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Psychological Expected Utility Theory and Anticipatory Feelings

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Abstract

We extend expected utility theory to situations in which the prizes include feelings about living with uncertainty. We provide two examples to show the impact of these anticipatory feelings on decision making.

Key Words: anticipation, expected utility, gambling.

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1 Introduction

We all experience feelings, such as fear, hopefulness, anxiety, and suspense, that are related to uncertainty about the future. While the importance of such anticipatory emotions has long been recognized in psychological theory, they have not been incorporated into the economic theory of decision making under uncertainty. This is unfortunate, since they may have a profound impact on decision making. An individual may commit resources to future projects to enhance feelings of anticipation. An individual with a nervous disposition may avoid risky situations that trigger excessive anxiety. Conversely, some individuals may seek out risk in order to enhance feelings of

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suspense. This paper extends expected utility theory to cover these and other emotional responses to future uncertainty.

As classically presented, expected utility theory is atemporal, and employs a very narrow interpretation of the prize space. The only pure prizes allowed are those in which all external uncertainty has been resolved. The substitution axiom is assumed to apply to lotteries over these deterministic outcomes. In this way, the theory implicitly rules out all feelings that might arise in periods prior to the resolution of uncertainty.[1] In fact the date of resolution of uncertainty is not even encoded in standard applications of expected utility theory. One implication is that an expected utility maximizer is of necessity indifferent concerning the date of resolution of uncertainty. This is clearly false both for an anxious individual who would like uncertainty to be resolved as soon as possible, and for the reader of a mystery novel who would resent being told who did it on page 3.

It was precisely the atemporal nature of expected utility theory that motivated Kreps and Porteus to introduce their model of preferences over the date of resolution of uncertainty, and we borrow heavily from their approach (Kreps and Porteus [1978], [1979a]). Given our desire to capture such diverse phenomena as anticipatory excitement and fear, our model allows for a far more general relationship between evolving uncertainty and personal preferences than do Kreps and Porteus. We introduce a very general space of personal psychological states to capture the emotional content of the situation under consideration, such as excitement, hope, or fear. We see these psychological states as the real prizes that are at stake in any underlying lotteries, and we apply the standard substitution axiom to lotteries over such personal states.

We believe that our approach of expanding the prize space and retaining the substitution axiom is the natural follow up to some suggestions of Samuelson [1952] and Machina [1989]. Machina wrote:

> "For my part, I will grant that separability[2] may be rational provided the descriptions of consequences are sufficiently deep to incorporate any relevant emotional states, such as disappointment (e.g. at having won $0 when you might have won $5 million), regret ..... and so on." (Machina [1989] p.1662).

[1]The inability to consider feelings in prior periods forms the heart of the provocative critique of Pope [1985].

[2]Separability is the same as the substitution axiom.

It is precisely this program of incorporating relevant emotional states that we formulate in this paper. The first two sections of the paper provide general machinery allowing the model-builder to capture whatever features of a given lottery might generate personal feelings worthy of attention. In section 2 we present the general psychological expected utility model in its most abstract form. In section 3 we present a special class of models with additional structure designed to make the framework more readily applicable, and define a broad class of associated decision problems.

In section 4, we apply the psychological expected utility framework to analyze the connection between anticipatory excitement and commitment, in the context of a decision on an upcoming vacation. The model illustrates that it may be worthwhile to invest resources up front to commit to a decision that will yield anticipatory excitement. In section 5, we provide a second application in which we explore the connection between suspense and gambling. The model illustrates that it may be worthwhile to gamble to enhance feelings of suspense. Section 6 discusses our work in relation to various connected literatures in decision theory, and the work of Kreps and Porteus in particular. While there are important linkages between our work and theirs, there are also critical differences. One important difference lies in their requirement of temporal consistency, which is extremely restrictive in a setting with anticipatory feelings. Section 7 contains concluding remarks.

2 General Model of Anticipatory Feelings

The psychological expected utility model has three critical components. The first is a definition of the relevant prize space, which we take to be a very abstract space of personal mental states on which we impose minimal structure. This is the domain of individual preferences. The second component is a formal description of the space of lotteries in the physical world, and of the manner in which uncertainty about these lotteries may be resolved over time. The third component is a mapping that connects the evolution of uncertainty with the space of psychological states. We take these up in turn.

2.1 Psychological States

We replace the standard space of prizes in the actual lottery with a more personal space of "psychological states", comprising a set of individual states of mind. An individual can experience many different types of mental state, and it is these that we connect to the agent's level of utility and well-being. We view the agent's underlying preference order as being defined on lotteries over sequences of states of mind. This preference order will be assumed to have all of the standard properties of completeness, transitivity, and continuity.

Formally, the first datum of the model is a discrete time horizon, $T > 0$, and a series of spaces X_t, for $1 \leq t \in \{1, \ldots, T\}$, that represent the possible psychological states in the given period.

Axiom 1 *Each set of psychological states, X_t, is a separable metric space, $t \in \{1, \ldots, T\}$.*

From these spaces we construct the space of psychological lotteries as follows. First we define the product spaces $X = \Pi_{t \in \{1,\ldots,T\}} X_t$ and $X^t = \Pi_{s=t}^T X_t$, $1 \leq t \leq T$,each with the product topology. We then define the psychological lottery space to be the space of all Borel probability distributions on X together with the topology of weak convergence: we denote this space $P(X)$. More generally, given any space S, the space $P(S)$ is used to denote the space of all Borel probability measures on S, with the topology of weak convergence. Given an element $p \in P(X)$, we let $(p_1, \ldots, p_T)$ denote the corresponding marginal distribution on the spaces X_t,so that $p_t \in P(X_t)$, $1 \leq t \leq T$.

Since each space X_t is separable, X is separable and $P(X)$ is separable and metrizable by the Prohorov metric. Note also that $P(X)$ is a mixture space: for $\lambda \in [0,1]$ and $p, q \in P(X)$, there exists an element of $P(X)$, which we shall denote $p\lambda q$, which assigns to each Borel measurable subset $A \subset P(X)$ the probability $\lambda p(A) + (1-\lambda)q(A)$.

We assume that the decision maker has a preference relation $\succeq$ on $P(X)$ that satisfies standard axioms:

Axiom 2 *The preference order $\succeq$ is complete and transitive.*

Axiom 3 *Given p, q, $r \in P(X)$ and $\lambda \in [0,1]$, $p \succeq q$ implies $p\lambda r \succeq q\lambda r$.*

Axiom 4 *Given p, q, $r \in P(X)$ such that $p \succeq q$ and $q \succeq r$, there exist λ_1, $\lambda_2 \in [0,1]$ such that $p\lambda_1 r \succeq q$ and $p\lambda_2 r \succeq q$.*

Axiom 5 *$\succeq$ is continuous in the topology of weak convergence.*[3]

[3] A binary relation $\succeq$ defined on a mixture space X is continuous if for all $x, y, z \in X$:

$$\{\lambda \in [0,1] : x\lambda y \succeq z\}$$

These conditions are sufficient to ensure that there exists an expected utility representation of $\succeq$ (See Fishburn [1982], Theorem 10.1). Let $E_r f$ denote the expectation of the random variable f with respect the measure r.

Lemma 1 *(A1) through (A4) are necessary and sufficient for there to exist a bounded, continuous function $U : X \to R$ such that for $p, q \in P(X)$, $p \succsim q$ if and only if $E_p U \geq E_q U$.*

Finally, we assume that preferences are time separable.

Axiom 6 *For all p, $q \in P(X)$, $(p_1, \ldots, p_T) = (q_1, \ldots, q_T)$ implies $p \sim q$.*

With this assumption, we know that the function $U : X \to R$ in Lemma 1 has a time additive representation (see Fishburn [1982], p.73):

$$U(x) = \sum_{1}^{T} u_t(x_t).$$

where $u_t : X_t \to R$, for $1 \leq t \leq T$.

2.2 The Space of Temporal Lotteries

We now introduce lotteries over outcomes in the physical world, and formalize the manner in which uncertainty about the prizes evolves over time. In order to capture this unfolding of uncertainty, we use the space of temporal lotteries due to Kreps and Porteus [1978]. A temporal lottery summarizes the process by which information about the future will arrive. The basic data comprises a sequence of spaces describing all prizes that the agent may receive in each period, Z_t, $t \in \{1, \ldots, T\}$.

Axiom 7 *Each of the spaces Z_t is a separable metric space, $t \in \{1, \ldots, T\}$.*

From these we construct the space of temporal lotteries, L_t, as follows. In the last period, let L_T denote the space of Borel probability distributions over Z_T. For earlier periods we define L_t recursively. Given L_{t+1}, let $Y_t = Z_t \times L_{t+1}$

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and

$$\{\lambda \in [0,1] : z \succeq x\lambda y\}$$

are closed.

and define L_t to be the space of Borel probability measures over Y_t. We endow Y_t with the product topology, and L_t with the topology of weak convergence. The sets Y_t can be thought of as the outcomes in period t; these outcomes include a prize from Z_t and a lottery over future prizes, L_{t+1}. It can be shown that both L_t and Y_t are separable, metrizable spaces. Finally, for notational convenience, we define $Y_T \equiv Z_T$.

2.3 Connecting the Two Spaces

Other basic data are needed to link the space of temporal lotteries with psychological states, and thereby link them with overall utilities. In the end, we are interested in deriving a series of preference orderings and associated utility functions on the various spaces of temporal lotteries,

$$V_t : L_t \longrightarrow R.$$

While these are indeed part of the data for the problem, the conditions that we assume of them can only be understood once we have explained how these functions are built up from a number of underlying building blocks. Given that they end up being derived from more basic data, we refer to the V_t as induced utility functions.

The key building block for the induced utility functions V_t is a sequence of functions that describe the contemporaneous feelings associated with temporal lotteries. We think of the actual period t temporal lottery as being realized at the beginning of period t, so that there is no time for feelings of anticipation concerning this lottery in period t. While uncertainty over future prizes may influence current feelings, uncertainty over current temporal lotteries does not. This means that the raw data upon which we construct the induced utility functions are a series of functions, $\phi_t : Y_t \rightarrow P(X_t)$ that describe the lottery over current psychological states implied by a pure temporal lottery over prizes, $y_t \in Y_t$. Each of these functions maps into $P(X_t)$ rather than X_t to allow for the possibility that there might be some uncertainty in the agent's mind about the mental state associated with the pure temporal lottery y_t.

Axiom 8 *Each function ϕ_t is continuous relative to Y_t and $P(X_t)$.*[4]

[4]The topologies on $P(X_t)$ and Y_t are the topologies of weak convergence.

As we will see in the examples in sections 4 and 5, it is the ϕ_t mappings that give the theory structure, and it is in these mappings that different psychological attitudes to uncertainty are captured. It is also important to note that the nature of the model as a formulation of anticipatory feelings is captured in the assumption that the period t feelings are based purely on the current physical prize and uncertainty that is as yet unresolved. The past prizes and past psychological states play no role in determining current feelings.[5]

We use these functions to build up the lotteries over current and future feelings associated with any $l_t \in L_t$,

$$\Phi_t : L_t \longrightarrow P(X^t).$$

The construction procedure is recursive. In period T,

$$\Phi_T(l_T) = E_{l_T}[\phi_T] \in P(X^T).$$

Given Φ_{t+1} we continue the inductive procedure,

$$\Phi_t(l_t) = E_{l_t}[(\phi_t, \Phi_{t+1})] \in P(X^t).$$

Given the functions $\Phi_t : L_t \longrightarrow P(X^t)$, we can immediately derive the induced utility functions $V_t : L_t \longrightarrow R$. Given $l_T \in L_T$,

$$V_T(l_T) = E_{l_T}[u_T(\phi_T)].$$

Given the function V_{t+1}, the function V_t is defined by,

$$V_t(l_t) = E_{l_t}[u_t(\phi_t) + V_{t+1}].$$

Note that these induced utility functions are continuous, and obey the expected utility property on the space of temporal lotteries, in the sense that we need only derive the function $V_t : Y_t \longrightarrow R$, and are then able to make all comparisons among temporal lotteries. Given $l_t, l'_t \in L_t$,

$$\Phi_t(l_t) \succeq \Phi_t(l'_t) \Longleftrightarrow E_{l_t}[V_t] \succeq E_{l'_t}[V_t].$$

For this reason, we refer to the functions $V_t : Y_t \longrightarrow R$ as induced expected utility functions.

[5]We believe that there are several interesting generalizations possible in which one includes a richer set of determinants of the current psychological state.

3 A Simple Special Case

We consider two period models, $T = 2$. The data corresponding to psychological states are standard, comprising the two period state spaces, X_1 and X_2, and the corresponding expected utility functions, $u_1 : X_1 \longrightarrow R$ and $u_2 : X_2 \longrightarrow R$. The space L_2 comprises all lotteries on the space $Y_2 = Z_2$ of second period pure prizes. As usual, we define the set Y_1 to be the set of period 1 pure temporal lotteries, $Y_1 = Z_1 \times L_2$, and L_1 to be the set of all lotteries over these objects. In addition, we assume that the feelings associated with pure temporal lotteries are themselves deterministic, so that the function that describes period t feelings maps Y_t into the set X_t rather than $P(X_t)$,

$$\phi_t : Y_t \longrightarrow X_t,$$

for $t = 1, 2$.

The induced expected utility functions are also standard:

$$V_2(y_2) = u_2(\phi_2(y_2)).$$

Given $y_1 = (z_1, l_2) \in Y_1$,

$$V_1(y_1) = u_1(\phi_1(y_1)) + E_{l_2}[u_2].$$

In applications, the construction of the function $\phi_1 : Y_1 \rightarrow X_1$ can be a somewhat subtle affair, as we will see in sections 4 and 5 below.

3.1 The Decision Theoretic Framework

The decision problems that we consider all have the same general structure. Given an initial state, the agent in the first period chooses an action from a feasible set. This action determines the physical payoff in the first period. Some uncertainty then resolves between the first and the second period. In the second period, the agent chooses a lottery over second period prizes from a feasible set which may depend upon both the earlier action choice, and the new information that has arrived. After this, all uncertainty is resolved and the second period payoff is realized.

There are several important new spaces to introduce in order to work with well-defined decision problems. There are the first and second period exogenous state spaces, and a Markov transition operator from one to the other. There are the first and second period global choice sets, as well as a description of how the actual choice sets for a given decision maker evolve as a function of earlier choices and exogenous shocks. Then there is a description of how all of these exogenous and endogenous factors determine the sequence of temporal lotteries experienced, and therefore get associated with utilities. Finally, we must define the set of feasible strategies, and show how to compute the expected present value of any given strategy.

The exogenous state spaces are denoted S_1 for period 1 and S_2 for period 2, with generic elements s_1 and s_2 respectively. The Markov transition operator is denoted $Q(B, s) = \Pr\{s_2 \in B | s_1 = s\}$. We also take as given a particular initial state, $s_1 \in S_1$.

Axiom 9 *The set S_2 is a compact metric space.*

The action space in period 2 is L_2, with the topology of weak convergence. We assume that Z_2 is a compact metric space, so that L_2 is also a compact metric space. The action space in period 1 is a compact metric space A with generic element $\alpha \in A$. The choices available to the agent in each period are assumed to depend on the value of that period's exogenous state variable, and the prior choice of action. We denote the period 1 choice set $\Gamma_1(s_1) \subset A$, and the period 2 choice set $\Gamma_2(s_2, \alpha_1) \subset L_2$.[6]

Axiom 10 *The sets Z_2 and A are compact metric spaces, and the choice correspondences Γ_1 and Γ_2 are compact valued and continuous.*

A policy in period 2 is a measurable function $\pi_2 : S_2 \times A \longrightarrow L_2$, with $\pi_2(s_2, \alpha) \in \Gamma_2(s_2, \alpha)$. Let Π_2 be the set of such period 2 policies. A strategy from the initial condition s_1 combines a feasible first period choice with a second period policy.

Definition 2 *A **feasible** strategy from the initial state $s_1 \in S_1$, $\pi = (\alpha, \pi_2)$, is a combination of an element $\alpha \in \Gamma_1(s_1)$ and a measurable function π_2 : $S_2 \times A \longrightarrow L_2$. The set of such strategies is denoted $\Pi(s_1)$.*

It is important to understand how strategies map into first period temporal lotteries. We assume that the first period prize is a function $\eta(s_1, \alpha)$ of the first period action and the first period state, and is continuous in the first period action.

[6] Whether or not mixed strategies are allowed in period 2 is determined by the nature of the correspondence $\Gamma_2(s_2, \alpha)$.

Axiom 11 *The first period prize* $\eta : S_1 \times A \longrightarrow Z_1$ *is continuous in* α *for any given* $s_1 \in S_1$.

In addition, note that associated with s_1, α, and π_2 is a lottery over second period prizes, $\lambda(s_1, \alpha, \pi_2) \in L_2$, defined by,

$$\lambda(s_1, \alpha, \pi_2) = \int \pi_2(s_2, \alpha) Q(ds_2, s_1).$$

3.2 Optimal Strategies: Definition and Existence

We begin the search for optimal strategies in period 2, and proceed by backward induction. Let $J_2(s_{2,}\alpha)$ denote the value of an optimal policy conditional on the second period state,

$$J_2(s_{2,}\alpha) = \sup_{l_2 \in \Gamma_2(s_{2,}\alpha)} E_{l_2}[u_2(\phi_2)].$$

We define the second period choice correspondence in the natural manner as:

$$G_2(s_{2,}\alpha) = \{l_2 \in \Gamma_2(s_{2,}\alpha) | J_2(s_{2,}\alpha) = E_{l_2}[u_2(\phi_2)]\}.$$

An optimal policy in period 2 is a measurable selection from $G_2(s_{2,}\alpha)$. The set of period 2 optimal strategies is denoted Π_2^C. Note that with our assumptions, application of the theorem of the maximum and a standard measurable selection theorem (Hinderer [1970]) guarantees that Π_2^C is non-empty.

One obvious possibility in this class of models is that preferences over lotteries need not be time consistent. The best lottery over final prizes from a period 2 viewpoint may be different than preferences over these lotteries from the period 1 viewpoint. This lack of time consistency gives rise to some subtleties in the definition of optimal strategies in period 1. As usual, the key point is that some of the strategies that are feasible in principle will not be able to be carried out, since they will not be optimal from a second period viewpoint. In order to characterize and analyze optimal strategies, we will need to bear in mind the constraint that future behavior must also be optimal. The tricky issue here is that there may be more than one optimal policy in the second period, so that we will have to address issues of selection. We allow the agent in period 1 to select as they prefer among the second period optimal strategies, $\pi_2 \in \Pi_2^C$. However they will not be allowed to utilize strategies that are not optimal in period 2. We define a strategy that obeys this criterion of obeying second period optimality a consistent strategy.

Definition 3 *A **consistent** strategy from the initial state $s_1 \in S_1$, $\pi = (\alpha,\pi_2)$, is a feasible strategy with the additional property that $\pi_2(s_{2,}\alpha) \in G_2(s_{2,}\alpha)$. The set of such strategies is denoted Π^C.*

For an arbitrary period 1 choice $\alpha \in \Gamma(s_1)$, we can now define the supremum of what can be achieved starting from this choice as given.,

$$K(s_1,\alpha) = \sup_{\pi_2 \in \Pi_2^C} \left[u_1(\phi_1(\eta(s_1,\alpha),\lambda(s_1,\alpha,\pi_2)) + E_{\lambda(s_1,\alpha,\pi_2)}[u_2(\phi_2)] \right].$$

We then define the period 1 value function as,

$$J_1(s_1) = \sup_{\alpha \in \Gamma(s_1)} K(s_1,\alpha)$$

The optimal strategies from the initial state $s_1 \in S_1$ are those in which α is selected from the set $G_1(s_1)$ defined by,

$$G_1(s_1) = \{\alpha \in \Gamma(s_1) | J_1(s_1) = K(s_1,\alpha)\},$$

and the second period strategy π_2 is selected from the set $G_2^C(s_1)$ defined by,

$$G_2^C(s_1) = \{\pi_2 \in \Pi_2^C | K(s_1,\alpha) = u_1(\phi_1(\eta(s_1,\alpha),\lambda(s_1,\alpha,\pi_2)) + E_{\lambda(s_1,\alpha,\pi_2)}[u_2(\phi_2)]\}.$$

The fact that optimal strategies exist can be shown using a slight adaptation of a theorem of Harris [1985].

Proposition 4 *An optimal strategy exists.*

Proof: If we consider the decision maker in period 1 and in period 2 as two separate individuals, our solution concept is equivalent to selecting the subgame perfect equilbrium that is best from the period 1 perspective. It is easy to confirm that our model satisfies all of the assumptions of Theorem 1 in Harris [1985], which implies that set of subgame perfect equilibra is compact. Hence an optimum exists.□

4 Example 1: Alaska or Hawaii? Anticipation and Commitment

As Loewenstein [1987] has pointed out, economic theorists have long been aware of the importance of anticipatory feelings, and in our first example we extend some early ideas of Jevons [1905] to a stochastic environment. Jevons made several hypotheses concerning the intensity of "anticipal pleasure" derived from an impending vacation:

> "The intensity of the anticipation will be greater the longer the holiday; greater, also, the more intensely one expects to enjoy it when the time comes. ... Again, the nearer the date fixed for leaving home approaches, the greater does the intensity of anticipal pleasure become." (Jevons [1905], p.64)

In addition to these plausible impacts of time on anticipatory utility, we believe that the degree of certainty is a key determinant of anticipatory pleasure. Consider an individual who is contemplating a summer vacation that they will spend either in Alaska or in Hawaii. We believe that even if they are indifferent between the two vacation spots ex ante, they may find that a lottery offering an equal probability of either vacation is less readily anticipated than is a fixed decision in favor of either Alaska or Hawaii. More generally, the more certain they can be ahead of time about which trip they will take, the greater their anticipatory pleasure. We explore the implications of this pattern of anticipatory feelings in a two period environment, in which an individual is considering a trip to Hawaii or to Alaska in the second period. One motivating hypothesis is that the desire for certainty about the impending vacation may motivate the decision maker to employ commitment devices to guard against future changes in preferences. The decision-theoretic set up that we employ in this example is designed to explore this possibility.

4.1 The Structure of the Model

There are two periods. An agent has choices concerning whether or not to take a vacation at the end of the second period, and if so, whether to go to Alaska or to Hawaii. They are born into the first period with a deterministic level of wealth W_0, and with fixed beliefs about the probability that the weather will be good in Alaska. To be concrete, we take their prior to assign equal probability to the state of good weather in Alaska as opposed to bad weather in Alaska. They are certain that the weather in Hawaii will be good. If the weather in Alaska was sure to be good, then they would prefer the Alaska vacation to the Hawaii vacation. If it was sure to be bad, they would prefer the Hawaii vacation to the Alaska vacation. The weather is taken to be the only form of uncertainty influencing the enjoyment of the vacation.

The only decision allowed during period 1 is how much money to put down as a deposit on the trip to Hawaii: this is allowed to be any proportion of the total cost of the vacation, which we take to be \$2,000 in total for either vacation. For simplicity we preclude deposits on Alaska. First period anticipatory utility is realized after the deposit has been put down.

At the beginning of the second period, nature provides an updated forecast of the probability of good weather in Alaska, s'. This new signal is drawn from a uniform distribution on the interval $[0, 1]$. With this new information, the agent must choose among the three options of completing their payment and taking the trip to Hawaii; paying \$2,000 to take the trip to Alaska; or staying at home and forfeiting any deposit put down on Hawaii.[7] Any vacation that is to be taken occurs immediately after making this second period decision. Finally we assume that $W_0 \geq 4,000$ so that the agent can afford to pay for either or both vacations.

4.2 Psychological states and psychological expected utility

The second period psychological prize space, X_2, comprises two distinct elements. There is a wide set of possible feelings concerning the impending vacation, and there are also feelings associated with the level of residual wealth. Each of these is measured as a scalar, with the obvious interpretation that higher values of the scalar correspond to more intense psychological prizes, which are assumed to be better for the agent. For wealth, the natural dollar yardstick is used in order to be concrete. It is convenient to limit the numerical representation of these feelings to be non-negative, and to record the vacation first:

$$X_2 = \left\{(f, w) \in R^2 : f, w \geq 0\right\}.$$

With respect to the psychological expected utility function $u_2 : X_2 \longrightarrow R$, it is convenient to assume that this is separable as between the vacation and the level of wealth, and also to assume that the underlying psychological component has been scaled to allow for the simplest of possible representations,

$$u_2(f, w) = f + w.$$

[7]For simplicity of exposition, we ignore mixed strategies in presenting the model. It is easy to confirm that the results are unchanged when such strategies are admitted.

Note that we have already fixed the units of measurement of the wealth variable as dollars, so that this assumption on functional form implies that the agent is risk neutral.

In period 1, the psychological prize is a feeling of anticipation. Again this is assumed to be representable as a simple non-negative scalar, with higher values corresponding to greater levels of anticipation,

$$X_1 = \{a \in R : a \geq 0\}.$$

We assume that utility is increasing in the level of anticipation, and that the identity mapping can be used to represent preferences on such anticipatory states,

$$u_1(a) = a.$$

4.3 External Lotteries

The second period pure prize space, Y_2, consists of the final level of wealth and the nature of the final vacation. Let H denote the outcome of a vacation to Hawaii, G and B denote the outcomes of vacations in Alaska in good and bad weather respectively, $\emptyset$ the outcome of no vacation at all, and $w \in R_+$ denote the final level of wealth. With this notation we can specify the prize space as,

$$Y_2 = \{(h, w) : h \in \{G, B, H, \emptyset\}, 0 \leq w \leq W_0\}.$$

There is no consumption in the first period, so that Z_1 is degenerate. This implies that the set of temporal lotteries in the first period Y_1 corresponds to the set of lotteries over outcomes in the set Z_2. Each such lottery $y_1 \in Y_1$ specifies the probability of a good vacation in Alaska, a bad vacation in Alaska, a vacation in Hawaii, and no vacation, as anticipated at the end of period 1, following the payment of the deposit, but prior to receiving the updated weather forecast: we denote these probabilities p_G, p_B, p_H, and $p_\emptyset$ $= 1 - (p_G + p_B + p_H)$ respectively. The lottery also specifies a finite list of possible wealth levels, W^F, together with an assignment of probabilities to the elements of this set, $q : W^F \longrightarrow S^{\#W^F-1}$, where this is a mapping into the simplex of dimension $\#W^F - 1$, where $\#W^F$ denotes the cardinality of the set of possible wealth levels. With this understanding we can write Y_1 in the simple manner,

$$Y_1 = \{(p_G, p_B, p_H, p_\emptyset, W^F, q) : 0 \leq p_G, p_B, p_H, p_\emptyset \leq 1, p_G + p_B + p_H + p_\emptyset \leq 1\}.$$

Given any element $y_1 \in Y_1$, we use the notation $p_G(y_1), p_B(y_1), p_H(y_1), p_\emptyset(y_1), W^F(y_1)$, and $q(y_1)$ in the obvious manner to denote the corresponding component of the lottery. We also use the notation $E_{y_1}(w)$ to denote the expected value of the wealth lottery associated with any given $y_1 \in Y_1$.

4.4 Mapping External Lotteries to Psychological States

With respect to the connection between these two spaces and the underlying psychological states, the second period mapping, $\phi_2 : Y_2 \longrightarrow X_2$ is assumed to take a particularly simple form:

$$\phi_2(h,w) = \begin{cases} (0,w), & \text{if } h \in \{B,\emptyset\}; \\ (4000,w), & \text{if } h = H; \\ (6,000,w), & \text{if } h = G. \end{cases}$$

This corresponds to the idea that a vacation in Alaska in bad weather is equivalent to no vacation in terms of its level of the vacation enjoyment variable, while the trip to Hawaii yields utility equivalent to \$4,000, and the trip to Alaska in good weather is worth \$6,000 in monetary terms.

What remains is to specify which features of a given lottery over second period outcomes determines the level of anticipatory feelings in period 1, $\phi_1 : Y_1 \longrightarrow X_1$. There are several key properties that we require of this function. In the first place, we wish to capture the idea that anticipation is tied to the taking of a vacation: if no vacation is to be taken, there will be no anticipation. In the second place, we wish to capture the idea of Jevons that the degree of anticipation is closely tied to beliefs about the pleasure that will be derived in the vacation. Finally we wish to capture the idea that anticipatory pleasure is higher the more certainty there is about which of the vacations will be taken. Our specific assumption on functional form is,

$$\phi_1(y_1) = \beta.\max[6,000.p_G(y_1), 4,000.p_H(y_1)].$$

In this formulation the parameter $\beta > 0$ is a scale factor that sets the importance of anticipatory utility relative to the utility of the prizes themselves. The first term inside the maximum expression represents the utility of the good trip to Alaska multiplied by the probability that the agent will take this good trip. The second term represents the utility of the trip to Hawaii multiplied by the probability that this trip will be taken. Finally, the maximum operator is used to represent the idea that feelings depend simply on which of these anticipatory possibilities is more intense, rather than on any form of summation operation.

Finally we turn to the induced expected utility functions on the spaces Y_2 and Y_1. The second period function $v_2 : Y_2 \longrightarrow R$ is given by,

$$v_2(h,w) = \begin{cases} w, \text{ if } h \in \{B, \emptyset\}; \\ 4,000 + w, \text{ if } h = H; \\ 6,000 + w, \text{ if } h = G. \end{cases}$$

The first period function $v_1 : Y_1 \longrightarrow R$ is given by,

$$\begin{aligned} v_1(y_1) &= \phi_1(y_1) + E_{y_1}(v_2). \\ &= \phi_1(y_1) + 4,000p_H(y_1) + 6,000p_G(y_1) + E_{y_1}(w) \end{aligned}$$

4.5 The Decision Problem and the Solution

In the second period, the agent chooses among a feasible set of lotteries over second period prizes. This feasible set of lotteries is determined as a result of the first period choice of a fractional down-payment, $\alpha \in A = [0,1]$ (where $\alpha = 1$ corresponds to a full down-payment of $2,000), and the new signal on the weather, s'. The second period choice correspondence, $\Gamma(s',\alpha) \subset L_2$, allows for the three possible choices of completing the down-payment and going to Hawaii, taking the trip to Alaska, and cancelling the vacation, as well as mixed strategies among these three options. Note that the only real uncertainty concerns the nature of the vacation in Alaska. We let $L_A(s')$ denote the vacation to Alaska with probability s' of good weather:

$$\Gamma(s',\alpha) = \{(H, W_0 - 2,000);\ (L_A(s'), W_0 - 2,000(1+\alpha); (\emptyset, W_0 - 2,000\alpha)\}.$$

Given this simple choice correspondence in the second period, it is straightforward to compute the second period value function, $J_2(s',\alpha)$. From a second period viewpoint, a probability s' that the Alaska trip will be good is worth $(6,000s' - 4,000)$ dollars more than the Hawaii trip, which in turn dominates the option of not going. This means that the unique optimum is for the agent to go to Alaska if and only if the signal is sufficiently high for the second period utility differential exceeds the cost difference of $2,000\alpha$,

$$s' > \frac{2+\alpha}{3}.$$

If the strict inequality is reversed, the unique optimum is to go to Hawaii. In the case of equality, either option is optimal. This allows us to solve for the second period value function as,

$$J_2(s',\alpha) = \begin{cases} 6,000s' + W_0 - 2,000(1+\alpha), & \text{if } s' \geq \frac{2+\alpha}{3}; \\ 4,000 + W_0 - 2,000\alpha, & \text{if } s' < \frac{2+\alpha}{3}. \end{cases}$$

With this we are in a position to compute the expected payoff to any strategy α in the first period. Given $\alpha \in [0,1]$, and a period 2 policy selected from the optimal choice correspondence[8], the probability of going to Hawaii is $p_H(\alpha) = \frac{2+\alpha}{3}$. If the trip is taken instead to Alaska, the probability that the weather is good depends on the signal s', and ranges uniformly over the range $[\frac{2+\alpha}{3}, 1]$, so that the overall probability of going to Alaska and experiencing good weather is,

$$p_G(\alpha) = \left[\frac{1-\alpha}{3}\right] \cdot \left[\frac{5+\alpha}{6}\right] = \frac{5-4\alpha-\alpha^2}{18}$$

With this we have identified the complete lottery $y(\alpha) \in Y$ associated with any first period strategy choice $\alpha \in [0,1]$,which we then substitute into the overall expression to derive the first period expected present value associated with the strategy,

$$K(\alpha) = \beta.\max[6,000p_G(\alpha), 4,000p_H(\alpha)] + 4,000p_H(\alpha) + 6,000p_G(\alpha) + E_{y(\alpha)}(w).$$

Note that $4,000p_H(\alpha) \geq 6,000p_G(\alpha)$ for all $\alpha \in [0,1]$, so that $K(\alpha)$ reduces to,

$$\begin{aligned} K(\alpha) &= \frac{4,000\,(2+\alpha)\,(\beta+1)}{3} + \frac{6,000\,(5-4\alpha-\alpha^2)}{18} + W_0 - \frac{2,000[3+\alpha(1-\alpha)]}{3}. \\ &\sim 7 + \frac{3W_0}{1,000} + 8\beta - 2\alpha + 4\alpha\beta + \alpha^2, \end{aligned}$$

where the relation $\sim$ denotes that the expressions have the same sign. Taking derivatives,

$$K'(\alpha) \sim -2 + 4\beta + 2\alpha.$$

[8]Note that the issue of selection is irrelevant here since there is only one level of the shock for which there is more than one optimal strategy, and this shock has probability zero.

To understand the sign of this derivative, note that additional down-payments serve to reduce final period expected utility both by increasing the expected cost of the vacation, and by potentially making it unworthwhile to go to Alaska even if it is anticipated that the vacation there would be superior. On the other hand, additional down-payments increase the probability of going to Hawaii and thereby add to the anticipatory utility in period 1. Whether or not they add to total utility depends on how powerful are the anticipatory gains relative to the second period loss of utility.

Since the function $K(\alpha)$ is convex, the optimum is at one of the two corners, depending on the value of $\beta > 0$. For the two alternatives of a full deposit and no deposit to be indifferent requires $\beta = \frac{1}{4}$. Therefore the unique optimal decision is to put down no deposit, $\hat{\alpha} = 0$, if $\beta < \frac{1}{4}$, and to put down a full deposit, $\hat{\alpha} = 1$, if $\beta < \frac{1}{4}$. At the cutoff value $\beta = \frac{1}{4}$, either one of these two choices is optimal: however note that these strictly dominate all intermediate choices $\alpha \in (0, 1)$.

4.6 Implications and Extensions

A critical implication of the above example is that an individual may find it worthwhile to put money down in the first period in order to raise the probability that they will go to Hawaii in the second period. This form of personal commitment is a response to the time inconsistency of the preferences over lotteries. From a second period viewpoint, the down-payment looks like a very bad idea, since it both lowers expected wealth and makes the agent less responsive to valuable information concerning where they will enjoy the vacation more. But from a first period viewpoint, this period 2 inflexibility is desirable, since it may make it possible to more intensely savor the vacation in Hawaii. Another implication of the lack of time consistency is that for high values of the parameter β, the agent would prefer not to receive the second period signal, so that they could know for sure that the second period vacation would involve going to Hawaii.

There are many ways to generalize the example. Consider cases in which there is a potentially unpleasant experience in period 2, in which case the relevant emotional response may be dread. In this case one might expect the corresponding model to produce some desire to undertake actions in period 1 designed to make the period 2 outcome less readily predictable, in order to dilute the focus on the negative outcome. This connects to other interesting issues that would arise if we were to endogenize the timing of the lottery.

Just as in the experiments reported in Loewenstein [1987], one would expect to find a greater incentive to delay an experience that carries anticipatory pleasure than one that brings anticipatory pain.

5 Example 2: Ali or Frazier? Suspense and Gambling

For many sports fans, part of the pleasure of watching an event derives from feelings of suspense. There is pleasure associated with not knowing the outcome of an important event, and watching the uncertainty resolve in front of their eyes. We believe that the extent of this suspense is connected to the personal stake, both monetary and psychic, in the outcome. This connection ties belief and pleasure together in a complex and interesting manner that may shed light on such phenomena as why there is so much gambling associated with sports events.

An example from one of our personal histories exemplifies the importance of suspense. In the United Kingdom, the second Ali-Frazier fight was shown with a tape delay. A close friend spent the whole day in almost complete isolation to avoid learning the outcomeand thereby enjoy the excitement of watching the fight. He clearly believed that his pleasure would be adversely impacted if anyone told him the outcome of the fight. Unfortunately for him, half an hour before the fight was shown, his father walked into the room and said "what are you so worried about? Ali won." While this was the preferred outcome, it was clear that he was crushed by having the joy taken out of watching the fight. This simple example suggests that decisions such as which sporting events to watch are best understood inside a framework that allows for enjoyment of suspense.

In the decision theoretic model that we build of this phenomenon, our basic hypothesis is that the more an individual cares about the outcome of a sporting event, the greater the degree of pleasure that they can derive from the process of watching the outcome unfold. With regard to behavior, this raises the possibility that it may be worthwhile to place a bet on a sporting event merely to heighten the feelings of suspense and pleasure to be derived from watching the event.

5.1 The Structure of the Model

We model the feeling of suspense of a person watching Muhammad Ali fight Joe Frazier. There are two periods. At the beginning of the first period nature draws a probability that Ali will win the fight: for simplicity we take this probability to be fixed at 1/2. There are two decisions to be taken in period 1. One decision concerns how much money to bet on the outcome of the fight. If the monetary bets in period 1 are $(b^A, b^F) \geq 0$ on Ali and Frazier respectively, then the payoff in period 2 is $2b^i$ if agent i wins, since we make the simple assumption that the bet is fair. The total amounts that can be bet are constrained to be no more than the level of initial wealth, $W_0 > 0$. In addition to placing their bets, the agent gets to decide whether or not to watch the fight at the end of period 1. It is in this period that the agent experiences any feelings of suspense.

In period 2, the outcome of the fight becomes known whether or not the fight was watched, and all bets are settled. We rule out draws, so that either Ali wins and becomes champion, or Frazier wins and remains champion. The subtle point in the model is how to capture the interaction between feelings of suspense, the external uncertainty concerning the outcome of the fight, the endogenous uncertainty concerning the amount at stake in the bet, and the decision on whether or not to watch the fight.

5.2 Psychological states and psychological expected utility

The second period psychological prize space, X_2, comprises two distinct elements. There is a wide set of possible feelings concerning who is the champ, and there are also feelings associated with the level of residual wealth. Each of these is measured as a scalar, with the obvious interpretation that higher values of the scalar correspond to more intense psychological prizes, which are assumed to be better for the agent. For wealth, the natural dollar yardstick is used. We represent these feelings as non-negative scalars, and record the feelings concerning who is champion first:

$$X_2 = \{(f, w) \in R^2 : f, w \geq 0\}.$$

With respect to the psychological expected utility function $u_2 : X_2 \to R$, it is convenient to assume that this is separable as between the champion and the level of wealth, and also to assume that the underlying psychological component has been scaled to allow for the simplest of possible representations,

$$u_2(f, w) = f + w - \beta w^2.$$

Since we have already fixed the units of measurement of the wealth variable as dollars, note that this assumption on functional form implies that the agent is risk averse, and has a quadratic utility function. The parameter $\beta > 0$ is included to allow for an appropriate degree of risk aversion.

In period 1, one possible psychological prize is a feeling of anticipatory suspense. Again this is assumed to be representable as a simple non-negative scalar, with higher values corresponding to greater levels of suspense. The second prize derives from some other activity, which for concreteness we shall consider to be a day of gardening. We treat this as a discrete prize, with a dummy variable $\gamma = 1, 0$ recording the presence or absence of this prize respectively,

$$X_1 = \{(a, \gamma) | a \in R_+, \gamma \in \{0, 1\}\}.$$

We assume that utility is increasing in the level of suspense, that gardening is enjoyable, and that the identity mapping can be used to represent preferences on anticipatory states,

$$u_1(a, \gamma) = \begin{cases} a \text{ if } \gamma = 0; \\ a + g \text{ if } \gamma = 1, \end{cases}.$$

where the parameter $g > 0$ reflects the pleasure of gardening.

5.3 External Lotteries

The second period pure prize space consists of the final outcome of the fight and the final level of wealth. We let A denote the feelings associated with a victory by Ali, F a victory by Frazier, and $w \in R_+$ the final level of wealth,

$$Z_2 = Y_2 = \{(c, w) : c \in \{A, F\}, 0 \leq w \leq 2W_0\}.$$

The prize in the first period is either to watch the match on television (T) or to engage in other activities ($\emptyset$),

$$Z_1 = \{T, \emptyset\},$$

where T denotes the activity of watching television and Φ represents the decision to engage in other activities. This implies that any element $y_1 \in Y_1$ must specify the corresponding first period prize, $z_1(y_1)$, and a simple probability distribution over outcomes in the set Z_2. Given the even odds that Ali wins, this comes down to specifying the (deterministic) final wealth levels contingent on Ali winning and on Frazier winning, $W_A(y_1)$ and $W_F(y_1)$ respectively.

5.4 Mapping External Lotteries to Psychological States

With respect to the connection between these two spaces and the underlying psychological states, the second period mapping, $\phi_2 : Y_2 \longrightarrow X_2$ is assumed to take a particularly simple form:

$$\phi_2(c, w) = \begin{cases} (0, w) \text{ if } c = F; \\ (x_A, w) \text{ if } c = A. \end{cases}$$

We assume that $x_A > 0$, corresponding to the idea that the individual is happier when Ali wins than when Frazier wins.

What remains is to specify which features of a given lottery over second period outcomes determines the level of anticipatory feelings in period 1, $\phi_1 : Y_1 \longrightarrow X_1$. We make the simple assumption on the nature of suspense that it is increasing in the absolute difference in the sum of the utility giving arguments in the second period when Ali wins as opposed to when Frazier wins,

$$\Delta(y_1) = |x_A + W_A(y_1) - W_F(y_1)|.$$

The specific assumption on functional form is,

$$\phi_1(y_1) = \begin{cases} (s.\Delta(y_1), 0), \text{ if } z_1(y_1) = T. \\ (0, 1); \text{ if } z_1(y_1) = \emptyset \end{cases}$$

Recall that the first argument in the period 1 psychological state space is the level of suspense, and the second is the gardening prize. The simple assumption is made that if the fight is not watched, there are no feelings of suspense. The parameter $s > 0$ scales the feelings of suspense.

Finally we turn to the induced expected utility functions that give present values of utility associated with each of the two different spaces, Y_2 and Y_1. The second period function $v_2 : Y_2 \longrightarrow R$ is given by,

$$v_2(c, w) = \begin{cases} w \text{ -}\beta w^2 \text{ if } c = F; \\ u_A + w - \beta w^2 \text{ if } c = A. \end{cases}$$

The first period function $v_1 : Y_1 \longrightarrow R$ is given by,

$$v_1(y_1) = \phi_1(y_1) + \frac{u_A}{2} + \frac{W_A(y_1) + W_F(y_1)}{2} - \beta\frac{W_A^2(y_1) + W_F^2(y_1)}{2}.$$

5.5 The Decision Problem and The Solution

We solve the decision problem in as direct a manner as possible. A simplifying feature is that there are no choices in period 2. The first period choice problem involves jointly deciding on an optimal bet level and making the optimal choice between watching television and gardening. It is immediate that if the decision is made not to watch the fight, then the optimal bet is zero, since the bet leaves first period utility unaffected, and lowers second period utility through risk aversion.[9] Even if the decision is made to watch television, it is clear that it cannot be optimal to bet a strictly positive amount on both fighters, since any shared component of the bet can only raise the level of risk without increasing the level of excitement in the first period. In addition, it is clearly preferable to place money on Ali rather than Frazier, since the final wealth implication does not depend on the fighter picked, but the utility at stake that determines period 1 excitement is strictly higher if the bet is placed on the favored fighter. This means that we can simplify notation and consider only bets of amount $b \in [0, W_0]$ placed on Ali.

The decision on whether or not to bet depends on the trade off between possible gains in first period utility from watching, betting, and anticipating as opposed to gardening, and the second period loss in expected utility associated with the bet. Given that the first period probability of Ali winning is $1/2$, we can compute the loss in second period expected utility associated with the bet $b \geq 0$ on Ali as βb^2. The gain in first period utility associated with betting amount $b \in [0, W_0]$ and watching the game can be straightforwardly computed as,

$$G(b) = s\,(u_A + 2b) - g.$$

The optimal bet conditional on watching the game can be found by maximizing the net gains function,

$$\max_{b \geq 0} \left[s\,(u_A + 2b) - g - \beta b^2 \right].$$

[9]This simple result follows from the assumption that there is no suspense associated with gambling unless the event is watched. A more realistic case would allow for a non-zero impact of gambling on suspense even if the fight could not be watched.

The first order condition solves for the optimal bet conditional on watching, b^*, as:

$$b^* = \frac{s}{\beta} > 0.$$

Whether or not it is worthwhile to bet depends on whether the net gain function is positive at this optimal level of betting. The unique optimum is to watch television and to bet amount b^* if and only if,

$$g < s\left(u_A + \frac{s}{\beta}\right).$$

The unique optimum is to garden if the strict inequality is reversed. In the case of equality, the two options of gardening or watching television and betting amount b^* are both optimal choices, and there are no other optima. Note that the basic comparative statics implied in this solution are natural. Watching television and betting is more attractive the more the individual cares who wins, the higher the suspense parameter, the more the the lower the level of risk aversion, and the smaller is the pleasure involved in the alternative activity.[10]

5.6 Implications and Extensions

The major qualitative prediction of the model is that it is possible for an agent to bet on a sporting event **not** because of its favorable implications for their wealth, but rather because it heightens feelings of suspense. This form of behavior is strongly tied to the time inconsistency of the preferences over lotteries. In the final period, the gamble is not a good idea, since the individual is risk averse. The bet is undertaken because of its interaction with the unfolding uncertainty, not because of its impact on the ultimate outcome of the lottery. In this sense our theory of gambling ties strongly with some early methodological comments of Samuelson:

> "...I am satisfied that a large fraction of the sociology of gambling and of risk taking will never significantly be discernible in terms of the money prizes alone, as distinct from elements of suspense...." (Samuelson [1952], p.676-77)

[10]Note that while in this simple case watching television implies gambling, these choices would be de-coupled if the bet was less than fair.

The broader theory of gambling implicit in the example has a number of testable implications. The basic prediction is that the degree of involvement matters to the decision to gamble, so that the theory may help explain why there is so much gambling on the outcome of sporting events. The theory also predicts a preference for betting on the favored outcome, and that the decision to gamble is linked with the decision on watching the event. This leads to the hypothesis that there will be more gambling on events that are broadcast than those that are not broadcast. We believe that practitioners of the art of design of lotteries and other avenues for gambling are exploiting these results in an intuitive manner already. After all, off-track betting outlets typically show the races on television, and even lottery selections are commonly televised.

One simple extension would be to allow for a broader initial condition, in the form of an arbitrary probability of Ali winning the fight. One way to do this would be to assume that the variance of the probability that Ali wins is a valid measure of the impact of prior beliefs on the level of suspense. In this case the level of suspense would be lower as the fight moved away from being evenly balanced, in favor of one fighter or the other. The immediate implication would be that both the level of gambling and the size of the audience would be increasing functions of the prior uncertainty concerning the outcome.

In contrast to our model, previous theories of gambling are essentially static, and are based on the properties of the utility function over final prizes. This is most transparent in the model of Friedman and Savage [1948], where the final prize is simply the level of wealth. It is also true of the non-expected utility model of Conlisk [1993], discussed in section 6.1 below.

There are many ways to generalize the example. One set of generalizations involves a multi-period setting in which it may be known that the outcome will not be determined until close to the end, which may well impact both the size of the audience and the extent of the gambling on the outcome. Other generalizations involve changing the domain of the decision problem to include broader settings in which suspense may be important. One such setting is the auction house: here it is of interest to explore whether the bidding and valuation process may be influenced by the auction design, specifically by its impact on the manner in which the uncertainty concerning who will get the object and at what price unfolds.[11] Another area in which suspense

[11] We thank Simon Gilchrist for suggesting this example.

matters is in the presentation of mystery novels, soap operas, etc. Indeed it has been argued that the build-up of suspense is a basic element in all story-telling (Brewer and Lichtenstein [1982]). Finally, there is the intriguing issue of identifying when and why some uncertainty is experienced as pleasurable suspense, while other forms of uncertainty inspire fear and anxiety.

6 Literature: Past, Present, and Future

Is our model really needed to cover anticipatory feelings, or can they be adequately treated in existing models? To answer this question, we discuss our ideas in the context of static non-expected utility theory in section 6.1, and the dynamic theory of Kreps and Porteus in section 6.2. In section 6.3 we discuss prospects for further development of the psychological expected utility theory.

6.1 Static Non-expected Utility Theory

In contrast with our approach, much recent progress in the theory of choice under uncertainty has involved the rejection of the substitution axiom for lotteries, and the development of various non-expected utility theories. Many of these theories have been developed in response to the observed violations of the substitution axiom, such as the Allais paradox. In fact, the chief goal of many non-expected utility theories is to relax the substitution axiom as little as possible, while nevertheless allowing for some limited class of violations.

Given this goal, most non-expected utility theories give little explicit consideration to the psychological origins of the implied departures from the classical substitution axiom.[12] As an example, consider the rank-dependent expected utility model of preferences over monetary lotteries (see Quiggin [1982]). For any monetary lottery X with associated cumulative distribution function F_X, the rank-dependent expected utility $V(X)$ can be expressed as:

$$V(X) = \int_{m=0}^{M} u(m) d[f \circ F_X(m)],$$

where $u : R_+ \longrightarrow R$ is a form of utility function, and the function $f : [0,1] \longrightarrow [0,1]$ acts to distort the assignment of probabilities to prizes. This

[12] Exceptions include the work of Bell [1985] and Loomes and Sugden [1986].

class of non-expected utility models has been further analyzed by Allais [1988], Green and Jullien [1989], and Grant and Kajii [1994], among others. As Green and Jullien point out, the model can be interpreted as arising from a version of expected utility theory in which the entire context of the lottery impacts the utility of an individual prize. For example, it allows an individual to value the prospect of \$100 at the 80th percentile more than they value this same prize at the 40th percentile.

What are the proposed psychological origins of this way of viewing lottery outcomes? On this point there is no consensus. In fact there is no consensus on whether the feelings that account for the probability transformation are anticipatory or retrospective. In his original article, Quiggin [1982] referred to the model as the anticipatory utility model, while Grant and Kajii [1994] define their special case of the model as a theory of disappointment aversion. It is not surprising that there can be disagreement on this point, since the model is silent on the connection between lotteries and psychological states.

This points to one vital difference between these classes of non-expected utility model and our psychological expected utility model. In these non-expected utility theories, rather than model the psychology of preferences directly, any such psychological forces that may be at work are left to be implicitly defined by the properties of the final utility function. This is recognized by Machina in his survey article on non-expected utility theory. After his comment (quoted at length in the introduction) on the appeal of the substitution axiom "provided the descriptions of consequences are sufficiently deep to incorporate any relevant emotional states", he goes on to add:

> "...preferences over observables such as monetary outcome levels .... could legitimately be non-separable. In other words, the various non-expected utility models of Table 1[13] could legitimately represent risk preferences when the consequences consist of monetary outcome levels."(Machina, [1989], p.1663).

Given the lack of explicit psychological rationale, it is not easy to apply non-expected utility theories to settings such as the anticipated vacation and the big fight introduced in sections 4 and 5 above. Application is all the more difficult in light of the second major qualitative difference between our model and non-expected utility theory, which is that we allow for dynamic

[13]In Table 1 Machina lists a wide variety of non-expected utility models, including the rank- dependent expected utility model.

phenomena, including time inconsistency of preferences. The static nature of non-expected utility theories is especially limiting in any application in which the evolution of personal feelings toward the lottery space is critical.

The model of gambling due to Conlisk [1993] illustrates some of the limitations inherent in attempts to apply static non-expected utility theory to such problems. For a given monetary lottery X with associated cumulative distribution function F_X, Conlisk introduces a function $V(X)$ to summarize the "gambling utility" associated with the lottery. In order to get the overall utility of the gamble for the agent with initial wealth W_0, Conlisk adds the two functions together, adding a parameter $\varepsilon > 0$ for purposes of scaling:

$$E(X : W_0) = \int_{m=-M}^{M} u(W_0 + m) dF_X(m) + \varepsilon V(X)$$

With respect to the function $V(X)$, Conlisk argues that it may depend on the standard deviation of the lottery winnings, or some such measure related to the possible dispersion of outcomes.

It is clear that there are some close analogies between the Conlisk model of gambling and our model of suspense in section 5. Indeed, our first period induced expected utility function can take precisely the same form as that given above, and for exactly the same reason:

> "The term $\varepsilon V(G, p)$[14] might be thought of as the utility of the excitement and suspense felt between the time he accepts the prospect, and the time that the uncertainty is resolved." [Conlisk [1993], p.262]

Despite these good intentions, the static nature of the Conlisk model makes it impossible to capture the importance of time. The result is a theory of gambling that is not as rich as the theory that we presented in section 5. There is no connection to other activities that heighten suspense, such as watching television; there is no theory that suggests which events will be worth gambling on; and there is no theory of which team will be supported. To capture these forces requires the explicit introduction of dynamic psychological forces.

More broadly, we believe that one of the advantages of our formulation over static non-expected utility models is that the latter theories attempt to

[14]The arguments of the V function correspond to the restrictive class of fair bets offering a gain of G with probability p.

telescope a dynamic pattern of feelings into a single static utility function. We do not believe that it will prove possible to develop psychologically rich theories of risk-taking behavior in a static context. This applies not only to the issue of suspense and gambling, but also to such phenomena as disappointment, as modelled by Bell [1985] and Loomes and Sugden [1986]. While their models are explicit concerning the way in which a given lottery may produce feelings of disappointment, they remain static. Among other things, this means that the models do not include the anticipatory phase. Have you ever felt disappointed about an outcome without having experienced prior feelings of hopefulness?[15]

6.2 The Kreps-Porteus Model

Our approach to the dynamic choice problem has borrowed heavily from the dynamic non-expected utility model introduced by Kreps and Porteus ([1978], [1979a], [1979b]). They provided the original definition of the space of temporal lotteries, and we have followed their lead in using this as the domain for our model of evolving uncertainty. Another important similarity is that they characterize preferences over these temporal lotteries that, within any given period, satisfy a substitution axiom and that therefore admit of an expected utility representation, just as do the preferences that we analyze. A third point of similarity is that the Kreps-Porteus model allows for preferences over the date of resolution of uncertainty, as does our model. In intuitive terms, it is clear that the example of the anticipated vacation in section 4 involves a preference for early resolution of uncertainty, since the agent prefers to have all information about where they would travel realized as soon as possible. Conversely, the example of suspense and gambling in section 5 involves a preference for late resolution of uncertainty, since the agent prefers to remain uninformed about the outcome of the fight for as long as possible.

Despite these similarities, our model differs in fundamental ways from that of Kreps and Porteus. One significant difference is that they link together the expected utility functions in distinct periods with the assumption of time consistency. What this means is that the agent's current preferences over future lotteries must remain unchanged as time passes. This condition

[15] Psychological recognition of the link between these two emotions can be found in the very interesting book of Ortony, Clore, and Foss [1988].

is far from natural in most interesting dynamic settings. In the two examples that we have presented, the preferences over lotteries are time inconsistent: indeed the most qualitatively interesting aspects of these examples derives precisely from the time inconsistency. In the setting with anticipation, the deposit is put down as a commitment device against the future changes in preferences. In the setting with suspense, the bet is placed in period one, despite the agent's risk aversion with respect to lotteries in period 2. Time consistency is only satisfied in our model under exceptional circumstances.

It is hardly surprising that time consistency is highly restrictive in settings with anticipatory feelings, since earlier feelings become bygones as the period of anticipation shrinks, and thereby lose their direct role in determining preferences. In fact, the restrictive nature of time consistency in settings with anticipation is clear even in a deterministic setting, as noted by Loewenstein [1987]. Loewenstein analyzed the impact of anticipatory feelings on the timing of consumption, arguing that an event that provides pleasurable anticipatory utility may optimally be delayed in order to enrich the overall experience. But if it is better to delay consumption until next month, won't such a delay also appear optimal in the next period? This gives rise to some very sophisticated questions concerning how to model the optimal timing of consumption, such as whether or not it may be optimal for the agent to use mixed strategies to create their own uncertainty.

The second difference between our model and that of Kreps and Porteus lies in the fact that, for us, the expected utility function on the space of temporal lotteries is induced by a more fundamental description of the psychological response to uncertainty. By adding in the additional data that is necessary to encode this psychological response, we believe that we are providing applied theorists with a highly flexible tool, which can be used in different ways depending on the underlying context. We are currently using this flexibility to expand the framework in several interesting (at least to us!) directions.

6.3 Prospects for Psychological Expected Utility Theory

In this section we offer preliminary comments on some of the most promising avenues for further exploration of the psychological expected utility theory, and also some of the thorny theoretical issues that will have to be resolved in order for the theory to achieve its full potential.

6.3.1 Other Anticipatory and Retrospective Feelings, and Moods

In addition to enriching the theory of suspense and of anticipatory excitement sketched out in sections 4 and 5 above, there are other anticipatory emotions that can be analyzed, including fear, anxiety, worry, nervousness, etc. Each of these will have its own set of associated decision-theoretic phenomena, just as feelings of suspense connect to decisions on gambling. In this manner, we believe the theory opens the door to considering some of the issues raised by Elster [1996] in his plea for theoretical consideration of the emotions.

There are other important emotional responses to uncertainty that occur in the periods following the resolution of uncertainty. These include disappointment, relief, pleasant surprise, satisfaction, etc. These are also candidates for study, and we believe that the model can be usefully extended to provide insight into the nature and implications of these feelings. Of even greater interest are examples in which there are both anticipatory and retrospective feelings, as when anticipation turns to disappointment if things do not turn out as hoped, or when feelings of fear give rise to feelings of relief if the feared outcome does not happen. One interesting possibility is that extending the theory in this direction will offer a dynamic insight into the construction of reference points. In this manner, our approach may ultimately offer an alternative to existing static versions of prospect theory (see Kahnemann and Tversky [1979]). However, it is important to note that the development of a general theory in this direction will not be trivial, since a number of subtle issues are raised by the simultaneity involved in determining feelings when there are both forward and backward linkages. What exactly happens when feelings today depend upon anticipated future feelings, and vice versa?

6.3.2 Time Inconsistency and the Existence of Optimal Choices

There are several distinct approaches to the existence of optimal choices in decision problems in which there is time inconsistency. Our approach in section 3 follows the suggestion of Strotz [1955], in which the agent in each period restricts attention to the constraints on future choices implied by future optimality, and then picks the best current strategy in light of these constraints. The difficulty of proving existence of optimal solutions in the spirit of Strotz was pointed out by Peleg and Yaari [1973], and is explicitly referred to by Kreps and Porteus [1978] as a partial justification for their assumption of time consistency. In cases of non-existence, the suggestion of Peleg and Yaari was to re-cast the decision problem as a game, and to look for Nash equilibria of the game, and variants of this approach have been followed in most of the recent literature.[16]

The approach that we follow in the two period model of section 3 is to select the best sub-game perfect equilibrium from the period 1 perspective. This approach can be generalized to the T-period model. However, we know of no proof of the existence of such an optimum when there are more than two periods. In this light, our discussion of the existence of an optimal consistent strategy in the two-period decision problems of section 3 merely scratches the surface of a very intricate set of questions concerning the formulation and solution of decision problems when preferences are not time consistent. We are actively researching this area, and are confident that progress can be made.

6.3.3 Feelings about Feelings and the Substitution Axiom

With respect to the psychological state space, our model makes the substitution axiom very close to an axiomatization of deliberative personal rationality. Suppose you list all of your possible sequences of personal psychological states, and describe them in a detailed enough manner to pin down just how pleasurable each such sequence would be to you in any future period. Now you are asked to compare two lotteries over such sequences of personal states, and are asked whether or not your preferences can be altered when you mix each lottery probabilistically with a third lottery over sequences of psychological states, i.e. you are asked whether your preferences obey the substitution axiom. If you answer that such a substitution could indeed alter your preferences, then it would be legitimate to ask you on which personal feelings such a reversal of preference was based. Any answer that you gave would necessarily reveal an incompleteness in your original description of your psychological state: the feelings upon which you differentiate between these two lotteries should already have been encoded in the list of psychological states, and therefore should not "spill over" and impact your preferences over composite lotteries.

[16] The work of Caillaud, Cohen, and Jullien [1994] is a notable exception.

In our view, the real difficulty that this discussion points to is that the mapping from external lotteries to psychological states may be extremely complex. Issues of this sort were foreshadowed in the early comments of Samuelson [1952]. He noted that the substitution axiom must always be applied to a fixed set of prizes, such as monetary lotteries plus feelings of suspense, and that it need not impose restrictions on preferences over simpler entities, such as monetary prizes alone. He went on to worry about how to stop the state space from expanding out of all control.

> "If every time you find my axiom falsified, I tell you to go to a space of still higher dimensions, you can legitimately regard my theories as irrefutable and meaningless..." (Samuelson [1952], p.677.)

Our view of the matter is that the psychological expected utility model is a useful general framework, with clear areas of application as already shown. The issue of just how far one must go in elaborating these feelings is a pragmatic one that will vary from application to application. In the thorniest of problems, one can imagine the kind of infinite regress suggested by Samuelson, in which lotteries over any fixed set of prizes necessarily add feelings that were ignored in the last iteration. However, these subtle issues are not present in all applications of the theory, as we have already seen.

6.3.4 State and Trait

One intriguing issue concerns how to specify the universe of problems in which a given feeling, such as anticipatory excitement (or dread) are experienced by a given individual. This may be intuitively obvious in such (almost) universal cases as the upcoming vacation, or the trip to the dentist. But in other cases, the issue will call for a great deal more thought, and there may be important interactions between the individual's personality type, and the nature of their emotional response to uncertainty. The issue of finding the domain of applicability of a particular emotional response to uncertainty is such an important issue that we believe it will warrant its own body of thought.

Examples of how subtle this may be can already be found in the psychological literature on anxiety. Early on, anxiety was seen in universal terms, as induced by particular features of the environment, down-playing the impact of the individual's character traits. The pioneer in the analysis of the impact of character traits was Spielberger [1985], whose more well-rounded approach is known as the "state and trait" approach. Pursuit of this approach leads to the need to think somewhat more deeply about the structure of the personality. Specifically, there are now a number of theories in which the nature of the feelings induced by uncertainty are seen as central to the individual's character-type (this is a common-place in everyday language, as when we differentiate between optimistic and pessimistic people).[17] In the longer term, we hope that our work helps build a bridge between economic theory and some of these stimulating psychological literatures.

7 Concluding Remarks

In this paper we have introduced the psychological expected utility model, and used it to analyze the impact on decision-making of anticipatory feelings, such as suspense and anticipatory excitement, for decision-making. The broader goal of our research agenda is to open up a variety of psychologically interesting phenomena to rational analysis, and in this respect our work has just begun.

[17] One recent approach that is particularly relevant has been pursued by Sorrentino [1996], who has developed a psychological theory of uncertainty aversion.

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