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Can Capital Mobility be Destabilizing?*

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Abstract

In a standard two-sector neoclassical model with distortions, capital mobility can render the steady state indeterminate, in the sense that there exist infinitely many convergent paths. In the closed economy with no international capital mobility, the utility function must be linear or close to it for indeterminacy to occur, while in the open economy the shape of the utility function makes no difference. The reason is that in the no mobility case changes in aggregate investment must be matched by changes in aggregate consumption, while in the case of full capital mobility they can simply be financed by borrowing abroad. The paper provides some solid theoretical underpinnings to the concerns that de-regulating the capital account may be destabilizing.

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1 Introduction

Can increased capital mobility be destabilizing? This question is often asked in the policy literature. Analysts fret over the consequences of large waves of capital inflows and outflows, and worry that such cycles may be the result of self-fulfilling prophecies. Some go as far as to advocate restrictions on capital mobility, or at least gradualism in de-regulating the capital account.¹ Yet the theoretical foundations for such concerns aren't always clear.

It is well understood by now that under some conditions closed economy versions of standard neoclassical models can be subject to indeterminacy, in the sense that there are multiple converging paths to the steady state.² It is seldom discussed what role, if any, capital mobility plays in making indeterminacy possible. In this paper we show that capital mobility can indeed cause indeterminacy in situations where, and under parameter values for which, it could otherwise not occur. This mechanism could provide one kind of theoretical underpinning for the concerns that de-regulating the capital account may be “destabilizing.”

The reason why capital mobility can contribute to indeterminacy is simple. Indeterminacy in investment and savings decisions is possible if, while going along an equilibrium path, a representative agent decides to invest more –and then asset prices and returns move in such a way as to make this decision optimal.³ In the closed economy, if the representative agent wants to invest more she must first curtail consumption. If the elasticity of intertemporal substitution in consumption is sufficiently low, doing so will be very costly, and the desire to smooth consumption will dominate the incentive to invest more. Hence, in the closed economy indeterminacy can only occur if the utility function is linear or close to it. This, of course, runs counter to all recent empirical evidence on elasticities of intertemporal substitution.

In the open economy matters are very different. An agent who wants to invest more can

¹For the latest in policy thinking on capital account liberalization and its consequences, see Eichengreen et al (1998) and Fischer et al (1998).

²For examples, see the survey paper by Benhabib and Farmer (1997).

³If there are increasing marginal returns to capital, for instance, holding more of it will raise its marginal product.

always borrow from the outside world, and hence need not reduce her consumption level. The curvature of the utility function does not affect investment decisions, and indeterminacy can occur for any degree of intertemporal substitution. Only certain technological conditions have to be satisfied.

Note also that in all the examples of indeterminacy we construct below, the dynamic paths for investment and consumption are not uniquely determined. As a result, in the open economy the current account and the capital flows that finance it are not uniquely determined either. If sunspots moved the economy from one equilibrium trajectory to another feasible equilibrium trajectory, then the capital and current accounts and the real exchange rate could be subject to sudden and potentially large movements guided exclusively by “animal spirits.” This would give credence to the concerns of policymakers about volatile capital flows and relative prices.

We formalize this idea in a perfectly standard setup. Our point of departure is the recent two sector model of Benhabib and Nishimura (1998). Aside from being relatively simple, this framework does not rely on increasing returns to generate indeterminacy. Given the current state of play in empirical estimates of technological increasing returns, this is an added advantage.⁴ Small externalities or other distortions (see below) are all that is required to generate multiplicity in the closed economy.⁵

The role of capital mobility in generating indeterminacy is an issue that only recently begun to be examined in the literature. To our knowledge, the first relevant paper was Lahiri (1997). Unlike us, Lahiri is concerned with multiple growth trajectories in growth models with human capital. His model also relies on increasing technological returns to generate indeterminacy, a feature that limits its empirical plausibility.⁶

⁴On empirical estimates, see Hall (1990), Basu and Fernald (1994a,b) and Burnside, Eichenbaum and Rebelo (1995). These papers find little evidence of increasing marginal returns.

⁵The Benhabib-Nishimura framework does not assume increasing marginal returns. It does allow for profits, which result from decreasing private returns (which coexist with constant social returns because of externalities). If one assumes a fixed cost of entry to determine the number of firms, then that sector would have increasing average returns, which is in line with current empirical findings.

⁶After competing a first draft of this paper we also became aware of the work by Weder (1999). Like

The paper is structured as follows. We start in Section 2 below by replicating the result of Benhabib and Nishimura (1998) that indeterminacy can occur only if the utility function is linear or close to it. In section 3 we then open the economy to perfect capital mobility. For simplicity we assume that the consumption good is traded while the investment good is non-traded. In that setup we prove our main result: in the economy with capital mobility indeterminacy can occur regardless of the degree of intertemporal substitution in consumption. Only a condition on the technology must be satisfied: the non-traded good must be labor intensive from the private perspective, but capital intensive from the social perspective.

In Section 4 we show that this result is not the artifice of particular number of goods considered nor the choice of which are tradeable and which are not. We generalize the underlying structure by allowing for a traded consumption good and a traded capital good such as equipment. We also allow the nontraded goods sector to produce both a nontraded consumption good and nontraded capital such as structures. Strikingly, the fundamental characteristics of this generalized model (and hence the conditions for indeterminacy to occur) are determined exclusively by the relative sectorial intensities in *nontraded* capital, just as in Section 3.

Finally, in section 5 we show that externalities are not necessary either to obtain our main indeterminacy result. Other distortions that introduce a wedge between private and social returns have the same effect. We study the role of factor taxation—in particular, policies that tax (or subsidize) factors in an asymmetric way across sectors. Once again, in the open economy the shape of the utility function plays no role in ensuring determinacy or indeterminacy. If the wedge created by tax rates causes the non-traded good to be labor intensive from the private perspective, but capital intensive from the social perspective, the steady state is again indeterminate.

Lahiri', Weder relies on increasing returns to obtain his results. His analysis is also limited to the case in which the consumption good is tradeable and the capital good is not, in contrast to the more general specification we study in section 4 below.

2 The Two-Sector Closed Economy

Consider a closed economy⁷ inhabited by an infinite-lived representative agent who maximizes the intertemporal utility function

$$\int_0^{\infty} [U(C_t) - V(L_t)]e^{-\rho t} dt \quad (1)$$

where C_t is consumption, L_t labor supply, ρ the parameter of time preference. As usual, assume that $U(\cdot)$ is concave, the consumption good is normal, and that $V(\cdot)$ is convex and not linear in L .

On the production side, there are two sectors with one producing consumption goods (Y) and the other investment goods (Y_N). The production functions are assumed to be Cobb-Douglas with externality components. The agent's decisions are to choose $C_t, L_t, K_t, K_{N,t}$ to maximize (1), and subject to

$$Y_t = L_t^{\alpha_0} K_t^{\alpha_1} \overline{L_t^{\alpha_0} K_t^{\alpha_1}}, \text{ where } \alpha_0 + \alpha_1 + a_0 + a_1 = 1 \quad (2)$$

$$Y_{N,t} = L_{N,t}^{\beta_0} K_{N,t}^{\beta_1} \overline{L_{N,t}^{\beta_0} K_{N,t}^{\beta_1}}, \text{ where } \beta_0 + \beta_1 + b_0 + b_1 = 1 \quad (3)$$

$$\dot{K}_t = Y_t - \delta K_t \quad (4)$$

$$L_{N,t} + L_t = L, \quad K_{N,t} + K_t = K \quad (5)$$

and $C_t = Y_t$, with initial capital stock K_0 as given. δ is the depreciation rate of capital. The components of the production functions, $\overline{L_t^{\alpha_0} K_t^{\alpha_1}}$ and $\overline{L_{N,t}^{\beta_0} K_{N,t}^{\beta_1}}$ represent output effects that are external, and are viewed as functions of time by the agent.

The Hamiltonian is

$$H = [U(L_t^{\alpha_0} K_t^{\alpha_1} \overline{L_t^{\alpha_0} K_t^{\alpha_1}}) - V(L_t)] + \lambda_t (\overline{L_t^{\alpha_0} K_t^{\alpha_1}} - \delta K_t) + \mu_t (\overline{L_{N,t}^{\beta_0} K_{N,t}^{\beta_1}} - \delta K_{N,t}) \quad (6)$$

⁷In this section, we follow Benhabib and Nishimura (1998).

$$+\bar{w}(L - L_N - L) + \bar{z}(K - K_N - K)$$

where \bar{q} , \bar{z} and \bar{w} are the utility price of the capital good, the rental rate of capital goods, and the wage rate of labor, all in terms of the price of the consumption good.

The first order conditions of this problem are given in Appendix A. Define

$$q = \frac{\bar{q}}{U'}, \quad z = \frac{\bar{z}}{U'}, \quad w = \frac{\bar{w}}{U'}, \quad (7)$$

Then, it turns out that the dynamics of the solution can be described by a pair of differential equations in K and q . These equations can be written as

$$\begin{bmatrix} \dot{K} \\ \dot{q} \end{bmatrix} = \begin{bmatrix} N Y - \delta K \\ E_2^{-1} \left[(\rho + \delta)q - z(q, K) + E_1 \left(\frac{-}{T} \right) \left(\frac{-}{T} \right) (Y(q, K) - \delta K) \right] \end{bmatrix} \quad (8)$$

where $E_1 = [-U''(C)C/U'(C)]$, and where we have implicitly expressed the rental rate z as a function of q and K . Below we derive the partial derivatives of this function.

This system can be readily linearized around its unique steady state. Appendix A also shows that the elements of the Jacobian matrix $[J]$ corresponding to the linearized system are:

$$[J] = \begin{bmatrix} -N - \delta & -N \\ E_1 E_2^{-1} \left(\frac{-}{T} \right) \left(\frac{-}{T} \right) (-N - \delta) & E_2^{-1} \left[\left(- + \rho + \delta \right) + E_1 \left(\frac{-}{T} \right) \left(\frac{-}{T} \right) \left(\frac{-}{T} \right) \right] \end{bmatrix} \quad (9)$$

where $E_2 = [1 - E_1(q/C)(\partial C/\partial q)]$.

By using the following instantaneous utility function

$$\frac{C^{1-\sigma}}{1-\sigma} - \frac{L^{1+v}}{1+v} \quad (10)$$

we have $E_1 = \sigma$. Using this, one can obtain the expressions

$$N \frac{\partial Y}{\partial K} - \delta = \frac{(\rho + \delta)\alpha_0 \left(1 + \left(\frac{-}{T}\right)\right)}{(\alpha_0\beta_1 - \alpha_1\beta_0 + \alpha_0\beta_1\alpha_0\left(\frac{-}{T}\right))} - \delta \quad (11)$$

and

$$\left(-\frac{\partial z}{\partial q} + \rho + \delta\right) = \frac{(\rho + \delta)(\beta_0 + b_0)}{(\alpha_1 + a_1)(\beta_0 + b_0) - (\alpha_0 + a_0)(\beta_1 + b_1)} \quad (12)$$

Consider now what happens if $\sigma = 0$ (i.e., utility is linear in consumption). The first consequence is that (11) becomes

$${}^N \frac{\partial Y}{\partial K} - \delta = \frac{(\rho + \delta)\alpha_0}{\alpha_0\beta_1 - \alpha_1\beta_0} - \delta \quad (13)$$

We also have $E_1 = 0$ and $E_2 = 1$, so that the two eigenvalues of the Jacobian matrix are

$$\gamma_1 = \frac{\rho\alpha_0 + \alpha_0\delta(1 - \beta_1) + \alpha_1\beta_0}{(\alpha_0\beta_1 - \alpha_1\beta_0)}$$

and

$$\gamma_2 = \frac{\rho(\beta_0 + b_0)}{(\alpha_1 + a_1)(\beta_0 + b_0) - (\alpha_0 + a_0)(\beta_1 + b_1)}$$

Notice finally that K is a state variable (cannot jump) while q is a jump variable. Hence, the existence of a saddle path requires that one eigenvalue be positive and the other negative. Two positive eigenvalues mean that the system is unstable, while two negative ones mean indeterminacy. We therefore have:

Proposition 1 *In the two-sector closed economy with $\sigma = 0$, i) if the investment good sector is labor intensive (or capital intensive) from both the private perspective and social perspectives ⁸ ($\gamma_1\gamma_2 < 0$), the transitional dynamics exhibits saddle-path stability; ii) if the investment good sector is capital intensive from the private perspective ($\gamma_1 > 0$) and labor intensive from the social perspective ($\gamma_2 > 0$), the system is unstable; and iii) if the investment good sector is labor intensive from private perspective ($\gamma_2 < 0$), but capital intensive from social perspective ($\gamma_1 < 0$), then there are multiple (an infinite number of) convergent paths toward the steady state.*

⁸The first-order conditions imply that $\frac{L_T}{K_T} - \frac{L_N}{K_N} = \frac{\alpha_0\beta_1 - \alpha_1\beta_0}{\alpha_1\beta_0} \frac{L_N}{K_N}$ (see equation (A6) in the appendix). Similarly for the social planner's solutions, $\frac{L'_T}{K'_T} - \frac{L'_N}{K'_N} = \frac{(\alpha_0 + a_0)(\beta_1 + b_1) - (\alpha_1 + a_1)(\beta_0 + b_0)}{(\alpha_1 + a_1)(\beta_0 + b_0)} \frac{L'_N}{K'_N}$.

Note that in absence of externalities the two eigenvalues have opposite signs, so that the system has a unique saddle path. With externalities, there are three possibilities. We are interested in case iii), in which the two eigenvalues are negative and equilibrium solutions are indeterminate.

It is clear that examples satisfying the above conditions for indeterminacy can be constructed with arbitrarily small external effects. A simple example is:

$$\alpha_0 = 0.66, \alpha_1 = 0.34, a_0 = 0.00, a_1 = 0.00;$$

$$\beta_0 = 0.65, \beta_1 = 0.30, b_0 = 0.00, b_1 = 0.05;$$

As Benhabib and Nishimura (1998) show, in this case indeterminacy arises because of the presence of external effects. Note that the sign of the eigenvalues will depend crucially on the expressions, $\frac{N}{K} - \delta$ and $(-\frac{N}{K} + \rho + \delta)$. The former depends on factor intensities reflecting the Stolper-Samuelson theorem (see(12)), and the latter also depends on factor intensities reflecting the Rybczynski effect (see (11)). Without external effects we have $\frac{N}{K} = -\frac{N}{K}$, so that the roots of $[J]$ are of opposite sign, and convergence to the steady state is unique. What is needed, then, is something to break the reciprocal relation between the Rybczynski and the Stolper-Samuelson effects. Externalities or other distortions accomplish this.

To understand the intuition behind this result, notice that in the case of $\sigma = 0$ the following equation holds (see Appendix A)

$$\dot{q} = (\rho + \delta)q - z(q, K) \tag{14}$$

Assume now the conditions in (iii) above are satisfied. Starting from an arbitrary equilibrium, consider an increase in the rate of investment above the level of its initial equilibrium, induced by an instantaneous increase in q . Since the capital good is labor intensive from the private perspective, an increase in the capital stock decreases its output at constant prices through Rybczynski effect (see (11)). This keeps the output of capital goods from exploding. The Stolper-Samuelson effect, on the other hand, operates through social factor intensities. If

the capital good is capital intensive from the social perspective, the initial rise in q causes an increase in the returns to capital z , and requires a price decline to maintain the overall returns to capital equal to the world interest rate or discount rate (as required by (14)). This offsets the initial rise in q and it causes it to reverse direction and move toward the steady state. Therefore, indeterminacy of equilibria happens here because the duality between the Rybczynski and Stolper-Samuelson effects is broken by the presence of market imperfections.

But in the closed economy if the representative agent wants to invest more she must first curtail consumption. If there is some curvature on the utility function, the desire to smooth consumption can overwhelm the effects described above, doing away with indeterminacy. We showed that indeterminacy can occur in the polar case of linear utility ($\sigma = 0$). But it can also occur for small values of σ . Indeed, Benhabib and Nishimura (1998) simulate the model and find that if the externalities are small (as in the parameterized examples given above), indeterminacy arise only for values of σ in a very narrow range above 0. Such values of σ are inconsistent with most empirical estimates of the elasticity of intertemporal substitution in consumption.

3 The Two-Sector Economy with Capital Mobility

Assume now the economy above is open to full international capital mobility, so that the domestic representative agent can borrow from and lend to the outside world freely. In order to facilitate the comparison between the results in this section to those in the previous section, we retain the same economic structure of a consumption good and an investment good. In the interest of realism we assume that the consumption good is tradable and the capital good nontradeable. In next section we generalize the model to two consumption and two investment goods (one tradable and one non-tradable in each case).

The agent now has access to net foreign bonds b , denominated in units of the tradable good, that pay an exogenously given world interest rate r . The agent's budget constraint becomes

$$\dot{b} = rb + Y - pY - C - pI \quad (15)$$

where p is the relative price of the investment or nontraded good to the traded good. Sometimes this price is referred to as the real exchange rate. Note that in (15) the traded good is taken to be the numeraire. Note also that p is taken as exogenously given by the agent, but is determined by market-clearing conditions. The variable I denotes gross investment, so that the law of motion for capital is

$$\dot{K} = I - \delta K \quad (16)$$

Equations (15) and (16) can be consolidated into

$$\dot{a} = ra + Y - pY - C + K(\dot{p} - rp - \delta p) \quad (17)$$

where $a = b + pK$.

The agent's problem is to choose C , L , I , K and b to maximize (1), subject to (2), (3), (5) and (17), and given K_0 and b_0 .

The Hamiltonian is

$$H = U(C) - V(L) + \lambda(ra + Y - pY - C + K(\dot{p} - rp - \delta p)) + \lambda_1(K - K_0) + \lambda_2(L - L_0) \quad (18)$$

where λ is a costate; λ_1 and λ_2 are the rental rate of capital goods and the wage rate of labor, all in terms of the price of the consumption good. First-order conditions are

$$U'(C) = \lambda \quad (19)$$

$$V'(L) = \beta_N \lambda p L^{\alpha-1} K^{1-\alpha} \quad (20)$$

$$\lambda_2 = \lambda \alpha_0 L^{0+0-1} K^{1+1} = \lambda \beta_0 p L^{0+0-1} K^{1+1} \quad (21)$$

$$\lambda_1 = \lambda \alpha_1 L^{0+0} K^{1+1-1} = \lambda \beta_1 p L^{0+0} K^{1+1-1} \quad (22)$$

$$\dot{\lambda} = \lambda(\rho - r) \quad (23)$$

$$\dot{p} = p(r + \delta - \beta_1 L^{0+0} K^{1+1-1}), \quad (24)$$

together with the transversality conditions

$$\lim_{t \rightarrow \infty} \lambda b e^{-\rho t} = \lim_{t \rightarrow \infty} \lambda p K e^{-\rho t} = 0 \quad (25)$$

As is standard in international macroeconomics, we impose $\rho = r$, a condition that ensures a well-defined steady-state with constant bond-holdings. This assumption will also imply, by (23), that marginal utility remains constant over all time –that is, $\lambda = \bar{\lambda}$. Substituting $\lambda = \bar{\lambda}$ into all other first-order conditions, by (19), we have

$$C = C(\bar{\lambda}) = \bar{C} \quad (26)$$

which means that consumption is completely smoothed.

Dividing (22) by (21) yields

$$\frac{\alpha_1 L}{\alpha_0 K} = \frac{\beta_1 L}{\beta_0 K} \quad (27)$$

Using (22) and (27) to solve for $\frac{L}{K}$ we have

$$\frac{L}{K} = \xi p^{\frac{1}{\alpha_0 + a_0 - \beta_0 - b_0}} = \xi p^{\frac{1}{(\alpha_0 + a_0)(\beta_1 + b_1) - (\alpha_1 + a_1)(\beta_0 + b_0)}} \equiv g(p) \quad (28)$$

where $\xi = \frac{1}{\alpha_1} \left(\frac{\alpha_0}{\beta_0} \right)^{0+0} > 0$.

Substituting (5), (27) and (28) into (20), we can solve for K :

$$K = \frac{\alpha_0\beta_1}{\alpha_0\beta_1 - \alpha_1\beta_0}K + h(p) \quad (29)$$

where

$$h(p) = -\frac{\alpha_1\beta_0}{\alpha_0\beta_1 - \alpha_1\beta_0} \frac{V'^{-1}(\beta_0\bar{\lambda}pg(p))}{g(p)} \quad (30)$$

Notice that $V'^{-1}(\cdot)$ is the inverse function for $V'(\cdot)$. Such an inverse function exists, since $V'(\cdot)$ is monotonic. In addition to the above first-order conditions, the market clearing condition for the investment (nontraded) good and the economy's current account are, respectively

$$\dot{K} = L K^{1+\theta} - \delta K \quad (31)$$

$$\dot{b} = rb + L K^{1+\theta} - C \quad (32)$$

If we substitute (28) and (29) into (24) and (31), we have the following dynamic equations for K and p :

$$\dot{p} = p(r + \delta - \beta_1 g(p)) \quad (33)$$

$$\dot{K} = \frac{\alpha_0\beta_1}{\alpha_0\beta_1 - \alpha_1\beta_0} g(p) K + h(p)g(p) - \delta K \quad (34)$$

These two equations describe the dynamics of the open economy. The solution to this system can then be used, in conjunction with the other conditions laid out above, to solve for all variables of interest.

Linearizing about the steady state, the dynamics of K and p can be approximated by

$$\begin{pmatrix} \dot{p} \\ \dot{K} \end{pmatrix} = \begin{pmatrix} a_{11} & 0 \\ a_{21} & a_{22} \end{pmatrix} \begin{pmatrix} p - p^* \\ K - K^* \end{pmatrix} \quad (35)$$

where asterisks denote steady-state values and

$$a_{11} = \frac{\beta_1(\beta_0 + b_0)p^*g(p^*)}{(\alpha_1 + a_1)(\beta_0 + b_0) - (\alpha_0 + a_0)(\beta_1 + b_1)} \quad (36)$$

$$\begin{aligned} a_{22} &= \frac{\alpha_0\beta_1}{\alpha_0\beta_1 - \alpha_1\beta_0}g(p^*) - \delta \\ &= \frac{\alpha_0r + \alpha_0\delta(1 - \beta_1) + \alpha_1\beta_0}{\alpha_0\beta_1 - \alpha_1\beta_0} \end{aligned} \quad (37)$$

$$a_{21} = \left[\frac{\alpha_0\beta_1}{\alpha_0\beta_1 - \alpha_1\beta_0}g(p)K + h(p)g(p) - \delta K \right]^* \quad (38)$$

We do not spell out the expression for a_{21} , since its sign or value are not important for our analysis.

Using these expressions one can arrive at the following results:

Proposition 2 *In the two-sector open economy with nontraded capital, i) if the nontraded good sector is labor intensive (or capital intensive) from both the private perspective and the social perspective ($a_{11}a_{22} < 0$), the transitional dynamics exhibits saddle-path stability; ii) if the nontraded good sector is capital intensive from the private perspective ($a_{22} > 0$) and labor intensive from the social perspective ($a_{11} > 0$), the system is unstable; and iii) if the nontraded good sector is labor intensive from the private perspective ($a_{22} < 0$), but capital intensive from the social perspective ($a_{11} < 0$), then there are multiple (an infinite number of) convergent paths toward the steady state.*

The same parametrized example given in the previous section, of course, applies to the indeterminacy case in the above proposition. Similar intuition for indeterminacy can also be given. Note that

$$a_{11} = -\frac{p}{\lambda} \frac{d(\frac{\cdot}{p})}{dp} \quad (39)$$

and

$$a_{22} \stackrel{N}{=} \frac{\partial Y}{\partial K}, \quad (40)$$

while (24) implies

$$\frac{\dot{p}}{p} + \frac{\lambda_1}{\lambda p} = \rho + \delta \quad (41)$$

both evaluated at the steady state. Note that λ_1 is the rental price of capital. Assume the conditions for indeterminacy in Proposition 2 hold. Starting from an arbitrary equilibrium, consider an increase in the rate of investment above the level of its initial equilibrium, induced by an instantaneous increase in real exchange rate p . As in the closed economy, the Rybczynski effect (see(40)) keeps the output of nontraded goods from exploding, while the Stolper-Samuelson effect keeps the real exchange rate p from exploding (see (41)). Again, market imperfections break the duality between the Rybczynski and Stolper-Samuelson effects.

But there are the essential differences between the closed economy and open economy environments. The most striking of them is that the conditions for indeterminacy in Proposition 2 are completely independent of the degree of intertemporal substitution in consumption (σ does not appear). By contrast, the closed economy results in Proposition 1 required the extreme assumption of $\sigma = 0$.

The intuition for the central difference between the closed and open economy is straightforward. In the former case, if the representative agent wants to invest more she must first curtail consumption. With enough curvature, the desire to smooth consumption prevails over all other effects. But in the open economy, the curvature on the utility function does not affect the investment decision, since the investor can always borrow from the outside world without reducing her consumption level. That is why the dynamics of the open economy

described in system (35) depends exclusively upon technologies, and so do the conditions for indeterminacy.

The dynamics of system (35) can best be appreciated in Figures 1-3. The first two of these figures depict what happens when a unique saddle path exists. There are two cases to consider. When the non-traded investment good is capital intensive from both the private and social perspectives (Figure 1), the saddle path coincides with the horizontal $\dot{p} = 0$ schedule. In that case the real exchange rate is constant along all transitions. If the non-traded investment good is labor intensive from both the private and social perspectives (Figure 2), the saddle path is downward sloping and steeper than the $\dot{K} = 0$ schedule. In that case the real exchange rate and the stock of capital move in opposite directions along all transitions. The case of indeterminacy is depicted in Figure 3. There are an infinite number of trajectories that converge to the steady state. The real exchange rate and the stock of capital can move together or in opposite directions along the transition.

Note finally that if the stock of capital and the real exchange rate are indeterminate in the transition, so are the other variables of the system. In particular, the current account and the capital flows that finance it are also indeterminate. This creates the potential for large swings governed by expectations alone.

4 The Generalized Two-Sector Economy with Capital Mobility

We now generalize the model and show that the results of Section 3 carry over to the general setup with traded and non-traded consumption and investment goods. We assume that using the same technology the traded goods sector now produces both the traded consumption good (C) and a traded capital good such as equipment (E). We also allow the nontraded goods sector to produce both a nontraded consumption good (C) and nontraded capital such as structures (S).⁹ The model includes both traded and nontraded investment expenditure,

⁹A similar setup was used by Brock and Turnovsky (1994) for a different purpose.

so that the production structure uses three factors (nontraded capital, traded capital, and labor) in two sectors (traded and nontraded). Strikingly, the fundamental characteristics of this generalized model (and hence the conditions for indeterminacy to occur) are determined exclusively by the relative sectorial intensities in *nontraded* capital, just as in Section 3.

We set up the model briefly and state our result in Proposition 3. Mathematical derivations are relegated to Appendix B. The agent maximizes

$$\int_0^{\infty} [U(\bar{C}, C) - V(L)] e^{-\rho t} dt$$

by choosing consumption levels (\bar{C}, C) , labor supply L and its allocation (L^N, L^T) , capital allocation decisions (K^N, K^T, S^N, S^T) , rates of investment (I^N, I^T) , and the rate of accumulation of bonds (\dot{b}) , subject to

$$\dot{b} = rb + Y^N + pY^T - C^N - pC^T - I^N - pI^T \quad (42)$$

$$\dot{E} = I^T - \delta E \quad (43)$$

$$\dot{S} = I^N - \delta S \quad (44)$$

$$Y^T = L^T \alpha E^T S^T \overline{L^T \alpha E^T S^T} \quad (45)$$

$$Y^N = L^N \alpha E^N S^N \overline{L^N \alpha E^N S^N} \quad (46)$$

$$E^N + E^T = E \quad (47)$$

$$S^N + S^T = S \quad (48)$$

$$L_N + L = L \quad (49)$$

where

$$\alpha_0 + \alpha_1 + \alpha_2 + a_0 + a_1 + a_1 = \beta_0 + \beta_1 + \beta_2 + b_0 + b_1 + b_2 = 1 \quad (50)$$

In Appendix B we solve the agent's problem. The optimality conditions and law of motions for the economy boil down to a system of two differential equations:

$$\begin{pmatrix} \dot{p} \\ \dot{S} \end{pmatrix} = B \begin{pmatrix} p - p^* \\ S - S^* \end{pmatrix} \quad (51)$$

From the solution of this system one can infer the behavior of the other variables of interest. For instance, it is easy to show that $C = C(p)$ and $C_N = C(p)$, where $C' \geq 0$, $C'' < 0$. Therefore consumption levels are not completely smoothed, but fluctuate along with real exchange rate. Moreover, investment decisions are again independent of intertemporal effects in consumption.

The matrix B has exactly the same property as the matrix in (35), in that its eigenvalues have the same sign as a_{22} (or $\alpha_0\beta_2 - \alpha_2\beta_0$) and a_{11} (or $(\alpha_2 + a_2)(\beta_0 + b_0) - (\alpha_0 + a_0)(\beta_2 + b_2)$). We therefore have:

Proposition 3 *In the two-sector open economy with traded and nontraded capital, i) if the nontraded good sector is labor intensive (or nontraded capital intensive) relative to nontraded capital from both the private and the social perspective ($a_{11}a_{22} < 0$), transitional dynamics exhibits saddle-path stability; ii) if the nontraded goods sector is nontraded capital intensive from the private perspective ($a_{22} > 0$) and labor intensive relative to the nontraded capital from the social perspective ($a_{11} > 0$), the system is unstable; and iii) if the nontraded goods sector is labor intensive from the private perspective ($a_{22} < 0$), but nontraded capital intensive from the social perspective ($a_{11} < 0$), then there are multiple (an infinite number of) convergent paths toward the steady state.*

5 Factor Taxation and Indeterminacy

We show in this section that in the economy with international capital mobility, indeterminacy may still arise even in the absence of externalities in the production functions. Other distortions that introduce a wedge between private and social returns have the same effect. Here we study the role of factor taxation—in particular, policies that tax (or subsidize) factors in an asymmetric way across sectors. Velasco (1993) and Schmidt-Grohé and Uribe (1997) have already shown that certain kinds of taxation can induce “fiscal increasing returns” and hence indeterminacy (or even multiple steady states) in dynamic models. In our setup there are no such “fiscal increasing returns,” but taxation nonetheless causes indeterminacy.¹⁰ In order to simplify matters we conduct the analysis in the simpler framework of Section 3. However, it is straightforward to extend the analysis and the results to the general setup of Section 4.

Assume factor taxation consists of four different tax rates $\tau_T, \tau_N, \tau_T, \tau_N$ on the earnings from factors used in the two sectors. The production functions for both sectors are now

$$Y = L K^{1-\beta} \quad (52)$$

$$Y_N = L_N K_N^{1-\beta} \quad (53)$$

In order to abstract from issues related to the choice of public spending and to facilitate comparison with results of previous sections, we assume that the revenue from factor taxes is transferred back to households in lump sum fashion. Then it can be shown that the dynamic equations for p and K become (see Appendix C):

$$\dot{p} = p(r + \delta - (1 - \beta)(1 - \tau_N)p^{\frac{\beta}{\alpha-\beta}}) \quad (54)$$

¹⁰Fiscal increasing returns occur when the tax rate has to be varied in the opposite direction to the taxable factor in order to keep revenue fixed and equal to a constant exogenous level of expenditure. In our examples below fiscal expenditures are endogenous, so that fiscal increasing returns cannot occur.

$$\dot{K} = \frac{W(p)}{\left(\frac{1-\alpha}{1-\alpha} \frac{1-\tau_{LT}}{1-\tau_{KT}} - \frac{1-\beta}{1-\beta} \frac{1-\tau_{LN}}{1-\tau_{KN}}\right)} K + V(p) \quad (55)$$

where $W(p) (> 0)$ and $V(p)$ are functions of p . The two eigenvalues of the Jacobian matrix corresponding to the system linearized around the steady state are $\chi_1 = -\frac{(1-\alpha)(1-\tau_{LN})}{(1-\beta)} p^{*\frac{\beta}{\alpha-\beta}}$ and $\chi_2 = \frac{(*)}{\left(\frac{\alpha}{1-\alpha} \frac{1-\tau_{LT}}{1-\tau_{KT}} - \frac{\beta}{1-\beta} \frac{1-\tau_{LN}}{1-\tau_{KN}}\right)}$.

We then have the following proposition:

Proposition 4 *In the two-sector open economy with nontraded capital, i) if the nontraded goods sector is labor intensive (or capital intensive) from both the private and social perspectives ($\chi_1 \chi_2 < 0$), transitional dynamics exhibits saddle-path stability; ii) if the nontraded goods sector is capital intensive from the private perspective ($\chi_2 > 0$) and labor intensive from the social perspective ($\chi_1 > 0$), the system is unstable; and iii) if the nontraded goods sector is labor intensive from the private perspective ($\chi_2 < 0$), but capital intensive from the social perspective ($\chi_1 < 0$), then there are multiple (an infinite number of) convergent paths toward the steady state.*

The intuition behind Proposition 4 for the indeterminacy case is simple. In previous sections the presence of externalities broke the duality between the *Rybczynski* and the *Stolper-Samuelson* effects, allowing for alternative but non-explosive equilibrium paths. The key is to have a wedge between private and social returns. Here that wedge is created by the tax rates.

Note that if $\tau_T = \tau_N, \tau_T = \tau_N$, or generally $\frac{1-\tau_{LN}}{1-\tau_{KN}} = \frac{1-\tau_{LT}}{1-\tau_{KT}}$, the two inequalities for indeterminacy or instability can not hold simultaneously. On the other hand, there are a variety of policy combinations (if at least one of $\tau_T = \tau_N, \tau_T = \tau_N$ does not hold) such that the two eigenvalues are both negative or both positive. Indeed, we can give one useful example which may have important implications. In the case that nontraded goods sector is capital intensive from social perspective ($\alpha > \beta$), if government implements policies such that $\tau_T = \tau_N$ but $\tau_T < \tau_N$, the steady state is indeterminate. This example

implies, strikingly, government policies that are preferential for traded goods sector (even slightly) may be another source of economic fluctuations. Again, unfettered capital mobility makes this occur much more easily for the same reason discussed in previous sections. In a parallel analysis made in the closed economy environment as in Section 2, it is easy to show that indeterminacy can not arise if government implements tax policies which are slightly preferential to one of the consumption and capital goods sector.

6 Conclusions

Capital mobility makes it easier for the steady state of a standard neoclassical model to be indeterminate. In the closed economy the utility function must be linear or close to it for indeterminacy to occur, while in the open economy the shape of the utility function makes no difference. The reason is that in the autarchic case changes in aggregate investment must be matched by changes in aggregate consumption, while in the case of full capital mobility they can simply be financed by borrowing abroad.

Indeterminacy depends only on technology and factors affecting it. In this model without increasing marginal returns, distortions must exist that drive a wedge between private and social returns. Those wedges can arise from externalities, but also from the presence of taxes or equivalent distortions.

This general principle should prove useful in many applications. One future task is to see whether plausible parametrization can generate the kinds of capital movements we observe in real-life economies.

7 Appendix

7.1 Appendix A

For the closed economy in Section 2, the first-order conditions with respect to L , K , and K_N yield

$$\bar{w} = U' \alpha_0 L^{\theta_0} K^{\theta_0 - 1} K^{\theta_0 - 1} = \bar{q} \beta_0 L^{\theta_0} K^{\theta_0 - 1} K^{\theta_0 - 1} \quad (56)$$

$$\bar{z} = U' \alpha_1 L^{\theta_0} K^{\theta_0 - 1} K^{\theta_0 - 1} = \bar{q} \beta_1 L^{\theta_0} K^{\theta_0 - 1} K^{\theta_0 - 1} \quad (57)$$

$$\frac{d(U'q)}{dt} = U'(\rho q + \delta q - z) \quad (58)$$

while law of motion (4) in the text becomes

$$\dot{K}_N = L^{\theta_0} K^{\theta_0 - 1} - \delta K \quad (59)$$

Equation (58) can be written as

$$\begin{aligned} \frac{dq}{dt} &= (\rho + \delta)q - z(q, K) - q \frac{U''(C)}{U'(C)} \left[\frac{-\tau}{1-\tau} + \frac{-\tau}{1-\tau} \right] \\ &= \left[1 - q \left(\frac{U''(C)}{U'(C)} \right) \frac{\partial C}{\partial q} \right]^{-1} \left[(\rho + \delta)q - z(q, K) - q \frac{U''(C)}{U'(C)} \left[\frac{-\tau}{1-\tau} \right] \right] \\ &= E_2^{-1} \left[(\rho + \delta)q - z(q, K) + E_1 \left(\frac{q}{C} \right) \left(\frac{\partial C}{\partial K^N} \right) (Y(q, K) - \delta K) \right] \end{aligned} \quad (60)$$

which appears in Section 2. It is then easy from here to obtain the Jacobian matrix $[J]$.

If the utility function takes the form of (10) in the text, the first-order condition with respect to L gives the labor market equilibrium condition

$$C^{1-\alpha_0} L^{-1} = L \quad (61)$$

which, along with other first-order conditions, can be used to derive the expressions for $\frac{L}{K}$ and $\frac{Y}{C}$ that are used in Section 2. For more derivations in detail, see Benhabib and Nishimura (1998).

Note that (56) and (57) imply

$$\frac{\alpha_1 L}{\alpha_0 K} = \frac{\beta_1 L}{\beta_0 K} \quad (62)$$

7.2 Appendix B

Here we provide the necessary derivations for Section 4. The consolidated budget constraint is

$$\dot{a} = ra + Y - pY - C - pC - E(r + \delta) + S(\dot{p} - rp - \delta p) \quad (63)$$

where $a = b + E + pS$. Optimality conditions are now

$$U(G, C) = \bar{\lambda} \quad (64)$$

$$U_N(G, C) = \bar{\lambda}p \quad (65)$$

$$V'(L) = \beta_N \bar{\lambda} p L^{0+0-1} E_N^{1+1} S^{2+2} \quad (66)$$

$$\alpha_0 L^{0+0-1} E_N^{1+1} S^{2+2} = \beta_0 p L^{0+0-1} E_N^{1+1} S^{2+2} \quad (67)$$

$$\alpha_1 L^{0+0} E_N^{1+1-1} S^{2+2} = \beta_1 p L_N^{0+0} E_N^{1+1-1} S^{2+2} \quad (68)$$

$$\alpha_2 L^{0+0} E_N^{1+1} S^{2+2-1} = \beta_2 p L_N^{0+0} E_N^{1+1} S^{2+2-1} \quad (69)$$

$$\beta_2 p L_N^{0+0} E_N^{1+1} S^{2+2-1} = p(r + \delta) - \dot{p} \quad (70)$$

$$\beta_1 p L \frac{0^+}{N} \frac{0}{E} \frac{1^+}{1^+} \frac{1^-}{1^-} S^{2^+} = r + \delta \quad (71)$$

Dividing (68) by (67) and (69) respectively, we have

$$\frac{\alpha_1}{\alpha_0} \frac{L}{E} = \frac{N\beta_1}{N\beta_0} \frac{L}{E} \quad (72)$$

$$\frac{\alpha_1}{\alpha_2} \frac{S}{E} = \frac{N\beta_1}{N\beta_2} \frac{S}{E} \quad (73)$$

If we write (68) and (71) as

$$\alpha_1 \left(\frac{L}{E} \right)^{0^+} \frac{0}{E} \left(\frac{S}{E} \right)^{2^+} = \beta_1 \frac{N}{N} \left(\frac{L}{E} \right)^{0^+} \frac{0}{N} \left(\frac{S}{E} \right)^{2^+} \quad (74)$$

$$\beta_1 \frac{N}{N} \left(\frac{L}{E} \right)^{0^+} \frac{0}{N} \left(\frac{S}{E} \right)^{2^+} = r + \delta, \quad (75)$$

then the four equations (72), (73), (74) and (75) can be used to solve $\frac{N}{N}, \frac{N}{N}$ (and $\frac{T}{T}, \frac{T}{T}$) as functions of p :

$$\frac{N}{N} \frac{L}{E} = \Delta_1 p^{-\frac{\alpha_2 + a_2}{(\alpha_2 + a_2)(\beta_0 + b_0) - (\alpha_0 + a_0)(\beta_2 + b_2)}} \quad (76)$$

$$\frac{N}{N} \frac{S}{E} = \Delta_2 p^{-\frac{\alpha_0 + a_0}{(\alpha_2 + a_2)(\beta_0 + b_0) - (\alpha_0 + a_0)(\beta_2 + b_2)}} \quad (77)$$

where Δ_1, Δ_2 are positive constants. Substituting (76) and (77) into (70), we can get a \dot{p} equation which is the counterpart of equation (33) in Section 3.

Dividing (72) by (73) gives

$$\frac{\alpha_2}{\alpha_0} \frac{L}{S} = \frac{N\beta_2}{N\beta_0} \frac{L}{S} \quad (78)$$

We can use the same procedure in Section 3 to solve for S (using (49), (48), (76) and (78)), and then substitute it and (76), (77) into the market-clearing condition for good S , so as to obtain the \dot{S} equation.

7.3 Appendix C

Here we derive (54) and (55) in the text. The household budget constraint is now

$$\begin{aligned} \dot{b} = & (1 - \tau_T)w_T L + (1 - \tau_T)r_T K + p[(1 - \tau_N)w_N L \\ & + (1 - \tau_T)r_N K] + rb - C - pI + T \end{aligned} \quad (79)$$

where

$$T = \tau_T w_T L + \tau_T r_T K + p[\tau_N w_N L + \tau_T r_N K] \quad (80)$$

is the government budget constraint: the RHS is government tax revenue, and the LHS is the government's transfer to the household.

By solving the firm's problem we have

$$w_T = \alpha L^{-1} K^{1-\alpha}, \quad r_T = (1 - \alpha)L^{-\alpha} K^{-\alpha}, \quad w_N = \beta L_N^{-1} K^{1-\beta}, \quad r_N = (1 - \beta)L_N^{-\beta} K^{-\beta} \quad (81)$$

In addition to other similar first-order conditions, the counterparts of (21) and (22) in this case are

$$(1 - \tau_T)\alpha L^{-1} K^{1-\alpha} = (1 - \tau_N)\beta p L_N^{-1} K^{1-\beta} \quad (82)$$

$$(1 - \tau_T)(1 - \alpha)L^{-\alpha} K^{-\alpha} = (1 - \tau_N)(1 - \beta)p L_N^{-\beta} K^{-\beta}, \quad (83)$$

which imply that after-tax returns to each factor across sectors should be equal).

Therefore we have

$$\frac{1 - \alpha}{\alpha} \frac{(1 - \tau_T) L}{(1 - \tau_T) K} = \frac{1 - \beta}{\beta} \frac{(1 - \tau_N) L}{(1 - \tau_N) K} \quad (84)$$

and

$$\frac{L}{K} = \eta p^{\frac{1}{\alpha-\beta}}, \quad \eta > 0 \quad (85)$$

Note that now $\alpha(1 - \tau_T)$, $(1 - \alpha)(1 - \tau_T)$, $\beta(1 - \tau_N)$ and $(1 - \beta)(1 - \tau_N)$ play the same role as $\alpha_0, \alpha_1, \beta_0$ and β_1 respectively in the model with externalities. The rest is straightforward if we follow the same derivation procedure as in Section 3.

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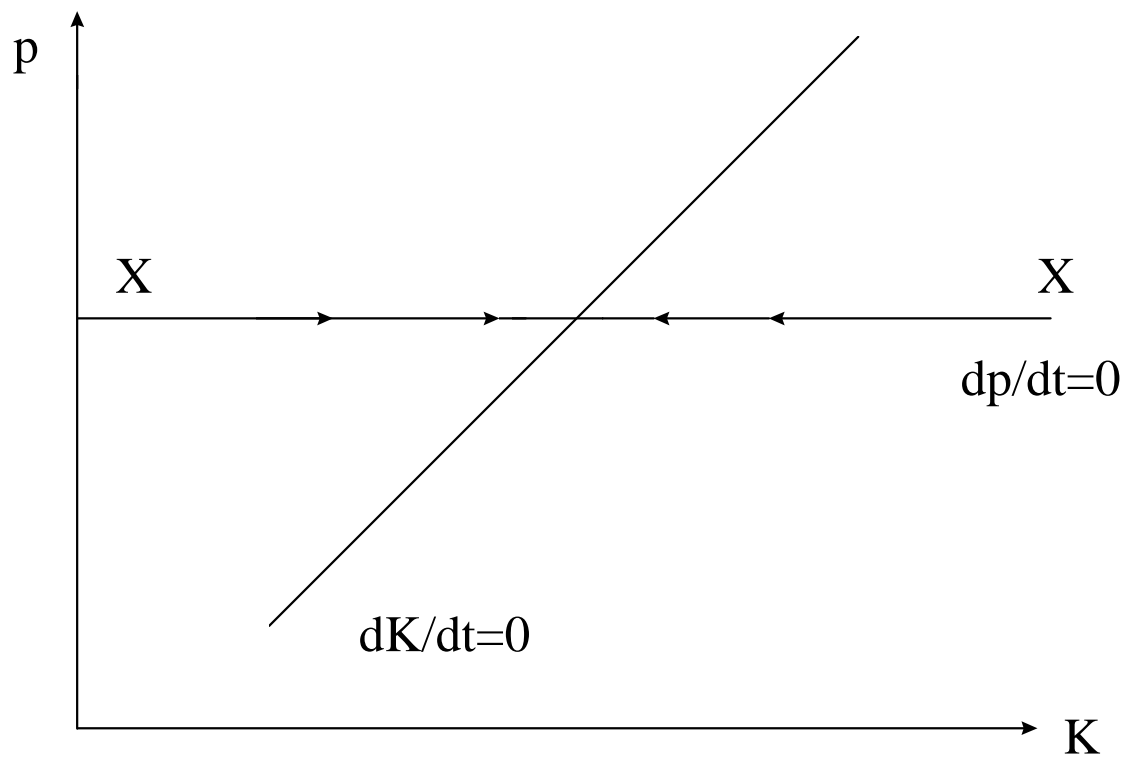


Figure 1: When the nontraded good is capital intensive from both private and social perspective with or without externalities. XX is the saddle path.

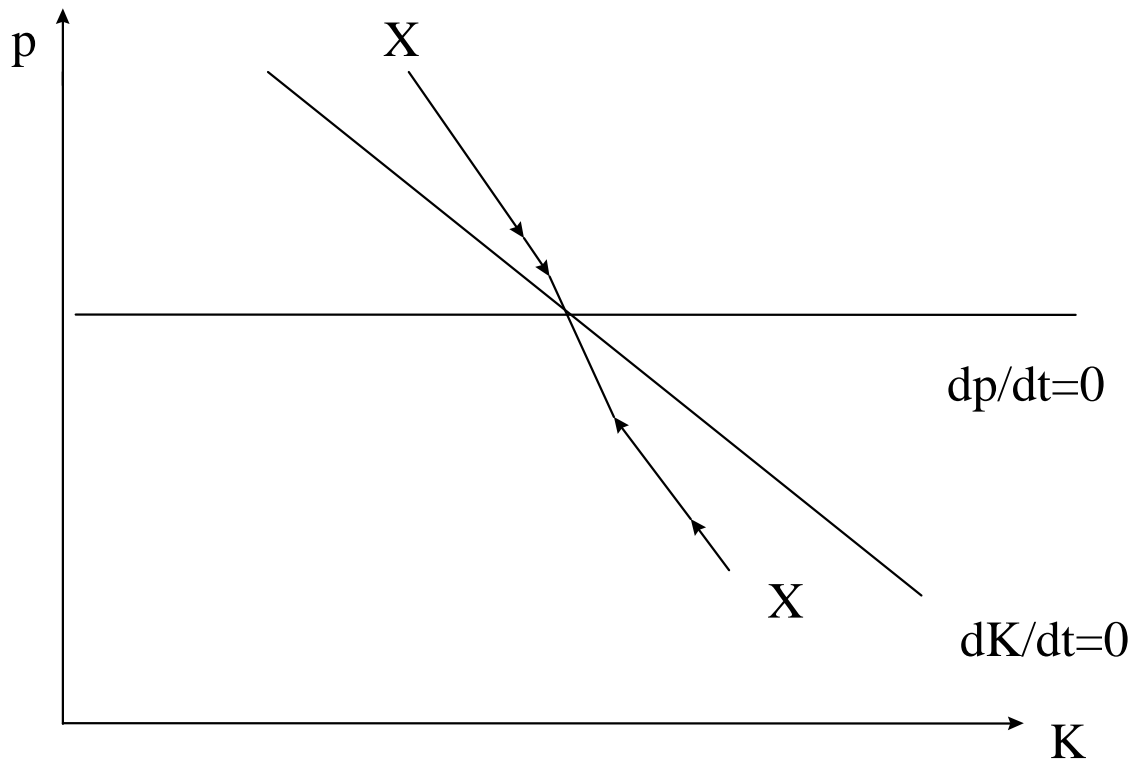


Figure 2: When the nontraded good is labor intensive from both private and social perspective with or without externalities. XX is the saddle path.

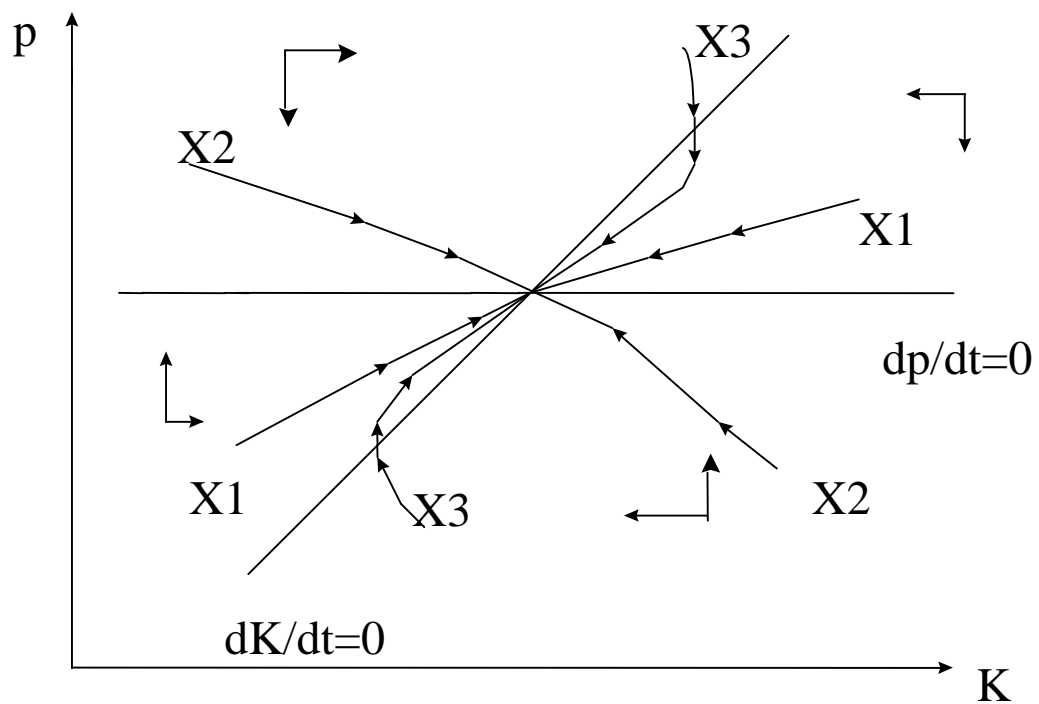


Figure 3: When the nontraded good is labor intensive from private perspective, but capital intensive from social perspective with externalities. There are an infinite number of convergent paths toward the steady state. $X1X1$, $X2X2$, $X3X3$ are just examples.