\tau), \tau)$; therefore: $D' = \mathfrak{D}_2/(1 - \mathfrak{D}_1)$. Substituting D' and using the fact that $\psi(\tau^*, \Delta^*) = 0$ at a steady–state, this becomes:

$$f'(\tau^*) < 0 \;\Leftrightarrow\; \psi_1(\tau^*, \Delta^*)\,(1 - \mathfrak{D}_1(\Delta^*, \tau^*)) + \psi_2(\tau^*, \Delta^*)\mathfrak{D}_2(\Delta^*, \tau^*) < 0$$

which is the same as (A.13), hence the result in terms of the slope of f. Its translation into the condition that $T'(\Delta^*) > (D^{-1})'(\Delta^*)$ is immediate. ‖

We now come to actually solving for steady–states. It will be more convenient here to work with the variable $x \equiv \beta\gamma(1 - \tau) \in [0, \infty)$. Accordingly, let us define:

$$\text{(A.14)} \qquad \Delta(x) \equiv \sqrt{\frac{w^2 + x^2 v^2/\gamma^2}{1 - (\alpha + x)^2}} = D(\tau),$$

and rewrite the equation $f(\tau) = \lambda$ as:

$$\text{(A.15)} \qquad \varphi(x) \equiv x\left(\Delta(x) + \frac{Bv^2/\gamma^2}{\Delta(x)}\right) - \left(\frac{\beta\delta}{\eta\gamma\Delta(x)}\right)\left(\frac{\beta\gamma - x}{x}\right) = \lambda\beta.$$

A stable steady–state is now an intersection of the function $\varphi(x)$ with the horizontal $\lambda\beta$, from below. Since $\varphi(0) \leq 0$ (it equals $-\infty$ for $1/\eta > 0$, or 0 for $1/\eta = 0$) while $\varphi(1 - \alpha) = +\infty$, for any $\lambda > 0$ there is always at least one stable equilibrium $x \in (0, 1 - \alpha)$, with $0 < \Delta(x) < +\infty$. Moreover, the total number of intersections must always be odd, with n intersections from below (stable equilibria), alternating with $n - 1$ intersections from above (unstable equilibria). Multiple intersections ($n > 0$) will actually occur, for some non-empty interval of values of λ, if and only if $\varphi(\cdot)$ *is non–monotonic.* Indeed, since $\varphi'(0) > 0$ (this is easily verified from (A.15)) and $\varphi(1 - \alpha) = +\infty$), non–monotonicity is equivalent to the property of having at least one strict local maximum, followed by one strict local minimum, in $(0, 1 - \alpha)$. Given the boundary values of φ, multiple equilibria then occur *if and only if* λ belongs to the range $[\underline{\lambda}, \bar{\lambda}]$, where:

$$\text{(A.16)} \qquad \begin{cases} \underline{\lambda} & \equiv & \min\{\varphi(x)\,|\,x \text{ is a strict local minimum of } \varphi(x)\} > 0 \\ \bar{\lambda} & \equiv & \max\{\varphi(x)\,|\,x \text{ is a strict local minimum of } \varphi(x)\} > 0. \end{cases}$$

That $\bar{\lambda}$ and $\underline{\lambda}$ are both always positive follows from the fact that for $1/\eta = 0$, $\varphi(x) > 0$ for all $x > 0$ (see (A.15)), while for $1/\eta > 0$, if $\varphi(x) < 0$ then $\varphi'(x) > 0$ necessarily. This last property can be verified directly from (A.15) and (A.17) below, or more intuitively by observing that if it were not true, there would be a subinterval of values of x where $\varphi(x) < 0$ and φ is not monotonic. This, in turn, would imply that there exists values of $\lambda < 0$ for which (A.15) has at least two solutions. But such solutions are also intersections of the curves $\Delta = D(\tau)$ and $\tau = T(\Delta)$; the former is always decreasing, and we saw earlier that, for all $\lambda < 0$, the latter is always increasing. Multiple intersections are therefore impossible.

Theorem 1 will now be proved by characterizing the set $\mathfrak{B} \equiv \{B \geq 0 \,|\, \varphi \text{ is non-monotonic on } (0, (1-\alpha)/\beta\gamma)\}$, then studying its variations with the parameters v, w, and η.

Lemma 2 *Let $1-\alpha < 2\beta\gamma$. The set $\mathfrak{B} \equiv \{B \geq 0 \,|\, \exists\, x \in (0, 1-\alpha),\, \varphi'(x) < 0\}$ is a non–empty interval of the form $\mathfrak{B} = (\underline{B}, +\infty)$ with $\underline{B} > 0$, or $\mathfrak{B} = [0, +\infty)$.*

Proof: Let us differentiate (A.15):

$$\begin{aligned}
\varphi'(x) &\equiv \frac{\varphi(x)}{x} + x\Delta'(x)\left(1 - \frac{Bv^2/\gamma^2}{\Delta^2(x)}\right) - \frac{\beta\delta}{\eta\gamma}\left[\left(\frac{1}{x} - \frac{2\beta\gamma}{x^2}\right)\frac{1}{\Delta(x)} - \left(\frac{\beta\gamma}{x} - 1\right)\frac{\Delta'(x)}{\Delta^2(x)}\right] \\
&= \Delta(x) + \frac{Bv^2/\gamma^2}{\Delta(x)} - \left(\frac{\beta\delta}{\eta\gamma\Delta(x)}\right)\left(\frac{\beta\gamma - x}{x^2}\right) + x\Delta'(x)\left(1 - \frac{Bv^2/\gamma^2}{\Delta^2(x)}\right) \\
&\quad - \frac{\beta\delta}{\eta\gamma}\left[\left(\frac{1}{x} - \frac{2\beta\gamma}{x^2}\right)\frac{1}{\Delta(x)} - \left(\frac{\beta\gamma}{x} - 1\right)\frac{\Delta'(x)}{\Delta^2(x)}\right].
\end{aligned}$$

Grouping terms, $\varphi'(x) < 0$ if and only if:

$$\text{(A.17)} \qquad \frac{Bv^2}{\gamma^2}\left(\frac{x\Delta'(x)}{\Delta(x)} - 1\right) > \left(\frac{x\Delta'(x)}{\Delta(x)} + 1\right)\Delta^2(x) + \left(\frac{\beta\delta}{\eta\gamma}\right)\left(\frac{\beta\gamma}{x^2} + \left(\frac{\beta\gamma}{x} - 1\right)\frac{\Delta'(x)}{\Delta(x)}\right).$$

We now establish the lemma through two intermediate claims.

- *Claim 1:* If (A.17) is satisfied for some $B \geq 0$ at some $x \in (0, 1-\alpha)$, then $x\Delta'(x)/\Delta(x) > 1$. As a consequence, (A.17) is satisfied at x for all $B' > B$.

Proof: If $\Delta'(x)/\Delta(x) - 1/x \leq 0$ the left–hand–side of (A.17) is non–positive, so on the right–hand side it must be that $\beta\gamma/x - 1 < 0$. But then:

$$\frac{\beta\gamma}{x^2} + \left(\frac{\beta\gamma}{x} - 1\right)\frac{\Delta'(x)}{\Delta(x)} > \frac{\beta\gamma}{x^2} + \left(\frac{\beta\gamma}{x} - 1\right)\frac{1}{x} = \frac{2\beta\gamma - x}{x^2}.$$

Since $x < 1-\alpha < 2\beta\gamma$, this implies that the right–hand side of (A.17) is positive, a contradiction.

- *Claim 2:* There exists an $\hat{x} \in (0, 1-\alpha)$ such that $x\Delta'(x)/\Delta(x) > 1$ if and only if $x > \hat{x}$. As a consequence, for any $x > \hat{x}$, (A.17) holds for B large enough.

Proof: Let us denote from here on $\omega \equiv v/\gamma w$. Since $\Delta^2(x) = w^2(1+\omega^2x^2)/(1-(\alpha+x)^2)$,

$$\text{(A.18)} \qquad \frac{\Delta'(x)}{\Delta(x)} = \frac{\omega^2 x}{1+\omega^2x^2} + \frac{\alpha + x}{1-(\alpha+x)^2}.$$

Therefore $x\Delta'(x)/\Delta(x) > 1$ if and only if

$$\omega^2x^2(1-(\alpha+x)^2) + x(\alpha+x)(1+\omega^2x^2) \quad > \quad (1+\omega^2x^2)(1-(\alpha+x)^2) \Leftrightarrow$$

$$\begin{aligned} \omega^2 x^3(\alpha+x) - (1-(\alpha+x)^2) + x(\alpha+x) &> 0 \Leftrightarrow \\ \omega^2 x^3(\alpha+x) + (\alpha+x)(\alpha+2x) - 1 &> 0. \end{aligned}$$

This last expression is clearly increasing in x on $(0, 1-\alpha)$, from $\alpha^2 - 1 < 0$ at $x = 0$ to $\omega^2(1-\alpha)^3 + 1 - \alpha > 0$ at $x = 1-\alpha$. This proves Claim 2 which, together with Claim 1, establishes that the set $\mathfrak{B}$ is a non-empty interval of the form $(\underline{B}, +\infty)$ or $[\underline{B}, +\infty)$. Moreover, note that its complement, $\mathbb{R}_+ \setminus \mathfrak{B} \equiv \{B \geq 0 \mid \forall\, x \in (0, 1-\alpha),\, \varphi'(x) \geq 0\}$, is a closed set because φ' is continuous in B at every point. This implies that either $\mathfrak{B} = (\underline{B}, +\infty)$ with $\underline{B} > 0$, or else $\mathfrak{B} = [0, \infty)$, and finishes to establish Lemma 2. From (A.17)–(A.18), moreover, it is clear that $\underline{B}$ is a function $\underline{B}(v/w; \eta v^2)$. $\|$

To conclude the proof of Theorem 1 as well as the additional claims in footnote 23 concerning the exact number of steady–states, we shall make use of a last lemma.

Lemma 3 *For $B < \underline{B}(v/w; \eta v^2)$, there is a unique stable steady–state. For $B > \underline{B}(v/w; \eta v^2)$ the same is true if $\lambda \notin [\underline{\lambda}, \bar{\lambda}]$, while for $\lambda \in [\underline{\lambda}, \bar{\lambda}]$ there are $n \in \{2, 3, 4\}$ stable steady–states. Moreover, if $1/\eta$ is small enough then $n \leq 3$, and if α is also small enough then $n = 2$.*

Proof: By definition, for $B < \underline{B}(v/w; \eta v^2)$ the function φ is strictly increasing everywhere, so the steady–state is unique. The same is true for $B = \underline{B}(v/w; \eta v^2) > 0$, since then $\mathfrak{B} = (\underline{B}, +\infty)$. The other measure–zero case, $B = \underline{B}(v/w; \eta v^2) = 0$, is too special to be of interest. Now, for $B > (v/w; \eta v^2)$, we saw earlier that there must be n stable equilibria and $n-1$ unstable ones in the interval $(0, 1-\alpha)$. To examine what values n can take, rewrite $\varphi(x) = \lambda\beta$ as:

$$S(x) \equiv \left[x^2 \left(\Delta^2(x) + \frac{Bv^2}{\gamma^2} \right) - \left(\frac{\beta\delta}{\eta\gamma} \right) (\beta\gamma - x) \right]^2 - [\lambda\beta x \Delta(x)]^2 = 0. \tag{A.19}$$

By (A.14), $\Delta^2(x)$ is a polynomial fraction in x whose numerator and denominator are both of degree 2. Multiplying the whole equation(A.19) by the squared denominator of $\Delta^2(x)$, we therefore obtain a polynomial $S^*(x)$ of degree $2 \times (2+2) = 8$, which can have at most 8 real roots. But we saw in the discussion following (A.15) that $\varphi(x) = \lambda\beta$ must have an odd number of solutions on $(0, 1-\alpha)$, with n intersections from above and $n-1$ from below. The numbers of stable and unstable equilibria can therefore only be $(1,0), (2,1), (3,2)$ or $(4,3)$.

When $1/\eta = 0$ we can simplify (A.19) by x^2, leaving for $S^*(x)$ only a polynomial of degree 6; this rules out $n = 4$. When, in addition, $\alpha = 0$, note from (A.14) that $\Delta(x)$ depends on x only through x^2. As a consequence, the sixth–degree polynomial $S^*(x)$ is also a polynomial of degree 3 in x^2, so it has at most three real roots. This rules out $n = 3$. Finally, since the polynomial $S^*(x)$, like (A.19), is continuous with respect to $1/\eta$ and α, so is (generically with respect to the other parameters) its number of real zeroes in the interval $(0, 1-\alpha)$. The preceding results therefore also apply for $1/\eta$ and α small enough. $\|$

This concludes the proof of Theorem 1. ■

Proof of Proposition 8 When $1/\eta = 0$ the condition for $\varphi'(x) < 0$, namely (A.17), becomes:

$$\text{(A.20)} \qquad \frac{Bv^2}{\gamma^2} > \left(\frac{x\Delta'(x)/\Delta(x)+1}{x\Delta'(x)/\Delta(x)-1}\right)\Delta^2(x),$$

with the requirement that the denominator must be positive. Using (A.18), this can be rewritten as

$$B > \left(\frac{x^2+\omega^{-2}}{x^3(\alpha+x)-\omega^{-2}(1-(\alpha+x)(\alpha+2x))}\right)\left(\frac{x^2(2-(\alpha+x)(\alpha+2x))+\omega^{-2}(1-\alpha(\alpha+x))}{1-(\alpha+x)^2}\right) \equiv \Gamma(x,\omega)$$

where $\omega \equiv v/w$. It is easily verified that the each of the bracketed functions is increasing in ω^{-2}, therefore:

$$\text{(A.21)} \qquad \underline{B}(\omega) \equiv \inf\left\{\Gamma(x,\omega)\,|\, x \in (0, 1-\alpha) \text{ and } x^3(\alpha+x) > \omega^{-2}(1-(\alpha+x)(\alpha+2x))\right\}$$

is strictly positive, and decreasing in ω. Observe next that as ω tends to infinity $\Gamma(x,\omega)$ approaches $\Gamma(x,\infty) = x^2/(1-(\alpha+x)^2)$, whose infimum value on $(0, 1-\alpha)$ is zero; therefore, $\lim_{\omega\to\infty} \underline{B}(\omega) = 0$. Finally, for any x with $x^3(\alpha+x) > \omega^{-2}(1-(\alpha+x)(\alpha+2x))$, note that $\Gamma(x,\omega) > \omega^{-2}/(1-\alpha^2)$, which tends to infinity as ω tends to 0. Therefore $\lim_{\omega\to 0} \underline{B}(\omega) = \infty$. ‖

Proof of Proposition 9 Let us fix $B \in \mathfrak{B}$ and make the function φ's dependence on η explicit, by denoting it $\varphi(x;\eta)$. Consider now any η such that, for some value of λ, there are two steady–states with $0 < \tau_1 < \tau_2 < 1$. Let $0 < x_2 < x_1 < \beta\gamma$ then denote the corresponding values of $x = \beta\gamma(1-\tau)$. We know that φ must have at least one consecutive local maximum x' and local minimum x'', with $x_2 < x' < x'' < x_1$. Now, for all $x \in (x', x'')$, $\varphi'(x;\eta) < 0$; since $x < x_1 < \beta\gamma$, the last bracketed in term (A.17) is positive, so the whole right–hand side of that equation is decreasing in η. As a result, for all $\hat{\eta} > \eta$, we still have $\varphi'(x;\hat{\eta}) < 0$ on (x', x''). Since $\varphi'(0;\eta') > 0$, the function $\varphi(x;\eta')$ must therefore have a local maximum $\hat{x}'$ in $(0, \beta\gamma)$. Since it must eventually increase back towards $+\infty$ as x tends to $1-\alpha$, it must also have a local minimum $\hat{x}''$ in $(x', 1-\alpha)$. There exists therefore a range $[\underline{\lambda}', \bar{\lambda}'] \supseteq [\varphi(\hat{x}''), \varphi(\hat{x}')]$ of λ's, each of which is cut from below by $\varphi(x;\eta')$ at no less than three points, one of them at least being to the left of $\hat{x}' < \beta\gamma$. Hence the existence of multiple stable steady–states, with one of them at least for a positive value of τ. ‖

Proof of Proposition 10 Given log–normality, (29) becomes $\ln \kappa_t = m_t + \gamma\Delta_t^2/2 = (\ln y_t - \delta l_t)/\gamma$, so substituting $\ln\tilde{\kappa}$ into the growth equation (15) yields:

$$\text{(A.22)} \quad \ln y_{t+1} - (\alpha+\beta\gamma+\omega)\ln y_t = \ln\bar{\kappa} + \delta(\ln l_{t+1} - (\alpha+\omega)\ln l_t) - \mathfrak{L}_v(\tau_t)v^2/2 - \mathfrak{L}_\Delta(\tau_t)\gamma^2\Delta_t^2/2 \equiv g_t$$

with $\ln\bar{\kappa} \equiv \beta\gamma\ln\mathfrak{s} - \gamma(1-\gamma)w^2/2$. In a steady state, if $\alpha+\beta\gamma+\omega < 1$ the left hand side equals $(1-\alpha-\beta\gamma-\omega)$ times the output level $\ln y_\infty$; when $\alpha+\beta\gamma+\omega = 1$ it becomes equal to the asymptotic growth rate, $\lim_{t\to\infty}\ln(y_{t+1}/y_t)$. As to the right–hand side, in a steady–state with $\tau_t = \tau$ it becomes:

$$\text{(A.23)} \qquad g_\infty(\tau) \equiv \ln\bar{\kappa} + \beta\gamma\delta\ln l(\tau) - \mathfrak{L}_v(\tau)v^2/2 - \mathfrak{L}_\Delta(\tau)\gamma^2 D(\tau)^2/2$$

For $\alpha = 0$ we saw that $\mathfrak{L}_\Delta(\tau)$ becomes equal to $\mathfrak{L}_v(\tau) = \beta\gamma(1-\beta\gamma)(1-\tau)^2$; therefore $\mathfrak{L}_v(\tau)v^2/2 + \mathfrak{L}_\Delta(\tau)\gamma^2 D(\tau)^2/2$ is strictly decreasing in τ. Now, with $1/\eta = 0$ the labor supply term is constant, therefore $g_\infty(\tau)$ is strictly increasing in τ; this proves the proposition's first claim. Conversely when $\beta\gamma = 1$ then $\mathfrak{L}_v(\tau) = \mathfrak{L}_\Delta = 0$; with $1/\eta > 0$, $g_\infty, (\tau)$, like $l(\tau)$, is then decreasing in τ; hence the second claim. ‖

Proof of Proposition 11 Once agent i knows his productivity z_t^i, hence also his income $y_t^i = z_t^i(k_t^i)^\gamma(l_t^i)^\delta$ and his investment subsidy rate $\hat{e}_t^i/e_t^i = (\tilde{y}_t/\hat{y}_t^i)^{\tau_t}$, his decision problem takes the form:

$$
\begin{aligned}
\text{(A.24)}\quad \ln V_t^i &= \max_{l,\nu}\Big\{(1-\rho)[\ln((1-\nu)y_t^i) - l^\eta] + (\rho/r')\ln[E_t(k_{t+1}^i)^{r'}] \mid k_{t+1}^i = \kappa\xi_{t+1}^i(k_t^i)^\alpha(\hat{e}_t^i)^\beta\Big\} \\
&= \max_\nu\{(1-\rho)\ln(1-\nu) + \rho\beta\ln\nu\} + \max_l\{-(1-\rho)l^\eta + (1-\rho+\rho\beta(1-\tau_t))\delta\ln l\} \\
&\quad +\rho(\ln\kappa - (1-r')w^2/2) + [\rho\alpha + (1-\rho+\rho\beta(1-\tau_t))\gamma]\ln k_t^i \\
&\quad +(1-\rho+\rho\beta(1-\tau_t))\ln z_t^i + \rho\beta\tau_t\ln\tilde{y}_t,
\end{aligned}
$$

where $\nu_t^i \equiv e_t^i/y_t^i$ is the savings rate. Strict concavity in ν and l is easily verified, and the first–order conditions directly yield the stated results**.** ‖

Proof of Proposition 12 Substituting the optimal $l_t^i = l_t$ and $\nu_t^i = \mathfrak{s}$ into (A.24) and denoting $\ln\kappa' \equiv \ln\kappa - (1-r')w^2/2$ yields:

$$
\begin{aligned}
\text{(A.25)}\qquad \ln V_t^i &= \rho\ln\kappa' + (1-\rho)\ln(1-\mathfrak{s}) + \rho\beta\ln\mathfrak{s} - (1-\rho)l_t^\eta + (1-\rho+\rho\beta(1-\tau_t))\delta\ln l_t \\
&\quad + [\rho\alpha + (1-\rho+\rho\beta(1-\tau_t))\gamma]\ln k_t^i + (1-\rho+\rho\beta(1-\tau_t))\ln z_t^i + \rho\beta\tau_t\ln\tilde{y}_t.
\end{aligned}
$$

Thus *conditional* on k_t^i, $\ln V_t^i$ is normally distributed, with variance $(1-\rho+\rho\beta(1-\tau_t))^2v^2$. This implies:

$$
\text{(A.26)}\qquad \ln U_t^i \equiv \frac{1}{r}\ln E\left[\left(V_t^i\right)^r \mid k_t^i\right] = E\left[\ln V_t^i \mid k_t^i\right] + r(1-\rho+\rho\beta(1-\tau_t))^2v^2.
$$

Therefore, to obtain $\ln U_t^i$ one simply needs to replace in (A.25) the term in $\ln z_t^i$ by $-(1-\rho+\rho\beta(1-\tau_t))\left[1-r(1-\rho+\rho\beta(1-\tau_t))\right]v^2/2$. Finally, substituting in the value of $\tau_t\ln\tilde{y}_t$ from (A.3) yields the claimed result, with:

$$
\text{(A.27)}\quad \bar{u}_t \equiv \ln[(1-\mathfrak{s})^{1-\rho}\mathfrak{s}^{\rho\beta}] + \rho(\ln\kappa - (1-r')w^2/2) + (\rho\alpha + (1-\rho+\rho\beta)\gamma)m_t + \rho\beta\gamma^2\Delta_t^2/2. \ \|
$$

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Table 1a: Political Participation by Income							
			Representation ratios by				p^*
Political Activity:	Total %		percentile family income				(in
(Electoral Politics, 1952-1988)	taking part	0-16	17-33	34-67	68-95	96–100	%)
Vote	66.1	.76	.90	1.00	1.16	1.27	55.5
Try to influence others	26.7	.63	.79	.98	1.25	1.54	60.6
Contribute money	8.9	.25	.51	.80	1.54	3.25	73.6
Attend meetings	7.8	.49	.73	.93	1.31	2.27	65.7
Work on campaign	4.6	.48	.74	.85	1.37	2.42	67.6

Source: Rosenstone and Hansen (1993), Table 8-2, plus own computation of p^*.

Table 1b: Political Participation by Education							
		Representation ratios by years of education					p^*
Political Activity:	Total	(with corresponding % of population)					
	%	0-8	9-11	12	13-15	16+	(in
	taking part	(20.1%)	(16.3%)	(32.8%)	(16.8%)	(14%)	%)
Electoral Politics, 1952-1988							
Vote	66.1	.85	.83	1.00	1.12	1.26	55.8
Try to influence others	26.7	.61	.75	.94	1.33	1.61	63.5
Contribute money	8.9	.33	.51	.87	1.37	2.41	73.7
Attend meetings	7.8	.48	.50	.85	1.43	2.14	72.2
Work on campaign	4.6	.48	.50	.87	1.33	2.25	72.0
Governmental Politics, 1976-1988							
Sign Petition	34.8	.34	[←	.87	→]	1.44	69.5
Attend local meeting	18.0	.31	[←	.78	→]	1.46	73.3
Write congress	14.6	.38	[←	.72	→]	1.56	75.9

Source: Rosenstone and Hansen (1993), Tables 8-1 and 8-2, plus own computation of p^*.