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Abstract

In 1972 Harsanyi and Selten characterized a one parameter asymmetric Nash solution. In this note I do the analog for the Kalai-Smorodinsky (KS) solution. Replacing Symmetry with a restricted version of Independence of Irrelevant Alternatives in the set of axioms that lead to the KS solution, I characterize an asymmetric version of that solution that depends only on one parameter.

1 Introduction

In 1950 Nash solved the first two person cooperative bargaining problem: "The economic situations of monopoly versus monopsony, of state trading between two nations, and of negotiation between employer and labor union may be regarded as bargaining problems". Generally, the primitives are: a set that describes the feasible outcomes of the bargaining, the *disagreement point* -an element of the set that is the outcome if no agreement is reached- and a function -the *solution* F- that assigns to each bargaining problem an outcome, a point in the set. The main issue in the field is characterizing solutions: state certain desirable properties that F should satisfy (the axioms) and find the unique functional form such that F satisfies the axioms if and only if it has that functional form.

Nash's classic result is that under certain axioms there is a unique solution to the bargaining problem. However, in the characterization he used the axiom of Independence of Irrelevant Alternatives (IIA) which was later criticized because it failed to take into account

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important features of the bargaining sets. Apart from this shortcoming, IIA is a reasonable axiom, so I will define Restricted IIA, a weaker axiom than IIA that partially overcomes its criticisms.

In 1975 Kalai and Smorodinsky noted that Nash's solution, in addition to being based on the problematic IIA, failed to satisfy certain properties they felt were desirable. Consequently, they postulated a new set of axioms and characterized a new solution.

Another important step in the field was given in 1972 by Harsanyi and Selten. They relaxed Symmetry -one of the properties used by both Nash (1950) and Kalai and Smorodinsky (1975)- from the set of axioms that leads to the Nash solution and, adding Strong Individual Rationality, found a one parameter class of asymmetric Nash solutions. Since then, some objections to Symmetry (Sym) have been raised. In particular, if the bargaining problem is interpreted as a robust representation of some more complicated non-cooperative game, Sym becomes a problematic axiom. In this setting, several features of the original situation may not be modeled in the problem. Imposing Sym can then mean, for example, assuming equality of bargaining skill between the parties (Harsanyi, 1977).[1] Hence, if in a bargaining situation within a family it is the case that the man and the woman do not have the same bargaining power, assuming Sym is unreasonable. Since, in fact, predictions based on symmetric solutions disagree with the empirical evidence on bargaining problems within families, Dasgupta (1993, p.342) argues against Sym. Thus, allowing for asymmetric solutions makes the theory more flexible.

Moreover, within the class of asymmetric solutions, one parameter families of solutions are important for at least two reasons. First, if it is known both that a solution belongs to a one parameter family and what the outcome dictated by the solution is in a certain problem, it is likely that the solution can be identified. Put differently, it is likely that the outcome in any other problem can be computed. This condition is not satisfied by large families of solutions: it may well be the case that a continuum of solutions within a large family would yield a given outcome. If this is the case, knowing the outcome in a problem is not very informative about the outcomes in other problems. Second, when a bargaining problem is just one stage of a complicated model, as in the literature on unemployment surveyed by Azariadis (1979), an easy-to-compute solution is needed. A one parameter class of solutions is likely to be simple.

If a solution is interpreted as the outcome an arbitrator would choose, IIA seems reasonable, for it is equivalent to requiring that the arbitrator's choices satisfy the Weak Axiom of

[1] Of course, if all the relevant information to the problem is contained in the bargaining set, there is nothing unspecified, in particular, there is no such thing as bargaining power.

Revealed Preference. However, if cooperative baragining theory is about how agents actually bargain, and not about "rational" arbitration, the axioms used by Kalai and Smorodinsky are more adequate than those used by Nash. Then, the criticisms to Sym and the need of simple and identifiable solutions call for a one parameter asymmetric Kalai-Smorodinsky (KS) solution. Nevertheless, relaxing Symmetry from the set of axioms that leads to the KS solution does not yield the desired result: Peters and Tijs (1985) showed that, given the outcome in one problem, there are more than continuum many solutions that satisfy all of KS axioms -exept for Sym- that would yield that outcome. So, to characterize the one parameter class of solutions, I will use the axiom of Restricted IIA which preserves the desirable features of IIA, and overcomes some of its problems.

As by products of this work I get three new characterizations of the KS solution that do not rely on Symmetry. Also, two of the characterizations do not depend on Scale Invariance either. That is, I recover those two properties from seemingly unrelated axioms. This work will shed some light on the classification of continuous solutions, an issue that, according to Kalai and Smorodinsky (1975), would lead to a better understanding of the bargaining problem. Finally, I extend my work to non-convex problems and address the issue of more than two players.

2 Preliminaries

As is standard, I denote a *two person bargaining problem,* or simply a *problem,* by a set $S \subseteq \mathbf{R}^2$ such that $\mathbf{0} \in S$. The usual interpretation is that S is the set of all utility profiles that a bargaining process could possibly yield, and $\mathbf{0}$ is the disagreement point.

For all $x, y \in \mathbf{R}^2$: $x \geq y$ if and only if $x_i \geq y_i$ for $i = 1, 2$; $x > y$ if and only if $x_i \geq y_i$ for $i = 1, 2$ and $x_j > y_j$ for some j. Finally, $x \gg y$ if and only if $x_i > y_i$ for $i = 1, 2$.

I say that S is *comprehensive* if $y \in S$ whenever $x \in S$ and $x \geq y \geq \mathbf{0}$. S is said to be *strictly comprehensive* if it is comprehensive and there exists a $z \in S$ such that $z \gg y$ whenever $x, y \in S$ and $x > y \geq 0$. If a set is strictly comprehensive its boundary in $\mathbf{R}^2_{++}$ does not have any vertical or horizontal flats.

The *comprehensive hull* of a set $S \subseteq \mathbf{R}^2$ is the smallest comprehensive set containing S. I will denote it *comp*(S). The *convex comprehensive hull* of a set $S \subseteq \mathbf{R}^2$, *cch*(S), is the smallest convex and comprehensive set containing S. I denote by Σ the class of compact and comprehensive sets $S \subseteq \mathbf{R}^2_+$ for which there is an $x \in S$ such that $x \gg \mathbf{0}$. The set of all convex elements of Σ is denoted Σ^c.

Any function that chooses for each set $S \in \Sigma$ an element of S is called a *solution.* A generic solution is denoted by F. For any set $S \in \Sigma$, I define:

$a_i(S) \equiv \max\{x_i : (x_1, x_2) \in S\}$, $i = 1, 2$

$P(S) \equiv \{x \in S : y > x \Rightarrow y \notin S\}$ and $WP(S) \equiv \{x \in S : y \gg x \Rightarrow y \notin S\}$

For any problem S, $a(S)$ is the "utopia" point, the utility profile in which each individual achieves the greatest utility level he can get from the bargaining problem. $P(S)$ and $WP(S)$ are the sets of strongly and weakly Pareto Optimal utility allocations.

3 Background and Motivation

Nash (1950) required that his solution satisfied:

Independence of Irrelevant Alternatives (IIA): For all $S, T \in \Sigma$, if $S \subseteq T$ and $F(T) \in S$ hold, then $F(S) = F(T)$.

Pareto Optimality (PO): For all $S \in \Sigma$, $F(S) \in P(S)$.

Symmetry (Sym): Given $e : \mathbf{R}^2 \to \mathbf{R}^2$ defined by $e((x_1, x_2)) = (x_2, x_1)$, we must have $F(e(S)) = e(F(S))$ for all bargaining problems S.

Scale Invariance (SI)[2]: For all $T \in \Sigma$ and affine $\sigma : \mathbf{R}^2 \to \mathbf{R}^2$, we have that $F(T) = x$ iff $F(\{\sigma(t) |\ t \in T\}) = \sigma(x)$.

As is well known, Nash's result is that there is one and only one function N that satisfies these axioms on Σ^c. For any S, $N(S)$ maximizes the mapping $x \mapsto x_1 x_2$ on S.

If cooperative baragining is about rational arbitration, IIA is a desirable axiom. However, if the theory is about how agents actually bargain, IIA is less acceptable.[3] Nash's motivation for IIA assumed that the geometry of the bargaining sets involved did not affect what would be the "fair bargain". For example, if for $T = cch(\{(1, 0), (0, 1)\})$, the solution dictates $F(T) = \left(\frac{1}{2}, \frac{1}{2}\right)$, IIA implies that for $S = cch\left(\left\{(1, 0), \left(\frac{1}{2}, \frac{1}{2}\right)\right\}\right)$, we must have $F(S) = \left(\frac{1}{2}, \frac{1}{2}\right)$.

The "un-fairness" of the choice dictated by IIA in this example comes from the fact that although in passing from T to S individual 2 sees his best choices disappear, and individual 1 does not, the solution is unchanged. It is possible to overcome this problem by weakening IIA in the following way:

[2] In the bargaining literature, a *problem* is usually defined as a pair (S, d) where $d \in S$ is called the disagreement point. In my formulation, $d = 0$ is without loss of generality since I confine attention to Scale Invariant solutions -for which translation of the origin is irrelevant-.

[3] Although Nash (1950) justified IIA for the case where the solution was interpreted as the outcome of a bargaining process, he said that the interpretation of the axiom was "more complicated" than that of the others he used.

Restricted IIA (RIIA): For all $T, S \in \Sigma$, if $S \subseteq T$, $F(T) \in S$ and $\beta a(S) = a(T)$ for $\beta \in \mathbf{R}_{++}$ hold, then $F(T) = F(S)$.

In contrast with IIA, Restricted IIA takes into account some features of the problems that it relates. In particular, RIIA requires that a_2/a_1, a proxy for the relative individual standings, remains constant. Consider two problems S, T such that $S \subseteq T$. It is easy to imagine that if the relative standings of the parties does not change (when passing from T to S) and the original choice remains available in the smaller set, they would choose again the same point. That is precisely RIIA. The requirement that a_2/a_1 does not change between two sets makes the allowed variation (in the problems to be related) small, and so one of the most common criticisms to IIA is partially overcome. Although RIIA is a new axiom, the idea of controlling for the utopia point in an IIA-type axiom is not new: Roth (1977a) and Imai (1983) use RIIA with $\beta = 1$. Also, since RIIA is weaker than IIA, it is satisfied by the Nash solution. In Section 4 I give alternative axioms that, taking for granted Pareto Optimality, imply RIIA.

In addition to the objections to IIA, some criticisms were raised directly against Nash's solution. In particular, Kalai and Smorodinsky argued that in any two problems S and T such that $S \subseteq T$, $a_i(T) = a_i(S)$ and $a_j(T) \geq a_j(S)$, player j has good reason to demand that he gets more in problem T than he gets in S, and Nash's solution fails to satisfy that requirement. To overcome these problems, they introduced the following axiom:

Individual Monotonicity (IM): if $S \subseteq T$, $a_i(T) = a_i(S)$ and $a_j(T) \geq a_j(S)$ then $F_j(T) \geq F_j(S)$ for $i, j \in \{1, 2\}$, $i \neq j$.

Kalai and Smorodinsky proved that the unique solution that satisfies SI, Sym, PO and IM is KS : for all $S \in \Sigma^c$

$$KS(S) \equiv \left\{ x \in \mathbf{R}_+^2 : x_2 = \frac{a_2(S)}{a_1(S)} x_1 \right\} \cap WP(S)$$

Note that while KS does not satisfy IIA, it does satisfy RIIA. Thus, IM and RIIA are compatible.

The criticisms to both IIA and Nash's solution and the acceptability of IM make the KS type of solutions compelling. Moreover, the criticisms to Sym -presented in the Introduction- and the need of simple and identifiable solutions call for a definition and characterization of a one parameter asymmetric KS solution.

The first step is to define a new asymmetric one parameter function that generalizes the KS solution in a natural way. For notational simplicity, let $\left\{ x \in \mathbf{R}_+^2 : x_2 = \infty x_1 \right\}$ denote the y axis throughout. Then, for $\lambda \in \mathbf{R}_+ \cup \{\infty\}$ define what Thomson (1994) calls the *weighted*

KS solution by $D_\lambda(S) \equiv \left\{x \in \mathbf{R}^2_+ : x_2 = \lambda \frac{a_2(S)}{a_1(S)} x_1\right\} \cap WP(S)$. The KS solution is nested within $\{D_\lambda : \lambda \in \mathbf{R}_+ \cup \{\infty\}\}$: $D_1 = KS$. However, for $\lambda \neq 1$ the solution D_λ is not Pareto Optimal.[4] Hence, by taking the Lexicographic extension of D_λ, I define a new one-parameter asymmetric solution that satisfies PO. I call it the *asymmetric KS solution*: for all $S \in \Sigma$ and some $\lambda \in \mathbf{R}_+ \cup \{\infty\}$

$$KS_\lambda(S) \equiv \{x \in \mathbf{R}^2_+ : x \geq D_\lambda(S)\} \cap P(S)$$

The second step, which is my aim in this paper, is to characterize the solution. As was noted earlier, relaxing Sym from the KS set of axioms is not enough, since this leads to the class of individually monotonic solutions.[5] The problem is that there are too many individually monotonic solutions: given $F(S)$ for any $S \in \Sigma^c$, there are more than continuum many individually monotonic solutions that would yield $F(S)$ as an outcome.

4 The Results

4.1 Convex bargaining problems

I will just prove the *only if* parts of the propositions, the *if* parts are straightforward.

4.1.1 The Asymmetric Kalai-Smorodinsky solution KS_λ

If a non dictatorial solution satisfies PO and SI on Σ^c, it can not satisfy both IM and IIA. However, in this section I prove that on the domain of convex sets, IM and RIIA are compatible, and characterize KS_λ using a tight set of axioms.

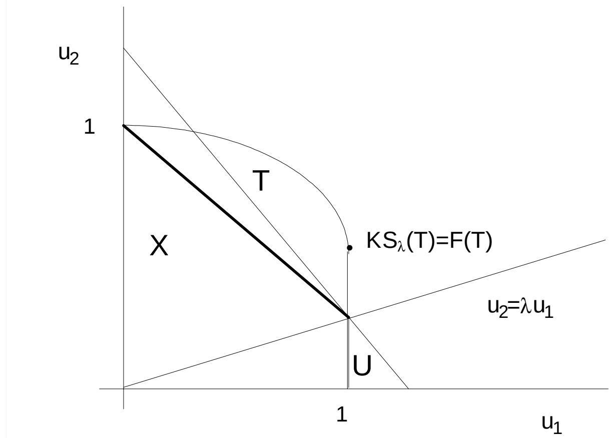
Theorem 1 *A solution F satisfies SI, IM, PO, and RIIA on Σ^c if and only if there exists a unique $\lambda \in \mathbf{R}_+ \cup \{\infty\}$ such that for all $S \in \Sigma^c$*

$$F(S) = KS_\lambda(S) \equiv \{x \in \mathbf{R}^2_+ : x \geq D_\lambda(S)\} \cap P(S)$$

[4]For $\lambda \neq 1$, $D_\lambda(cch(\{(1,1)\})) \neq (1,1)$ the only Pareto Optimal point in $cch(\{(1,1)\})$.

[5]Peters and Tijs (1985) proved that a solution F satisfied IM, SI and PO if and only if it had an associated *monotonic curve* $\theta : [1,2] \rightarrow cch(\{(1,1)\})$ such that for all $s, t \in [1,2]$ if $s \leq t$, holds, then $\theta(s) \leq \theta(t)$ and $\theta_1(s)+\theta_2(s) = s$ hold. Then, for each solution F satisfying the axioms, there exists an individually monotonic solution π^θ such that $\pi^\theta(S)$ is the unique point of $P(S)$ that lies in $\{\theta(t); t \in [1,2]\}$ for all $S \in \Sigma^c$ such that $a(S) = (1,1)$.

Proof: Let $\Delta \equiv \left\{x \in \mathbf{R}_+^2 : x_1 + x_2 \leq 1\right\}$. Given $F(\Delta)$, there is a unique $\lambda \in \mathbf{R}_+ \cup \{\infty\}$ such that $F(\Delta) = \{x : x_2 = \lambda x_1\} \cap WP(\Delta)$ (see figure). For any set $S \in \Sigma^c$, define $T = \left\{\left(\frac{y_1}{a_1(S)}, \frac{y_2}{a_2(S)}\right) \mid y \in S\right\}$ and find $D_\lambda(T) = (D_\lambda(T)_1, D_\lambda(T)_2)$.



Now let $as_\lambda(T) \equiv D_\lambda(T)_1 + D_\lambda(T)_2$, and $U \equiv \{as_\lambda(T)y \mid y \in \Delta\}$. Then SI implies that $F(U) = as_\lambda(T)F(\Delta) = D_\lambda(T)$. Let $X \equiv cch(\{D_\lambda(T), (1,0), (0,1)\})$. Since $a(U) = as_\lambda(T)a(X)$ and X is contained in U, RIIA implies that $F(X) = D_\lambda(T)$. Applying IM twice I get $F(T) \geq D_\lambda(T)$. Then, PO yields $F(T) = KS_\lambda(T)$, and by SI, $F(S) = KS_\lambda(S)$. ■

I have managed to preserve desirable axioms like IM, PO and SI, and by adding a version of IIA that overcomes its major criticisms, I get a simple, one parameter asymmetric KS solution. As desired, given the choice KS_λ assigns to any strictly comprehensive $S \in \Sigma^c$, λ can be identified, and the solutions in any problem can be found.

Note that Nash's solution, $F(S) = \mathbf{0}$ and $LexEg(S) = \{x : x \geq \left\{x \in \mathbf{R}_+^2 : x_2 = x_1\right\} \cap WP(S)\} \cap P(S)$ fail to satisfy, respectively, IM, PO and SI while they satisfy the rest of the axioms. An individually monotonic solution that does not satisfy RIIA is $H(S) = \{x : x \geq \left\{x \in \mathbf{R}_+^2 : x_2 = \frac{a_2(S)}{2}\right\} \cap WP(S)\} \cap P(S)$.[6]

4.1.2 Three New Characterizations of the KS Solution

I now use the results of the previous section and give three new characterizations of the KS solution that do not rely on Sym. Moreover, the last two do not depend on SI either. For my first characterization I need to define continuity.

Continuity (C): for all sequences $\{S_n\}_1^\infty$, $S_n \in \Sigma$ for all n, such that S_n converges to S in the Hausdorff topology, $F(S_n)$ converges to $F(S)$

[6] To see this, note that $H(\Delta) = \left(\frac{1}{2}, \frac{1}{2}\right)$and $H\left(cch\left(\left\{\left(\frac{3}{4}, 0\right), \left(0, \frac{3}{4}\right), \left(\frac{1}{2}, \frac{1}{2}\right)\right\}\right)\right) \neq \left(\frac{1}{2}, \frac{1}{2}\right)$

Proposition 2 *A solution F satisfies SI, IM, PO, C and RIIA on Σ^c if and only if for all $S \in \Sigma^c$*

$$F(S) = KS(S)$$

Proof: For any $S \in \Sigma^c$, pick a sequence of strictly comprehensive sets $\{S_n\}_1^\infty$, that converges to S. Since for all n, $WP(S_n) = P(S_n)$, $F(S_n) = D_\lambda(S_n)$ for some fixed $\lambda \in \mathbf{R}_+ \cup \{\infty\}$. Since F is continuous, $F(S) = D_\lambda(S)$. Then PO and the fact that D_λ satisfies PO only for $\lambda = 1$ yield $F(S) = D_1(S) \equiv KS(S)$. ■

Apart from giving a new characterization of the KS solution, this Proposition addresses the issue of continuity of solutions studied by Kalai and Smorodinsky. In Σ^s the domain of strictly comprehensive problems, KS_λ is continuous for all λ, so the requirement of KS_λ being continuous is binding only on $\Sigma^c \backslash \Sigma^s$, a "small" set. In the sets where KS_λ is discontinuous, it is assigning to some player a payoff strictly bigger than that D_λ would give.

For the next result, recall the axiom of Restricted Monotonicity.

Restricted Monotonicity ($\mathbf{IM_2}$): If $S \subseteq T$ and $a(S) = a(T)$, then $F(S) \leq F(T)$.

Peters and Tijs (1985) showed that a solution F satisfies IM if and only if it satisfies IM_2. Now, IM can be interpreted as follows: if the relative standings of the two individuals do not change from one problem to another and the set of possible outcomes is enlarged, they should both get more in the larger problem. I now define a similar notion:

General Monotonicity (GM): If $S \subseteq T$ and $a(S) = \beta a(T)$ for some $\beta \in \mathbf{R}_{++}$, then $F(S) \leq F(T)$.

The interpretation is the same as the one just given for IM_2. To show that GM implies IM, set $\beta = 1$ in GM and use the Peters and Tijs's result of the last paragraph. The following lemma shows that, taking PO for granted, RIIA can be derived from an axiom similar to IM. In addition, it provides my second new characterization of the KS solution, this time based on GM. Note that SI is not needed.

Proposition 3 *A solution F satisfies GM and PO on Σ^c if and only if for all $S \in \Sigma^c$*

$$F(S) = KS(S)$$

Proof: For any $S \in \Sigma^c$, PO implies $F(comp(\{KS(S)\})) = KS(S)$. Then, GM yields $F(S) \geq KS(S) \in P(S)$ and PO completes the proof. ■

Raiffa (1953) discusses the KS function as a possible solution when interpersonal comparisons of utility are allowed. The following axiom and the results that follow give a characterization of KS in that spirit.

Relative Monotonicity (RM): For all $S, T \in \Sigma^c$, if $S \subseteq T$ and $a_j(T)/a_j(S) \geq a_i(T)/a_i(S)$ hold, then $F_j(T)/F_j(S) \geq F_i(T)/F_i(S)$ for $i, j \in \{1, 2\}$, $i \neq j$ must hold.

RM states that given two problems, if individual j improves his situation more than i in passing from situation S to T, he should get relatively more in T than in S. Note that this axiom is similar to IM. Before the third characterization of KS, I derive RIIA from RM and WPO.

Lemma 4 *If a solution F satisfies WPO and RM on Σ, it also satisfies RIIA*

Proof: Pick any S, $T \in \Sigma^c$ such that $S \subseteq T$, $F(T) \in S$ and $\beta a(S) = a(T)$ for $\beta \in \mathbf{R}_{++}$. Then, $\beta a(S) = a(T)$ and RM applied twice imply that $F(S) = \gamma F(T)$ for some $\gamma > 0$, so WPO implies $F(T) = F(S)$. ■

I now give my third characterization of KS. As before, SI is not needed.

Proposition 5 *A solution F satisfies PO and RM on Σ^c if and only if for all $T \in \Sigma^c$*

$$F(T) = KS(T)$$

Proof: For any set $T \in \Sigma^c$ there exists $x \in T$ such that $x \gg 0$, so there exists a $\beta > 0$ for which $S = \beta comp(a(T)) \subseteq T$. *PO* implies that $F(S) = \beta a(T)$, so *RM* applied twice yields $F(T) = \gamma a(T)$ for some $\gamma > 0$. *PO* then yields $F(T) = KS(T)$. ■

Since the KS solution is Symmetric and Scale Invariant, I have recovered Sym and SI from other axioms that do not imply them directly.[7]

A few words about the significance of these results are in order. Characterizations of solutions in cooperative bargaining are meant to be in the utility space because of the use of SI. Hence, if the primitives of a problem are monetary payoffs, bargaining theory cannot be used. That is the reason why empirical applications (for example, testing if Nash's solution or the KS solution yield correct predictions) are forced to assume specific forms of utility functions. Proposition 5 gives a characterization of the KS solution that does not assume SI, so that applications can now use directly data on money to test predictions. Moreover, RM can be reinterpreted as a comparison of relative monetary, and not utility, gains.

[7] A result of this kind was obtained by Roth (1977b). He recovered Pareto Optimality from other axioms.

4.2 Non-Convex Bargaining Problems

The assumption of convexity of the bargaining problem arises naturally if randomizations between outcomes are allowed, even if the original problem is non-convex. However, randomizations are not always allowed in bargaining problems, as in state trading between two nations (Ok and Zhou, 1997). Moreover, even if randomizations are permitted, the problem may be settled at a lottery, and the axioms that characterize the solution may only be satisfied in expectation (Conley and Wilkie, 1996). Hence, if I want the axioms to be satisfied *ex-post* it is important to see how the results on the convex domain extend to Σ, the non-convex domain. To do so, I first introduce an axiom:

Strong Individual Rationality (SIR): For all $S \in \Sigma$, $F(S) \gg 0$.

This axiom, used by both Roth (1977b) and Harsanyi and Selten (1972), states that if there is some chance of benefitting from bargaining, the outcome will leave both parties strictly better off than in the no agreement situation. In particular, SIR rules out dictatorial solutions. Harsanyi and Selten argued that since in any agreement the parties will choose something greater than 0, eliminating that utility value for each agent should not change the solution.

The first result in this section is that three desirable axioms like SIR, PO and IM -that KS_λ satisfies for $\lambda > 0$ in the convex domain- cannot be preserved in non convex sets.

Lemma 6 *There does not exist a solution F that satisfies IM, PO and SIR on Σ.*

Proof: Suppose there exists such a solution F. Then, for any strictly comprehensive problem S pick a point $p \in P(S)$ such that $p \neq F(S)$. Let $X \equiv comp(\{p, (a_1(S), 0), (0, a_2(S))\})$. Since $P(X) \cap \mathbf{R}^2_{++} = p$, PO and SIR imply that $F(X) = p$. Apply IM twice and get $F(S) \geq F(X) = p$, so $p \in P(S)$ implies $F(S) = p$, a contradiction. ■

Moreover, in the domain of non convex problems, the KS_λ solution is not a function, but a correspondence, so I will chararacterize D_λ. Since there are too many solutions that satisfy WPO, IM and SI, the set of axioms needs to be strengthened in order to get a one parameter class of solutions.

Proposition 7 *A solution F satisfies SI, WPO and RM on Σ if and only there exists a unique $\lambda \in \mathbf{R}_+ \cup \{\infty\}$ such that for all $S \in \Sigma^c$*

$$F(S) = D_\lambda(S)$$

Proof: Given $F(\Delta)$ there exists a unique $\lambda \in \mathbf{R}_+ \cup \{\infty\}$ such that $F(\Delta) = D_\lambda(\Delta)$. For any set $S \in \Sigma$, and any fixed $p \in \mathbf{R}_{++}$, define $T_p(S) = \left\{ \left(\frac{y_1}{a_1(S)} \frac{1}{p}, \frac{y_2}{a_2(S)} \frac{1}{p} \right) \mid y \in S \right\}$. Since for some p, $T_p(S) \subseteq \Delta$ and $a(T_p(S)) = \frac{1}{p} a(\Delta)$, RM applied twice implies $F(T_p(S)) = \gamma D_\lambda(\Delta)$. WPO then implies $F(T_p(S)) = D_\lambda(T_p(S))$ and SI completes the proof. ■

5 Concluding Remarks

Given the criticisms to IIA, I have defined a new axiom, a restricted version of IIA (RIIA) that overcomes its major difficulties. It turned out that, as desired, this weaker version of IIA is compatible with Individual Monotonicity (IM). Also, I derived RIIA from axioms related to IM.

Since I was concerned with finding a one parameter asymmetric version of the KS solution, I first defined D_λ, a non-Pareto Optimal asymmetric version of KS. Based on it I defined KS_λ, the new one parameter *asymmetric KS solution*, and characterized it for the convex domain using RIIA.

Also, I gave three new characterizations of KS. The axioms used in the characterizations seem unrelated to Sym and Scale Invariance and both are recovered. It then seems that there is much to be learnt from the relationships between the axioms that have been used in the literature. Finally, I characterized D_λ on the non-convex domain.

For $n > 2$ players, and $\lambda \in (\mathbf{R}_+ \cup \{\infty\})^{n-1}$, both D_λ and IM_2 are easily defined. Then, D_λ can be characterized (for strictly comprehensive sets) following the steps of Theorem 1 using IM_2 instead of IM. Like the KS solution, a limitation of the asymmetric KS solution is that it requires a lexicographic extension to satisfy PO (see Thomson, 1994). Then, the characterization becomes too technical.

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