

The Paradox of Disconnected Coalitions

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November 18, 1999

Abstract

The paradox of disconnected coalitions is that “disconnected” coalitions can arise from the “connected” (i.e., single-peaked) preferences of as few as five or six players.

Two models of coalition formation are analyzed in which the paradox can occur, in both of which players coalesce when they all find each other mutually acceptable:

- *Fallback (FB)*: Players seek coalition partners by descending lower and lower in their preferences until a majority coalition emerges.
- *Build-Up (BU)*: Same as FB, except that when nonmajority subcoalitions form, they fuse into composite players, and the descent restarts.

FB better reflects the unconstrained, or nonmyopic, possibilities of coalition formation, whereas BU, because all subcoalition members must be included in any majority coalition that forms, restricts combinatorial possibilities considerably and tends to produce larger and less tight majority coalitions.

If player perceptions of the single-peaked ordering are sufficiently similar (“spatially” single-peaked), the paradox is precluded. But the strange bedfellows frequently observed in legislative coalitions and military alliances suggest that even when players agree on, say, a left-right ordering, their perceptions of exactly where players stand in this ordering may differ substantially. In such situations a player, while acceptable to a coalition, might not find every member in it acceptable, which causes that player not to join and creates a “hole” in an otherwise connected coalition. Applications of the models are discussed.

JEL Classification: C61, C78, D72

Keywords: Coalition formation; dynamic analysis; single-peakedness; legislatures; military alliances.