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Negatively Interdependent Preferences*

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Abstract

We develop a theory of representation of interdependent preferences that reflect the widely acknowledged phenomenon of *keeping up with the Joneses* (i.e. the preferences of the individuals whose well-being depends on their "relative standings" in the society as well as on their material consumption). The principle ingredient of our analysis is the assumption that the individuals desire to occupy a better position than their peers. This is quite a primitive starting point in that it does not give any reference to what is actually regarded as "status" in the society. We call this basic postulate *negative interdependence*, and observe that it has unexpectedly far reaching implications. In particular, combining this assumption with some other basic postulates that are widely used in a number of other branches of the theory of individual choice, one is able to "derive" the famous relative income hypothesis, and in fact, obtain a quite definitive representation of interdependent preferences. Some extensions and applications of our representation theory are also discussed.

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1 Introduction

The idea that the welfare of an individual depends on the relative as well as the absolute income (or consumption) goes at least as far back as to Thorstein Veblen:

> "... the desire for wealth can scarcely be satiated in any individual instance ... the ground of [this need] is the desire of everyone to excel everyone else in the accumulation of goods." Veblen (1899, p. 32)[1]

This basic insight has later been reformulated in several distinct ways to capture the notion that one's well-being is determined not only by the intrinsic utility of her material consumption, but also by one's relative standing (status) in the society or in her peer group (cf. Duesenberry, 1949, Easterlin, 1974, Layard, 1980, and Frank, 1985, *inter alia*).[2] In the economics literature, this notion is sometimes referred to as *the relative income hypothesis*, or as the phenomenon of *keeping up with the Joneses.*

The significance of one's "relative position" has also been prevalent in the socio-psychological approaches to subjective well-being. For instance, the *judgement theory* in psychology claims that happiness of an individual results from a comparison between her actual level of income and some reference level which is, according to the *social comparison theory*, formed by observing other individuals' incomes.[3] In economics, on the other hand, the dependence of one's well-being on others' consumption (or wealth) is usually introduced either by postulating envious (or altruistic) preferences (Hochman and Rodgers, 1969, Layard, 1980, and Oswald, 1983), or by adhering to relative deprivation considerations (Runciman, 1966, Yitzhaki, 1982, and Sen, 1983). Furthermore, it is noted by several authors that, in some cases, the individuals may care about their relative standings in the society simply because a higher status implies better access to goods which the individuals value (Cole, Mailath and Postlewaite, 1992, 1995).

It is also evident that there are serious implications of entertaining this idea in economic models. Indeed, most of the conventional results in public finance, general equilibrium theory, and macroeconomics have been either completely overturned or significantly modified once the reliance of agents' welfare on their relative standings is formally recognized.[4]

[1] Also cited in Becker (1974).

[2] There is a large body of both direct and indirect empirical evidence which supports this fundamental hypothesis. See, for instance, Easterlin (1974), Kapteyn and Wansbeek (1982), van de Stadt, et al. (1985), Tomes (1986), and Clark and Oswald (1996). Frank (1985, p.5) notes that "... abundant evidence suggests that people do in fact care much more about how their incomes compare with those of their peers than about how large their incomes are in any absolute sense. Most poor citizens of the United States enjoy an absolute consumption standard that would be the envy of all but the richest citizens of, say, India. Yet the poor here are often said to be much less content with their lot than are the upper-middle class citizens of many poorer nations."

[3] See Diener (1984) for a survey of the related psychological literature.

[4] There is now a sizable literature studying the effects of building the "relative position" concerns of the individuals into their utility functions within various economic models. In the context of optimal income taxation and public policy, for instance, the implications of the relative income hypothesis are analyzed by Boskin and Sheshinski (1978), Layard (1980), Oswald (1983), Ng (1987), Lommerud (1989), Tuomala (1990), and Blomquist (1993). Related applications to the theory of asset pricing include

Nevertheless, to the best of our knowledge, the fundamental problem of "representing the preferences of the individuals who care about their relative standings" is not addressed in the literature. Most authors appear to have simply postulated some particular ways of formalizing the notion of "the relative position" (such as one's relative income, or rank order in the society), and accordingly modified the utility functions of the constituent agents in a more or less *ad hoc* manner. Clearly, such an approach carries a considerable level of arbitrariness in the way it alters the individual preference relations, and therefore, it may well result in models lacking a satisfactory degree of predictive power. This may, in fact, explain the reluctance of the majority of the economists in departing from the conventional framework where individuals are assumed to care *only* about their absolute levels of consumption; it is, after all, a commonplace criticism that "one can explain anything by changing the domain of utility functions appropriately!"

In this paper, we shall attempt to offer a resolution of this issue by developing a theory of representation of interdependent preferences which starts from the primitives. Our present purposes are: (i) to axiomatically analyze the notion of one's relative standing in the society; (ii) to provide an operational way of devising *interdependent* preference relations which would reflect the individuals' concern for their relative standings; (iii) to offer a unified way of formalizing the relative income hypothesis; and (iv) to uncover and critically examine the implicit assumptions behind some 'standard' ways of modifying the utility functions to incorporate the "interdependence" of the economic agents.

We build our analysis on the basic presumption that individuals desire to occupy a better position than others in the social hierarchy, that is, they simply like to be happ*ier* than others (or than their peers). This is indeed a primitive starting point in that one can formalize it without giving any reference to what counts as *status* in the society. Suppose that we confront an individual with two income (or consumption vector) distributions such that the level of her own income is identical in both distributions. Suppose also that this individual evaluates subjectively that all other agents in the society are happier in the former distribution. Which distribution will this individual choose to live in? Clearly, if her well-being is independent of that of the rest of the society, she would be indifferent between these two distributions. On the other hand, if she really wishes to be "happier than the others," then it seems quite reasonable that she would prefer the latter distribution; we then say that this individual has *negatively interdependent* preferences.[5]

Abel (1990), Galí (1994), Campbell and Cochrane (1995), Bakshi and Chen (1996), and Ljungqvist and Uhlig (1996). Moreover, while Frank (1984) and Akerlof and Yellen (1990) study the theory of wage determination with interdependent preferences, Gregory (1980) and Robson (1992) reexamine the Friedman-Savage theory of attitudes towards risk when individuals care about their wealth also via the relative standing that wealth entails. Finally, there are a number of papers which explore the analytical properties of the Walrasian equilibrium in the presence of consumption externalities; see, for instance, Schall (1972), Scott (1972), Rader (1980), Villar (1988) and Parks (1991).

[5]It is imperative that one should not dismiss negatively interdependent preferences because they seem 'unethical.' It may, after all, justly be argued that the ethicality of things cannot be decided upon independently of the preferences of the constituent individuals; this is one of the basic themes of the general theory of social choice. Moreover, if negative interdependence is simply an aspect of human nature, and thus, if it dictates some of the economic decisions of the individuals, it is only natural that we include it in the realm of economic analysis.

We note that this particular way of conceptualizing one's concern for her relative position adheres only to the *subjective* evaluations of the individuals so that it abstracts from what "status" really is. (We view this as an appealing feature, for the *perceived* relative position of an agent induced by a given distribution of goods may well differ across individuals.) As a matter of fact, the property of negative interdependence that we advance here seems so general that one's immediate reaction is that it cannot possibly bring any structure to individual preference relations at all; it ostensibly appears as hardly anything different than positing arbitrarily that one's well-being is decreasing in others' consumption (or welfare). Perhaps somewhat surprisingly, however, this contention turns out to be unwarranted; as we shall formally demonstrate in the sequel, negatively interdependent preferences are something altogether distinct from (and in fact considerably more restrictive than) the standard envious (malevolent) preferences.

Having established how one can formalize the keeping up with the Joneses effect in general, the second part of the paper deals with developing an operational representation theory for negatively interdependent preferences. Towards this end, we study here the additive representations of such preferences, and observe that the additivity property and negative interdependence together essentially entail a particular status factor: *the relative income.* Therefore, we basically "derive" the relative income hypothesis from more primitive notions in the present paper, as opposed to postulating it at the outset. Moreover, we show that a natural refinement of the class of all additive and negatively interdependent preferences leads one to a complete characterization of a potentially useful set of preference relations which reflect the relative income concerns of individuals in an analytically convenient way. Put somewhat informally, our study culminates in axiomatically characterizing the preference relations of individuals over income distributions which are represented by utility functions of the form

$$u(a, y) = af(a/\mu),$$

where a is the income level of the subject individual, y is the income distribution of the rest of the society, μ is the average income, and f is an arbitrary continuous, strictly increasing and positive real-valued function. This particular representation could be thought of as interesting in that it clearly constitutes a compromisation between the standard case where one cares only about her absolute income and the extreme case in which one is concerned only with her relative income. In addition, while this representation result provides one with a rich class of preference relations (and hence leaves some room for choice in applications), it clearly brings a unified structure to how one can introduce the relative income hypothesis into individual utility functions in economic models.[6] What is more, due to the axiomatic nature of our inquiry, the adoption of the relative income hypothesis through this representation is clearly less arbitrary than what seems to be the practice. Consequently, our prescription renders certain kinds of individual behavior inconsistent with the specification of the interdependent preferences (so long as they are represented by utility functions of the form described above), thereby preserving a certain degree of predictive power.

[6]We must note that a surprisingly large number of papers have actually used explicit preference relations which are *not* in the class that we characterize here. So, this particular characterization result has some strength of selection; see footnote 22.

The paper is organized as follows. In Section 2 we introduce a preliminary framework where individual preference relations are defined over distributions of a single good (income) in a society of a given size. Section 3 is devoted to a detailed discussion of the negative interdependence property. In this section, we motivate this property as a fundamental axiom, present several examples to provide a better sense of its formal characteristics, and formulate the standard properties of continuity and decomposability for the well-being orderings under consideration. In Section 4, we introduce the additivity axiom, and examine its implications with regard to the relative income hypothesis. Our main findings are reported in Section 5 where we outline the characterizations of the Duesenberry utility function and the utility functions prescribed above. In Section 6, we discuss certain ways of extending the present development to the cases of many commodities and populations of varying sizes. The present paper concludes with a section outlining our final remarks about future research, and an appendix containing the proofs of our main results.

2 Preliminaries

We consider $\mathbf{R}^m_{++}$ as the space of all distributions of a single good in a society with population $m \geq 2$. For referential convenience, we shall refer to the commodity in question as *income*. Thus, we interpret $y = (y_1, ..., y_m) \in \mathbf{R}^m_{++}$ as an income distribution where y_i is the level of income of the ith individual at a given point in time. We shall later extend our analysis to the case of the distributions of many commodities.

Let $\mathfrak{P}_0$ stand for the class of all complete preorders (transitive and reflexive binary relations) on $\mathbf{R}^{1+n}_{++}$ (where n is an arbitrary positive integer). We think of the members of $\mathfrak{P}_0$ as preference relations (or well-being orderings) of individuals over income distributions of the same population size. As usual, the asymmetric and symmetric factors of a $\succsim \in \mathfrak{P}_0$ are denoted by $\succ$ and $\sim$, respectively. We shall thus henceforth refer to all relations in $\mathfrak{P}_0$ as *preference relations* or as individual *well-being orderings.*

For any $(a, y), (b, x) \in \mathbf{R}^{1+n}_{++}$ with $a, b > 0$, we think of the statement

$$(a, y) \succsim (b, x)$$

as revealing that the individual (characterized by $\succsim$) is better off at the income distribution (a, y) where her income is a and the income distribution of the *rest of the society* is y, than at the income distribution (b, x) where her income is b and the income distribution of the rest of the society is x.[7]

Given the present interpretation, it is natural to confine attention to preference relations $\succsim$ in $\mathfrak{P}_0$ which are *monotonically increasing* in the first argument and *symmetric* in the rest of the arguments. We denote the class of all such relations by $\mathfrak{P}$. That is, $\succsim \in \mathfrak{P}$ if, and only if, $\succsim \in \mathfrak{P}_0$,

$$(a, y) \succ (b, y) \quad \text{for all } y \in \mathbf{R}^n_{++} \text{ and } a > b > 0,$$

[7]It is important to note that the "rest of the society" here need not refer to the society at large. One may think of (a, y) as a subdistribution where y is the income distribution of a particular group of agents the individual associates herself with, that is, of her *peer* or *social reference group* (Duesenberry, 1949, Kapteyn and van Herwaarden, 1980, Yitzhaki, 1982, Kapteyn and Wansbeek, 1982, 1985).

and

$$(a, y) \sim (a, y\Pi) \quad \text{for all } (a, y) \in \mathbf{R}^{1+n}_{++}$$

for any $n \times n$ permutation matrix Π. In words, if $\succsim \in \mathfrak{P}$ is the preference relation of a given agent, we understand that this agent always prefers more money to less, all other things being constant. Moreover, the external effect that is imposed by others on her well-being is independent of the identities of these other individuals. The symmetry property is thus nothing but an *anonymity* condition with respect to the rest of the society.

3 Negative Interdependence

3.1 Definition and Motivation

It is clear that if an individual with $\succsim \in \mathfrak{P}$ does *not* care about the rest of the population in assessing her well-being, then

$$(a, y) \succsim (b, x) \quad \text{if and only if} \quad a \geq b$$

must hold for all $x, y \in \mathbf{R}^n_{++}$ and $a, b > 0$. If, on the other hand, the individual's well-being is indeed affected by the level of others' earnings, then there must clearly exist at least one pair $((a, y), (b, x))$ such that $b > a$ and $(a, y) \succsim (b, x)$. Therefore, so long as there exists an $a > 0$ such that

$$(a, y) \nsim (a, x) \quad \text{for some } x, y \in \mathbf{R}^n_{++},$$

we may view $\succsim$ as exhibiting at least some degree of "interdependence."

One obvious channel through which an individual's well-being could depend on others' incomes is one's altruistic or envious feelings towards others. For example, the well-being of a parent might be positively influenced by the incomes of his/her children, or the earnings of a colleague could create negative externalities. In other words, the notion of "interdependence" takes a particular form through this channel which may, in fact, be called (positive or negative) *dependence* on others' incomes. Formally speaking, *negative dependence*, for instance, would require the preference relation $\succsim$ of an individual to be monotonically decreasing in the incomes of others. Nevertheless, this appears to be too general (and perhaps somewhat *ad hoc*) a formulation which may lead to economic models of only limited predictive power (Cole et. al, 1992, 1995).

On the other hand, there is a natural way of bringing further discipline to the analysis by studying some of the primitives behind *envious* preferences. It is argued by many authors that one such primitive is the desirability of higher status than others', or more generally, the attraction towards being better off than others. There is indeed a voluminous social science literature which supports the claim that one's relative standing in the society is one of the major factors behind her well-being; these studies indicate very clearly that many agents prefer to be "*a big enchanted prince in a small pond than* *a small enchanted prince in a big pond.*"[8] In what follows we shall try to present a particular formalization of this basic insight.

Take an individual with preference relation $\succsim$ and income a, and consider two income distributions (a, y) and (a, x). Since the subject individual has the same level of income in both distributions, if she 'likes' one distribution more than the other, this must be due solely to the external effect that is imposed on her by the rest of the society, or alternatively, due to the difference in the agent's status in the two distributions.

Now, suppose $(x_i, (a, x_{-i})) \succsim (y_i, (a, y_{-i}))$ for agent i.[9] This means that the subject individual would prefer to be in agent i's position in the distribution (a, x) over that in (a, y). In other words, she simply 'feels' that individual i is better off in the former distribution. Therefore, if $(x_i, (a, x_{-i})) \succsim (y_i, (a, y_{-i}))$ holds for all i, then the individual in question would *subjectively* assess everybody else's relative standing as being better in (a, x) than in (a, y); in a sense, therefore, the individual regards (a, y) as "the smaller pond." But then, if this person wishes to occupy a better relative position in any given society (that is, if she values being better off than others positively), it follows that her well-being would be higher in distribution (a, y) than in (a, x).

This reasoning leads us to the following definition.

Definition 1. Let $\succsim \in \mathfrak{P}$. $\succsim$ is said to be **negatively interdependent** if, for all $a > 0$ and $x, y \in \mathbf{R}^n_{++}$, $(a, y) \succsim (a, x)$ holds whenever

$$(x_i, (a, x_{-i})) \succsim (y_i, (a, y_{-i})) \quad \text{for all } i = 1, ..., n, \tag{1}$$

and $(a, y) \succ (a, x)$ holds whenever (1) and at least one of the relations in (1) hold strictly. (**Positively interdependent** preferences are defined dually.)

There are several things to comment on this particular conceptualization of the notion of "interdependence." First, the primitives behind our formulation is not 'love' or 'hate' one might have towards others, but rather the desire *to occupy a better position than the others.* We maintain that this is a less *ad hoc* assumption than simply positing that one has envious preferences.[10] Second, negative interdependence relies exclusively on the *subjective* evaluation of "the relative position" by the agent in question. Indeed, our formulation does not even require the knowledge of precisely what this individual regards as "status"; this information is implicit in the agent's preference relation $\succsim$. Finally, again due to the subjective nature of our formulation, negative interdependence is a concept which abstains from the problem of interpersonal preference comparisons;

[8]For a very lucid account of this issue, we refer the reader to Frank (1985, Chapters 1 and 2). Frank (1985), and Clark and Oswald (1996) provide extensive sets of references from the biology and social psychology literatures which provide ample evidence in support of the "interdependence" hypothesis that we advance here.

[9]Throughout this paper, z_{-i} shall denote the vector $(z_1, ..., z_{i-1}, z_{i+1}, ..., z_m) \in \mathbf{R}^{m-1}$, for any $z \in \mathbf{R}^m$.

[10]As noted earlier, the appeal of "doing better than others" may even be a direct consequence of the primitives of an economic model. One such example is provided by Cole, et. al. (1992) within the context of a matching model where (independent) utility maximization leads to the desire of having a better standing (in terms of relative wealth) in the society than others.

it employs only the subject individual's particular preference relation in comparing the well-being of others' in the society. In fact, the way this comparison is carried out is essentially identical to say that, from the perspective of the subject individual, agent i in (a, y) would 'envy' her position in (a, x) in the sense of Foley (1967).

3.2 Examples

To get a better sense of what is involved in negative interdependence, we now consider some interesting examples of preference relations in $\mathfrak{P}$, and check if they are negatively interdependent in the sense of Definition 1.

Example 1. Let us denote the *average income* in the distribution $(a, y) \in \mathbf{R}^{1+n}_{++}$ by $\mu(a, y)$; that is,

$$\mu(a,y) \equiv \frac{1}{n+1}\left(a + \sum_{i=1}^{n} y_i\right) \qquad \text{for all } (a,y) \in \mathbf{R}^{1+n}_{++},$$

and consider the preference relation $\succsim_{\text{Rel}} \in \mathfrak{P}$ defined as

$$(a,y) \succsim_{\text{Rel}} (b,x) \quad \text{if and only if} \quad \frac{a}{\mu(a,y)} \geq \frac{b}{\mu(b,x)}$$

for all $(a, y), (b, x) \in \mathbf{R}^{1+n}_{++}$. This preference relation envisages that the well-being of an agent is completely determined by her *relative income,* and it is thus a straightforward reflection of Duesenberry's relative income hypothesis. Intuitively, one would expect that $\succsim_{\text{Rel}}$ exhibits negative interdependence between individuals, and this indeed turns out to be the case.[11] We shall have opportunity to discuss this negatively interdependent well-being ordering at some length in Subsection 5.1. ||

To give a more revealing example, let us define the relation $\succsim_\theta \in \mathfrak{P}$ for an arbitrary real number θ as follows:

$$(a,y) \succsim_\theta (b,x) \quad \text{if and only if} \quad a - \theta\sum_{i=1}^{n} y_i \geq b - \theta \sum_{i=1}^{n} x_i$$

for all $(a, y), (b, x) \in \mathbf{R}^{1+n}_{++}$.[12] Evidently, any such $\succsim_\theta$ is negatively *dependent* in the sense that the well-being of an individual with the preference relation $\succsim_\theta$ is monotonically

[11]To see this, take any $(a, y), (b, x) \in \mathbf{R}^{1+n}_{++}$ such that (1) holds for $\succsim_{\text{Rel}}$, i.e., let $x_i/\mu(b,x) \geq y_i/\mu(a,y)$ for all $i = 1, ..., n$. It then follows that $by_i + y_i \sum_{j\neq i} x_j \leq ax_i + x_i \sum_{j \neq i} y_j$ for all i, and this yields

$$b\sum_{i=1}^{n} y_i + \sum_{i=1}^{n}(y_i \sum_{j\neq i} x_j) \leq a \sum_{i=1}^{n} x_i + \sum_{i=1}^{n}(x_i \sum_{j\neq i} y_j).$$

But by direct computation, $\sum_{i=1}^{n}\left(y_i \sum_{j\neq i} x_j\right) = \sum_{i=1}^{n}\left(x_i \sum_{j\neq i} y_j\right)$ so that we have $a/\sum_{i=1}^{n} y_i \geq b/\sum_{i=1}^{n} x_i$, that is, $a/\mu(a,y) \geq b/\mu(b,x)$.

[12]Any such $\succsim_\theta$ is in fact a special case of the parametric individual well-being function characterized by Gilboa and Schmeidler (1996).

decreasing in others' incomes. Yet, perhaps somewhat surprisingly, $\succsim_\theta$ need not be negatively *interdependent* depending on the value of θ:

Example 2. Let $\theta \geq 1$, $n = 2$, and take $x = (1, 2)$ and $y = (3, 2 + 2\theta)$. Clearly, for any $a > 0$, while $(a, x) \succ_\theta (a, y)$ holds, one can easily check that

$$(x_1, (a, x_2)) \succsim_\theta (y_1, (a, y_2)) \quad \text{and} \quad (x_2, (a, x_1)) \sim_\theta (y_2, (a, y_1)).$$

Therefore, $\succsim_\theta$ is *not* negatively interdependent on $\mathbf{R}^3_{++}$. ||

Example 2 clarifies that negative interdependence is something altogether different from saying that one's well-being is strictly decreasing in others' incomes. In other words, negative *dependence* on others' incomes does *not* entail negative *interdependence*; our formalization of interdependent preferences is thus fundamentally different from the usual formulation (such as in Boskin and Sheshinki, 1978, Layard, 1980, Oswald, 1983, Villar, 1988 and Galí, 1994.) In fact, the converse of this observation also holds: a preference relation which is negatively interdependent need not be *strictly* decreasing in others' incomes.[13] As we shall see later, however, combining negative interdependence with a weak separability condition ensures the negative dependence of one's well-being on others' incomes. So, for our present purposes, one may think of negative interdependence as a special (and as we shall see shortly, considerably more restrictive) case of negative dependence.

We conclude this section by noting the characterization of the class of all $\succsim_\theta$s which are negatively interdependent.

Proposition 1. *Let $n \geq 2$. The preference relation $\succsim_\theta$ is negatively interdependent if, and only if,* $0 < \theta < 1/(n-1)$.

This proposition demonstrates that negative interdependence puts an upper limit (within the class $\{\succsim_\theta : \theta > 0\}$) to the extent of externalities imposed by others on the well-being of an individual. Indeed, if $\theta > 1/(n-1)$, there are at least two distributions such that the subject individual prefers the one with the higher total income even though her own income is the same in both distributions. This, once again, underscores the distinction between negative dependence and our particular formalization of negative interdependence.

3.3 Decomposable Representation

In this subsection we aim to combine the property of negative interdependence with standard continuity and separability conditions, and hence introduce further structure

[13]For example, one can show that the preference ordering $\succsim^* \in \mathfrak{P}$ defined as

$$(a, y) \succsim^* (b, x) \quad \text{if and only if} \quad \#\{i : a > y_i\} \geq \#\{i : b > x_i\}$$

is negatively interdependent, while it is clear that $\succsim^*$ is *not* strictly decreasing in others' incomes. (This particular ordering is studied in a continuous setting by Robson, 1992, among others.)

to the present model.

Since our main objective in this paper is to obtain a useful *representation* of negatively interdependent preferences, it is natural to confine our attention at this point to *continuous* members of $\mathfrak{P}$. The motivation behind the continuity property is, of course, standard: infinitesimal changes in the distributions do not lead to preference reversals.

To motivate our decomposability assumption, let us take a case where $(a,y) \succsim (a,x)$ holds. Clearly, the individual feels better off in the former distribution relative to the latter one due *only* to the external effect imposed on her by the rest of the society. One may thus think of this as revealing that the subject individual prefers the externality (status) induced by the subdistribution y over the external effect that is imposed on her by x. But then how would this individual rank distributions (b,y) and (b,x)? If we wish to *decompose* the individual well-being of the agent into "the own income effect" and "the external effect of the society," it seems reasonable that the ranking would be $(b,y) \succsim (b,x)$. Indeed, while the own income effect is clearly the same in both distributions, by the hypothesis that $(a,y) \succsim (a,x)$, one may conceivably infer that the individual would 'prefer' the external effect imposed by the rest of the society in distribution (b,y) over that imposed in (b,x).

These considerations lead us to postulate the following weak *continuity* and *decomposability* conditions on $\succsim$.

Axiom CD. Let $\succsim \in \mathfrak{P}$.

(i) (Continuity) $\succsim$ is continuous, i.e. $\{(a,y) \in \mathbf{R}^{1+n}_{++} : (a,y) \succsim (b,x)\}$ and $\{(a,y) \in \mathbf{R}^{1+n}_{++} : (b,x) \succsim (a,y)\}$ are closed.

(ii) (Decomposability) For any $(a,y),(b,x) \in \mathbf{R}^{1+n}_{++}$,

$$(a,y) \succsim (a,x) \quad \text{if and only if} \quad (b,y) \succsim (b,x).$$

We note that decomposability is, in effect, a separability requirement which posits that an individual ranks the income distributions of the rest of the society *independently* of her own income.[14] The following observation demonstrates that this property indeed lets us *decompose* "the own income effect" and "the external effect of the society" in a formal sense.

Proposition 2.[15] *Let $\succsim \in \mathfrak{P}$. $\succsim$ satisfies Axiom CD if, and only if, there exist a continuous $\varphi : \mathbf{R}^n_{++} \to \mathbf{R}$ and a continuous and strictly increasing $H : \mathbf{R}_{++} \times R_\varphi \to \mathbf{R}$ such that*

$$(a,y) \succsim (b,x) \quad \textit{if and only if} \quad H(a,\varphi(y)) \geq H(b,\varphi(x)) \tag{2}$$

for all $(a,y),(b,x) \in \mathbf{R}^{1+n}_{++}$.[16]

[14]Of course, this does not mean that the *extent* of the external effect is independent of one's own income. In other words, decomposability is not necessarily an *additive* separability axiom; as we shall see later, a decomposable preference relation may well exhibit a case where the individual gets less and less concerned with the others as she gets richer (see Theorem 2).

[15]This proposition is essentially the same with Result A of Koopmans (1972); we nonetheless provide a brief proof of the proposition in the appendix for completeness.

[16]Here R_φ stands for the *range* of φ, i.e., $R_\varphi = \{\varphi(x) : x \in \mathbf{R}^n_{++}\}$.

4 Additive Representations of Negatively Interdependent Preferences

Since the additivity hypothesis is extensively used in the theory of individual choice, a natural inquiry at this preliminary stage of the theory of interdependent preferences would be to ascertain the implications of negative interdependence in additive settings. Towards this end, we shall study the representation of negatively interdependent preferences in this section under two different additivity assumptions.

A straightforward adoption of the standard additivity property in the present framework would read as follows:

Definition 2. The preference relation $\succsim \in \mathfrak{P}$ is said to be **(totally) additive** if, for any $(a, y), (b, x) \in \mathbf{R}^{1+n}_{++}$,

$$(a, y) \succsim (b, x) \quad \text{implies} \quad (a + c, y + z) \succsim (b + c, x + z)$$

for all $(c, z) \in \mathbf{R}^{1+n}$ such that $(a + c, y + z), (b + c, x + z) \in \mathbf{R}^{1+n}_{++}$.

Given Proposition 1, it is in fact an easy matter to uncover the precise form of negatively interdependent and (totally) additive preferences.

Proposition 3. *$\succsim \in \mathfrak{P}$ is an additive, continuous and negatively interdependent preference relation if, and only if, there exists a constant $\theta \in (0, 1/(n-1))$ such that*

$$(a, y) \succsim (b, x) \quad \textit{if and only if} \quad a - \theta \sum_{i=1}^{n} y_i \geq b - \theta \sum_{i=1}^{n} x_i \tag{3}$$

for all $(a, y), (b, x) \in \mathbf{R}^{1+n}_{++}$.

Proof. (Sketch) It is well-known that the hypotheses of monotonicity in the first argument, continuity and additivity guarantee that there exist $\rho > 0$ and $\rho_i \in \mathbf{R}$, $i = 1, ..., n$ such that $(a, y) \succsim (b, x)$ if and only if $\rho a + \sum_{i=1}^{n} \rho_i y_i \geq \rho b + \sum_{i=1}^{n} \rho_i x_i$ for all $(a, y), (b, x) \in \mathbf{R}^{1+n}_{++}$. Clearly, by symmetry of $\succsim$, we must have $\rho_i = \rho_j$, $i, j = 1, ..., n$ so that defining $\theta = -\rho_i / \rho$, we obtain (3). The claim then follows from Proposition 1. ||

The (total) additivity of $\succsim$ is, however, hardly an innocuous assumption in the present context. Indeed, as observed in Proposition 3, along with the continuity and negative interdependence, this assumption entails that the marginal external effect (the status) imposed by the rest of the society on an individual is completely independent of the wealth of the individual. But this of course go against the evident intuition that *one becomes less concerned with others as she gets richer.*[17] It then appears that there are strong intuitive grounds for studying interdependent preferences which are not (totally) additive.

[17] Put differently, negatively interdependent additive preferences exhibit *loss neutrality* and are subject to *constant sensitivity* to losses and gains of income (cf. Tversky and Kahneman, 1991).

This particular objection to additivity has, of course, no bearing on postulating that the imposed external effect of the society is additive. In fact, this seems quite reasonable in our context. Consider a situation where $(a, y) \succsim (a, x)$ so that the individual prefers the externality induced by the subdistribution y over the external effect that is induced by x. Then, so long as the individual is not at all concerned with the source of the external effect imposed on her by the society, and in addition, her income stays the same, it is quite reasonable that she would prefer the external effect invoked by the subdistribution $y + z$ over that induced by $x + z$.[18] This leads us to the following *partial additivity* postulate which is also widely used in other contexts:

Axiom A. (Additivity with respect to the External Effect) Let $\succsim \in \mathfrak{P}$ and $a > 0$. For any $x, y \in \mathbf{R}^n_{++}$,

$$(a, y) \succsim (a, x) \quad \text{implies} \quad (a, y + z) \succsim (a, x + z)$$

for all $z \in \mathbf{R}^n$ such that $y + z, x + z \in \mathbf{R}^n_{++}$.

As one would expect, the result one would obtain with this axiom is a *partially additive* representation of negatively interdependent preferences:

Theorem 1. *If $\succsim \in \mathfrak{P}$ is a negatively interdependent preference relation which satisfies Axioms CD and A, then there exists a continuous $F : \mathbf{R}^2_{++} \to \mathbf{R}$ which is strictly increasing (decreasing) in the first (second) argument such that*

$$(a, y) \succsim (b, x) \quad \textit{if and only if} \quad F\left(a, \sum_{i=1}^{n} y_i\right) \geq F\left(b, \sum_{i=1}^{n} x_i\right) \tag{4}$$

for all $(a, y), (b, x) \in \mathbf{R}^{1+n}_{++}$.

Theorem 1 may be thought of as an interesting result in that it clarifies the nature of "status" as perceived by an individual whose preferences satisfy negative interdependence and Axioms CD and A. Clearly, such an individual decides on her "relative standing" in the society by somehow comparing her income to a reference level of income which is derived from the total income of others. (It is in this sense we claim that Theorem 1 paves way towards 'deriving' the relative income hypothesis.) For instance, Theorem 1 eliminates the possibility that the subject individual is in fact concerned with her *relative rank* that she occupies in the society (as in Frank, 1984, and Robson, 1992). Axiom A, therefore, brings an important discipline to the model at hand. While we shall use this postulate in the rest of the paper, it must be clear that future work should focus on

[18] This interpretation is analogous to the interpretation of *additivity* of preferences for income distributions as a postulate of *independence of income source*; see Weymark (1981, p. 418). The property we have in mind simply states that if in two income distributions (which specify the same level of income for the subject agent) the incomes of others from all but one type of income are the same, then the judgement that one distribution imposes more external effect on the agent than the other is determined by comparing the incomes of the constituents of the society with respect to the *variable* source.

replacing this partial additivity property with alternative axioms which are capable of identifying various other types of "status" or "aspiration" concepts.

Theorem 1 is not, however, a complete characterization result; one may show that a preference relation of the form given in (4) need not be negatively interdependent. Moreover, the class of all well-being orderings specified in this result is quite large, so while Theorem 1 may be thought of as reporting an interesting observation on its own right, its applicability is clearly limited. We shall therefore study two refinements of this result by considering some other properties which are likely to prove suitable in potential applications of the theory.

5 Further Refinements

5.1 Negatively Interdependent Relative Preferences

Put informally, we say that an individual's preferences are *relative* whenever she is indifferent between two income distributions so long as her relative position (as measured by the level of her income in reference to that of the rest of the society) in the distributions are identical. In the theory of inequality measurement for instance, this is usually formalized as follows.

Definition 3. The preference relation $\succsim \in \mathfrak{P}$ is said to be **relative** if, for any $(a, y) \in \mathbf{R}^{1+n}_{++}$,

$$(a, y) \sim (\lambda a, \lambda y) \quad \text{for all } \lambda > 0.$$

A number of authors have indeed maintained that individual well-being orderings are completely relative, and claimed that an individual evaluates her income level solely by comparing it to that of the others. A well-known example is of course (an extreme version of) *the relative income hypothesis* of Duesenberry (1949). Another widely cited example is the cross-country comparisons of Easterlin (1974) who found that the correlation between the average self-rating of happiness per country and per capita income is not significantly different from zero. Kapteyn (1977) and Kapteyn et al. (1980), on the other hand, develop a model of preference formation where agents evaluate their earnings solely by their ranking in the (perceived) income distribution. (See Kapteyn and Wansbeek (1985) for a survey of the related studies.) Moreover, Kapteyn and Wansbeek (1982) and Van de Stadt et al. (1985) provide empirical evidence in support of the relativity hypothesis.

While it is clear that whether an individual's well-being is relative or not is an empirical issue in the final analysis, it is interesting to note here that there is only one negatively interdependent relative preference relation which satisfy our previous axioms. As one might expect, this is the ordering $\succsim_{\text{Rel}}$ introduced in Example 1.

Proposition 4. $\succsim \in \mathfrak{P}$ *is a negatively interdependent relative preference relation which satisfies Axioms CD and A if, and only if,*

$$(a, y) \succsim (b, x) \quad \textit{if and only if} \quad \frac{a}{\mu(a, y)} \geq \frac{b}{\mu(b, x)} \tag{5}$$

for all $(a, y), (b, x) \in \mathbf{R}^{1+n}_{++}$.

Proof. Suppose that $\succsim \in \mathfrak{P}$ satisfies the hypotheses of the proposition. By Theorem 1, there exists a continuous $F : \mathbf{R}^2_{++} \to \mathbf{R}$ which is strictly increasing (decreasing) in the first (second) argument such that (4) holds for all $(a, y), (b, x) \in \mathbf{R}^{1+n}_{++}$. By relativity, F must be homogeneous of degree zero, and hence $(a, y) \succsim (b, x)$ holds if and only if $a/\sum_{i=1}^n y_i \geq b/\sum_{i=1}^n x_i$ (which holds if and only if $\frac{a}{\mu(a,y)} \geq \frac{b}{\mu(b,x)}$) for all $(a, y), (b, x) \in \mathbf{R}^{1+n}_{++}$. To complete the proof, therefore, all we need to do is to show that the preference relation defined in (5) is, in fact, negatively interdependent. But this is already proved in Example 1. ||

The interpretation of the preference relation defined in (5) is obvious: the individual prefers the income distribution (a, y) to the income distribution (b, x) if, and only if, her *relative income* (that is, the ratio of her income to the average income) in (a, y) is higher than her relative income in (b, x). It is interesting to note that this relation is a special case of the interdependent preference relations formulated by Duesenberry (1949, p. 35), Easterlin (1974, p. 112), and Kapteyn and Wansbeek (1980, p. 139).

In passing, we note that the preference relation $\succsim_{\text{Rel}}$ constitutes an extreme alternative to the independent preferences where one is concerned only with the absolute level of income that she has. It may thus be used as a benchmark in checking the robustness of the results obtained in the standard economic models with respect to the relaxation of the usually implicit "independence" assumption. Indeed, if such a result remains intact when independent preferences of the agents are replaced with $\succsim_{\text{Rel}}$, then its formal support will be considerably improved. If, on the other hand, a particular finding fails to hold when individuals behave according to $\succsim_{\text{Rel}}$, then it becomes clear that the validity of the result would be suspect in an environment where the individuals exhibit at least some degree of interdependence.[19]

5.2 Negatively Interdependent Homothetic Preferences

Although there may be reasons to argue that individual well-being is a completely relative notion, there are of course strong counter-arguments. Consider the following two income distributions: (a, y) and $(10a, 10y)$. Although the relative income of the subject individual is trivially the same in both distributions, anyone who believes that the level of income that goes to one's pocket is also important to a person would argue that the individual is better off in the latter distribution. The strength of the argument becomes clearer if one thinks of the agent in question as a very poor person who can hardly make the ends meet. It seems obvious that, other things being constant, this agent would be better off when

[19]As noted earlier, the literature provides a variety of economic contexts where some well-known results are overturned when a certain degree of interdependence is introduced to the model. (See, for example, Boskin and Sheshinski, 1978, Oswald, 1983, and Akerlof and Yellen, 1990.) Nevertheless, it is conceivable that there would be some cases where the findings are robust with respect to the introduction of interdependence, and even with respect to the complete relativity of the well-being orderings. One such example is provided by Mitra, Ok and Koçkesen (1996) within the theory of voting over income taxes where it is shown that some standard results of the theory remain valid when the agents engage in voting according to $\succsim_{\text{Rel}}$ (and hence according to any negatively interdependent homothetic preference relation; see Subsection 5.2).

she makes ten times more money than she does now, even though so does everybody else in the society. Of course, this argument does not say that the individual's preferences are not interdependent, but rather that she attaches a positive welfare weight to "the own income effect" in addition to "the external effect".[20]

This intuition leads one to posit the following axiom:

Axiom AIE. (Absolute Income Effect) Let $\succsim \in \mathfrak{P}$. For all $(a,y) \in \mathbf{R}^{1+n}_{++}$,

$$(\lambda a, \lambda y) \begin{cases} \succ (a,y), & \text{if } \lambda > 1 \\ \\ \prec (a,y), & \text{if } \lambda < 1. \end{cases}$$

A preference relation that satisfies Axiom AIE is of course not relative, but it is possible to formulate weaker relativity properties which would not contradict this reasonable axiom. The following definition provides one such compromisation:

Definition 4. The preference relation $\succsim \in \mathfrak{P}$ is said to be **homothetic** if, for any $(a,y),(a,x) \in \mathbf{R}^{1+n}_{++}$ and all $\lambda > 0$

$$(a,y) \sim (b,x) \quad \text{if and only if} \quad (\lambda a, \lambda y) \sim (\lambda b, \lambda x).$$

Homotheticity demands that the indifference between two income distributions be preserved in response to an equiproportional change in all incomes. It is, therefore, a strictly weaker requirement than relativity (and in fact, it is compatible with Axiom AIE). Nevertheless, homotheticity still carries an intuitive sense of "relativity," and moreover, it is a property that is widely used in the theories of individual choice, interpersonal comparability of well-being, and income inequality measurement.

Another motivation for the homotheticity postulate is that it provides a compromisation between relativity and Axiom AIE. Indeed, while AIE requires one to put at least some emphasis on the absolute income, homotheticity (when combined with some of the properties introduced earlier) acts as a checking device not allowing a preference relation that satisfies AIE to be completely ignorant of the individual's relative income. The following result illustrates the case in point.

Theorem 2. $\succsim \in \mathfrak{P}$ *is a negatively interdependent homothetic preference relation which satisfies Axioms CD, A, and AIE if, and only if, there exists a continuous and strictly increasing* $f : (0, n+1) \to \mathbf{R}_{++}$ *such that, for all* $(a,y),(b,x) \in \mathbf{R}^{1+n}_{++}$,

$$(a,y) \succsim (b,x) \quad \textit{if and only if} \quad af\left(\frac{a}{\mu(a,y)}\right) \geq bf\left(\frac{b}{\mu(b,x)}\right). \tag{6}$$

[20]Frank (1985, pp. 32) is very clear on this point: "... the conclusion that absolute income does not matter at all appears just as spurious as the notion that absolute income is the *only* income concept that matters. Granted, ... people tend to feel dissatisfied in proportion to how far their incomes fail to match those of their peers. But that does not mean that people would be indifferent if everyone's income suddenly became twice what it is today. After all, people are in competition not only with one another but with the external environment as well."

We maintain that Theorem 2 provides a concrete compromisation between the two extreme cases we have considered so far, the standard independent preference relations which focus only on the "own income effect," and the relative preference relations which are concerned only with one's "relative position" in the society (as measured by the relative income). It is obvious that, a preference relation that is defined by (6) cannot neglect either of these effects. It is precisely in this sense that we view any such well-being ordering as a *compromise* preference relation.[21]

In passing, we note that Theorem 2 characterizes a *class* of negatively interdependent preferences, and thus, it leaves some room for choice in economic applications. For instance, the characterized class is rich enough to include several *loss averse* preference relations which exhibit *diminishing sensitivity* with the average income acting as the reference point (cf. Tversky and Kahneman, 1991).[22]

6 Extensions

6.1 An Extension to the Case of Many Commodities

In this subsection, we shall provide an extension of our previous development to a case where there are more than one good that the individuals are interested in. For expositional purposes, we shall assume that there are two goods in the economy, but generalization to n-good case is straightforward. Moreover, for brevity, we shall only provide sketches of the formal arguments.

Let $\mathbf{R}^{2m}_{++}$ be the space of all distributions of the two commodities in a society with population $m \geq 2$. By a preference relation $\succsim$ of an individual in this case we mean a complete preorder on $\mathbf{R}^{2(1+n)}_{++}$ (where $n \geq 1$ is arbitrary). The interpretation of the notation

$$\left([a^1, y^1], [a^2, y^2]\right) \succsim \left([b^1, x^1], [b^2, x^2]\right)$$

for any $(a^i, y^i), (b^i, x^i) \in \mathbf{R}^{1+n}_{++}$, $i = 1, 2$, must be straightforward: the individual is better off at the distribution where her endowment vector is (a^1, a^2) and the agent i's endowment vector is (y^1_i, y^2_i), than at the distribution $([b^1, x^1], [b^2, x^2])$. Naturally, we assume that $\succsim$

[21]To derive this point home, let $\alpha > 0$ and choose $f(t) = t^\alpha$, $0 < t < n + 1$ in (6). Clearly, this preference relation would be represented by the utility function $u(a, y) = \log a + \alpha \log\left(\frac{a}{\mu(a,y)}\right)$ for all $(a, y) \in \mathbf{R}^{1+n}_{++}$, and is thus a weighted average of independent and relative preference relations (cf. Boskin and Sheshinski, 1978, and Tuomala, 1990). As α gets smaller, the "independence" of this particular preference relation increases, and as α gets larger, the "relative income effect" becomes more pressing.

[22]Of course, this does not mean that Theorem 2 has no eliminating power. For example, one can easily show that the preference relations specified in Abel (1990), and Ljungqvist and Uhlig (1996) cannot be of the form given in (6). (In the case of Abel (1990), we have in mind the utility function $u(a, y) = \frac{1}{1-\alpha}\left(\frac{a}{\mu(a,y)}\right)^{1-\alpha}$ defined on $\mathbf{R}^{1+n}_{++}$ for some $\alpha > 1$; see Gali, 1990, p. 7.) On the other hand, the parametric specification of Gali (1990) may or may not be of form (6) (i.e. satisfy all the postulates of Theorem 2) depending on the values of its parameters.

is strictly increasing in the first and $(m+1)$th arguments, and is symmetric in the levels of consumption of the rest of the society.

The analysis is considerably simplified by assuming that the levels of satisfaction induced by goods 1 and 2 are independent from each other in the sense formalized below.

Definition 5. The preference relation $\succsim$ on $\mathbf{R}_{++}^{2(1+n)}$ is said to be **separable** if

$$\left([a^1,y^1],\omega\right) \succsim \left([b^1,x^1],\omega\right) \quad \text{if and only if} \quad \left([a^1,y^1],\omega^*\right) \succsim \left([b^1,x^1],\omega^*\right)$$

and

$$\left(\omega,[a^2,y^2]\right) \succsim \left(\omega,[b^2,x^2]\right) \quad \text{if and only if} \quad \left(\omega^*,[a^2,y^2]\right) \succsim \left(\omega^*,[b^2,x^2]\right)$$

for all $(a^i,y^i),(b^i,x^i),\omega,\omega^* \in \mathbf{R}_{++}^{1+n}$, $i=1,2.$[23]

Given a separable preference relation $\succsim$ on $\mathbf{R}_{++}^{2(1+n)}$, we may define the ordering $\succsim_1$ on $\mathbf{R}_{++}^{1+n}$ as

$$\left(a^1,y^1\right) \succsim_1 \left(b^1,x^1\right) \quad \text{if and only if} \quad \left([a^1,y^1],\omega\right) \succsim \left([b^1,x^1],\omega\right)$$

for an arbitrarily chosen $\omega \in \mathbf{R}_{++}^{1+n}$. Clearly, the separability of $\succsim$ ensures that $\succsim_1$ is independent of the choice of ω. We define the relation $\succsim_2$ on $\mathbf{R}_{++}^{1+n}$ similarly.

Now, assume that $\succsim$ is separable and let $\succsim_1$ be a negatively interdependent relative preference relation which satisfies Axioms CD and A. Let us first consider the case where the subject individual's preferences are *independent* with respect to the second good, that is, between two distributions which differ only in the amount of the second good specified for her, she always prefers the one which gives her more of good 2. It then follows (from Proposition 4 and the separability property) that, for all $(a^i,y^i),(b^i,x^i) \in \mathbf{R}_{++}^{1+n}$, $i=1,2$,

$$\left([a^1,y^1],[a^2,y^2]\right) \succsim \left([b^1,x^1],[b^2,x^2]\right) \text{ if and only if } G\left(\frac{a^1}{\mu(a^1,y^1)},a^2\right) \geq G\left(\frac{b^1}{\mu(b^1,x^1)},b^2\right)$$

for some strictly increasing and continuous $G:(0,n+1)\times\mathbf{R}_{++}\rightarrow\mathbf{R}$. (Of course, the converse of this proposition is easily checked to hold true as well.)

This seems like a useful representation of the preferences of an individual who cares only about her relative wealth with respect to the first good, and only about her absolute level of consumption of the second good. If the first good is "income" (or a composite commodity) and the second good is "leisure," for instance, the individual in question would be portrayed as adhering to the relative income hypothesis while not being interested at all in how much other members of the society actually work.

However, there is no *a priori* reason why an individual should have independent preferences with respect to one commodity while she has interdependent preferences

[23] While this assumption may or may not be acceptable depending on the context within which "interdependence" plays a role, it is interesting enough to warrant its study. We shall see in this section that such a separability requirement leads one to a very precise representation result.

with respect to another commodity.[24] Indeed, so long as one entertains the presumption that individuals desire to establish a better standing relative to the others in general, it would be natural to postulate that they might well behave in a status-seeking manner with respect to both of the goods. In any event, our present development is capable of producing representations of such preferences as well.

Proposition 5. **(i)** *If $\succsim$ is separable, and both $\succsim_1$ and $\succsim_2$ are negatively interdependent relative preference relations which satisfy Axioms CD and A, then, and only then, there exists a continuous and strictly increasing $G : (0, n+1)^2 \to \mathbf{R}$ such that $([a^1, y^1], [a^2, y^2]) \succsim ([b^1, x^1], [b^2, x^2])$ if and only if*

$$G\left(\frac{a^1}{\mu(a^1, y^1)}, \frac{a^2}{\mu(a^2, y^2)}\right) \geq G\left(\frac{b^1}{\mu(b^1, x^1)}, \frac{b^2}{\mu(b^2, x^2)}\right)$$

for all $(a, y), (b, x) \in \mathbf{R}^{1+n}_{++}$.

(ii) *If $\succsim$ is separable, and both $\succsim_1$ and $\succsim_2$ are negatively interdependent homothetic preference relations which satisfy Axioms CD, A and AIE, then, and only then, there exist continuous and strictly increasing $g^i : (0, n+1) \to \mathbf{R}_{++}$, $i = 1, 2$, and continuous and strictly increasing $G : \mathbf{R}^2_{++} \to \mathbf{R}$ such that $([a^1, y^1], [a^2, y^2]) \succsim ([b^1, x^1], [b^2, x^2])$ if and only if*

$$G\left(a^1 g_1\left(\frac{a^1}{\mu(a^1, y^1)}\right), a^2 g_2\left(\frac{a^2}{\mu(a^2, y^2)}\right)\right) \geq G\left(b^1 g_1\left(\frac{b^1}{\mu(b^1, x^1)}\right), b^2 g_2\left(\frac{b^2}{\mu(b^2, x^2)}\right)\right)$$

for all $(a, y), (b, x) \in \mathbf{R}^{1+n}_{++}$.[25]

Proposition 5 is likely to be useful in studying the impact of interdependent preferences in specific economic models which posit more than one commodities. Examples include Mirleesian economies that are widely used for studying the problem of optimal income taxation (Blomquist, 1993, and Ljungqvist and Uhlig, 1996).

6.2 An Extension to the Case of Populations with Varying Sizes

We have so far focussed on individual preferences over income distributions of a fixed size. There are, however, various ways of extending the present analysis to cover individual preferences over income distributions the sizes of which are allowed to vary. We shall briefly discuss one such way in this subsection.

Let us define the set of all income distributions as

$$\mathcal{A} \equiv \bigcup_{m=1}^{\infty} \mathbf{R}^m_{++}.$$

[24]Surprisingly, however, the majority of the studies assume that this is indeed so; see, Oswald (1983), Tuomala (1990) and Ljungqvist and Uhlig (1996), for instance. One exception to this is Blomquist (1993). See Persson (1995) for a discussion of this issue.

[25]The modification of this result when $\succsim_1$ is relative and $\succsim_2$ is homothetic is straightforward.

By a preference relation in this subsection, we mean any complete preorder $\succsim$ on $\mathcal{A}$. For any such relation $\succsim$ and $n \geq 1$, the ordering

$$\succsim^{(n)} \equiv \succsim \cap \left(\mathbf{R}_{++}^{1+n} \times \mathbf{R}_{++}^{1+n}\right)$$

is called a *sector* of $\succsim$. Clearly, our development prior to Section 6 was couched in terms of arbitrary sectors of $\succsim$. Therefore, there is a natural way of applying the properties we have considered earlier to $\succsim$ by using the sectors of $\succsim$. We say that the preference relation $\succsim$ satisfies a property (applicable to relations defined on $\mathbf{R}_{++}^{1+n}$) if *all* sectors of $\succsim$ satisfy this property.

Informally put, we now need a way of "connecting" the sectors of $\succsim$ to each other. This is usually done either by specifying a notion of replication invariance or postulating a pattern of expanding the economy. Here we adopt the second route, for there seems to be natural way of specifying a pattern of expansion in our context.

Recall that in Section 4 we have identified "wealth" as the notion with respect to which "status" is perceived. Moreover, in Section 5, we have arrived at a status formulation in terms of "relative wealth." But then it appears reasonable to assert that an individual (whose preferences over income distributions of a fixed size satisfy the properties considered in Proposition 4 and Theorem 2) would be indifferent to an expansion of the society so long as her relative wealth stays the same. Therefore, a natural way of expanding the society in a way compatible with our previous development is simply to posit the indifference of the subject individual with respect to adding a new individual to this population with an income level precisely equal to the average income of the society.[26]

Put formally, we postulate the following property.

Axiom NE. (Neutral Expansion) For any $(a, y) \in \mathcal{A}$, $(a, y) \sim (a, (y, \mu(a, y)))$.

This axiom leads us to obtain precise generalizations of Proposition 4 and Theorem 2. The following is the main result of this subsection.

Theorem 3. **(i)** *$\succsim$ is a negatively interdependent relative preference relation on $\mathcal{A}$ which satisfies Axioms CD, A and NE if, and only if,*

$$(a, y) \succsim (b, x) \quad \text{if and only if} \quad \frac{a}{\mu(a, y)} \geq \frac{b}{\mu(b, x)}$$

for all $(a, y), (b, x) \in \mathcal{A}$.

[26] It may at first seem compelling that if the mean income is higher than that of the individual, she would feel "worse off" by the addition of a new agent with mean income. This may indeed be a sensible property to posit in certain environments where "status" is thought of as, say, the rank of the individual in the society. However, it seems less plausible in the present context where the reference income for an individual is deteremined as the average income. In fact, one can show that such a property is formally *incompatible* with our previous axioms.

(ii) $\succsim$ *is a negatively interdependent homothetic preference relation on $\mathcal{A}$ which satisfies Axioms CD, A, AIE and NE if, and only if, there exists a continuous and strictly increasing* $f : \mathbf{R}_{++} \to \mathbf{R}_{++}$ such that

$$(a, y) \succsim (b, x) \quad \textit{if and only if} \quad af\left(\frac{a}{\mu(a, y)}\right) \geq bf\left(\frac{b}{\mu(b, x)}\right)$$

for all $(a, y), (b, x) \in \mathcal{A}$.

7 Concluding Comments

In this paper we have studied the individual preference relations which take into account one's relative standing (status) in the society. Our primitive, which we called the *negative interdependence* postulate, has been the basic idea that individuals desire to occupy a better position than others in whatever social hierarchy they belong to. Our axiomatic development has culminated in identifying "the relative position" of an individual with her *relative income* and in fully characterizing negatively interdependent preferences with and without own income effects. In doing so, our primary aim has been to provide a formal background to utility representations already used in various applications and to offer an operational representation for future applications.

It is clear that the introduction of negative interdependence modifies the structure of a very fundamental ingredient of economic models, the individual preference relations, and thus possess a wide applicability to various branches of economic theory with the promise of potentially interesting results. We have already mentioned some of the existing applications which have produced an improved perspective with respect to some of the standard results obtained in models of optimal taxation, asset pricing, etc.. It will also be interesting to explore the implications of the negative interdependence hypothesis in some other areas such as voting over income taxes, economic growth and the theory of endogenous stratification. The merit of our study is that it prescribes a precise way of how negatively interdependent preferences can be introduced into such models.

We conclude by stressing that the present analysis and alike must be supplemented by rigorous empirical tests of the negative interdependence hypothesis. In particular, experimental studies testing whether individuals really care about their relative wealth as opposed to some other status criteria will certainly prove useful in shaping future theoretical and applied work in this area.

8 Appendix: Proofs

As usual, we think of the elements of $\mathbf{R}^m$ as *row* vectors throughout this appendix. If $y \in \mathbf{R}^m$, the *transpose* of this vector is denoted by y^T. We let $\mathbf{1}_m$ and $\mathbf{0}_m$ denote the vectors $(1, ..., 1)$ and $(0, ..., 0)$ in $\mathbf{R}^m_+$, respectively, and use the following notation for vector inequalities: for all $x, y \in \mathbf{R}^m$, $x \geq y$ iff $x_i \geq y_i$ for all i; $x > y$ iff $x \geq y$ and $x \neq y$, and $x \gg y$ iff $x_i > y_i$ for all i.

8.1 *Proof of Proposition 1*

Take any $\succsim_\theta$ with $0 < \theta < 1/(n-1)$, and assume that (1) holds for arbitrarily fixed $a > 0$ and $x, y \in \mathbf{R}^n_{++}$. By definition of $\succsim_\theta$, we have

$$x_i - \theta \sum_{j \neq i} x_j \geq y_i - \theta \sum_{j \neq i} y_j \quad \text{for all } i = 1, ..., n \tag{7}$$

so that $\sum_{i=1}^n \left(x_i - \theta \sum_{j \neq i} x_j \right) \geq \sum_{i=1}^n \left(y_i - \theta \sum_{j \neq i} y_j \right)$, that is,

$$(1 - (n-1)\theta) \sum_{i=1}^n x_i \geq (1 - (n-1)\theta) \sum_{i=1}^n y_i.$$

Therefore, $0 < \theta < 1/(n-1)$ implies that $\sum_{i=1}^n x_i \geq \sum_{i=1}^n y_i$, and hence $a - \theta \sum_{i=1}^n y_i \geq a - \theta \sum_{i=1}^n x_i$, i.e., $(a, y) \succsim_\theta (a, x)$. Of course, if one of the inequalities in (7) held strictly, by the same chain of arguments, we would obtain $(a, y) \succ_\theta (a, x)$.

To see the necessity, assume that $\theta > 1/(n-1)$. As is evident from the definition of $\succsim_\theta$, we need to show that there exist $x, y \in \mathbf{R}^n_{++}$ such that

$$(y_i - x_i) - \sum_{j \neq i} \theta (y_j - x_j) \leq 0 \quad \text{for all } i = 1, ..., n \quad \text{and} \quad \sum_{i=1}^n (y_i - x_i) > 0.$$

Therefore, all we need to show is that there exists a $z \in \mathbf{R}^n$ such that $z_i - \sum_{j \neq i} \theta z_j \leq 0$ for all i and $\sum_{i=1}^n z_i > 0$, that is, such that

$$A(\theta) z^\top \leq \mathbf{0}_n^\top \quad \text{and} \quad \mathbf{1}_n z^\top > 0 \tag{8}$$

where $A(\theta) \equiv [a_{ij}(\theta)]$ is an $n \times n$ matrix with

$$a_{ij}(\theta) \equiv \begin{cases} 1, & \text{if } i = j \\ -\theta, & \text{if } i \neq j \end{cases}.$$

Clearly, $A(\theta)$ is a symmetric matrix. Moreover, by direct computation, one can verify that $A(\theta)$ is an invertible matrix for any $\theta \neq 1/(n-1)$, with $A^{-1}(\theta) = [b_{ij}(\theta)]$ where

$$b_{ij}(\theta) \equiv \begin{cases} \dfrac{(n-2)\theta - 1}{(n-1)\theta^2 + (n-2)\theta - 1}, & \text{if } i = j \\[2ex] \dfrac{-\theta}{(n-1)\theta^2 + (n-2)\theta - 1}, & \text{if } i \neq j \end{cases}. \tag{9}$$

Now, consider the linear equation system $A(\theta) w^\top = \mathbf{1}_n^\top$. Obviously, the unique solution of this system is $w^\top = A^{-1}(\theta) \mathbf{1}_n^\top$. Therefore, by (9),

$$w_i = \sum_{j=1}^n b_{ij}(\theta) = \frac{(n-2)\theta - 1 - (n-1)\theta}{(n-1)\theta^2 + (n-2)\theta - 1} = \frac{-(1+\theta)}{(n-1)\theta^2 + (n-2)\theta - 1}.$$

But the roots of the polynomial $(n-1)\theta^2 + (n-2)\theta - 1$ is easily checked to be -1 and $1/(n-1)$. Therefore, $\theta > 1/(n-1)$ implies that $(n-1)\theta^2 + (n-2)\theta - 1 > 0$, and we may thus conclude that the only solution $A(\theta)w^{\mathsf{T}} = \mathbf{1}_n^{\mathsf{T}}$ is in $\mathbf{R}^n_{--}$. But then by Farkas' lemma, there must exists a $z \in \mathbf{R}^n$ such that $A(\theta)z^{\mathsf{T}} \leq \mathbf{0}_n^{\mathsf{T}}$ and $\mathbf{1}_n z^{\mathsf{T}} > 0$, and (8) is established.[27]

To complete the proof, let $\theta = 1/(n-1)$ for some $n \geq 2$. But choosing $x = \mathbf{1}_n$ and $y = 2\mathbf{1}_n$, we have $x_i - \sum_{j\neq i} \theta x_j = 0 = y_i - \sum_{j\neq i} \theta y_j$ for all $i = 1, ..., n$, while $\sum_{i=1}^n x_i < \sum_{i=1}^n y_i$. Therefore, for any $a > 0$,

$$(1, (a, \mathbf{1}_{n-1})) \sim_\theta (2, (a, 2\mathbf{1}_{n-1})) \quad \text{for all } i = 1, ..., n, \qquad \text{and} \qquad (a, \mathbf{1}_n) \succ_\theta (a, 2\mathbf{1}_n),$$

and we conclude that $\succsim_\theta$ is not negatively interdependent when $\theta = 1/(n-1)$. This observation completes the proof since that negative interdependence implies $\theta > 0$ is obvious.

8.2 *Proof of Proposition 2*

By Debreu's representation theorem, there exists a continuous $v : \mathbf{R}^{1+n}_{++} \to \mathbf{R}$ such that $(a, y) \succsim (b, x)$ if and only if $v(a, y) \geq v(b, x)$ for all $a, b > 0$ and $x, y \in \mathbf{R}^n_{++}$. Choose an $\hat{a} > 0$ and define $\varphi(z) \equiv v(\hat{a}, z)$ for all $z \in \mathbf{R}^n_{++}$.

Claim: For all $a > 0$, $\alpha \in R_\varphi$ and all $x, y \in \varphi^{-1}(\alpha)$, $v(a, y) = v(a, x)$.

Proof of Claim: For any $x, y \in \mathbf{R}^n_+$ such that $\varphi(y) = \varphi(x) = \alpha$, the definition of φ entails that $v(\hat{a}, y) = v(\hat{a}, x)$, i.e., $(\hat{a}, y) \sim (\hat{a}, x)$. But then, by Axiom D, $(a, y) \sim (a, x)$, and hence $v(a, y) = v(a, x)$ for all $a > 0$. ||

Now define, for any $a > 0$ and $\alpha \in R_\varphi$,

$$H(a, \alpha) \equiv v(a, z) \quad \text{for some } z \in \varphi^{-1}(\alpha).$$

By the claim above, H is well-defined on $\mathbf{R}_{++} \times R_\varphi$. Moreover, since $z \in \varphi^{-1}(\varphi(z))$ for all $z \in \mathbf{R}^n_{++}$, we have $H(a, \varphi(z)) = v(a, z)$ by the above claim and the definition of H. (5) is thus established. By the strict monotonicity of $\succsim$, H is strictly increasing in the first argument. To see that H is also strictly increasing in the second argument, let $\alpha > \beta$ for some $\alpha, \beta \in R_\varphi$, and take any $(y, x) \in \varphi^{-1}(\alpha) \times \varphi^{-1}(\beta)$. Clearly, $v(\hat{a}, y) = \varphi(y) = \alpha > \beta = \varphi(x) = v(\hat{a}, x)$ so that, by Axiom D, $(a, y) \succ (a, x)$, that is, $H(a, \alpha) = v(a, y) > v(a, x) = H(a, \beta)$ for all $a > 0$. Finally, that φ and H are continuous follows readily from the continuity of v.

8.3 *Proof of Theorem 1*

We begin with the following

[27] *Farkas' Lemma:* Let A be an $n \times n$ real matrix, and let $c \in \mathbf{R}^n_+$. Either there exists a $w \in \mathbf{R}^n_+$ such that $A^{\mathsf{T}} w^{\mathsf{T}} = c^{\mathsf{T}}$, or (exclusive) there exists a $z \in \mathbf{R}^n$ such that $Az^{\mathsf{T}} \leq \mathbf{0}_n^{\mathsf{T}}$ and $cz^{\mathsf{T}} > 0$. (See Murata, 1977, Theorem 9, p. 284.)

Lemma A. *If $\succsim \in \mathfrak{P}$ satisfies Axiom A and is negatively interdependent, then*

$$x > y \quad \textit{implies} \quad (a, y) \succ (a, x)$$

for all $a > 0$, and $x, y \in \mathbf{R}^n_{++}$.

Proof. Take any $a > 0$ and pick any $x, y \in \mathbf{R}^n_{++}$ such that $x > y$. Clearly, there exists a $\delta = (\delta_1, ..., \delta_n) > \mathbf{0}_n$ such that $x = y + \sum_{i=1}^n \delta_i e^i$ where e^i is the ith unit vector. Define

$$x^k \equiv y + \sum_{i=1}^k \delta_i e^i, \quad k = 1, ..., n.$$

We claim that $(a, y) \succsim (a, x^1)$. If $\delta_1 = 0$, we trivially have $(a, y) \sim (a, x^1)$, so assume that $\delta_1 > 0$. Now if $(a, (y_1 + \delta_1, a\mathbf{1}_{n-1})) \succsim (a, (y_1, a\mathbf{1}_{n-1}))$ held, then since $(y_1 + \delta_1, a\mathbf{1}_n)) \succ (y_1, a\mathbf{1}_n)$ (by monotonicity), negative interdependence would yield $(a, (y_1 + \delta_1, a\mathbf{1}_{n-1})) \succ (a, (y_1, a\mathbf{1}_{n-1}))$, contradiction. Therefore, by completeness of $\succsim$, we have

$$(a, (y_1, a\mathbf{1}_{n-1})) \succ (a, (y_1 + \delta_1, a\mathbf{1}_{n-1})),$$

and applying Axiom A with $z = (0, y_2 - a, ..., y_n - a)$, we obtain $(a, y) \succ (a, x^1)$. Similarly, we observe that

$$(a, x^k) \begin{cases} \succ (a, x^{k+1}), & \text{if } \delta_{k+1} > 0 \\ \sim (a, x^{k+1}), & \text{if } \delta_{k+1} = 0 \end{cases}, \quad k = 1, ..., n-1,$$

and thus by transitivity, we have $(a, y) \succ (a, x^n) = (a, x)$ since $\delta_i > 0$ for some i. ‖

Let us now take any $n \geq 2$, $a > 0$ and define $\succcurlyeq_a$ on $\mathbf{R}^n_{++}$ as

$$y \succcurlyeq_a x \quad \text{if and only if} \quad (a, y) \succsim (a, x).$$

By Axiom D, $\succcurlyeq_a = \succcurlyeq_b$ for any $a, b > 0$ so that we write $\succcurlyeq$ for $\succcurlyeq_a$. By Axiom A, $\succcurlyeq$ satisfies

$$y \succcurlyeq x \quad \text{if and only if} \quad y + z \succcurlyeq x + z \quad \text{for any } x, y, z \in \mathbf{R}^n_{++}.$$

$\succcurlyeq$ is thus continuous and additive, and hence, it must have a linear representation, i.e., there must exist $\rho_i \in \mathbf{R}$, $i = 1, ..., n$, such that $y \succcurlyeq x$ if and only if $\sum_{i=1}^n \rho_i y_i \leq \sum_{i=1}^n \rho_i x_i$ for all $x, y \in \mathbf{R}^n_{++}$. By Lemma A, $\rho_i < 0$ for all i, and by symmetry of $\succcurlyeq$, $\rho_1 = \cdots = \rho_n$. We may thus conclude that $y \succcurlyeq x$ if and only if $\sum_{i=1}^n y_i \leq \sum_{i=1}^n x_i$ for all $x, y \in \mathbf{R}^n_{++}$. By Proposition 2, therefore, there exist a continuous $\varphi : \mathbf{R}^n_{++} \to \mathbf{R}$ and a continuous and strictly increasing $H : \mathbf{R}_{++} \times R_\varphi \to \mathbf{R}$ such that, for all $a > 0$ and $x, y \in \mathbf{R}^n_{++}$,

$$H(a, \varphi(y)) \geq H(a, \varphi(x)) \quad \text{if and only if} \quad \sum_{i=1}^n y_i \leq \sum_{i=1}^n x_i$$

so that $\varphi(y) \geq \varphi(x)$ if and only if $\sum_{i=1}^n y_i \leq \sum_{i=1}^n x_i$. There must then exist a strictly decreasing $\zeta : R_\varphi \to \mathbf{R}$ such that $\varphi(z) = \zeta\left(\sum_{i=1}^n z_i\right)$ for all $z \in \mathbf{R}^n_{++}$. Of course, since φ is continuous, so is ζ. Therefore, (4) is established by setting

$$F(a, \alpha) \equiv H(a, \zeta(\alpha)) \quad \text{for all } a > 0 \text{ and } \alpha \in R_\varphi.$$

That F is continuous and strictly increasing (decreasing) in the first (second) component is immediate from the corresponding properties of H and ζ.

8.4 *Proof of Theorem 2*

We shall first prove the necessity of the axioms. By Theorem 1, there exists a continuous $F : \mathbf{R}^2_{++} \to \mathbf{R}$ which is strictly increasing (decreasing) in the first (second) argument such that (4) holds for all $(a, y), (b, x) \in \mathbf{R}^{1+n}_{++}$. By Axioms AIE and H, respectively, we have

$$F(\lambda a, \lambda t) \begin{cases} > F(a,t), & \text{if } \lambda > 1 \\ \\ < F(a,t), & \text{if } \lambda < 1 \end{cases} \tag{10}$$

and

$$F(a,t) = F(b,t') \quad \text{if and only if} \quad F(\lambda a, \lambda t) = F(\lambda b, \lambda t') \tag{11}$$

for all $a, b, t, t' > 0$. We wish to show that there exists a strictly increasing $h : R_F \to \mathbf{R}$ such that $h \circ F$ is linearly homogeneous.[28] To establish this fact, we shall proceed by means of a number of claims.

Define $r : \mathbf{R}_{++} \to \mathbf{R}$ as $r(\alpha) \equiv F(\alpha, \alpha)$.

Claim 1: r is strictly increasing and continuous on $\mathbf{R}_{++}$.

Proof of Claim 1: For any $\beta > \alpha > 0$, (10) yields $r(\beta) = F(\beta, \beta) = F(\frac{\beta}{\alpha}\alpha, \frac{\beta}{\alpha}\alpha) > F(\alpha, \alpha) = r(\alpha)$. Continuity of r is immediate from that of F. ||

Claim 2: r^{-1} is well-defined and strictly increasing on $(F(0+, 0+), \lim_{\alpha\to\infty} F(\alpha, \alpha))$.

Proof of Claim 2: r is trivially onto $(F(0+, 0+), \lim_{\alpha\to\infty} F(\alpha, \alpha))$, and it is injective by Claim 1. Thus r^{-1} exists on $(F(0+, 0+), \lim_{\alpha\to\infty} F(\alpha, \alpha))$. Again by Claim 1, r^{-1} is strictly increasing. ||

Claim 3: For all $a, t > 0$, $F(a, t) > F(0+, 0+)$.

Proof of Claim 3: By (10),

$$F(a,t) > F\left(\frac{a}{2}, \frac{t}{2}\right) > \cdots > F\left(\frac{a}{2^m}, \frac{t}{2^m}\right) > \cdots$$

for all $a, t > 0$ and all $m \in \mathbf{N}$, so that by letting $m \to \infty$, we obtain the desired result. ||

Claim 4: For all $a, t > 0$, there exists an $\alpha > 0$ such that $r(\alpha) = F(a, t)$.

Proof of Claim 4: If $t > a > 0$, since F is strictly increasing in the first argument, and by Claim 3, $r(t) = F(t,t) > F(a,t) > F(0+, 0+) = r(0+)$. The claim then readily follows from the continuity of r. Let us consider then any $a > t > 0$. By strict increasingness of F in the first argument and Claim 3, we have $F(a,t) > F(t,t) > F(0+, 0+)$, and therefore, by continuity of F, $F(\lambda a, \lambda t) = F(t, t)$ for some $\lambda \in (0, 1)$. By (11), therefore, we obtain $F(a, t) = F(t/\lambda, t/\lambda)$. ||

Claim 5: $r^{-1} \circ F$ is well-defined and linearly homogeneous.

Proof of Claim 6: First notice that by Claims 1 and 2, $R_r = R_F$ so that $r^{-1} \circ F$ is well-defined. To see the homogeneity, take any $a, t > 0$. By Claim 4, there exists an $\alpha > 0$ such that $r(\alpha) = F(a, t)$. Thus, for any $\lambda > 0$, $\lambda r^{-1}(F(a,t)) = \lambda r^{-1}(F(\alpha, \alpha)) = \lambda r^{-1}(r(\alpha)) =$

[28]Notice that this does not follow immediately from the homotheticity of F since F is *not* a monotonic function (i.e. it is not strictly increasing (or decreasing) in *both* of its arguments).

$\lambda\alpha$, and by using (11), $r(\lambda\alpha) = F(\lambda\alpha, \lambda t)$ so that $r^{-1}(F(\lambda\alpha, \lambda t)) = r^{-1}(r(\alpha)) = \lambda\alpha$; Claim 5 follows. $\|$

We may now easily complete the proof of theorem by defining $f : (0, n+1) \to \mathbf{R}$ as

$$f(\alpha) \equiv r^{-1}\left(F\left(1, \frac{(n+1)}{\alpha} - 1\right)\right).$$

Indeed, by Claim 5, for any such f and all $a, t > 0$,

$$af\left(\frac{a}{\frac{a+t}{n+1}}\right) = ar^{-1}\left(F\left(1, \frac{a+t}{a} - 1\right)\right) = r^{-1}(F(a,t))$$

and since r^{-1} is strictly increasing, we obtain the representation noted in (6) by applying Theorem 1. Since r^{-1} and F are continuous, and since r^{-1} is strictly increasing and F is strictly decreasing in the second argument, f is continuous and strictly increasing. Moreover, by (10), $\lambda f(a/\mu(a,y)) > f(a/\mu(a,y))$ for all $\lambda > 1$ and $(a,y) \in \mathbf{R}^{1+n}_{++}$, and therefore, $f(\alpha) > 0$ for all $\alpha \in (0, n+1)$.

As for sufficiency, we only need to check the negative interdependence, for other properties are readily satisfied. Take any continuous and strictly increasing $f : (0, n+1) \to \mathbf{R}_{++}$ and consider any $(a,y), (a,x) \in \mathbf{R}^{1+n}_{++}$ such that

$$x_i f\left(\frac{x_i}{\mu(a,x)}\right) \geq y_i f\left(\frac{y_i}{\mu(a,y)}\right), \quad i = 1, ..., n \tag{12}$$

(that is, let (1) hold with respect to the preference relation defined in (6)). Assume for contradiction that $\mu(a,x) < \mu(a,y)$ so that by the positiveness of f, (12) yields

$$\frac{x_i}{\mu(a,x)} f\left(\frac{x_i}{\mu(a,x)}\right) > \frac{y_i}{\mu(a,y)} f\left(\frac{y_i}{\mu(a,y)}\right), \quad i = 1, ..., n.$$

Since $\alpha \mapsto \alpha f(\alpha)$ is a strictly increasing mapping on $(0, n+1)$ (by strict monotonicity and positivity of f), therefore, we have $x_i/\mu(a,x) \geq y_i/\mu(a,y)$ for all i. But then $\sum_{i=1}^n x_i/\mu(a,x) \geq \sum_{i=1}^n y_i/\mu(a,y)$ which yields $\mu(a,x) > \mu(a,y)$, contradiction. We conclude that (12) entails that $\mu(a,x) \geq \mu(a,y)$, and hence in view of the strict monotonicity of f, we have $af\left(a/\mu(a,y)\right) \geq af\left(a/\mu(a,x)\right)$. Of course, if one of the inequalities in (12) held strictly, we would obtain $af\left(a/\mu(a,y)\right) > af\left(a/\mu(a,x)\right)$ by the same chain of reasoning. The proof is thus complete.

8.5 *Proof of Theorem 3*

We provide here a proof of only part (ii) of the theorem, part (i) is proved analogously.

Define the function $d : \mathcal{A}^2 \to \mathbf{R}_+$ as

$$d(x,y) \equiv \begin{cases} \displaystyle\sum_{i=1}^{n(x)} \frac{1}{2^i}\left(\frac{|x_i - y_i|}{1 + |x_i - y_i|}\right), & n(x) = n(y) \\ 1, & n(x) \neq n(y) \end{cases}.$$

It can easily be checked that d is a distance function on $\mathcal{A}$; $(\mathcal{A}, d)$ is thus a metric space. Let $\mathcal{B} \equiv \cup_{n=1}^{\infty} \mathbf{Q}_{++}^{n}$. Since a countable union of countable sets is countable, $\mathcal{B}$ is countable. But $\mathcal{B}$ is clearly dense in $\mathcal{A}$, so $(\mathcal{A}, d)$ is a separable metric space. Since for metric spaces separability is equivalent to second countability (perfect separability), we may in fact conclude that $(\mathcal{A}, d)$ is second countable.

We now claim that $\succsim$ is a continuous ordering on $\mathcal{A}$ with respect to the topology induced by the metric d. To see this, take any $(a^m, y^m) \in \mathcal{A}$, $m \in \mathbf{N}$, such that $(a^m, y^m) \succsim (b, x)$ and $(a^m, y^m) \to (a, y)$ as $m \to \infty$. By definition of d, there must exist an $M \geq 1$ such that $n(y^m) = n(y)$ for all $m \geq M$. Suppose that $n(y) \leq n(x)$ holds. By using Axiom NE and the continuity (introduced in Axiom CD), we obtain

$$\begin{aligned}(a, y) &\sim (a, y, \mu(a, y)\mathbf{1}_{n(x)-n(y)}) = \lim_{k \to \infty} \left(a^{M+k}, y^{M+k}, \mu(a^{M+k}, y^{M+k})\mathbf{1}_{n(x)-n(y)}\right) \\ &\succsim (b, x).\end{aligned}$$

The case where $n(y) > n(x)$ is similarly settled; $\succsim$ is upper semicontinuous. Since the lower semicontinuity of $\succsim$ in $(\mathcal{A}, d)$ can be analogously demonstrated, we conclude that $\succsim$ is continuous.

We may now apply Theorem II of Debreu (1954) to conclude that there exists a continuous $\Phi : \mathcal{A} \to \mathbf{R}$ such that

$$(a, y) \succsim (b, x) \quad \text{if and only if} \quad \Phi(a, y) \geq \Phi(b, x) \tag{13}$$

for all $(a, y), (b, x) \in \mathcal{A}$. Clearly, for any $n \geq 1$, the restriction of Φ to $\mathbf{R}_{++}^{1+n}$ represents the sector $\succsim^{(n)}$ on $\mathbf{R}_{++}^{1+n}$. Therefore, by Theorem 2, there exist strictly increasing and continuous $g_n : \mathbf{R}_{++} \to \mathbf{R}$ and $f_n : (0, n+1) \to \mathbf{R}_{++}$ such that

$$\Phi(a, y) = g_n\left(a f_n\left(\frac{a}{\mu(a, y)}\right)\right) \qquad \text{for all } (a, y) \in \mathbf{R}_{++}^{1+n},\ n \geq 1. \tag{14}$$

Without loss of generality, we can normalize f_ns so that $f_n(1) = 1$ for all $n \geq 1$. By using (13) and (14), we obtain

$$(a, y) \succsim (b, x) \quad \text{if and only if} \quad g_{n(y)}\left(a f_{n(y)}\left(\frac{a}{\mu(a, y)}\right)\right) \geq g_{n(x)}\left(b f_{n(x)}\left(\frac{b}{\mu(b, x)}\right)\right)$$

for all $(a, y), (b, x) \in \mathcal{A}$. But then by Axiom NE, we must have

$$g_n\left(a f_n\left(\frac{a}{\mu(a, y)}\right)\right) = g_{n+1}\left(a f_{n+1}\left(\frac{a}{\mu(a, y)}\right)\right) \tag{15}$$

for all $(a, y) \in \mathbf{R}_{++}^{1+n}$. Choosing $y = a\mathbf{1}_n$, therefore, we have $g_n(a) = g_{n+1}(a)$ for all $a > 0$, and this, in turn, yields by (15) that $f_n(t) = f_{n+1}(t)$ for all $t \in (0, n+1)$, $n \geq 1$. The proof is completed by defining $f : \mathbf{R}_{++} \to \mathbf{R}_{++}$ as

$$f(t) \equiv \begin{cases} f_1(t), & 0 < t < 2 \\ & \\ f_n(t), & n \leq t < n+1 \end{cases}, \quad n = 2, 3, \ldots,$$

and by using (13) and (14) along with the fact that $g_n = g_{n+1}$.

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