

Abstract

In the fair-division problem addressed here, the indivisible goods to be divided among two or more players have divisible “bids” associated with them—namely, the prices the players must pay for them. A procedure, called the Gap Procedure, is proposed whereby players bid for the goods, but the bidding competition is balanced by fairness requirements that entitle the players to certain numbers of goods.

Under the Gap Procedure, the prices the players pay for the goods reflect the bids of not only the winners but also those of players that bid less. This market-oriented approach to fair division (1) ensures non-negative prices that never exceed a player’s winning bid, (2) is Pareto-optimal, precluding mutually beneficial trades, though not envy, (3) is monotonic in bids, so higher bids never hurt in obtaining a good, (4) encourages sincere bids, and (5) produces prices that are partially independent of the amounts bid (as in a Vickrey auction).

The analysis is developed in the context of the “housemates problem,” in which the rent for a house (the bad) must be apportioned among several housemates, each of whom is entitled to receive one room (the good). Each housemate is assumed to have the same endowment, so the auction of rooms is a “relativized” one—only relative, not absolute, bids for each room matter. Other applications of the Gap Procedure, in which player endowments and entitlements may be different, or the procedure may be carried out in rounds, are discussed.

JEL Classification: D44, D61, D63.

Keywords: Fair division; envy-freeness; allocative efficiency; bidding; Vickrey auction.