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***CATCH-22 AND KING-OF-THE-
MOUNTAIN GAMES: CYCLING,
FRUSTRATION, AND POWER***

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Catch-22 and King-of-the-Mountain Games: Cycling, Frustration, and Power

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Abstract

In his classic novel, *Catch-22* (1961), Joseph Heller describes a thoroughly frustrating situation faced by a combat pilot in WWII. This is generalized to a “generic” 2 x 2 strict ordinal game, in which whatever strategy the column player chooses, the best response of the row player inflicts on the column player a worst or next-worst outcome, and possibly vice versa. The 12 specific games subsumed by the generic game are called *catch-22 games*. These games, along with 4 *king-of-the-mountain games*, turn out to be the only games in which moving power is *effective*, based on the theory of moves (TOM): each player can induce a better outcome in a game when it possesses this power than when its opponent possesses it.

These 16 games constitute 28% of the 57 2 x 2 *conflict games*, in which there is no mutually best outcome. A specific *catch-22* game is used to model the conflict between the pilot in the Heller novel and a doctor who can certify his sanity; a different *catch-22* game is used to model medieval witch trials, in which people accused of consorting with the devil were condemned (and often executed) if they confessed, tortured if they did not. *King-of-the-mountain* games portray related situations in which there is a contest to come out on top, but the player who “loses” does not suffer as much as in a *catch-22* game.

In all these games, cycling is always possible and frequently observed; additionally, cycling often destroys focal points, including unique pure-strategy Nash equilibria. Ways to attenuate the frustration of players who continually move to try to escape inferior outcomes are discussed, including depriving them of the wherewithal to cycle and rendering uncertain their prospects of outlasting their opponents.

JEL Classification: C72 and C73. *Keywords:* Catch-22; frustration; cycles; power; focal points; theory of moves; witch trials.

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Catch-22 and King-of-the-Mountain Games: Cycling, Frustration, and Power

1. Introduction

In his classic novel, *Catch-22* (1961, p. 52), Joseph Heller describes the thoroughly frustrating situation a combat pilot faced in WWII: “If he flew them [more missions] he was crazy and didn’t have to; but if he didn’t want to he was sane and had to.” We generalize the pilot’s predicament to a class of situations that can be modeled by the following “generic” 2 x 2 strict ordinal game: whatever strategy the column player C chooses (c_1 or c_2), the best response of the row player R (r_1 or r_2) inflicts on C a worst or next-worst outcome, and possibly vice versa.¹ This game can be characterized by the following four properties, based on the *theory of moves*, or TOM (Brams, 1994):

1. Cyclicity. There is one and only one direction—clockwise or counterclockwise—in which neither player, when it has the next move, ever departs from its best outcome as the players alternately move and countermove around the matrix. Because each player must always move to try to attain this outcome, cycling in this direction is rational, making the game *cyclic*.

¹The novel, as we will argue later, offers a specific realization of the generic game, which seems roughly to fit the following dictionary definition of a catch-22: “A supposed law or regulation containing provisions which are mutually frustrating . . . ; a set of circumstances in which one requirement, etc., is dependent on another, which is in turn dependent upon the first” (*Oxford English Dictionary*, 2d ed., 1989). There is no such interdependence in another frustrating situation, called a “Hobson’s choice,” which is “the option of taking the one thing offered or nothing” (*Oxford English Dictionary*, 2d ed., 1989); it is named after Thomas Hobson (1544? - 1630), the English keeper of a livery stable, who told his customers they could “take either the horse nearest the stable door or none” (*American Heritage Dictionary of the English Language*, 3d ed., 1992).

2. Frustration for C. When it is R's turn to move during this cycling (by switching from r_1 to r_2 or from r_2 to r_1), these moves induce C's two worst outcomes. If forced to choose between them, it is rational for C to choose its next-worst outcome (call this outcome x).

3. Incentive of R to Frustrate C. R prefers x to either of the outcomes when it is its turn to move, giving R an incentive always to move to try to attain x .

4. Power of R to Frustrate C. R has *moving power*—it can continue the move-countermove process when C no longer has the wherewithal or will to continue and must, consequently, stop—forcing C to choose x .

Of the 57 distinct 2×2 strict ordinal games in which there is no mutually best outcome (*conflict games*),² 36 are cyclic and 12 of these are *catch-22s*. In 4 of these games, C can also induce a catch-22 if it, rather than R, has moving power.

The catch-22 games illustrate how frustration can arise in a dynamic setting, whereby players are free, according to TOM, to move and countermove from outcomes in 2×2 matrix games (as opposed to choosing strategies simultaneously, according to standard game theory). In sections 2 and 3, we will describe aspects of TOM relevant to the analysis of catch-22 and related games.

In section 4, we apply TOM to the catch-22 game in the Heller novel, which involves a pilot trying to avoid combat duty and a doctor who can declare him to be sane to fly or not sane to fly. Then we extend this analysis to all games that satisfy the aforementioned four conditions, proving that

²If the 21 games with a mutually best outcome are included, there are a total of 78 distinct 2×2 strict ordinal games (Rapoport and Guyer, 1966).

they are a subset of games in which moving power is *effective*: each player can induce a better outcome when it possesses moving power than when its opponent possesses it, which turns out to imply cyclicity (Theorem 1).

In section 5 we show that witches' trials in medieval times can be conceptualized as a catch-22 game different from the game in *Catch-22*. In these trials, men and women who were accused of consorting with the devil were condemned (and often executed) if they confessed, tortured if they did not. A contemporary example of this phenomenon arose after the 1997 O. J. Simpson civil trial for "wrongful death," following Simpson's acquittal in the 1995 criminal trial: the plaintiff, Fred Goldman, offered Simpson a choice between confessing to the murder of his son, Ron, and Nicole Simpson—and not paying \$33.5 million in compensatory and punitive damages that Goldman had just won in court—or not confessing and paying.³

Somewhat less frustrating, perhaps, than the catch-22 games are 4 other games in which moving power is effective that we call *king-of-the-mountain games*. In these games, which we analyze in section 6, whichever player has moving power can induce its best outcome, forcing on the other player its next-best outcome (instead of its next-worst outcome, as in catch-22 games).

We briefly discuss the class of conflicts that king-of-the-mountain games seem best to model and show that they, together with the 12 catch-22 games, exhaust the 2 x 2 cyclic games in which moving power is effective (also Theorem 1). These 16 games, which constitute 28% of all conflict

³Admitting his guilt and not paying may be interpreted as a kind of mental condemnation, whereas not admitting and paying has more the earmarks of physical deprivation. For more on Goldman's offer and Simpson's response to it (he turned it down), see "Goldman Makes an Offer; Simpson Refuses," *New York Times*, February 13, 1997.

games and 44% of the cyclic games, are precisely the games in which each player has a clear incentive to continue the cycling—if it thinks it has moving power—in order to try to come out on top.

Implications of our analysis for identifying focal points, such as pure-strategy Nash equilibria in a 2 x 2 game, are discussed. The fact that plausible dynamic rules of play, based on TOM, can lead to the annihilation of such focal points—caused by the frustration of players who continually move to try to escape inferior outcomes—illustrates not only how static equilibria may be dynamically unstable but also how a “good” focal point for one player may be “bad” for the other.

In effect, such focal points destroy each other, throwing players into a swirl. In the concluding section (7), we discuss possible ways of ameliorating this problem, including how players, caught in catch-22 and king-of-the-mountain games, might reach mutually satisfactory settlements. We also mention a few examples in which they seem to have done so.

2. Theory of Moves (TOM)

The starting point of TOM is a payoff matrix, or *configuration*, in which the order of play is not specified. In fact, players are assumed not even to choose strategies but, instead, to move and countermove from outcomes in a game of *complete information* (they know their opponent’s payoffs as well as their own).

Under one set of rules of TOM, players look ahead and use backward induction to determine the rationality of not only their moves but also those of an opponent. The backward-induction calculations assume that players do not cycle in a payoff matrix.

In the case of catch-22 and king-of-the-mountain games, however, we assume that cycling is possible and, accordingly, postulate rules of play that allow for cycling as well as the exercise of moving power. Implications of these alternative rules are developed in Brams (1994, ch. 4), which we shall summarize and illustrate in this section and section 3. First, however, we discuss the rules of play that are common to games that cycle and games that do not.

Because game theory assumes that players choose strategies simultaneously,⁴ it does not raise questions about the rationality of moving or departing from outcomes—at least beyond an immediate departure, à la Nash. In fact, however, most real-life games do not start with simultaneous strategy choices but commence at outcomes. The question then becomes whether a player, by departing from an outcome, can do better not just in an immediate or myopic sense but, rather, in an extended or nonmyopic sense.

In the case of 2×2 games, in which each of two players chooses between two strategies, TOM postulates four *rules of play*, which describe the possible choices of the players at different stages:

1. Play starts at an outcome, called the *initial state*, which is at the intersection of the row and column of a 2×2 payoff matrix.
2. Either player can unilaterally switch its strategy, and thereby change the initial state into a new state, in the same row or

⁴Strategies may allow for sequential choices, but classical game theory does not make endogenous who moves first, as TOM does, but instead specifies a fixed order of play (simultaneous or sequential). In the case of the alternative rules allowing for cycling that we apply here, however, which player moves first, and the initial state from which it moves, have no bearing on the outcome. But which player possesses moving power, and can thereby force termination of play, is critical, as we will show in section 3.

column as the initial state.⁵ The player who switches, who may be either R or C, is called player 1 (P1).

3. Player 2 (P2) can respond by unilaterally switching its strategy, thereby moving the game to a new state.
4. The alternating responses continue until the player (P1 or P2) whose turn it is to move next chooses not to switch its strategy. When this happens, the game terminates in a *final state*, which is the *outcome* of the game.

Note that the sequence of moves and countermoves is strictly alternating: first, say, R moves, then C moves, and so on, until one player stops, at which point the state reached is final and, therefore, the outcome of the game.⁶

The use of the word “state” is meant to convey the temporary nature of an outcome, before players decide to stop switching strategies. We assume that no payoffs accrue to players from being in a state unless it is the final state and, therefore, becomes the outcome (which could be the initial state if the players choose not to move from it).⁷

⁵We do not use “strategy” in the usual sense to mean a complete plan of responses by the players to all possible contingencies allowed by rules 2-4, because this would make the normal form unduly complicated to analyze. Rather, *strategies* refer to the choices made by players that define a state, and *moves and countermoves* to their subsequent strategy switches from an initial state to a final state in an extensive-form game, as allowed by rules 2-4. For another approach to combining the normal and extensive forms, see Mailath, Samuelson, and Swinkels (1993, 1994).

⁶An emendation in the rules of TOM that allows for backtracking would be appropriate in games of incomplete information, wherein players may make mistakes that they wish to rectify. For more on possible rules changes under TOM, see Brams (1994); on rules that allow for backtracking, see Willson (1997).

⁷However, players do suffer costs from moving, which is how moving power comes into play: it distinguishes which, if either, player can better endure these costs and, consequently, hold out longer (section 3).

Rule 1 differs radically from the corresponding rule of play in classical game theory, in which players simultaneously choose strategies in a matrix game, which determines an outcome. Instead of starting with strategy choices, we assume that players are already in some state at the start of play and receive payoffs from this state if they stay. Based on these payoffs, they decide, individually, whether or not to change this state in order to try to do better, which may involve either mental moves (illustrated in section 4) or physical moves (illustrated in section 5).

To be sure, some decisions are made collectively by players, in which case it would be reasonable to say that they choose strategies from scratch, either independently or by coordinating their choices. But if, say, two countries are coordinating their choices, as when they agree to sign a treaty, the most important strategic question is what individualistic calculations led them to this point. The formality of jointly signing the treaty is the culmination of their negotiations, which covers up the move-countermove process that preceded it. This is precisely what TOM is designed to uncover.

In summary, play of a game starts in a state, at which players accrue payoffs only if they remain in that state so that it becomes the outcome of the game. If they do not remain, they still know what payoffs they would have accrued had they stayed; hence, they can make a rational calculation of the advantages of staying versus moving. They move precisely because they calculate that they can do better by switching states, anticipating a better outcome if and when the move-countermove process finally comes to rest.

Rules 1–4 say nothing about what causes a game to end but only when: termination occurs when a “player whose turn it is to move next chooses not to switch its strategy” (rule 4). But when is it rational not to continue moving, or not to move in the first place from an initial state?

The rules of TOM that preclude cycling, and the return of play to the initial state, include two so-called rationality rules: a termination rule (rule 5) and a two-sidedness rule (rule 6), which lead to a definition of “nonmyopic equilibria,” based on backward induction. We shall not discuss this equilibrium concept here—it does not capture the cyclicity of catch-22 and king-of-the-mountain games. Instead, we define a different solution concept, based on alternative rules 5' and 6'; rule 5' permits players to cycle in a matrix, and rule 6' enables one player, if it possesses moving power, to terminate the cycling.

To try to model the cyclic aspect of certain conflicts, and give players the ability to make choices in which they repeat themselves (why they may do so will be considered shortly), we next define a class of games in which cycling is possible by precluding a class of games in which it is not. Rule 5' provides a sufficient condition for cycling *not* to occur:

- 5'. If, at any state in the move-countermove process, a player whose turn it is to move next receives its best payoff, it will not move from this state.

In the subsequent analysis, we will focus exclusively on 2 x 2 strict ordinal games, in which each player ranks the four possible outcomes as follows: 4 = best; 3 = next best; 2 = next worst; 1 = worst. Rule 5', which says that a player will never move from a 4-state, precludes cycling in 42 of the 78 distinct 2 x 2 strict ordinal games, 21 of which contain a mutually best (4,4) state. Excluding the latter games, there are 57 conflict games, 36 of which are cyclic, as defined by property 1 in section 1.

Because only 12 of the cyclic games (33%) are *catch-22 games*, properties 2 and 3 have bite—they confer catch-22 status on only a minority

of cyclic games. In these games, as we will see, the incentive that players have to cycle to try to attain their best outcomes creates frustration when the player with moving power uses it to force the other player to choose between its two worst outcomes.

As an illustration of one game that does not cycle and one that does, consider the two games shown in Figure 1 (these are games 22 and 35 in the

Figure 1 about here

Appendix of Brams, 1994). Starting from (4,2) in each of these games, neither player has an incentive to move, according to both standard game theory (it is a Nash equilibrium) and TOM (it is a nonmyopic equilibrium).⁸

But, in fact, there is a significant difference between these two games: game 35 is “cyclic,” whereas game 22 is not. To illustrate this distinction, first consider game 22. Cycling will not occur—in either a clockwise or counterclockwise direction—in this game because moves from any state always bring the process to a state where the player who moves next receives its best payoff of 4, making a move from this state irrational, according to rule 5'. Thus, for example,

- in a clockwise direction, the move by R from (2,1) to (1,4) gives C its best payoff, so C will not move from (1,4), as shown by the blocked arrow emanating from (1,4); and
- in a counterclockwise direction, the move by C from (2,1) to (4,2) gives R its best payoff, so R will not move from (4,2), as shown by

⁸Nash equilibria are usually defined in terms of the strategies that yield particular outcomes, not the outcomes themselves. Because only pure-strategy equilibria are defined in ordinal games, however, Nash-equilibrium strategies can be referenced by the outcomes they produce.

the blocked arrow emanating from (4,2).

Now consider game 35 in Figure 1. Although a counterclockwise move by C from (2,1) to (4,2) gives R its best payoff, preventing cycling in a counterclockwise direction—as shown by the blocked arrow emanating from (4,2)—moves in a clockwise direction never give a player its best payoff when it has the next move: R at (2,1), C at (1,3), R at (3,4), and C at (4,2) never receive payoffs of 4, making cycling in a clockwise direction possible, according to rule 5', as shown by the clockwise arrows in Figure 1.

The fact that clockwise moves around the payoff matrix of game 35 do not violate rule 5' renders this game *cyclic*. In cyclic games, it turns out, cycling can occur in only one direction—either clockwise or counterclockwise, but not both (Brams, 1994, Theorem 4.1, pp. 90-91).

Game 22, in which cycling in *both* directions runs amok of a player receiving its best payoff when it is its turn to move, makes it *noncyclic*. The 42 2×2 noncyclic games include all 12 *symmetric games* (e.g., Prisoners' Dilemma and Chicken), wherein the payoffs of the players can be arranged so that their ranks along the main diagonal are the same and those on the off-diagonal are mirror images of each other (Brams, 1994, Corollary 4.1, p. 91).

To summarize, no symmetric game is cyclic; if an asymmetric game is cyclic, cycling can go in only one direction. As we will illustrate in section 6, cyclic games can be divided into three classes—strongly cyclic, moderately cyclic, and weakly cyclic—but first we analyze moving power in these games.

3. Moving Power

In cyclic games, under what circumstances would players have an incentive to cycle to try to outlast an opponent? By “outlasting” we mean

that one (stronger) player can force the other (weaker) player to stop the move-countermove process at a state where the weaker player has the next move.

Forcing stoppage at such a state involves the exercise of moving power. One player (P1) has *moving power* if it can force the other player (P2) to stop, in the process of cycling, at one of the two states at which P2 has the next move. The state at which P2 stops, we assume, is that which P2 prefers.

Recall that rule 5' specified what players would *not* do—namely, move from a best (4) state when it was their turn to move. However, this rule did not say anything about *where* cycling would stop, which the exercise of moving power determines by enabling the player who possesses it (we assume there is at most one player who does) to break the cycle of moves.

Rule 6' ensures that there will be termination:

6'. At some point in the cycling, P2 must stop.

This is not to say that P1 will always exercise its moving power. In some games, as we shall see, it is rational for P1 to terminate play, even though it can always force P2 to stop first.

Moving power is *effective* if the outcome that a player can induce with this power is better for it than the outcome that the other player can induce. To illustrate when moving power is effective, consider game 56 in Figure 2a. The arrows shown in Figure 2a (ignore for now the distinction between the

Figures 2a and 2b about here

single and double arrows) illustrate the cyclicity of game 56 in a counterclockwise direction: starting in the upper right-hand state,

- C benefits by moving from (4,2) to (2,4);
- R does not benefit by moving from (2,4) to (1,1)—which is what we will later call an “impediment”—but it departs from a 2, not a 4, state so does not violate rule 5¹;
- C benefits by moving from (1,1) to (3,3); and
- R benefits by moving from (3,3) to (4,2).

Because no player, when it is its turn to move, ever departs from its best (i.e., 4) state, game 56 is cyclic.

To show what outcome R can induce if it has moving power, which might be thought of as greater stamina or endurance—in the sense that R can continue moving when C must eventually stop—let R’s moves (vertical, as illustrated on the left side of Figure 2a) be represented by double arrows. C, whose (horizontal) moves are represented by single arrows, must stop in the cycling at either (1,1) or (4,2), from where its single arrows emanate that indicate it has the next move. Since C would prefer to stop at (4,2) rather than (1,1), R can induce its best state of (4,2) as the outcome if it has moving power.

On the other hand, if C has moving power (right side of Figure 2a), it can force R to stop at either (2,4) or (3,3), whence its single arrows emanate that indicate it has the next move. Since R would prefer to stop at (3,3) rather than (2,4), C can induce its next-best outcome of (3,3) if it has moving power. Thus, the possession of moving power benefits the player who possesses it—compared with the other player’s possession of it—so it is effective in game 56.

This is not the case in a game in which 1 and 2 are interchanged for C in game 56, which defines game 49 in Figure 2b. Applying the foregoing

reasoning to game 49, we see that R can induce only (1,2), because C prefers this state to (4,1), the other state from which it can move.

On the other hand, C can induce (3,3), because R prefers this state to (2,4), the other state from which it can move. Hence, moving power is not effective in game 49: R *cannot* induce a better outcome when it has moving power than when C has it. Instead, moving power in game 49 is “irrelevant,” because it would be in R’s interest to stop at (3,3), even if it has moving power, rather than to force C to stop at (1,2). More generally, moving power is *irrelevant* when the outcome induced by one player is better for both.⁹

In many real-world conflicts, there may be no clear recognition of which, if either, player has moving power. In fact, there may be a good deal of uncertainty or misinformation. For example, if both players believe they can hold out longer in a game in which moving power is effective, cycling is likely to persist until one player succeeds in demonstrating its greater endurance, or both players are exhausted by the repeated cycling, which is a subject we shall return to later.

4. The Original Catch-22 Game and the Generic Game

In *Catch-22*, John Yossarian and the other men who fly combat missions in WW II have two strategies for avoiding combat duty: they can ask Doc Daneeka for psychiatric leave, or not ask him. Doc Daneeka can declare each man to be either insane or sane; a declaration of insanity would exempt a man from combat duty.

⁹There is a third possibility: moving power is *ineffective* for a player when the outcome its opponent can induce is better than the outcome the player can induce with this power. Moving power is ineffective in only 4 of the 36 cyclic games, whereas it is irrelevant in 16 games and effective in 16 games. As we will see later, 12 of the 16 games in which moving power is effective are catch-22 games; the remaining 4 games are king-of-the-mountain games.

Yossarian has two goals. His top priority is to survive the war, so he works assiduously to avoid combat duty. But once put on a combat mission, he does his best to avoid being shot down; as “the best man in the group at [taking] evasive action” after bombing runs (Heller, p. 56), Yossarian is well equipped to escape unharmed.

At the same time, Yossarian wishes to act honorably. When given a chance to escape the war by Colonels Cathcart and Korn, he says, “That’s a pretty scummy trick I’d be pulling on the men of the squadron, isn’t it?” (Heller, p. 393). Translating these thoughts into action, Yossarian declares, “I’m not making any deals with Colonel Korn . . . I’m breaking the agreement [with them to avoid combat duty], . . . which is best for Cathcart, Korn, and me, not for everyone” (Heller, pp. 405-406). Evidently, the pull of “everyone” overrides the pull of “me,” so a conscience-stricken Yossarian prefers not to bail out (figuratively speaking). In the end, however, Yossarian, searching desperately for a way out of his predicament, finds one—desertion.¹⁰

Doc Daneeka “was a very warm, compassionate man, who never stopped feeling sorry for himself” (Heller, p. 42). Lamenting his own bad luck, the doctor preferred not to have to render any medical judgment. If he had to make a judgment, he preferred to render the easiest one, which was not to buck the system. After all, “The system worked fine for everybody, especially for Doc Daneeka. . . . The only time Doc Daneeka went to the medical tent was when he began to feel that he was a very sick man” (Heller, p. 40).

¹⁰As we will see in section 5, some accused witches chose confession *after* being tortured, which they had not considered as an option previously. A catch-22, it seems, stimulates the search for new options to escape the very dilemma it poses.

We model the conflict between Yossarian (Y) and Doc Daneeka (D) by game 47, shown in Figure 3. This game is cyclic in a counterclockwise

Figure 3 about here

direction, as indicated by the arrows in the figure (we will justify shortly why we give Doc Daneeka the double arrows). Y either can ask to be relieved of combat duty or not ask. D either can declare him insane or not declare him insane. The rankings of the players for the four possible states are based on the following considerations:

I. Y asks, D declares insane: (1,4). This is the best state (4) for Yossarian, because he gets out of combat duty. Moreover, because Doc Daneeka certifies him as truly insane, leaving the squadron is not dishonorable. It is the worst state (1) for Doc Daneeka, because it requires him to render an opinion, and to file official papers, declaring Yossarian insane.

II. Y does not ask, D declares insane: (4,2). This is the next-worst state (2) for Yossarian, who, while not getting out of combat duty, at least acts honorably. Because he is declared insane, however, he is no hero when he goes willingly into battle. It is the best state (4) for Doc Daneeka, because he can show compassion without filing papers (since no request was made).

III. Y asks, D declares sane: (2,1). This is the worst state (1) for Yossarian, because not only does he not get out of combat duty but he also acts dishonorably by attempting, deceptively, to get psychiatric leave. It is the next-worst state (2) for Doc Daneeka, because he must respond

(negatively) to Yossarian's request, although he need not file papers declaring him insane.

IV. Y does not ask, D declares sane: (3,3). This is the next-best state (3) for Yossarian, because even though he must go into combat, a sane man who willingly risks death is a hero. Doc Daneeka is also quite happy (3), because, though he does not render a compassionate judgment, he does not have to file papers (since no request was made).

We assume that military regulations endow Doc Daneeka with moving power, as shown by the vertical double arrows in Figure 3. Thus, he can induce Yossarian to stop at either (4,2) or (2,1)—when it is Yossarian's turn to move—which are Yossarian's two worst states (II and III).

Yossarian makes the rational choice of II, but the process by which he reasons his way to this outcome requires him to understand Doc Daneeka's motives. These are revealed in two exchanges that Yossarian has (Heller, pp. 51-52), which provide further justification of Doc's ranking:

[Yossarian asks] "Can't you ground someone who's crazy?"
 [Doc Daneeka replies] "On sure. I have to. There's a rule saying I have to ground anyone who's crazy . . ."
 "Then ask the others, they'll tell you how crazy I am."
 "They're crazy."
 "Then why don't you ground them?"
 "Why don't they ask me to ground them?"
 "Because they're crazy, that's why."

In other words, if Yossarian chooses not to ask for psychiatric leave, Doc Daneeka concludes that he is crazy; hence, he will declare him insane, consistent with Doc's preferences in Figure 3. Yet this state (II) is the next-worst (2) for Yossarian.

If, on the other hand, Yossarian asks for psychiatric leave, he faces a quandary (Heller, p. 52):

"Is Orr crazy?"
 "He sure is," Doc Daneeka said.
 "Can you ground him then?"
 "I sure can, but first he has to ask me to, that's part of the rule."
 "And then you can ground him?"
 "No, then I can't ground him."
 "You mean there is a catch?"
 "Sure, there is a catch," Doc Daneeka replied. "Catch-22. Anyone who wants to get out of combat duty isn't really crazy."

Thus, anytime Yossarian asks for psychiatric leave, Doc Daneeka will conclude that he is sane and declare him so, consistent with Doc's preferences in Figure 3. Yet this state (III) is worst (1) for Yossarian, so Yossarian's better option is not to ask for psychiatric leave and receive a payoff of 2 in state II. Because this is Doc Daneeka's best state (4), he prefers it to either of the states (I and IV) at which he could halt the cycling; hence, Doc has no desire to halt the cycling.¹¹

Game 47 satisfies the four conditions given in section 1 and, we believe, accurately models the dilemma posed in *Catch-22*. But there are 11 other 2 x 2 games that also satisfy these conditions. We present them in Figure 4¹² and next show that they, along with 4 other games, exhaust all

Figure 4 about here

games in which moving power is effective. Surprisingly, if moving power is effective, a game is automatically cyclic, making condition 1 (cyclicity) in section 1 redundant:

¹¹The cycling is largely mental in this game, but in the witch persecutions analyzed in section 4, the moves were often physical, as we will show.

¹²To facilitate identification, the numbers of these 12 games in parentheses in Figure 4, and later of the 4 king-of-the-mountain games in Figure 6, are those given in the numbering scheme of Rapoport and Guyer (1966).

Theorem 1. *If moving power is effective, a 2 x 2 strict ordinal game is cyclic. There are exactly 16 specific games in which moving power is effective, 12 of which are catch-22 games that satisfy the four conditions in section 1.*

Proof. Consider the generic game in Figure 4. Assume moves are clockwise, as shown, but that the game is not necessarily cyclic. If R has moving power, it can induce either (x_{11}, y_{11}) or (x_{22}, y_{22}) ; without loss of generality, assume that C prefers (x_{11}, y_{11}) , so

$$y_{11} > y_{22}. \quad (i)$$

If C has moving power, it can induce either (x_{21}, y_{21}) or (x_{12}, y_{12}) ; without loss of generality, assume that R prefers (x_{21}, y_{21}) , so

$$x_{21} > x_{12}. \quad (ii)$$

Moving power will be effective if R prefers the state it can induce, (x_{11}, y_{11}) , to that which C can induce, (x_{21}, y_{21}) , or

$$x_{11} > x_{21}; \quad (iii)$$

and C prefers the state it can induce, (x_{21}, y_{21}) , to that which R can induce, (x_{11}, y_{11}) , or

$$y_{21} > y_{11}. \quad (iv)$$

Combining inequalities (ii) and (iii) yields

$$x_{11} > x_{21} > x_{12}, \quad (v)$$

which gives rise to four different orderings for R:

- I. $x_{11} = 3, x_{21} = 2, \text{ and } x_{12} = 1 \Rightarrow x_{22} = 4$
 II. $x_{11} = 4, x_{21} = 2, \text{ and } x_{12} = 1 \Rightarrow x_{22} = 3$
 III. $x_{11} = 4, x_{21} = 3, \text{ and } x_{12} = 2 \Rightarrow x_{22} = 1$
 IV. $x_{11} = 4, x_{21} = 3, \text{ and } x_{12} = 1 \Rightarrow x_{22} = 2.$

Analogously, combining inequalities (i) and (iv) yields

$$y_{21} > y_{11} > y_{22},$$

which gives rise to 4 different orderings for C:

- I. $y_{21} = 3, y_{11} = 2, \text{ and } y_{22} = 1 \Rightarrow y_{12} = 4$
 II. $y_{21} = 4, y_{11} = 2, \text{ and } y_{22} = 1 \Rightarrow y_{12} = 3$
 III. $y_{21} = 4, y_{11} = 3, \text{ and } y_{22} = 2 \Rightarrow y_{12} = 1$
 IV. $y_{21} = 4, y_{11} = 3, \text{ and } y_{22} = 1 \Rightarrow y_{12} = 2.$

Altogether, then, there are $(4)(4) = 16$ games in which moving power is effective.

Notice in these games that when it is R's turn to move (lower left-hand and upper right-hand states in the generic game), neither x_{21} nor x_{12} is R's best state (4); and when it is C's turn to move (upper left-hand and lower right-hand states in the generic games), neither y_{11} nor y_{22} is C's best state (4). Hence, the 16 games in which moving power is effective are necessarily cyclic (but, as we showed earlier, there are cyclic games in which moving power is not effective).

The easiest way to show that 12 of the 16 games are catch-22s is to single out the 4 games that are not. These are the games in which the moving-power outcome that R can induce, (x_{11}, y_{11}) , and that which C can induce, (x_{21}, y_{21}) , are at least next-best (3) for both players. They are defined by the orderings III and IV for R, and III and IV for C, which yield $(2)(2) = 4$ games.

For the remaining 12 games, condition 2 in section 1 (frustration for C—it could be either player) in the generic game is satisfied: the player with moving power (say, R) inflicts on C its two worst outcomes ($y_{11} = 2$ and $y_{22} = 1$), leading C to choose (x_{11}, y_{11}) that gives C its next-worst state. Condition 3 in section 1 [incentive of R to frustrate C by forcing the choice of (x_{11}, y_{11})] is also satisfied: that this outcome, (x_{11}, y_{11}) , is better for R than the two outcomes that C can induce, (x_{21}, y_{21}) and (x_{12}, y_{12}) , or

$$x_{11} > x_{21} \text{ and } x_{11} > x_{12},$$

is implied by inequality (v). Condition 1 (cyclicity), as we showed earlier in the proof, is implied by moving power's being effective, and condition 4 (power of R to frustrate C) is satisfied whenever R has moving power. Q.E.D.

In Figure 4, we distinguish the 8 games in which R can induce a catch-22 (class A) if it has moving power and the 4 games in which either player can induce a catch-22 if it has moving power (class B). In the class A games—which includes game 47 used to model *Catch-22*—R inflicts on C its next-worst state (2) when it exercises moving power, whereas C inflicts on R its next-best state (3) when it exercises moving power.¹³

By contrast, in the class B games, each player inflicts on its opponent its next-worst state (2) when it exercises moving power.¹⁴ In section 5 we

¹³Note that 6 of these 8 games have unique pure-strategy Nash equilibria, but R's moving power undermines all of them, resulting in a different outcome. C's moving power induces the Nash equilibrium in 4 games (38 - 41) but not two others (48 and 56). None of the 4 class II games has a pure-strategy Nash equilibrium.

¹⁴Class A and class B games can be defined in terms of the orderings in the proof of Theorem 1. The 8 class A games match (i) orderings III and IV for R against orderings I and II for C, which lead to C's two worst outcomes when R has moving power (4 games); and (ii) orderings III and IV for C against orderings I and II for R, which lead to R's two

model the witch persecutions by a class B game, although we argue that only one player in this game (the accusers) possessed moving power.

The 12 games in Figure 4 are not the only games in which moving power is effective (as proved in Theorem 1): there are 4 games (the king-of-the-mountain games), to be given in section 5, in which *each* player, when it exercises moving power, inflicts on its opponent its next-best (3) state. Invariably in these latter games, the player exercising moving power enjoys its best state (4), whereas this is not always true in either the class A or the class B catch-22 games.

5. The Witch Persecutions

In Europe during the Middle Ages, those accused of practicing witchcraft were placed in a judicial catch-22. Their accusers had a paramount goal: to accumulate wealth by accusing people of witchcraft, executing them, and confiscating their property:

So far, at length, did the madness of the furious population go in this thirst for blood and booty that there was scarcely anybody who was not smirched by some suspicion of this crime.

Meanwhile notaries, copyists, and innkeepers grew rich. The executioner . . . went clad in gold and silver (Burr, 1971, p. 13, quoting from *Gesta Trevirorum*).

Not only did the accusers desire to kill the accused witch in order to confiscate his or her property, but they also wished to provide themselves with future opportunities—usually by getting the accused witch to implicate others in his or her alleged crime.¹⁵

worst outcomes when C has moving power (4 games). (In the latter 4 games, however, we have reversed the roles of R and C in Figure 4 so that in *all* class A games it is R that inflicts on C its two worst outcomes.) The 4 class B games match orderings I and II for R against orderings I and II for C, which lead to each player's two worst outcomes when the other player has moving power.

¹⁵Confiscation of property was not the principal force driving the Salem witch trials in Massachusetts in 1632 (Konig, 1979, p. 174) but, rather, the perceived "survival of

The accused witches had three goals. First, they preferred to live rather than die at the hands of their accusers. Second, most preferred to die honorably rather than to besmirch their names by confessing to witchcraft while being killed anyway.

Knowing they would probably die, many were willing, at least at the outset, to endure torture rather than confess to their alleged crimes:

Then [the torturer] asks me, "Kinsman, how come you here" . . . I said, "I am no witch" . . . [Then] the executioner . . . put the thumb-screws on me . . . so that the blood ran out at the nails and everywhere . . . Thereafter they first stripped me, bound my hands behind me . . . Eight times did they draw me up and let me fall again, so that I suffered terrible agony (Burr, 1971, p. 26, quoting from *Beiträge zur Geschichte des Hexenwesen in Fraken*, 1883, on events that occurred in Bamberg, Germany, in 1628).

Manifestly, accused witches would have no reason to undergo such excruciating pain unless they placed their honor above a false confession.

As a third goal, accused witches preferred not to implicate others, whom they knew to be innocent, even though they were often pressed to do so by their accusers:

For not only is there in general no . . . escape, but she is also compelled to accuse others, of whom she knows no ill, and whose names are not seldom suggested to her by her examiners or by the executioner. These in their turn are forced to accuse others, and these still others, and so it goes on (Burr, 1971, p. 35, quoting Friedrich Spee, *Cautio Criminalis*, 1631).

To model the conflict between the accused witches and their accusers, consider first the strategies of each player. A person, once accused of witchcraft, could either confess or deny the charge. This person's accusers,

society," especially "at a time of profound insecurity" (Konig, 1979, p. 177) caused by, among other things, challenges to the church and French and Indian raids that threatened to erupt into war. Of the 26 people convicted of witchcraft in Salem in 1632, 19 were executed (Weisman, 1984, p. 118), compared with the thousands who were killed in witch hunts in Europe beginning in the 15th century. For a recent popular account of the Salem witch trials, see Hill (1995).

whom we treat as a single player, could either torture or not torture the accused, yielding the game shown in Figure 5. The rankings of the players

Figure 5 about here

for the four possible states are based on the following considerations:

I. Torture/Confess: (1,3). Despite the accused's confession, the accusers continue to torture him or her. Because this action would break medieval law, which had strict rules about the use of torture,¹⁶ it would bring punishment down on the accusers, making this their worst state (1). It is the next-best state (3) for the accused: whether this person lives or dies, he or she knows the accusers will suffer too.

II. Torture/Do not confess: (3,2). This is the next-best state (3) for the accusers, because their torture is "justified" by a recalcitrant suspect. It is the next-worst state (2) for the witch, who at least dies honorably without implicating others.

III. Do not torture/Confess: (4,1). This is the best state (4) for the accusers, because their allegations appear to have been justified and execution can occur immediately. It is the accused witch's worst state, because he or she dies dishonorably, implicating himself or herself as well as others without even being under duress.

¹⁶When torture is carried out, "the notary must write down everything in his record of the trial—how the prisoner is tortured, on what points he is questioned, and how he answers" (Burr, 1971, p. 12, quoting from *Malleus Maleficarum*, 1486). Later we discuss how accused witches, who confessed during torture, were later required to verify that the confession "is not due alone to the force of the torture." These "safeguards" for the accused, as we will see later, were quite empty, but they at least suggest that torturers would be penalized if they tortured a suspect after she had confessed.

IV. Do not Torture/Do not Confess: (2,4). This is the next-worst state (2) for the accusers, because they do not execute the suspect, despite his or her denial, and hence are unable to confiscate the accused's property. It is the best state (4) for the accused—tantamount to acquittal—because this person continues to live with honor and his or her possessions.

The game given in Figure 5 is game 45 in Figure 4 (with the roles of R and C reversed), which is a class II catch-22 game. We presume that only the accusers have moving power, in which case they can implement (3,2), which is quite unfortunate for the accused witch.

Evidence suggests that the players moved in cyclical fashion, as TOM predicts.¹⁷ These dynamics were buttressed by medieval law, which required that confessions obtained under torture be “verified” later without torture. But, ironically, the torture was allowed to resume if someone who confessed changed his or her mind:

And note that, if he confesses under the torture, he must afterward be conducted to another place, that he may confirm it and certify that it was not due alone to the force of the torture.

But, if the prisoner will not confess the truth satisfactorily, other sorts of tortures must be placed before him . . . (Burr, 1971, p. 12, quoting from *Malleus Maleficarum*, 1486).

Thus, the accused witch might confess in order to escape torture, but then he or she could revert to denial once the torture ceased. The torture would subsequently resume, however, yielding a cycle of physical moves.

¹⁷TOM does not make this prediction if there is a clear recognition that one player has moving power, in which case the other player should submit immediately. That this was not the case—because a “forced” confession was not acceptable, as we shall next explain—is why we often observe the exercise of moving power and a testing of wills.

The Jesuit poet, Friedrich Spee, who was a confessor of those sentenced to death for witchcraft, recognized the catch-22 nature of the accusers' moving power:

So, whether she confesses or does not confess, the result is the same. If she confesses . . . , she is executed. If she does not confess, the torture is repeated—twice, thrice, four times . . . There is no limit of duration or severity or repetition of the tortures (Burr, 1971, p. 34, quoting from Friedrich Spee, *Cautio Criminalis*, 1631).

Thus, the accused witch can never do better than his or her next-worst state, (3,2), because the accusers can move indefinitely, leaving “no stone unturned” (Burr, 1971, p. 33, quoting from Friedrich Spee, *Cautio Criminalis*, 1631), through the states which the accused prefers.¹⁸

Interestingly, the witch persecutions usually ended with confession rather than death by torture, even though the accused witches preferred denial to confession *before the torture*. There seem to be two possible explanations for this. First, the accused witches might have changed their preferences on experiencing the torture, caring less about honor and more about a quick death and, possibly, a reprieve, as some were led to believe (Burr, 1971, p. 111, quoting from *Malleus Maleficarum*, 1486).¹⁹ Second,

¹⁸If 2 and 3 are interchanged for the accused witch in game 45—under the presumption that he or she prefers “honorable death” to “accusers break law” (and are punished)—then the resulting game is 40 in Figure 4. With moving power, the accuser can again induce “honorable death,” which is now (3,3) instead of (3,2). Consequently, the accused witch does better according to this interpretation, but the moving-power outcome that the accusers can induce is the same.

¹⁹Game 45 reflects this preference on the part of the accused witch being tortured: (1,3), when the accused confesses, is better than (3,2) when the accused does not. What game 45 does not capture is the fact that once the confession is extracted—in the manner suggested by the aforementioned quotation of Friedrich Spee—and torture ceases, the outcome is not “dishonorable death,” yielding (4,1) in Figure 5. Instead, the outcome is more like a (3,3) “compromise,” in which both players get something of what they want (the accused survives with his or her honor partially intact for having endured some torture before succumbing; the accusers get their confession, and perhaps some new names, but at a cost if the inquisition takes long). In effect, the exercise of moving power

even if they continued to recognize that denial would produce an outcome they preferred, they nonetheless might have lost their ability to resist under the pain of torture, even though “it was all a lie” (Burr, 1971, p. 27, quoting from *Beiträge zur Geschichte des Hexenwesens in Franken*, 1883, on events that occurred in Bamberg, Germany, in 1628).

6. King-of-the-Mountain Games

The 4 games we call *king-of-the-mountain games* are given in Figure 6a.²⁰ Observe that in each game there is a (3,4) state that C can induce with

Figure 6 about here

moving power and a (4,3) state that R can induce with moving power. While the outcome predicted by standard game theory is the unique Nash equilibrium of (3,4)—associated with the dominant strategy of R and the best response of C—TOM predicts (4,3) as the outcome if R has moving power.

Instead of giving an extended example of a king-of-the-mountain game, and the exercise of moving power in it, we next discuss the class of situations that these games seem best to model. As depicted in the generic game in Figure 4b, R can choose to cooperate or not cooperate with C. C, in turn, can choose to hold out or not hold out against R.

in game 45 may change the choices of the players and their preferences over time—leading to a new game—but the *process* by which this new game evolves is, we would argue, well modeled by the TOM rules. Finally, it is worth noting that when the accused witches remains steadfast in their honor, the predicted outcome—their martyrdom at (3,2)—is accurate.

²⁰The appellation, “king of the hill,” is also used for these games, which are defined to be games “in which each person attempts to climb to the top of some point, as a mound of earth, and to prevent all others from pushing or pulling him off the top” (*Random House Dictionary of the English Language*, 1979).

If R is noncooperative, the outcomes are bad for both players (i.e., never better than next-worst, or 2), whereas if R is cooperative, the outcomes are good for both (3 or 4). Following standard game theory, this game is easy to solve: R will choose its dominant strategy of cooperation; anticipating this choice, C will hold out for its preferred state, resulting in the Nash equilibrium of (3,4).

But if R has moving power, according to TOM, it can force C to stop at (4,3), just as C can force R to stop at (3,4) if it has moving power. In other words, which of the Pareto-optimal outcomes will occur depends on which, if either, player can continue the move-countermove process when the other player is forced to “throw in the towel.” The latter player, it has been argued in game 37, was North Vietnam after repeated bombing campaigns by the United States in the Vietnam war (Brams, 1994, ch. 4), and Saddam Hussein after the air and ground attacks by the United States and its allies in the 1990-91 Persian Gulf war (Massoud, 1997).

These conflicts were rife with misperception and vulnerable to threats as well.²¹ But, as indicated in section 3, a simple lack of information as to which player has moving power may well lead to cycling, as each player strives to demonstrate—by continuing the move-countermove process—that it, in the end, can prevail (i.e., be the “king”).

There are some subtle differences in the four games. Call a move an *impediment* if it involves a player’s moving from a better to a worse state, as is the case in all the king-of-the-mountain games when R moves from the upper left-hand cell to the lower left-hand cell (i.e., from 3 to either 2 or 1).

²¹In particular, R can threaten D’s two worst outcomes, assuming it has “threat power” (Brams, 1994, ch. 5), to try to induce its preferred (4,3) outcome instead of the Nash equilibrium of (3,4).

In games 33 and 34, this is the only impediment, making these games *moderately cyclic*. In games 36 and 37, by comparison, there is a second impediment when C moves from the lower left-hand cell to the lower right-hand cell (i.e., from 2 to 1), making these games *weakly cyclic*.²²

Notice in the king-of-the-mountain games that the players must move through *two* states in which the players suffer their worst (1) or next-worst (2) outcomes, which is never the case in catch-22 games. Presumably, when a parent tells a child being punished that “it hurts me as much as it hurts you,” he or she is suffering at one of these states.

It is well known, of course, that such punishment does not always lead to the best outcome for the parent, nominally the more powerful figure. Likewise with such prepossessing historical figures as Julius Caesar, Napoleon Bonaparte, Adolf Hitler, Winston Churchill, Charles de Gaulle, and Margaret Thatcher, while kings and queens in all but name, each fell from grace abruptly and, sometimes, ignominiously when their power fizzled out.

7. Conclusions

It is not surprising that empirical instances of both catch-22 and king-of-the-mountain games abound because, altogether, they represent 28% of the 57 2 x 2 conflict games.²³ Although not as well-known as 2 x 2

²²Of the 12 catch-22 games, six are moderately cyclic (38 - 41, 48, and 56) and none is weakly cyclic; the remaining six games (42 - 47) are *strongly cyclic*—there are no impediments—making these games “frictionless.” It would seem that players would have the most incentive to cycle in the latter games, because a move always brings a player to an immediately better state.

²³To be sure, this theoretical percentage says nothing about the empirical relative frequency of these games. It is worth noting, however, that several of these games are used to model conflicts in Brams (1994), including two Bible stories (the exodus of the Israelites from Egypt, which involved twelve broken promises by Pharaoh about freeing the Israelites from bondage, each of which was followed by a plague visited on the

symmetric games like Prisoners' Dilemma, Chicken, and Stag Hunt, these 16 games nevertheless pose trying choices for the players, especially when viewed from a theory-of-moves (TOM) perspective.

The specific TOM perspective we offer in this paper is that of cyclic games (36 in all), in which moves in either a clockwise or a counterclockwise direction never require that a player move from its best state. But more than being cyclic, what the 12 catch-22 and 4 king-of-the-mountain games share is that moving power in them, and only in them, is effective. That is, a player does better if it possesses moving power than if its opponent possesses it in 44% of the cyclic games (moving power is irrelevant or ineffective in the remainder), so there is good reason for each player to try to outlast its opponent in these games as the players alternately move and countermove around the matrix.

Such cycling can lead to endless frustration on the part of the players,²⁴ but occasionally they may recognize the futility of cycling and resolve their differences. This seems to have occurred in the Egyptian-Israeli conflict between 1948 and 1979 as well as similar long-standing conflicts (e.g., between France and Germany after WWII, and within South Africa more recently). Although Egypt and Israel fought five wars in this period (1948, 1956, 1967, 1969-70, and 1973) at great cost to both sides, it still required considerable pressure from the United States to achieve the 1978 Camp

Egyptians by Moses and God; and Samson and Delilah, which involved three lies by Samson about the secret of his strength, each of which was followed by a thwarted attempt by Delilah and the Philistines to subdue him) and one theological game, also involving back-and-forth maneuvering, concerning belief in a superior being. Moving power is explicitly applied to the analysis of the latter game, as it is to a series of sanctions, continually imposed, lifted, and reimposed by the United States on Haiti in the 1980s and early 1990s (Simon, 1996).

²⁴For an alternative analysis of frustration, based on the concepts of nonmyopic equilibrium and threat power in TOM, see Brams (1997).

David accords that paved the way for the signing of a peace treaty between Egypt and Israel in 1979.

Likewise, similar outside pressure was exerted successfully in the South African conflict, but it has been less successful in other conflicts, including that in Northern Ireland.²⁵ On the other hand, the intervention of a major UN military force was decisive in persuading the warring sides in the former Yugoslavia to sign a peace treaty in November 1995, but only after four years of bitter conflict that cost some 250,000 lives.

Enduring international and national rivalries, in which learning can also be modeled using TOM (Maoz and Mor, 1996), take more poignant form at the organizational and personal level, as we illustrated in the fictional case of *Catch-22* and the real case of medieval witches' trials. Great frustration can build up, or destruction occur, unless there is a recognition that

- one side has moving power and, therefore, it is in the other side's interest to acquiesce in the more powerful player's preferred state; or
- neither side has a clear advantage, and some compromise can work to their mutual advantage.

In the former case, the focal point for a settlement is likely to be one-sided, but this is simply a reflection of the one-sidedness of the conflict—the two players are unequally matched.

In the latter case, it is harder to define what, if any, compromise will be achieved, because each side presumably wants to hold out as long as possible if there is still some reasonable chance that, by persisting, it can

²⁵A TOM analysis of this conflict, based on incomplete information and threat power, is given in Brams and Togman (1996/97).

improve on the outcome.²⁶ Thereby focal points that are mutually advantageous [e.g., (4,3) and (3,4) in king-of-the-mountain games]—one of which is a unique pure-strategy Nash equilibrium [(3,4) in these games]—may be destroyed by each player's striving to get still more if it can (4 rather than 3 in these games).

We see no ready solution to this problem, short of the mutual exhaustion of the players after long travail. Because cycling is rational in games in which moving power is effective, one must, to prevent cycling and its attendant frustration, deprive the players of the wherewithal that fuels the cycling. Limiting their resources by embargoes, sanctions, outside intervention, ostracism, and the like are obvious tools for this purpose, but they have not always been successful.

A less obvious means to bring such conflicts to a halt is to cast doubt that either player will "win" in the end. The more clouded the victory appears, the more the will of the combatants to continue fighting will be sapped. Undermining the players' faith in victory, especially if this victory is likely to be Pyrrhic, may well bring the two sides to a settlement that creates, rather than annihilates, focal points.

²⁶Refugee repatriation is a case in point, as modeled by TOM using moving power and threat power (Zeager and Bascom, 1996).

Figure 1
Cyclicity of Two Games

Noncyclic (Game 22)

<u>(4,2)</u>	←	(2,1)
↓		↓
(3,3)	←	(1,4)

Cyclic (Game 35)

<u>(4,2)</u>	←	(2,1)
	→	
↓↑		↓
(3,4)	←	(1,3)

Key: (x,y) = (payoff to R, payoff to C)

4 = best; 3 = next best; 2 = next worst; 1 = worst

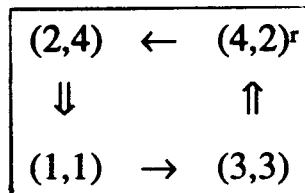
Nash equilibria underscored

Unblocked arrows indicate direction of cycling

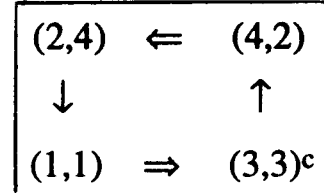
Figure 2
Moving Power in Two Cyclic Games

2a. Moving Power Is Effective in Game 56

R has moving power

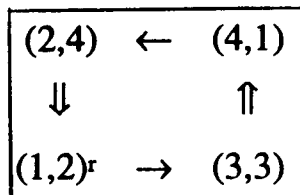


C has moving power

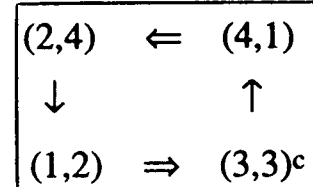


2b. Moving Power Is Irrelevant in Game 49

R has moving power



C has moving power



Key: (x,y) = (payoff to R, payoff to C)

4 = best; 3 = next best; 2 = next worst; 1 = worst

Double arrows indicate moves of player with moving power

Single arrows indicate moves of player without moving power

r = state that R can induce with moving power

c = state that C can induce with moving power

Figure 3
The Original Catch-22 (Game 47)

		Yossarian	
		<i>Ask</i>	<i>Do not ask</i>
<i>Declare insane</i>	I. No combat duty	(1,4)	← (4,2) ^d
	II. Combat duty and insanity, showing no heroism	↓	↑
Doc Daneeka			
<i>Declare sane</i>	III. Sane but dishonorable	(2,1)	→ (3,3)
	IV. Combat duty and sanity, showing heroism		

Key: $(x,y) = (\text{payoff to R, payoff to C})$

4 = best; 3 = next best; 2 = next worst; 1 = worst

Double arrows indicate moves of player with moving power

Single arrows indicate moves of player without moving power

d = state that player with moving power (Doc Daneeka) can induce

Figure 4
Generic Catch-22 Game and 12 Specific Games It Subsumes

Generic Game

		C	
		c_1	c_2
R	r_1	(x_{11}, y_{11})	\rightarrow (x_{12}, y_{12})
		\uparrow	\downarrow
	r_2	(x_{21}, y_{21})	\leftarrow (x_{22}, y_{22})

Class A: R can induce a catch-22 with moving power

38 (51)	39 (52)	40 (53)	41 (54)
$(\underline{3,4})^c$ $(4,2)^r$ $(2,1)$ $(1,3)$	$(\underline{3,4})^c$ $(4,2)^r$ $(1,1)$ $(2,3)$	$(\underline{3,3})^c$ $(4,2)^r$ $(2,1)$ $(1,4)$	$(\underline{3,3})^c$ $(4,2)^r$ $(1,1)$ $(2,4)$
46 (70)	47 (71)	48 (57)	56 (56)
$(3,4)^c$ $(2,1)$ $(4,2)^r$ $(1,3)$	$(3,3)^c$ $(2,1)$ $(4,2)^r$ $(1,4)$	$(\underline{2,3})$ $(4,2)^r$ $(1,1)$ $(3,4)^c$	$(\underline{2,4})$ $(4,2)^r$ $(1,1)$ $(3,3)^c$

Class B: R and C each can induce a catch-22 with moving power

42 (73)	43 (74)	44 (75)	45 (76)
$(2,4)^c$ $(4,1)$ $(3,2)^r$ $(1,3)$	$(2,4)^c$ $(3,1)$ $(4,2)^r$ $(1,3)$	$(2,3)^c$ $(4,1)$ $(3,2)^r$ $(1,4)$	$(2,3)^c$ $(3,1)$ $(4,2)^r$ $(1,4)$

Key: $(x,y) = (\text{payoff to R}, \text{payoff to C})$
 4 = best; 3 = next best; 2 = next worst; 1 = worst
 Nash equilibria underscored
 Arrows indicate direction of the cycling
 r = state that R can induce with moving power
 c = state that C can induce with moving power

Figure 5
The Witches' Game (Game 45)

		Accused Witch	
		<i>Confess</i>	<i>Do not confess</i>
Accusers	<i>Torture</i>	I. Accusers break law (1,3) ↓	II. Honorable death (3,2) ^a ↑
	<i>Do not torture</i>	III. Dishonorable death (4,1)	IV. Acquittal (2,4)

Key: (x,y) = (payoff to R, payoff to C)

4 = best; 3 = next best; 2 = next worst; 1 = worst

Double arrows indicate moves of player with moving power

Single arrows indicate moves of player without moving power

a = state that player with moving power (Accusers) can induce

Figure 6

6a. 4 King-of-the-Mountain Games

33 (19)	34 (20)	36 (49)	37 (50)
<u>(3,4)^c</u> (4,3) ^r	<u>(3,4)^c</u> (4,3) ^r	<u>(3,4)^c</u> (4,3) ^r	<u>(3,4)^c</u> (4,3) ^r
(1,2) (2,1)	(2,2) (1,1)	(2,1) (1,2)	(1,1) (2,2)

6b. Generic King-of-the-Mountain Game

		C	
		<i>Hold out</i>	<i>Do not hold out</i>
R	<i>Cooperate</i>	I. C prevails <u>(3,4)^c</u>	II. R prevails (4,3) ^r
		↓	↑
	<i>Do not cooperate</i>	III. Bad for both (1 or 2, 1 or 2)	IV. Bad for both (1 or 2, 1 or 2)

Key: (x,y) = (payoff to R, payoff to C)
 4 = best; 3 = next best; 2 = next worst; 1 = worst
 Nash equilibria underscored
 Arrows indicate direction of the cycling
 r = state that R can induce with moving power
 c = state that C can induce with moving power

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