ECONOMIC RESEARCH REPORTS

An Uncertainty-driven Theory of the Productivity Slowdown: Manufacturing

by

Diego Comin

RR#: 2000-16 **September 2000**

C.V. Starr Center for Applied Economics

Department of Economics

Faculty of Arts and Science

New York University

269 Mercer Street, 3rd Floor

New York, New York 10003-6687

An Uncertainty-driven Theory of the Productivity Slowdown: Manufacturing

Diego Comín[*]
Department of Economics, Harvard University.

First draft: July 1999, This draft: September 2000.

Abstract

This paper presents a theory of the productivity slowdown based on the effects that uncertainty has on the productivity of specialized capital. Uncertainty reduces the e±ciency of in°exible capital and generates a slowdown. It also increases the demand for °exible capital which reteins its productivity in the new volatile environment. The increase in the share of °exible capital explains the acceleration of the rate of productivity growth embodied in new capital observed by McHugh and Lane [1987]. This fact is di±cult to explain by the theories that emphasize the cost of implementing the new technologies as the cause of the slowdown. The model also highlights the positive e®ect that uncertainty has on the speed of di®usion of technologies, and on the rate of technological progress. These relationships are successfully tested in manufacturing and are used to explain the rapid di®usion of computers and the spectacular TFP growth rate of the computer producing sectors.

JEL: D2, D8, D9, O3.
Keywords: Productivity Slowdown, Uncertainty, Specialized Capital, Flexibility.

[*] I am very grateful to Philippe Aghion , Robert Barro, Elhanan Helpman and John Leahy for their guidance and dedicated advice. To Gady Barlevy, Francesco Caselli, Chris Foote, Dale Jorgenson, Michael Kremer, David Laibson, Albert Saiz and Jaume Ventura for very helpful discussions. To Randy Becker, Eli Berman, Bill Gullickson, Dale Jorgenson and Heather McMullen for providing me the data used in this paper. To the seminar participants at NYU, Rochester, Wharton and the Minnesota workshop in macro theory for their numerous comments and suggestions. All errors are my own. Financial assistance from the Banco de España, Harvard University and the Real Colegio Complutense is gratefully acknowledged. Please direct correspondence to diego.comin@nyu.edu

Because every client is unique,
we must be flexible.
Balducci's.

1 Introduction

Since the 1970's most OECD economies have experienced a very sizeable decline in the growth rate of total factor productivity (TFP). This decline is known as the productivity slowdown. In the U.S. the decline in the annual growth rate has been of about 1.8 percentage points.[1] In manufacturing, the decline in TFP growth during the 70's was followed by a recovery during the 80's, while in services the slow growth continued (figure 1).[2]

The most successful explanation of the evolution of TFP growth in manufacturing is the General Purpose Technology (GPT) approach (Hornstein and Krusell [1996], Greenwood and Yorukoglu [1997], Helpman and Trajtenberg [1998], Greenwood and Jovanovic [1998]). According to this theory, the advent of new general purpose technologies such as information technologies first lowers productivity growth as firms learn how to use the new technologies, and later raises productivity growth as the productive benefits of these technologies become fully realized. Whereas the GPT approach to the slowdown fits the general trend in manufacturing productivity, it does not fit with some of the details. In particular, it appears to be at variance with the contribution of investment in new capital to economic growth.

Solow [1959] introduced the distinction between embodied and disembodied productivity growth. His idea was that there are certain types of productivity gains that "can be introduced into the production process only through gross investment in new plant and equipment." He called this type of technological progress embodied productivity growth, and contrasted it with disembodied productivity growth which makes old and new capital

[1] These data correspond to the private business sector which represents 80% of the U.S. economy. Source: Bureau of Labor Statistics http://146.142.4.24/cgi-bin/surveymost?mp. The pre-70's period used in this calculation is 1947-73, and the post-70's is 1973-97. The slowdown is very robust to other partitions.

[2] This figure is constructed using data compiled by Jorgenson and available at his web page: http://www.economics.harvard.edu/faculty/jorgenson/data/35klem.html

equally more productive. In this paper, I argue that the evolution of embodied and disembodied productivity growth can shed new light on the causes of the slowdown.

There have been several attempts to estimate the relative contribution of embodied and disembodied TFP growth to the productivity slowdown (McHugh and Lane [1987], Hornstein and Krusell [1996] and Greenwood and Yorukoglu [1997]). All point to an acceleration in embodied technological progress since 1973 and simultaneous decline in disembodied productivity growth. As we shall see, it is hard to reconcile the GPT approach with this fact.

I propose an alternative explanation of the productivity slowdown, one that is consistent with the evidence on embodied and disembodied productivity growth. I argue that in the 1970's there was a general increase in the uncertainty that ¯rms faced. As the business environment became more unpredictable, ¯rms responded by adopting more °exible production technologies. In this view the productivity slowdown was concentrated in the old technologies, which were ill suited to rapidly changing conditions. New technologies retained their productivity because of their increased °exibility. The short run acceleration of the rate of embodied TFP growth was the result of this shift in investment from in°exible to °exible capital.

There are two parts to the story. The ¯rst part is that there was an increase in the uncertainty in the 70's. This hypothesis appears reasonable at ¯rst glance. The 70's witnessed a variety of events that contributed to ¯rm level uncertainty, including the oil shocks, stop and go monetary policy, the demise of Bretton Woods, and increased globalization. In section 4, I argue more formally that this increase in uncertainty is evident in a variety of measures of economic activity. I show that the average volatility of individual ¯rm stock returns increased between the 60's and the 70's, indicating the presence of shocks that a®ected ¯rms' pro¯tability. I also show that there has been an upward trend in the excess job reallocation rate. The excess job reallocation rate is the di®erence between gross and net job creation or the amount of simultaneous job creation and destruction. It is indicative of a shifting business climate. The upward trend also points to an increase in the uncertainty that ¯rms face.

The second step in the argument is the connection between uncertainty and TFP growth. In Section 3, I model the decision of a ¯rm to upgrade the quality of its capital stock. The ¯rm faces two decisions: when to replace its capital and what type of capital to choose.

Capital in the model is specialized. It is fabricated to e±ciently produce a particular good with particular characteristics. Capital, however, di®ers in its °exibility. The more °exible capital is, the more easily it can be adapted to produce products with di®erent characteristics. When the business environment is uncertain the characteristics of desired output are constantly changing. Old in°exible machines become less productive while the new °exible technologies are more desirable. By substituting old by new capital, ¯rms obtain an extra boost in their productivity. As a result the measures of embodied technological progress increase and measures of disembodied technological progress fall.

There is a vast amount of informal evidence showing that, since the 1970's, the di®usion of programmable automation technologies[3] (PA) has increased the °exibility and e±ciency of production in manufacturing.[4;5] In Section 4, I present econometric evidence in support of the e®ect of uncertainty on TFP. One implication of the model is that those sectors that experienced a larger increase in uncertainty should su®er a larger decline in TFP growth. This negative e®ect of uncertainty appears in the data. A second implication is that productivity slows less in those sectors that are able to implement more intensively the °exible technologies. I test this implication by looking at the effect of computers on TFP growth. I ¯nd a positive e®ect, which speaks in favor of the uncertainty-driven theory. Note that it is hard to reconcile this ¯nding with a model where the slowdown is caused by the implementation of computers.

The model also predicts that in more uncertain environments, equipment will be replaced more frequently. The reason is that when the economic environment changes more rapidly, the installed equipment becomes quickly ill-suited to producing the optimal commodity. An increase in the uncertainty therefore has important consequences for the speed of di®usion of technologies. I ¯nd empirical support for this implication of

[3] The main exponents of the PA are computer-aided design (CAD), robots, numerically controlled (NC) machine tools, °exible manufacturing systems (FMSs).

[4] Examples abound. Computerized process-control equipment allows steel producers to regulate the carbon content of steel more precisely and to add a sequence of di®erent alloys without interrupting the °ow of production (Greene [1982]). This together with the computer-controlled cutting-and-rolling equipment facilitates the changeover from one product to another. Advances in the 1970's in semiconductor and computer technology made possible the development of numerically controlled (NC) machines that could easily be reprogrammed to perform the wide range of simple tasks that make up the majority of machining jobs. For more evidence on the °exibility gains associated with the PA technologies see Piore and Sabel [1984], OTA [1984] and Milgrom and Roberts [1990].

[5] Although many of the improvements in °exibility have been possible thanks to the development of the computers and the semiconductors, many other have been quite independent of these technologies like, for example the just-in-time system or the mini-mills.

the model in Section 4: using as a proxy for uncertainty the average volatility of stock returns for the firms in each 4-digit manufacturing sector, I find that an increase in uncertainty leads to an increase in the speed of diffusion of computers.

Another implication of the model is that an increase in the speed of diffusion of technologies affects the value of innovations. With shorter diffusion lags, successful innovators capture the incumbent's market share sooner. This makes innovation activities more attractive in a way that is consistent with the experience of the OECD economies since the mid-1970's (figure 8). In equilibrium, the rate of capital embodied technological progress accelerates.

Both the increased demand for flexible technologies and the increased frequency with which firms upgrade their capital lead to an increase in the size of the market for flexible capital and raises the return to R&D activities directed to improving its productivity. As a result, there is an increase in the rate of embodied TFP growth of the sectors producing flexible capital (i.e. computers and IT's in general). Gordon [1999] and Jorgenson and Stiroh [1999] have recently shown that indeed the growth rate of various measures of productivity in the IT producing sectors (sic 35) has been spectacular since the 1970's.

The rest of the paper is organized as follows. Section 2 presents the evolution of embodied and disembodied TFP growth and shows why the GPT approach is not consistent with it. The model is presented in section 3 and I use it to analyze the impact of an increase in uncertainty on the speed of diffusion of new technologies, the rate of embodied TFP growth and the shift towards more flexible production processes. Section 4 is devoted to illustrate the increase in uncertainty and test the implications of the uncertainty-driven theory of the slowdown. Section 5 further discusses the GPT approach and some extensions of this analysis for the service sector, the experience premium, the design of organizations and the evolution of the stock market.

2 The decomposition of TFP growth

To understand the causes of the productivity slowdown it is useful to decompose TFP growth into its embodied and disembodied components. According to Solow [1959] the former can be introduced into the production process only through gross investment in new plant and equipment, while the latter make new and old capital goods equally more productive. In this section I review two approaches to this decomposition and show that the GPT theories where the slowdown is caused by the cost of implementing the new technologies are inconsistent with the decomposition of TFP growth.[6]

2.1 The Solow-Nelson approach

Richard Nelson [1964] used variation in the average age of capital to identify the two components of TFP in Solow's model. Nelson begins with the following production function:

$$Y(t) = Ae^{\theta t}\left(\int_0^t e^{\lambda v} K(v;t)dv\right)^{\alpha} L(t)^{1-\alpha} \quad (1)$$

Here $Y(t)$ represents output at time t and $L(t)$ represents labor at time t. Capital goods are distinguished by their vintage. $K(v;t)$ denotes the amount of capital of vintage v at date t (so that the age of the capital is $t - v$). Technological progress comes in two forms. $\alpha\lambda$ is the rate of embodied technological progress. It is the contribution to TFP that is associated with newer vintages of capital. θ is the rate of disembodied productivity growth.

[6] In section 4.2 I use cross-sectional data to test related predictions of the implementation-based GPT models. As we shall see, the results obtained are consistent with those obtained in section 2.3. In section 5, I discuss the other type of GPT models where the slowdown is caused by the cost of developing the new technologies and its complementary inputs (Helpman and Trajtenberg [1998]).

Nelson uses the average age of capital to approximate the effective capital stock, $\hat{K}(t)$:[7]

$$\begin{aligned}\hat{K}(t) &= \int_0^t e^{\lambda v} K(v;t)dv \\ &\approx e^{\lambda(t-G_t)} K_t\end{aligned} \tag{2}$$

where K_t is the aggregate capital stock and G_t is the average age of capital.

Substitution of (2) into (1) leads to the following decomposition of TFP which makes possible the identification of embodied and disembodied TFP growth:[8,9]

$$\ln TFP_{it} \equiv \ln\left(\frac{Y}{L}\right)_{it} - \alpha \ln\left(\frac{K}{L}\right)_{it} = \beta_{0i} + \beta_1 t + \beta_2 G_{it} \tag{3}$$

where $\beta_1 = \mu + \lambda\alpha$ and $\beta_2 = -\lambda\alpha$.

Using annual data from 1953 to 1979 for thirteen 2-digit manufacturing sectors, McHugh and Lane [1987] test for structural breaks in (3).[10] Their main finding is that the

[7] The approximations conducted in this section follow Nelson [1964] who argues that they are extremely accurate for the US.

$$\begin{aligned}\hat{K}(t) &= \int_0^t e^{\lambda v} K(v;t)dv \\ &= e^{\lambda t}\int_0^t e^{\lambda(v-t)} K(v;t)dv \\ &= e^{\lambda t} K(t) \int_0^t e^{\lambda(v-t)} \frac{K(v;t)}{K(t)} dv \\ &\approx e^{\lambda t} K(t) \int_0^t (1 - \lambda(t-v)) \frac{K(v;t)}{K(t)} dv \\ &= e^{\lambda t} K(t)[1 - \lambda G_t] \\ &\approx K(t) e^{\lambda(t-G_t)}\end{aligned}$$

[8] Nelson also includes a correction for cyclical variations in TFP due to labor hoarding or capital utilization.

[9] A similar approach is developed by Hobijn [1999]. He uses the time series variation in the investment output ratio to identify the rate of embodied TFP growth. Another interesting difference between Hobijn and the McHugh and Lane exercise is that Hobijn runs a regression for the aggregate economy instead of using a sectorial panel. This approach has the advantage of being inmune to the reallocation of embodied TFP from the industries that use to the industries that produce capital when this is adjusted for quality .

[10] The excluded industries are tobacco (21), apparel (23) , lumber and wood (24), furniture and

productivity slowdown in manufacturing was caused by a decline in the growth rate of disembodied TFP beginning in 1974, while the rate of embodied technological change actually accelerated in the post-73 period. More specifically, the average growth rate of embodied TFP prior to 1973 was of 1.5 percent, while after 1973 it has increased 0.6 percentage points. The rate of disembodied TFP growth, however, declined after 1973 from 2.9 percent to minus 0.2 percent (table 1).

2.2 The Jorgenson-Gordon approach

An alternative approach based on Jorgenson [1966] identifies the rate of embodied TFP growth with the rate of decline of the quality adjusted price of new equipment in terms of goods and services. To see why this is a sensible strategy consider an economy with two sectors one producing efficiency units of investment (i) and the other producing efficiency units of consumption (c) goods with the following technologies:

$$
\begin{aligned}
i_t &= e^{(\theta+\lambda)t} k_{i;t}^{\alpha} l_{i;t}^{1-\alpha} \\
c_t &= e^{\theta t} k_{c;t}^{\alpha} l_{c;t}^{1-\alpha}
\end{aligned}
$$

where $k_{x;t}$ and $l_{x;t}$ denote the efficiency units of capital and labor used at time t in sector x. In this accounting framework, the measure of capital is adjusted for changes in quality, so that embodied technological progress is subsumed into the measure of capital. A new machine is as productive as several old machines. Improvements in the quality of capital show up in the relative productivity of the investment goods producing sector and are captured by λ. Embodied technological progress is therefore $\alpha\lambda$.

In a perfectly competitive environment the price of c_t and i_t are equal to their respective marginal costs of production:

$$
\begin{aligned}
P_t^c &= e^{-\theta t} \left(\frac{r_t}{\alpha}\right)^{\alpha} \left(\frac{w_t}{1-\alpha}\right)^{1-\alpha} \\
P_t^i &= e^{-(\theta+\lambda) t} \left(\frac{r_t}{\alpha}\right)^{\alpha} \left(\frac{w_t}{1-\alpha}\right)^{1-\alpha}
\end{aligned}
$$

---

fixtures (25), printing and publishing (27), petrochemicals (29) and leather (31). They represent less than 20 percent of the manufacturing value added. For data sources and details on the construction of capital, and age series see McHugh and Lane [1987].

where r_t and w_t denote the cost of capital and labor at time t. If there is perfect mobility of the factors of production across sectors the factor prices are the same in both sectors and the price of investment goods in terms of consumption (P_t^i/P_t^c) is equal to $e^{-\gamma t}$. We can therefore compute the rate of embodied TFP growth from the rate of decline in the relative price of investment goods.

Gordon (1990) has constructed a quality adjusted price index for producer durable equipment (PDE) which can be used to compute the relative price P_t^i/P_t^c. Hornstein-Krusell [1996], Greenwood and Yorukoglu [1997] and Greenwood, Herzcowitz and Krusell [1997] estimate that on average the relative price of investment goods has declined 3 percent per year between 1954 and 1973. The first two papers also estimate that the rate of decline has increased by between 0.5 and 1 percentage points a year since 1973. This represents a 25 percent increase in embodied technological progress, or an increase of between 0.15 and 0.3 percentage points.

In summary, both approaches lead to the same conclusion: that the rate of embodied technological progress has accelerated since the early 1970's and that the productivity slowdown is completely explained by a decline in disembodied productivity growth. Further, note that the acceleration of the rate of embodied TFP growth had also some long-run component since it lasted long after the slowdown was over. To understand the productivity slowdown we need a model that can explain the evolution of its components.

2.3 The GPT approach and embodied technological change

According to the GPT approach, the advent of new general purpose technologies such as information technologies first lowers productivity growth as firms incur in the cost of implementing the new technologies, and later raises productivity growth as the productive benefits of these technologies become fully realized.[11] Intuitively, since it is investing in new capital what initially makes a firm relatively less productive, the GPT approach associates the productivity slowdown with a decline in embodied technological progress.

To see this more precisely, consider the following specification for the evolution of productivity which captures the implementation-based GPT literature. Denote the produc-

[11] Greenwood and Yorukoglu [1997], Greenwood and Jovanovic [1998] and Hornstein and Krusell [1996].

tivity of capital vintage v at time t by:

$$e^{\theta(t)t+\lambda(t;v)v} = \begin{cases} e^{\frac{\theta}{\alpha}t+\lambda v} & \text{for } v \leq t < \tilde{v} \\ e^{\frac{\theta}{\alpha}t+\lambda v-\frac{\zeta_o}{\delta_o(t-\tilde{v})+1}} & \text{for } v < \tilde{v} \leq t \\ e^{\frac{\theta}{\alpha}t+\hat{\lambda}(v-\tilde{v})+\lambda\tilde{v}-\frac{\zeta_n}{\delta_n(t-\tilde{v})+1}} & \text{for } v \geq \tilde{v} \end{cases}; \tag{4}$$

The new GPT arrives with vintage $\tilde{v}$. Prior to this moment, the efficiency of capital has grown over time at rate $\frac{\theta}{\alpha}$; and over the vintages at rate λ. At $t = \tilde{v}$, firms are not acquainted with the new GPT and must learn how to use it before its advantages are fully realized (i.e. $\zeta_i > 0$, for $i = o, n$). $\hat{\lambda} > \lambda$ ensures that the productivity under the new GPT eventually surpasses the trend under the old GPT. $\delta_i > 0$ captures the idea that as time goes by and the agents figure out how to use the new GPT, the learning cost disappears. The formulation in expression (4) is sufficiently rich to accommodate negative externalities from the new GPT to the productivity of the old vintages ($\zeta_o \geq 0$) and different speeds of integration of the new GPT in the production process with the old and the new vintages ($\delta_n \neq \delta_o$).

The level of potential output is equal to:

$$Y(t) = \left(\int_{v_0}^{t} e^{\theta(t)t+\lambda(t;v)v} K(v;t)\right)^{\alpha} L(v;t)^{1-\alpha},$$

where $v_0 > 0$ is the first vintage v for which $K(v;t)$ is positive.

To mimic the Solow-Nelson approach, expression (4) is plugged into the production function and then it is approximated by the average age of capital.

$$Y(t) = \begin{cases} e^{\theta t}\left(\int_{v_0}^{t} e^{\lambda v} K(v;t)dv\right)^{\alpha} L(t)^{1-\alpha} & \text{for } t < \tilde{v} \\ e^{\theta t}\left(\int_{v_0}^{\tilde{v}} e^{\lambda v-\frac{\zeta_o}{\delta_o(t-\tilde{v})+1}} K(v;t)dv + \int_{\tilde{v}}^{t} e^{\hat{\lambda}(v-\tilde{v})+\lambda\tilde{v}-\frac{\zeta_n}{\delta_n(t-\tilde{v})+1}} K(v;t)dv\right)^{\alpha} L(t)^{1-\alpha} & \text{for } t \geq \tilde{v} \end{cases}.$$

In the appendix (claim 1), I show that the result of this approximation is the following:

$$Y(t) \approx \begin{cases} e^{(\theta+\alpha\lambda)t-\alpha\lambda G_t} K(t)^{\alpha} L(t)^{1-\alpha} & \text{for } t < \tilde{v} \\ e^{(\theta+\alpha\lambda^1(t))t-\alpha\tilde{\lambda}(t)G_t} K(t)^{\alpha} L(t)^{1-\alpha} & \text{for } t \geq \tilde{v} \end{cases} \tag{5}$$

where $\lambda > \{\lambda^1(t), \tilde{\lambda}(t)\}$ for an interval after the new GPT arrives.

The rate of embodied TFP growth is the elasticity of the level of TFP with respect to the average age of capital. From equation (5) can be concluded that, with the arrival of the new GPT, the rate of embodied TFP growth declines from α_λ to $\alpha_{\tilde{\lambda}}(t)$. Note that if $\gamma_i > 0$; $\tilde{\lambda}(t)$ is smaller than λ for an interval after $\bar{t}$: This means that according to the GPT approach there is a productivity slowdown if and only if there is a decline in the rate of embodied TFP growth. The implementation-based GPT models are therefore inconsistent with the evidence presented above on the acceleration of the Solow-Nelson rate of embodied TFP growth during the productivity slowdown. [12]

There is a simple explanation for this inconsistency. According to the GPT theory, the implementation of computers causes the slowdown. Investing in the new GPT is particularly bad for productivity short after the arrival of the new GPT. Therefore a reduction in the age of capital stock in the 70's is going to be reflected in a smaller increase in TFP than what used to be the case under the previous GPT. In other words, the elasticity of TFP with respect to investment should decline in the 70's according to the GPT approach. Yet, we know from McHugh and Lane that it increased substantially.

To explain the evolution of embodied and disembodied technological progress, it is necessary to build a model in which the shock that causes TFP growth to slowdown makes investing in new capital particularly productive. This is what I do in the next section.

3 Uncertainty and TFP growth.

The model presented in this section analyzes the effect of uncertainty on the flexibility of capital and on the speed of diffusion of technologies. In a more uncertain business environment, old capital becomes obsolete. This generates the productivity slowdown and the short-run acceleration of the relative productivity of new vs. old capital documented by McHugh and Lane [1987]. Uncertainty also enhances the speed of diffusion of technologies and the long-run rate of embodied technological progress documented by Greenwood and Yorukoglu [1997] and Hornstein and Krusell [1996].

[12] Note however that, if the future increase in productivity is sufficiently large, the learning-based GPT models could be consistent with the acceleration of the rate of decline of the quality adjusted relative price of new capital (i.e. the Jorgenson-Gordon rate of embodied TFP growth).

3.1 The diffusion of product-specific capital

Consider a firm that rents capital to produce output. This capital comes in many designs, each of which is characterized by three attributes: its productivity, the product that it is optimally designed to produce, and its flexibility. In a frictionless world, the firm would always rent the design perfectly suited to its output. I assume, however, that there is a cost of adopting new designs so that firms upgrade their capital infrequently. The rest of this section is devoted to describing the costs and benefits of this adoption decision.

New vintages of capital are more productive than old vintages of capital. The productivity of a machine developed at date τ is $e^{\gamma\tau}$: The output produced at date t with $k_{t;\tau}$ units of capital adopted at date $\tau \leq t$, is

$$y_{t;\tau} = (k_{t;\tau} e^{\gamma\tau})^{\alpha}$$

where $\alpha \in (0; 1)$:

The firm faces an uncertain environment, because the unique product demanded by consumers evolves stochastically.[13] More specifically, the firm's output $y_{t;\tau}$ is characterized by a continuum of attributes r_j with $j \in [0; 1]$. The characteristics that consumers demand evolve over time. In particular, each of the r_j follows an independent driftless Brownian motion with infinitesimal variance equal to σ^2.

Machines are optimally designed for the production of one specific set of characteristics, which I will call the template. It is costly to produce a product that deviates from this standard. But, since from time t; the expected demanded at $t + s$ is the same as the demanded product at t; a firm that upgrades its capital at t will set the template equal to the demanded product that instant: Over time, if the machine design is not altered, the product that consumers demand will diverge from the template. After s periods the j^{th} characteristic will diverge by $r_{j;t+s} - r_{j;t}$. I assume that the cost of operating a

[13] All the results obtained in this paper generalize to other sources of uncertainty like variation in the prices of the inputs used in production (i.e. supply side) or in the mix of goods produced (i.e. other demand side specifications).

machine depends on the average of the square deviations from the template:

$$d_s \equiv \int_0^1 (r_{j,t+s} - r_{j,t})^2 \, dj = s\sigma^2$$

I will refer to d as the distance between the demanded product and the template.[14]

Some production systems are more flexible than others. Flexible capital is more easily employed to produce products that deviate from the template. Flexibility is captured by a parameter f which must be chosen at the time the firm adopts a new design. The more flexible a machine, the higher is f. The cost of operating a machine depends on its flexibility and the distance of demand from the template. Let $C(t; d; f)$ denote this costs. We have

$$C(t; d; f) = (a(f)\, e^d + c_m(f))e^{\lambda t}$$

where $a'(\cdot) < 0$; $a''(\cdot) > 0$, $c_m'(\cdot)$, $c_m''(\cdot) > 0$ and $\lim_{f \to \infty} c_m''(f) > 0$. $a(f)e^d$ represents the cost of operating a machine to produce a product that differs from the template by d units. It is decreasing in flexibility. $c_m(f)$ represents the cost of maintaining the capital. It is increasing in flexibility. $e^{\lambda t}$ scales the operating cost so that it does not become insignificant in the long run.

If the firm could, it would always want to rent a machine that was built to produce the precise characteristics that consumers demand. To prevent this, I assume that in order to adopt machines designed to produce a different bundle of characteristics, the firm must pay a fixed cost equal to $A_c e^{\lambda t}$: This cost represents the costs of learning to work with the new design. Given this adoption cost firms will upgrade their capital infrequently. moreover, since the cost is independent of the size of the upgrade, firms will always upgrade to the most productive available capital.

The firm chooses the amount of capital to rent, the dates at which to upgrade its templates, and the flexibility of the upgrade in order to maximize profits. Future profits are discounted at a rate ρ. This maximization problem can be expressed as

$$\Pi_0 \equiv \max_{k_{t,T_i};\, T_i;\, f_i} \sum_{i=0} \left[\int_{T_i}^{T_{i+1}} (\pi_{t,T_i} - C(t; d_{t-T_i}; f_i))e^{-\rho t} dt - A_c e^{-(\rho - \lambda)T_i} \right]$$

[14] Note that by assuming a continuum of characteristics this distance depends only on the variance of demand and the time since the machine was adopted. Assuming a single characteristic would have resulted in a state dependent rule.

Here $\pi_{t,T_i} = y_{t,T_i} - p_k\, k_{t,T_i}$, p_k is the rental price of capital, T_i represents the adoption times, and f_i denotes the associated choices of flexibility. Note that ρ must be greater than λ for the problem to be well defined.

Because the firm rents capital, the choice of k_{t,T_i} is a static decision. The profit maximizing choice is:

$$k_{t,T_i} = \left(\frac{\alpha e^{\alpha \tilde{\lambda} T_i}}{p_k} \right)^{\frac{1}{1-\alpha}} \tag{6}$$

It follows that

$$\begin{aligned} \pi_{t,T_i} &= y_{t,T_i} - p k_{t,T_i} \\ &= \left[\left(\frac{\alpha}{p_k} \right)^{\frac{\alpha}{1-\alpha}} - \left(\frac{\alpha}{p_k^{\alpha}} \right)^{\frac{1}{1-\alpha}} \right] e^{\frac{\alpha \tilde{\lambda} T_i}{1-\alpha}} \\ &\equiv \pi e^{\lambda T_i} \end{aligned}$$

where $\lambda \equiv \frac{\alpha \tilde{\lambda}}{1-\alpha}$.

The first order condition with respect to T_i is:

$$\overbrace{[\pi - a(f_i) - c_m(f_i)] - \left[\pi e^{-\lambda \Delta T_i} - a(f_{i-1}) e^{\sigma^2 \Delta T_i} - c_m(f_{i-1}) \right]}^{\text{Marginal Cost of Delay}} \tag{7}$$

$$= \underbrace{\frac{\lambda \pi}{\rho} \left[1 - e^{-\rho \Delta T_{i+1}} \right] + a(f_i)\sigma^2 \frac{e^{(\sigma^2 - \rho + \lambda)\Delta T_{i+1}} - 1}{\sigma^2 - \rho + \lambda} + A_c(\rho - \lambda)}_{\text{Marginal Benefit from Delay}}$$

The left-hand side of this expression represents the costs that the firm incurs by delaying the decision to upgrade its capital. By delaying an instant, the firm does not enjoy the higher net profits associated with a more sophisticated technology that is better suited to the product that consumers demand. The right-hand side represents the benefits of delay. By delaying the adoption decision, the new capital will be marginally more advanced and its template will be marginally closer to the demanded products. These effects correspond to the first and second terms of the RHS of (7). The third term represents the benefit of postponing the adoption costs.

The first order condition with respect to f_i is:

$$\overbrace{c_m'(f_i)\frac{1-e^{-(\rho-\lambda)\Delta T_{i+1}}}{\rho-\lambda}}^{\text{Marginal Cost of Flexibility}} = \underbrace{-a'(f_i)\frac{1-e^{-(\rho-\lambda-\sigma^2)\Delta T_{i+1}}}{\rho-\lambda-\sigma^2}}_{\text{Marginal Value of Flexibility}} \tag{8}$$

The flexibility of the manufacturing system is determined by the balance between the higher costs of maintaining flexible production systems and the lower cost of adopting flexible systems to a changing environment.

Equations (6), (7), and (8) characterize the solution to the firm's problem. In the next subsection, I study the steady state properties of this system.

3.2 Steady state analysis

The steady state of this economy is defined as a situation where the diffusion lags and the degree of flexibility are constant (i.e. $\Delta T_i \equiv T_i - T_{i-1} = \Delta T^*$ and $f_i = f^*; \forall i$). For a given rate of technological progress (λ) and level of uncertainty (σ^2), equations (A) and (F) define the steady state levels of ΔT^* and f^*.

$$\overbrace{\left[\pi - a(f^*)\right] - \left[\pi e^{-\lambda\Delta T^*} - a(f^*)e^{\sigma^2\Delta T^*}\right] - \frac{\lambda\pi}{\rho}\left(1-e^{-\rho\Delta T^*}\right)}^{\text{Net Marginal Cost of Delay}} - a(f^*)\sigma^2\frac{e^{(\sigma^2-\rho+\lambda)\Delta T^*}-1}{\sigma^2-\rho+\lambda} - A_c(\rho-\lambda) = 0 \tag{A}$$

$$\overbrace{-a'(f^*)\frac{e^{(\sigma^2-\rho+\lambda)\Delta T^*}-1}{\sigma^2-\rho+\lambda} - c_m'(f^*)\frac{1-e^{-(\rho-\lambda)\Delta T^*}}{\rho-\lambda}}^{\text{Net Marginal Value of Flexibility}} = 0 \tag{F}$$

To conduct the comparative statics exercises it is useful to represent equations (A) and (F) in the $\left(\Delta T^*; f\right)$ plane. I show in the appendix that both equations describe positive relationships between ΔT^* and f (claim 2). According to equation (A), if capital is more flexible, then deviations from the template are less costly, and the firm chooses to upgrade its capital less often. This relationship is illustrated by the AA curve in figure 2. According to equation (F), if the firm is upgrading less often, the value of flexibility increases since there is more time for desired product characteristics to deviate from the template. The FF curve in ¯gure 2 illustrates this relationship.

Sometime at the end of the 60's or the beginning of the 70's the uncertainty faced by the ¯rms increased substantially. We can use the model to show the e®ect of an increase in uncertainty on the °exibility and the speed of di®usion of technologies (i.e. $\frac{1}{4T}$). In a more uncertain world, the distance between the template and the demanded product will, ceteris paribus, increase faster. This means that the return from upgrading capital more frequently raises and, as a result, the di®usion lags are reduced. As shown in ¯gure 3, this implies that the AA locus shifts to the right. For a given speed of di®usion, uncertainty raises the marginal value of °exibility and ¯rms implement more °exible production systems. This translates into a shift to the right of the FF curve. As can be observed from ¯gure 3, the impact of uncertainty on °exibility and on the speed of di®usion is ambiguous.

Firms have two ways to react to the higher uncertainty: increasing the °exibility of the manufacturing systems and upgrading more frequently their template by installing new equipment. If uncertainty has a very large impact on the net marginal value of °exibility and the net marginal cost of delay is very sensitive to the degree of °exibility it may be the case that an increase in uncertainty is followed by an increase in °exibility and in the di®usion lags. Conversely, if a higher uncertainty reduces very much the di®usion lags and the net marginal value of °exibility is very sensitive to the faster di®usion of technologies, then an increase in uncertainty might accelerate the speed of di®usion and reduce the °exibility of the production systems. In less extreme situations, however, ¯rms will react to an increase in uncertainty both by implementing more °exible production processes and by adopting more frequently the state of the art technology. This intuition can be formalized with the following proposition.

Proposition 1 There exist two numbers $\hat{c}^0_{m\sigma}$ and $\hat{a}^0_{\sigma}$ such that if $c^0_m(\cdot)$ is bounded below by $\hat{c}^0_{m\sigma}$ and $-a^0(\cdot)$ is bounded above by $-\hat{a}^0_{\sigma}$ an increase in uncertainty will increase the °exibility and the speed of di®usion. This restriction will be denoted as condition 1.

Proof. : See appendix A.1.

As we shall see, the view implied by condition 1 seems to be consistent with the US experience. In section 4.3, I show that uncertainty has had a positive effect on the speed of diffusion of computers in US manufacturing. There is also a large body of evidence showing that since the 1970's the production processes have become more flexible (Piore and Sabel [1984], OTA [1984] and Milgrom and Roberts [1990]). Based on these results I assume for the rest of the paper that condition 1 holds.

A second exercise that has received some attention is to study the effect of the rate of technological progress (λ) on the speed of diffusion.[15] An increase in λ raises the net marginal cost of delaying adoption because the benefits of moving to the frontier technology are greater. Therefore, for a given level of flexibility, the diffusion lags will be shorter in the economy with faster technological progress. As illustrated in figure 4, this implies that the AA locus shifts to the right with λ. This effect has been highlighted by Chari and Hopenhayn [1991]. In my framework, however, since the cost associated with the divergence between the template and the demanded product is increasing in λ; technological progress also affects directly the value of flexibility. In particular, for a given speed of diffusion, I show in the appendix that an increase in λ raises the net marginal value of flexibility (claim 3). As shown in figure 4, this implies that the FF locus also shifts to the right with λ.

As with the effect of uncertainty, the resulting impact of technological progress on ΔT^1 and f is ambiguous. As above, this ambiguity can be fixed by bounding the elasticity of the net marginal value of flexibility with respect to the speed of diffusion and the elasticity of the net marginal cost of delay with respect to flexibility.

Proposition 2 There exist two numbers $\hat{c}'_{m\lambda}$ and $\hat{a}'_{\lambda}$ such that if $c'_m(\cdot)$ is bounded below by $\hat{c}'_{m\lambda}$ and $-a'(\cdot)$ is bounded above by $-\hat{a}'_{\lambda}$ an increase in the rate of technological progress raises the flexibility and the speed of diffusion. This restriction will be referred to as Condition 2.

Proof. : See appendix A.1.

The loci AA and FF can be subsumed into the locus AF in figure 5. AF represents the equilibrium diffusion lag consistent with all the possible levels technological progress (λ). Under condition 2, technological progress has a negative effect on the diffusion lags;

[15] See Rosenberg [1976] and Chari and Hopenhayn [1991].

and therefore AF is downward sloping in the $\left(\Delta T^{-1}, \lambda\right)$ plane. Moreover, under condition 1, uncertainty reduces the diffusion lag, for any given λ. This implies that an increase in uncertainty shifts AF to the left. These properties shall be useful to understand the interaction between technological progress and the speed of diffusion. Before conducting this analysis, I must put the firms problem in an equilibrium framework by endogenizing the rate of technological progress and the rental price of capital. The next section addresses this issue.

3.3 Endogenous technological change

To close the model, I need to specify a production function for capital, the possible uses of labor and the structure of the R&D process.

Capital is produced by combining a continuum [0,1] of intermediate good varieties. To produce capital of a given vintage τ, only the intermediate goods of vintage τ or earlier can be used. In particular, the technology for producing efficiency units of vintage τ capital takes the form:

$$\hat{k}_\tau = \left[\int_0^1 x_{j\tau}^{\beta} A_{j\tau}^{\beta}\, dj\right]^{\frac{1}{\beta}}; \tag{9}$$

where $x_{j\tau}$ is the amount of intermediate goods of the j^{th} variety developed before time τ, $A_{j\tau} \leq e^{\lambda\tau}$ is the associated productivity level of this variety and $\beta \in (0; 1)$. Both intermediate goods and capital depreciate instantaneously. Capital is sold in a perfectly competitive market.

The economy is populated by a continuum of agents with measure L. Labor has two competing uses. A measure n of the agents tries to develop improved versions of the intermediate goods. They are the innovators. Alternatively, an agent can also manage a final good firm. Managers earn the profits made by their firms. Innovators are granted perpetual patents over their innovations and use them to collect monopolistic rents from the sale of the intermediate goods that embody them. Intermediate goods can be duplicated at a fixed marginal cost of c units of final output.

Innovations randomly arrive with a Poisson arrival rate that depends on the intensity of R&D. In particular, when n_j innovators try to improve the productivity of the j^{th} intermediate good, the arrival rate is $\lambda(n_j)$; where $\lambda' > 0$ and $\lambda'' < 0$: Innovations incorporate the state of the art knowledge in society (A_{max}). Therefore, after an innovation occurs at the j^{th} variety, the productivity parameter of the j^{th} intermediate good jumps to A_{max} and becomes the leading edge technology. As equation (??) captures, the rate of accumulation of knowledge is increasing in the total measure of innovators.[16]

$$\frac{\dot{A}_{max}}{A_{max}} = \lambda(n)$$

The productivity of the leading edge technology (A_{max}) and equation (9) can be used to pin down the production function of capital (k).

$$k_\tau = \frac{\hat{k}_\tau}{A_{max}} = \left[\int_0^1 x_{j\tau}^{\beta}\left(\frac{A_{j\tau}}{A_{max}}\right)^{\beta} dj\right]^{\frac{1}{\beta}}$$

$$k_\tau = \left[\int_0^1 x_{j\tau}^{\beta} a_{j\tau}^{\beta} dj\right]^{\frac{1}{\beta}}; \tag{10}$$

where $a_{j\tau} \equiv \frac{A_{j\tau}}{A_{max}}$: As shown in Aghion and Howitt [1997], $a_{j\tau}$ is distributed at the steady state uniformly in the interval [0,1]. Using this fact, the production function (10) can be rewritten as:

$$k = \left[\int_0^1 x(a)^{\beta} a^{\beta} da\right]^{\frac{1}{\beta}}$$

This implies that, to produce k_v units of capital of vintage v, it will be used $x(\tau; v)$ units of the vintage $\tau\ (\leq v)$

$$x(\tau; v) = \left[\frac{k_v e^{\frac{\beta}{1-\beta}(\tau - v)} p^{\frac{-1}{1-\beta}}}{\int_0^1 i^{\frac{\beta}{1-\beta}} p(i)^{\frac{-\beta}{1-\beta}} di}\right]; \tag{11}$$

where p(i) is the price charged for the intermediate good with relative productivity i

[16] The law of motion of knowledge could take any other functional form at the only cost of changing the ergodic distribution of relative productivities.

and $p = p(e^{\lambda(\tau - v)})$: Plugging in k_v from expression (6), $x(\tau; v)$ can be rewritten as:[17]

$$x(\tau; v) = \frac{e^{\lambda\left(\frac{1-2\beta}{1-\beta}\right)v} e^{\frac{\beta}{1-\beta}\tau} p^{\frac{-1}{1-\beta}} \left(\frac{\alpha}{p_k}\right)^{\frac{1}{1-\alpha}}}{\left[\int_0^1 i^{\frac{\beta}{1-\beta}} p(i)^{\frac{-\beta}{1-\beta}} di\right]} \tag{12}$$

Equation (12) can be integrated over all the capital vintages (D) that use this intermediate good to obtain the instantaneous aggregate demand faced by the producer of an intermediate good with productivity τ ($X(\tau)$).

$$X(\tau) = \frac{\int_D e^{\lambda\left(\frac{1-2\beta}{1-\beta}\right)v} dv \, e^{\frac{\beta}{1-\beta}\tau} p^{\frac{-1}{1-\beta}} \left(\frac{\alpha}{p_k}\right)^{\frac{1}{1-\alpha}}}{\left[\int_0^1 i^{\frac{\beta}{1-\beta}} p(i)^{\frac{-\beta}{1-\beta}} di\right]} \tag{13}$$

A monopolist innovator facing this demand and a constant marginal cost c will set a price $p = \frac{c}{\alpha}$: In a perfectly competitive market for capital, the price of capital is equal to its marginal cost which is given by the following expression:

$$\begin{aligned} p_k &= \left[\int_0^1 v^{\frac{\beta}{1-\beta}} p(v)^{-\frac{\beta}{1-\beta}} dv\right]^{-(1-\beta)} \\ &= \left(\frac{c}{\alpha}\right)^{\beta} (1-\beta)^{-(1-\beta)} \end{aligned} \tag{14}$$

The expressions for p_k and p together with (13) define the demand faced by a successful innovator as a function of her intermediate good's vintage and the measure of firms that demand it.

In steady state, firms upgrade their capital with frequency $\frac{1}{\Delta T}$: This generates a gradual inflow of firms into the pool of users of capital that embodies a newly developed intermediate good and also a gradual outflow from this pool when a superior innovation arrives. The speed of diffusion of technologies ($1/\Delta T$) has a direct effect on the intensity of these flows. When firms upgrade their capital more frequently, both the inflow and

[17] This specification captures two situations depending on whether $x(\tau; v)$ is increasing or decreasing in v. In the first ($\beta > 0.5$), the users of capital vintages closer to τ demand more vintage-τ intermediate good: In the second ($\beta < 0.5$), the demand of the vintage-τ intermediate goods is larger from the users of more distant capital (i.e. larger v). In the intermediate case ($\beta = 0.5$); the demand of the intermediate good only depends on the productivity of the intermediate good (τ) and is independent of the capital vintage (v).

the outflow will be more intense. However, the faster inflow exactly cancels with the faster outflow and the total demand that faces an innovator is independent of the speed of diffusion of technologies (figure 7). Nevertheless, with a faster speed of diffusion of technologies the demand is concentrated earlier in time and since time is discounted at the rate ρ, the present discounted demand will be increasing in the speed of diffusion of technologies.[18,19] Let's denote the expected present discounted value of demand faced by an innovator of a $\tau-$ vintage intermediate good by $e^{\lambda\tau}\chi(\Delta T^{-1})$.

Innovators earn a margin $(\frac{1-\alpha}{\alpha})c$ per intermediate good sold. The measure of firms is equal to $L - n$, and the marginal probability of being successful at the j^{th} intermediate good when there are n_j innovators working at improving this variety is $\lambda'(n_j)$. The resulting expected value of being an innovator at time τ at the j^{th} variety is

$$V(\underset{+}{\tau}; \underset{-}{n_j}; \underset{-}{n}; \underset{-}{\Delta T^{-1}}) \equiv \lambda'(n_j)\ \overbrace{(\frac{1-\alpha}{\alpha})c}^{\text{Profit Margin}}\ \overbrace{(L-n)}^{\text{Measure of firms}}\ \overbrace{e^{\lambda\tau}\underset{-}{\chi(\Delta T^{-1})}}^{\text{P.D.V. Demand}}$$

The value of being an entrepreneur from τ onwards is $\Phi_\tau \equiv e^{\lambda\tau}\int_\tau^\infty \pi e^{-(\rho-\lambda)(t-\tau)}dt$, where $\pi e^{\lambda\tau}$ are the average instantaneous profits net of the cost of uncertainty and the adoption costs. The opportunity cost of innovation activities at time τ is therefore $\pi e^{\lambda\tau}$, where π is decreasing in σ^2 and A_c. At any interior equilibrium, the expected value of an innovation is equal to its opportunity cost. In equilibrium, the value of innovating is also equalized across varieties. This occurs when $n_j = n, \forall j$. The resulting arbitrage condition is:

$$\pi(\underset{-}{\sigma^2}; \underset{-}{A_c})e^{\lambda t} = V(\underset{+}{\tau}; \underset{-}{n}; \underset{-}{n}; \underset{-}{\Delta T^{-1}}) \qquad \text{(L)}$$

$$\pi(\underset{-}{\sigma^2}; \underset{-}{A_c}) = \lambda'(n)(\frac{1-\alpha}{\alpha})c(L-n)\underset{-}{\chi(\Delta T^{-1})}$$

[18] More formally, it is easy to show that for any time τ where innovation 2 (superior to innovation 1) is developed, a. the total cumulative demand of innovation 1 is independent of ΔT^{-1} and b. the cumulative demand of innovation 1 at time t (cd(t)) is decreasing in ΔT^{-1} (i.e. $\frac{\partial cd(t)}{\partial \Delta T^{-1}} \leq 0, \forall t$, and strictly negative $\forall t$ where the demand is strictly positive). If $\rho > 0$, it follows immediately that $\underset{-}{\chi(\Delta T^{-1})}$.

[19] In this simple model the positive impact of the speed of diffusion of technologies on the value of innovations comes from the time-discounting. More generally, there are more substantial mechanisms that might strengthen this result. If there is imitation, most of the benefits from innovating will be accrued shortly after the innovation is developed and a fast diffusion of the innovation may greatly enhance the profits that it generates.

When technology di®uses faster (i.e. $4T^{-1}$ is reduced), the present discounted demand faced by an innovator is also larger and the expected return to developing an improvement in capital raises. Agents react to this by °owing to the R&D sector, and in equilibrium the measure of innovators (n) increases. Equation (L) de¯nes, therefore, a negative relationship between $4T^{-1}$ and n which is represented in ¯gure 5 as LL: Equations AF and L determine the measure of innovators (n) and the size of the di®usion lags ($4T$).

Following an increase in uncertainty, LL shifts to the left because the opportunity cost of innovation activities falls (see ¯gure 6). The AF locus shifts to the right because, from condition 1, the net value of adopting a better and more appropriate machine increases. If L is steeper than AF,[20] higher uncertainty implies shorter di®usion lags and higher rate of technological progress.

The intuition for this result is straightforward. In a more uncertain environment technology di®uses faster. This raises the present discounted demand faced by innovators and therefore the value of innovations. Uncertainty also accelerates the rate of divergence between templates and demanded products reducing the value of managing a ¯rm. Both mechanisms increase the incentives to conduct R&D activities. In equilibrium, the rate of technological progress is higher in a more uncertain environment and technology di®uses faster. Since technological progress is embodied in new capital, a given amount of investment in physical capital will lead to a larger increase in productivity in the more uncertain environment than in the less uncertain environment. This is precisely the way Nelson identi¯es an acceleration in embodied TFP growth.

The model also predicts that uncertainty accelerates the rate of embodied TFP growth when this is identi¯ed following the Jorgenson-Gordon approach. To see this, note that the price of a ¯nal good has been normalized to 1. The rental price of a physical unit capital is p_k units of ¯nal good: A unit of capital at time t is as productive as $e^{¸t}$ units of capital at time 0. Therefore, the price of an e±ciency unit of capital at t in terms

[20] If the locus AF is steeper than LL the equilibrium is not stable. To see this note that, if an entrepreneur decides to become an innovator there are going to happen two things. On the one hand, the size of the market for innovations declines by one, and value of an innovation falls. On the other, the size of the innovation will also raise. This enhances the speed of di®usion of technologies and as a result the value of an innovation. If AF is steeper than LL, this second channel is stronger than the ¯rst, and therefore the former entrepreneur will be better o® working as an innovator. Hence the equilibrium was not stable.

of the final good is $p_k e^{-\lambda t}$. According to the Jorgenson-Gordon approach, the rate of embodied TFP growth is the rate of decline of the relative price of an efficiency unit of capital times the capital share. That is exactly $\alpha\lambda$. Therefore, when λ raises due to the increase in uncertainty so does the rate of embodied TFP growth.

At a more basic level, the evolution of the amount of resources devoted to R&D activities has been consistent with the acceleration predicted by the uncertainty-driven theory. As we shall see in section 4, there is strong evidence that the uncertainty of the business environment increased at the end of the 60's or the beginning of the 70's. Figure 8 presents the evolution of the ratio of non-defense R&D expenditures to GDP for the G-7 countries between 1970 and 1992. In all of them there seems to be a significant positive trend in the ratio. More specifically, the increments since the mid 1970's have been substantial ranging from a 14 percent in Germany to 47 percent in Italy. In the US the ratio of non-defense R&D expenditures to GDP has risen from about 1.6 in 1973 to 2.1 in 1992. This represents a 27 percent increase. Therefore the timing of the increase in uncertainty and of the increase in resources devoted to R&D activities seems to support the predictions of the model.

After having shown that the uncertainty-driven approach explains the long-run acceleration of embodied TFP growth, the next section shows that it also accounts for the deceleration of disembodied TFP growth.

3.4 Disembodied TFP-growth and short run dynamics

Up to now the analysis has been restricted to the steady state of the economy. However, the decline in the growth rate of TFP in manufacturing has been a transitory phenomenon. To show that the uncertainty-driven approach accounts for the pattern observed in figure 1, it is necessary to study the adoption decision along the transition from the low to the high uncertainty steady state. For simplicity, the rate of technological progress is assumed to be constant.[21,22]

[21] In the short run, this assumption may be defended by the relatively long gestation lags of new technologies.

[22] Beyond this assumption, there are other reasons to interpret some of the results that I derive below with caution in a general equilibirum context. In particular, during the interval when no firm upgrades its capital, there is a smaller net supply of final goods, because of the higher costs of operating capital,

In the low uncertainty state, firms adopt new technology with periodicity $\Delta\bar{T}_l$ and the flexibility of the installed capital is $\bar{f}_l$. At date $\hat{t}$ uncertainty increases unexpectedly from σ^2 to $\bar{\sigma}^2$. Firms react to this shock by modifying the technology upgrading plans that they had made. The first order conditions of this reoptimization problem are given by the following equations:

$$[\pi - a(f_{\hat{t}}) - c_m(f_{\hat{t}})] - \left[\pi e^{-\lambda \Delta T_{\hat{t}}} - a(\bar{f}_l) e^{\bar{\sigma}^2 (T_{\hat{t}} - \hat{t}) + \sigma^2 (\hat{t} - T_{\hat{t}-1})} - c_m(\bar{f}_l)\right] \quad (15)$$

$$= \frac{\lambda \pi}{\rho}\left(1 - e^{-\rho \Delta T_{\hat{t}+1}}\right) + a(f_{\hat{t}})\bar{\sigma}^2 \frac{e^{(\bar{\sigma}^2 - \rho + \lambda)\Delta T_{\hat{t}+1}} - 1}{\bar{\sigma}^2 - \rho + \lambda} + A_c(\rho - \lambda)$$

$$c_m'(f_{\hat{t}}) \frac{1 - e^{-(\rho - \lambda)\Delta T_{\hat{t}+1}}}{\rho - \lambda} = -a'(f_{\hat{t}}) \frac{e^{(\bar{\sigma}^2 - \rho + \lambda)\Delta T_{\hat{t}+1}} - 1}{\bar{\sigma}^2 - \rho + \lambda} \quad (16)$$

where $T_{\hat{t}}$ is the time of the first technological upgrade after $\hat{t}$ and $f_{\hat{t}}$ is the selected level of flexibility.

The increase in uncertainty surprises firms at different stages in their adoption phase. Some have just upgraded their technology while others were about to do it. Their reaction to the more uncertain environment is also going to be heterogeneous. However, it is easy to show that this heterogeneity only lasts for one adoption phase. After that, all firms will adopt technologies with the periodicity and flexibility of the high uncertainty steady state (i.e. $\Delta\bar{T}_h$ and $\bar{f}_h$, respectively).[23] Equation (17) defines a function between the age of installed capital at $\hat{t}$ and the length of the diffusion lag $\Delta T_{\hat{t}}$:

$$\pi e^{-\lambda \Delta T_{\hat{t}}} - a(\bar{f}_l) e^{\bar{\sigma}^2 (T_{\hat{t}} - \hat{t}) + \sigma^2 (\hat{t} - T_{\hat{t}-1})} - c_m(\bar{f}_l) = \pi e^{-\lambda \Delta \bar{T}_h} - a(\bar{f}_h) e^{\bar{\sigma}^2 \Delta \bar{T}_h} - c_m(\bar{f}_h). \quad (17)$$

Figure 9 plots the distribution of diffusion lags for the transition period for a particular parameterization described in appendix A.2.

From equation (15), firms that upgraded their capital just before $\hat{t}$ (i.e. $T_{\hat{t}-1} = \hat{t}$) will renew it in less than $\Delta\bar{T}_h$ periods because installed capital is too inflexible for the new environment. Therefore, uncertainty increases their net marginal cost of delay. Firms

but a constant demand of final output, because the amount of final output devoted to rent capital is constant. Therefore, unless agents have linear preferences, the price of final output should increase during this period.

[23] To see this, note that if $\Delta T_{\hat{t}+1}$ is equal to $\Delta\bar{T}_h$ and $f_{\hat{t}}$ equals $\bar{f}_h$ then equation (15) holds if condition (17) holds. Note further that $f_i = \bar{f}_h$ and $\Delta T_{i+1} = \Delta\bar{T}_h$ is by definition of steady state consistent with equations (7) and (16) for all $i \geq \hat{t}$. Therefore the only binding condition is equation (17). But this will always hold by selecting the appropriate adoption time $T_{\hat{t}}$.

that were about to upgrade at $\hat{t}$ (i.e. $T_{\hat{t}-1} = \hat{t} - \Delta T_l$) will delay adoption because the expected distance in the next phase increases with uncertainty, and so does the marginal value of delay. The adoption behavior of the other firms falls in between these two extremes. As a result, the distribution of adoption times is compressed.

The evolution of aggregate TFP is presented in figure 10. The delay in upgrading at $\hat{t}$ generates an interval at which no firm adopts new equipment. This reduces the level of TFP because gross output remains constant while the costs of operating the equipment ($C(t; d; f)$) increase over time. When firms start to adopt new flexible capital the rate of decline of net output decreases. Depending on the relative magnitude of the rate of technological progress and the increase in uncertainty, TFP may continue to fall (although at a slower rate) or may bounce back. This diversity of possibilities arises because there are two forces operating in opposite directions. On the one hand, firms are starting to upgrade their technologies and renewing their templates. On the other, for those firms that have not upgraded since $\hat{t}$; the costs of operating equipment are increasing because of the more rapidly changing environment. When this second force dominates, TFP will continue to decline for a while. Eventually, a sufficiently large number of firms has upgraded the capital stock and make it more flexible. At this point, aggregate TFP starts to grow.

After showing that the increase in uncertainty generates a decline in the rate of TFP growth it is natural to wonder whether it can explain the short-run acceleration of the Solow-Nelson rate of embodied TFP growth. This is studied next. In section 4.2, I show that in the cross-section, investment in computers helped alleviate the slowdown during the 70's. The last goal of this subsection is to show that the uncertainty-driven theory can explain this fact very naturally.

Since in the production function used in the model there is no spillover across the productivities of different capital vintages, the rate of embodied TFP growth is approximately proportional to the relative productivity of new vs. old capital. For illustrative purposes, I approximate the productivity of new capital by the average level of TFP of the 25 percent of the firms that have adopted new technology most recently. Analogously, the productivity of old capital is calculated by the average over the 25 percent of the firms that adopted new equipment least recently. These times series are plotted in figure 11.

At the time of the increase in uncertainty, the relative level of TFP of new vs. old capital rises.[24] As some ¯rms start to upgrade their capital stocks and make them more °exible their productivity increases relative to those who have not upgraded yet. This increase in the share of °exible capital explains the acceleration of embodied TFP growth along the transition. Note that if the increase in uncertainty is su±ciently large, at this stage the growth rate of TFP growth will still be smaller than in the low uncertainty environment. Therefore, the model explains the simultaneous decline in disembodied TFP growth and acceleration of embodied found by McHugh and Lane [1987] and Hobijn [1999].

Production processes di®er in the elasticity of the costs of operating capital with respect to °exibility and in the degree of development of the applications of the °exible technologies for the di®erent production processes. Both of these factors generate di®erences in the intensity of investment in °exible technologies. To analyze the e®ect of °exible technologies on the magnitude of the slowdown, consider two sectors one that is unable to increase the °exibility of the manufacturing system and another that can by adopting more °exible capital. Figure 12 illustrates the evolution of TFP growth for both sectors.

Being able to implement °exible equipment helps the ¯rm to reduce the higher costs of operating machines associated with a more uncertain environment. Therefore, according to the model, the implementation of °exible reduces the size of the slowdown.

The next subsection extends the steady state analysis to a multisectorial framework to explain the spectacular performance of the information technologies (IT) producing sectors.

3.5 A miracle in Silicon Valley

Robert Gordon [1999] has pointed the economists' attention towards the spectacular productivity growth rates achieved by the IT producing sectors.[25] Between 1973 and 1991 the average annual TFP growth rate in non-electrical machinery was 3.7 percent, while between 1948 and 1973 had been only 0.8 percent. The ¯gures for total man-

[24] This is an uninteresting e®ect that appears because the cost of operating the equipment in a changing environment are convex in the distance d. Therefore when no ¯rm adopts, the costs are going to increase at the same rate for all the ¯rms, but the average over the ¯rms with higher costs (those that adopted least recently) grows faster than the average over the ¯rms that adopted most recently.

[25] Most IT's are produced in the sectors classi¯ed under sic 35, that is non-electrical machinery.

ufacturing were 2.2 percent between 1948 and 1973 and 1.3 between 1973 and 1991. The comparison is even more spectacular when we focus only on computers. Gordon reports that the annual growth rate of output per hour in the production of computers between 1972 and 1995 was about 20 percent while for total manufacturing it was 2.58 percent. In this subsection I extend the model presented above to argue that an increase in uncertainty may explain this fact.

I introduce three modifications in the model. First, instead of a unique final good, there are M: Consumers have symmetric Cobb-Douglas preferences defined over the M goods. The share of income allocated to each good is therefore $\frac{1}{M}$: Second, instead of a continuum of possible levels of flexibility, now there are only two ways to organize production. With the flexible manufacturing system the degree of flexibility is f_f while with the non-flexible it is f_{nf} (smaller than f_f). The third and most important variation is that technological progress capital is specific to the flexibility of capital. This means that innovations that enhance the productivity of the flexible capital cannot be adapted to the non-flexible and viceversa.[26] One implication is that the rates of technological progress in flexible (λ_f) and inflexible (λ_{nf}) production systems will generally differ.

Let's normalize every instant the value of nominal GDP to 1. From the Cobb-Douglas assumption, this implies that nominal sectorial GDP ($p_i y_i$) is equal to $\frac{1}{M}$: It follows immediately that the growth rate of the price at the i^{th} sector (γ_{p_i}) is equal to minus the rate of technological progress at the i^{th} sector. Since technology is common to all the sectors using a given manufacturing system, γ_{p_i} is equal to $-\lambda_f$ or to $-\lambda_{nf}$ depending on whether the i^{th} sector uses flexible or inflexible capital.

The growth rate of the general price level (γ_P) is an average of the growth rate of the prices of the M products. Let sh_f denote the fraction of sectors using flexible capital. Then γ_P is equal to:

$$\begin{aligned} \gamma_P &= sh_f \, \gamma_{p_f} + (1 - sh_f)\, \gamma_{p_{nf}} \\ &= -\lambda_{nf} - sh_f \, (\lambda_f - \lambda_{nf}) \end{aligned}$$

The growth rate of the relative price of flexible ($\gamma_{\tilde{p}_f}$) and inflexible ($\gamma_{\tilde{p}_{nf}}$) capital goods are therefore equal to:

$$\gamma_{\tilde{p}_f} = (1 - sh_f)\,(\lambda_{nf} - \lambda_f)$$

[26] This is the only critical assumption in this section. The other two are just simplifying assumptions.

$$\gamma_{p_{nf}} = sh_f(\lambda_f - \lambda_{nf})$$

In terms of the problem of the firm, the only difference that these modifications introduce is that now there will be a trend in the relative price of the final good. This affects the value of output over time. In particular, a positive trend in the relative price of a product is equivalent to a reduction in the discount rate. Equation A_i, for $i = f$ and nf, expresses the first order condition associated with the selection of diffusion lags.

$$[\pi - a(f_i)] - \left[\pi e^{-\lambda_i \Delta\tilde{T}^{1}} - a(f_i)e^{\sigma^2 \Delta\tilde{T}^{1}}\right] = \frac{\lambda_i \pi}{\rho - \gamma_{p_i}}\left(1 - e^{-(\rho-\gamma_{p_i})\Delta\tilde{T}^{1}}\right) + a(f_i)\sigma^2 \frac{e^{(\sigma^2-\rho+\gamma_{p_i}+\lambda_i)\Delta\tilde{T}^{1}} - 1}{\sigma^2 - \rho + \gamma_{p_i} + \lambda_i} + A_c(\rho - \gamma_{p_i} - \lambda_i) \qquad (A_i)$$

As in the one sector model, equation A_i defines a downward sloping locus in the $\Delta\tilde{T}^{1} - \lambda_i$ space. An interesting property of the A_i loci is that, for any given level of technological progress, the cost of delay is higher when firms use more inflexible equipment. In graphical terms, this means that the A_f locus is located to the right of the A_{nf} locus (see figure 13).

Production processes differ in how much flexibility is achieved by installing flexible capital. In some sectors the cost of having an inappropriate capital (i.e. one that is not designed for producing the demanded product) can be reduced substantially with flexible capital while in others it may be quite insensitive to the flexibility of capital. As shown in the one sector model, the value of flexibility increases with uncertainty. This makes firms more willing to adopt flexible capital in a more uncertain environment. Therefore, the fraction of sectors using flexible capital is increasing in the level of uncertainty (i.e. $sh_f(\underset{+}{\sigma^2})$).

In this capital-specific world there are two R&D sectors; one that improves the productivity of flexible capital which employs a measure n_f of innovators and another where n_{nf} innovators work to enhance the productivity of non-flexible capital. The return to innovation activities is equalized in equilibrium to the value of managing a firm.

$$\tilde{\pi}(\underset{-}{\sigma^2}; \underset{-}{A_c}) = \lambda'(\underset{-}{n_f})\, sh_f(\underset{+}{\sigma^2})\, (L - (n_f + n_{nf}))\left(\frac{1-\alpha}{\alpha}\right) c\chi(\underset{-}{\Delta\tilde{T}^{1}_{nf}}) \qquad (L_f)$$

$$\tilde{\pi}(\underset{-}{\sigma^2}; \underset{-}{A_c}) = \lambda'(\underset{-}{n_{nf}})\, (1 - sh_f(\underset{+}{\sigma^2}))\, (1 - (n_f + n_{nf}))\left(\frac{1-\alpha}{\alpha}\right) c\chi(\underset{-}{\Delta\tilde{T}^{1}_{f}}) \qquad (L_{nf})$$

Equation L_i de¯nes a negative relationship between $4\hat{T}_i$ and n_i; for i 2 f f ; n_f g: Equations A_f ; A_{nf} ; L_f and L_{nf} characterize the equilibrium speeds of di®usion and rates of technological progress for both types of technology. Proposition 3 describes the consequences of an increase in uncertainty from $¾^2$ to $\hat{¾}^2$.

Proposition 3 There exists a positive number $\bar{sh}_f$ such that if $sh_f(\hat{¾}^2) > \bar{sh}_f > sh_f(¾^2)$; $¸_f$ will be higher than $¸_{nf}$.

Proof. : See appendix A.1.

The intuition for this proposition is represented in ¯gure 14. In a more uncertain world, ¯rms want to adopt more often new equipment because installed equipment becomes inappropriate faster. Capital becomes more obsolete the more in°exible it is. In graphical terms this translates into a shift of the A_i loci to the left, but the shift of the A_{nf} locus is larger. Uncertainty also reduces the value of a ¯nal output ¯rm and therefore the opportunity cost of innovation. This second e®ect shifts to the right both L_i loci. In a world with speci¯c technological progress, an increase in uncertainty has a third e®ect of the equilibrium. More speci¯cally, some ¯rms that in the low uncertainty environment used in°exible capital now are going to implement °exible production systems. This increase in the scale of the market for °exible capital enhances the return to innovation in °exible equipment and reduces the value of innovations in in°exible. Graphically, this third e®ect implies a shift to the right the L_f curve while the L_{nf} shifts to the left.

If the increase in the scale of the market for °exible equipment is su±ciently large the rate of technological progress embodied in °exible capital ($¸_f$) will be higher than the rate of technological progress embodied in in°exible ($¸_{nf}$). In quality adjusted data sets like Gordon's these improvements in e±ciency of capital show up in the productivity of the technology-producer sectors like O±ce Computing and Accounting machines (OCAM) where most IT's are manufactured. Therefore the increase in uncertainty can potentially explain the spectacular TFP growth rate observed by Gordon and others in the sectors that produce computers and in general IT's.

Next I turn to the empirics of the uncertainty-driven approach.

4 Testing the role of uncertainty and computers on the slowdown

This section has two goals. First, it provides evidence of an increase in the uncertainty of the business environment. Second, it tests several of the implications of the model presented above. In particular, the effect of computers and uncertainty on the slowdown and the impact of uncertainty on the speed of diffusion of computers.

4.1 Evidence on the increase in uncertainty

The cause of the productivity slowdown in the uncertainty-driven theory is the increase in the uncertainty of the business environment. In the model presented in section 3, I have used, for simplicity, a particular source of uncertainty, namely, the characteristics of the product demanded by the firms. However, uncertainty can take many forms. Firms may be uncertain about the future evolution of variables such as input and output prices, market structure, technologies available, degree of liquidity, relative demand of each of the products, trade tariffs, ... It is important to stress that the model could be rewritten using these other sources of uncertainty and the predictions would be unchanged. What is crucial for the main results is that 1. the efficiency of the firm is tied to some changing attributes about the state of the world and 2. on average, as time goes by and the managers do not respond to these changes in the state of the world, the efficiency of the firm declines. The first property yields the slowdown when uncertainty increases while the second is sufficient for the acceleration of the adoption frequencies.

In principle, there are two ways to illustrate the increase in uncertainty. One is to identify and measure the specific sources of uncertainty. This strategy is partially pursued at the end of this subsection. An alternative approach consists in tracking the evolution of general measures of the uncertainty faced by the firms. Next I propose three of these general measures.

As Leahy and Whited [1996] argue, one proxy for the uncertainty faced by the firms is the volatility of the (monthly) stock returns. Asset returns should capture the effects of any aspect of a firms's environment that investors deem important.[27] The data on stock returns for all the stocks in the NYSE, AMEX and NASDAQ comes from the CRSP data set. For each individual stock and decade I compute the standard deviation of monthly returns, and then the average over all the stocks for each decade. Column 1 of table 2 illustrates an important increase in the average volatility of individual stock returns which could be dated somewhere between the mid 60's and the mid 70's. In manufacturing, this is mostly a within 4-digit sector phenomenon since 84% of the sectors (representing around 90% of the manufacturing value added) experienced the increase in uncertainty between the 1960's and the 70's. As shown in the rest of the columns this finding is robust to many variations.

Column 2 only considers those stocks with more than two years of data. Column 3 computes the median of the standard deviation of individual stock returns. Column 4 computes the average across stocks of the standard deviation of the deviations from a stock and decade specific time trend. Column 5 computes the average standard deviation of yearly individual stock returns. This measure is probably more immune to fads, bubbles and other non-fundamentals sources of return variability. Reassuringly, its pattern is the same as in the other columns, therefore, one can conclude that the measured increase in the volatility of asset returns mostly reflects an increase in the uncertainty of fundamentals.[28]

Another concern that might be raised is that the increase in measured uncertainty might be due to an increase in the share of small (more volatile) firms in the US stock markets. To control for this composition effect columns 6 and 7 compute the average standard deviation of individual stock returns for the firms in sample in the 50's and 60's respectively. Note that this approach could a priori bias the results against the increase in volatility because of a selection effect. Finally, column 8 reports standard deviations of the individual stock returns weighted by the share in total capitalization over the decade.

[27] One caveat is that movements in asset returns may be quite noisy, reflecting not only changes in fundamentals, but also bubbles, fads and the influence of noise traders. However, to the extent that what matters for the theory is not the level of uncertainty but its evolution over time this should not be an important concern.

[28] One could also argue that the increase in short term volatility is due to the faster trading methods available since the 1970's. However, the increase in volatility is robust to the length of the periods over which the returns are computed and this limits very much any potential concern about changes in the trading technology.

A second general measure of uncertainty can be obtained by studying the volatility of the return to other factors of production, for example labor. Gottschalk and Mo±t [1994] compute the standard deviation of the real wage rates for all the individuals in the PSID during the 70's and 80's.[29] They observe a 35 percent increase in the variance of the logarithm of the weekly wage rate. The increased volatility is observed for all education, experience and income levels both for workers that did and did not change jobs. From a sectorial point of view, the higher volatility is mostly (88%) due to an increase in the within 2-digit sectors wage rate volatility. Indeed, the average volatility of real wage rates increases in all 2-digit sectors. Gottschalk and Mo±t [1994] also report an important increase in the average volatility of hours worked which is also robust to the same partitions of the data.

The third measure of uncertainty comes from the literature on job creation and destruction. The excess job reallocation rate measures the degree of simultaneous job creation and destruction in a given industry. In the light of the canonical model of Mortensen and Pissarides [1994], the excess job reallocation rate is a proxy for the variance of the value of jobs. The data for excess job reallocation at the 2-digit level for manufacturing comes from Davis, Haltiwanger and Schuh [1997] and covers the period between 1973 and 1993. Table 3 reports the existence of a strong positive trend in excess job reallocation when the left hand side includes net employment growth (to control for its cyclical variability) and sector speci¯c dummies. This upward trend is robust to di®erent weighting schemes.

Once illustrated the increase in the uncertainty of the business environment, the follow up question concerns the forces that drive it. These may be the higher volatility of energy prices, the movement to a °oating exchange-rate system (and the associated volatility of relative prices), or the e®ect of globalization on the volatility of demand (both within and across varieties) and on the market structure. To assess the relative importance of these and other potential causes goes beyond the purposes of this paper, however, I conduct a preliminary exploration at two levels. First, I try to explain the volatility of stock returns at the 4-digit manufacturing sectors with some measures of openness and of the intensity of adoption of new technologies. Second, I look directly at the volatility of intermediate input and output prices to show that they become more volatile in the 70's.

[29] The exact periods are 1970-78 and 1979-87.

Table 4 reports the contemporaneous correlation between several measures of openness and investment in computers and stock market volatility at the four digit manufacturing level. The degree of openness is measured by the degree of import penetration,[30] the ratio of exports to shipments and the sum of the two. The first three columns of table 4 show that the increment in the three variables is positively (and significantly) correlated with the increment in the average volatility of individual stock returns at the 4-digit manufacturing level. Nevertheless, the correlation between the increment in volatility and the increment in computer share is weak and not statistically significant.

In table 5 and figure 14, I study the 2-digit sectorial volatility in manufacturing of the prices of energy, output and materials deflated by some aggregate producers price index for the whole economy. It is evident that in the 70's there was an important (and quite permanent) increase in the volatility of manufacturing output and energy prices.

4.2 Contrasting the two theories

What was the effect of computers on the TFP slowdown? Did they cause it as suggested by the GPT approach, or did they ease it, as predicted by the uncertainty-driven theory? I answer these questions by testing the cross-sectional implications of the two approaches.

4.2.1 Did computers cause the Slowdown?

According to the GPT approach, the slowdown in the 70's is the consequence of the costs associated with the adoption of computers. It follows almost immediately that those sec-

[30] This equals value of imports divided by the sum of value of shipments and value of imports.

tors which invest more intensively in computers should experience larger slowdowns.[31,32]

The uncertainty-driven approach predicts that the sectors with the flexible technologies were developed , by using two proxies for the intensity of implementation of the computer technologies. The first is the Berman, Bound and Griliches [1994] share of computers in total investment (BBG, henceforth) and the second is the share of expenditures in Office, Computing and Accounting Machines (OCAM) in total investment (BEA [1975], [1985] and BEA web site).

Suppose that the logarithm of TFP at the i^{th} sector (tfp_{it}) follows the law of motion

$$tfp_{it} - tfp_{it-1} = \gamma_1 + \gamma_2 csh_{it} + \varepsilon_{it}; \tag{18}$$

where csh_{it} represents the share of computer expenditures in total investment at the i^{th} sector and ε_{it} is an iid error term. Since I only have measures of csh for some years I can aggregate (18) in the following way:

$$tfp_{it_j} - tfp_{it_{j-1}} = \gamma_1 + \gamma_2 \widehat{csh}_{it_j} + v_{it_j}; \tag{19}$$

where $\widehat{csh}_{it_j}$ denotes the average share of investment in computers at the i^{th} sector between the periods t_{j-1} and t_j; $tfp_{it_j} - tfp_{it_{j-1}}$ is the average growth rate of TFP in

[31] To see this, let $B_i(csh_i) = b_i\, csh_i$ denote the benefit derived, at sector i, from a share of investment in computers csh_i. This benefit takes place in the 80's. Let $C_i(csh_i) = C_{0i} + c_{1i}(csh_i)^{\gamma}$ ($\gamma > 1$) be the cost associated with the same investment intensity in the 70's. A representative unit facing this cost will devote a share $csh_i = \left(\frac{\beta b_i}{c_{1i}\gamma}\right)^{\frac{1}{\gamma-1}}$ of total investment to computers, where β is the discount factor. Therefore, the cost of adopting the new technologies is $C_i = C_{0i} + b_i^{\frac{1}{\gamma-1}}\left(\frac{\beta}{c_{1i}\gamma}\right)^{\frac{\gamma}{\gamma-1}}$. In particular C_i is higher in those sectors with higher csh_i.

In order to reach this conclusion I have assumed that C_{0i} is uncorrelated with csh_i. If the initial cost of investing in computers was negatively correlated with the variables that determine the intensity of investment the conclusion reached above would not be so unambiguous. Is this an important concern?

Not really. One may think of C_{0i} as planning costs. I show below that there is no evidence on the effect of planning costs on the slowdown. Second, the costs of using computers for certain basic tasks like accounting, inventory management or building an intranet are probably very similar accross sectors therefore C_{0i} are not only small but also uncorrelated with the variables that determine csh_i.

[32] Several authors have tested similar relationships. Morrison [1997] finds that the rate of return to office and information technology capital in the 70's was very large, by the end of the 80's this had decline and in the 90's it was large again. She uses 2-digit manufacturing data and her definition of office and information technology equipment is more inclusive than mine because she also considers investment in communications equipment and scientific and engineering instruments. Chun [2000] pools data from 47 2-digit industries both in services and manufacturing from 1960 to 1997 and finds an inverted U-shaped relationship between the share of OCAM capital in total capital and the log of TFP. This is consistent with Morrison [1997] and with my findings if, as it is the case, the share of OCAM capital was relatively low in the 70's.

this interval and v_{it_j} is an iid error term. In the first two columns of table 6, I estimate this regression in a cross-section of the 4-digit manufacturing sectors[33] with $t_j = 1980$ and $t_{j-1} = 1973$ and $\widehat{csh}_{it_j}$ approximated by the computer share in 1977 as measured by BBG and the BEA. The results for the proxies are very similar and, in both, computers seem to have a very strong and highly significant positive effect on TFP growth.

There are several potential problems that this exercise may suffer and that prevent me to interpret the relation between computers and TFP as causal at this stage. The first is that the share of computers expenditures over total investment might be endogenous. This concern is clearly much less important than if the regressor used was the total investment in computers. Yet, it still might be the case that because computers were a relatively risky investment in the 70's, those sectors with more liquidity (and probably also higher TFP growth rates) were more prone to undertake these investments. At first glance, one trouble that this objection encounters is that if investing in computers was relatively risky, there should be a positive correlation between computer share and the sectorial volatility of the stock returns. However, above I have shown that this correlation is insignificant.

To make sure that the effect of TFP growth on the composition of investment is not driving the coefficients in the first two columns of table 6, I instrument the computer share in 1977 with the computer share in 1967 (columns 3 and 4) and with the average sectorial standard deviation of the stock returns in the 60's (columns 5 and 6). Past investment shares in computers are a priori a good instrument because the degree of liquidity in 1977 should not be affected by the share of investment in computers in 1967 provided the very small share that these represented.[34] The volatility of stock returns in the 60's is also a good instrument because 1. as the model predicts and we shall see in the next subsection, uncertainty accelerates the speed of diffusion of computers and 2. as we will see below, for this measure of uncertainty and TFP-growth are uncorrelated.

From columns 3 to 5 it is clear that the positive effect that computers had on TFP growth in the 70's is very robust to the instrumentation and therefore the share of computers in investment is not endogenous.

The second econometric problem that may face regression (18) is that the error structure

[33] The data on TFP comes from the NBER manufacturing database (Bartelsman and Gray [1996]).

[34] For OCAM, which is a larger set, the average share in total investment in 1967 was about 5 percent.

may be missespecified. If $\epsilon_{it} = \gamma_i + \varepsilon_{it}$; where γ_i is a sector-specific component of TFP growth, the estimated effect of computers on TFP growth might be inconsistent. To see this suppose that those sectors that on average have higher TFP growth rates (i.e. high γ_i) are those that invest more intensively in computers (i.e. $cov(\widehat{csh}_i; \gamma_i) = c_\circ > 0$). Then the $p \lim \hat{\gamma}_2 = \gamma_2 + c_\circ$ which can be positive even with γ_2 negative: To get rid of the fixed effects term I differentiate (19).[35] In column 7 of table 6 we can see that, after correcting for the fixed effects, computers still have a major impact on TFP growth. Therefore I conclude that the increase in computer usage reduced the productivity slowdown in the 70's. It is hard to reconcile this result with the implementation-based GPT models of the productivity slowdown.

The proponents of the GPT theory might still argue that the relevant costs in the implementation of computers are the costs of planning how to adapt the production processes to the new technologies. If this is the case, the current share of investment in computers might be a poor proxy for the intensity of the current planning costs. These however, should be positively correlated with the share of investment in computers in the future. In column 8 of table 6 I show that the sectors that increased by more the investment share of computers in the 80's did not experienced larger declines in TFP growth in the 70's. Therefore, the cost of planning the investment in computers did not cause the slowdown.

4.2.2 Did uncertainty cause the slowdown?

The uncertainty-driven approach argues that an increase in the uncertainty faced by the firms caused the slowdown. Moreover, as shown in section 3.4, those sectors where the flexible technologies were more available should use them to dampen the negative effect of uncertainty.

To test these predictions I use the following specification

$$\begin{aligned} gtfp_{i70} - gtfp_{i60} = {} & \gamma_1(u_{i70} - u_{i60}) + \\ & \gamma_2(csh_{i77} - csh_{i67}) + v_{i70} \end{aligned} \quad (20)$$

[35] That means that the dependent variable is the difference between the average TFP growth rate during 1973-80 and 1958-73, and the independent variable is the difference between the conputer share in investment in 1977 and 1967.

which takes care of sector-specific fixed effects in the growth rates. $gtfp_{ix}$ is the average growth rate of TFP in sector i during decade x; u_{ix} is the sectorial uncertainty in decade x, csh_{ix} is the share of investment in computers in year x, and v_{i70} is an iid error term.

In the estimation of equation (20) I use three different measures of uncertainty all of them at the 4-digit aggregation level. These are the average standard deviation of stock returns, the standard deviation of the growth rate of shipments and the standard deviation of tfp growth. The intensity of investment on flexible technologies is proxied by the share of investment in OCAM over total investment as measured by the BEA.

The first three columns of table 7 report the basic results. First, computers have an strong and positive effect on TFP-growth during the slowdown. This means that, as predicted by the uncertainty-driven approach, they alleviated the slowdown. Second, for two out of the three measures of uncertainty, there is a strong negative effect of uncertainty on TFP-growth during the slowdown.

One might be concerned that sectors with low TFP-growth rates are more volatile (i.e. uncertainty is endogenous) and this generates the negative coefficient in columns 2 and 3 in table 7. Yet, the relationship between TFP growth and the two measures of uncertainty used in these columns is positive as reflected in columns 4 and 5, where I estimate this relationship pooling the data for the three decades (60's, 70's and 80's).[36] Hence it was the increase in uncertainty what generated the slowdown and not the other way around.

Finally, the uncertainty-driven theory also predicts that the negative effect of uncertainty on TFP-growth will be milder in the 80's and 90's because by then many firms will have adopted the flexible technologies. This result is confirmed empirically in columns 6 and 7 of table 7.

[36] The same results are obtained by estimating the relationship for each individual decade.

4.3 Does Uncertainty accelerate diffusion?

According to the uncertainty-driven theory, uncertainty accelerates the speed of diffusion of new technologies (proposition 1).[37] To test this prediction, I build a panel at the four digit manufacturing level. The typical regression that I run takes the form:

$$csh_{it} = \alpha_0 + \alpha_1 sd_{it-1;t-10} + \varepsilon_{it}$$

where the dependent variable is the share of computers on total investment and measures the speed of diffusion of technologies,[38] while $sd_{it-1;t-10}$ is the average standard deviation of the stock returns in the previous decade and measures uncertainty. The use of this lagged effect is very natural provided that the model developed above generates slow diffusion of technologies. Moreover, it is a way of avoiding the potential endogeneity of uncertainty, although above it has been shown that, contemporaneously, the increase in uncertainty and in computer share are very weakly correlated.

The regressions reported in table 8 differ in the assumptions made about the error term ε_{it}: In the first two columns, I use a random effects estimator to show that uncertainty accelerates the speed of diffusion of computers both for the BEA and the BBG measures.

In the model presented in section 3, technology diffuses linearly, however there is a large body of evidence showing that the diffusion of technologies follows S-curves (Griliches [1957], Davies [1979]). In this case, the speed of diffusion should depend of the date at which technology is available and on the ceiling of the diffusion process. Since in the first two columns I am using cross-sectional variation, the positive impact of uncertainty of the speed of diffusion might be caused by the fact that high uncertainty sectors started adopting computers earlier or have a higher ceiling (Griliches [1957]).

In columns 3 and 4, I use only time series variation by introducing sector specific dummies. This should take care of cross-sectional differences in availability dates and ceilings. As predicted in section 3.3, uncertainty still has a strong and significant effect on the speed of diffusion of computers. Therefore, the acceleration associated with uncertainty goes beyond the normal acceleration that would be observed due to the S-curve pattern.

[37] In a recent paper, Rich and Tracy [1999] show that uncertainty reduces the duration of employment contracts.

[38] Note that, if the level of diffusion of a technology z (X_z) is defined as the capital stock of z (K_z) adjusted by the size of the sector (S), the speed of diffusion $v_z = \frac{d}{dt}\left(\frac{K_z}{S}\right)$ is equal to $v_z = \frac{I_z}{I}$; when S is measured by the level of investment at the sector (I) and this does not vary too much.

5 Concluding remarks

This paper has proposed an explanation for the evolution of TFP growth in manufacturing. According to the uncertainty-driven approach, during the 70's the business environment became more uncertain. The existing capital was too inflexible to cope with a rapidly changing environment and its productivity declined generating a productivity slowdown. Firms reacted to the increase in uncertainty by investing in new flexible capital that could be adapted to the new environments. The higher adaptability made possible that new capital retained its productivity. As a result, the rate of embodied productivity growth accelerated.

I have also shown that the slowdown was not caused by the arrival of a new GPT. Both in the cross-section and in the time series, the elasticity of TFP with respect to investment in new equipment was specially high in the 70's. I have also noticed that it is very hard to rationalize these facts with a model that emphasizes the costs of implementing the new technologies as the cause of the slowdown.

There is one scenario where the new GPT could have caused the slowdown and still be consistent with this evidence. In this scenario is one where the relevant cost is not an implementation cost but a development cost as in Helpman and Trajtenberg [1998]. If this is the case, the 70's should have witnessed a very large decline in TFP growth in the sectors that develop the new GPT and its secondary innovations. These are mainly localized in sic 35 (non-electrical machinery). However, as reported in section 3.5, this sector has experienced almost miraculous growth rates of TFP and productivity since the 70's.

Computers (and more generally IT's and PA machinery) are a crucial element in my story. In this sense, this paper is not a criticism to the GPT literature in general but only to the explanations of the slowdown based on the costs of implementing the GPT. In this broader literature, this paper provides an explanation for the rapid development and diffusion of the new GPT. One in which the development of the new GPT is endogenous. This view argues that the return from developing and adopting the PA's and information technologies increased as uncertainty raised. The empirical section has supported this hypothesis.

The increase in the uncertainty of the business environment has other implications that I plan to pursue in future research. Some of them are quite straightforward. The last two decades have witnessed unprecedented rates of return in the stock market that followed a very poor performance in the 70's. The latter can naturally be accounted by the e®ect that uncertainty had on the productivity of old capital while the former may be a consequence of the acceleration in the speed of di®usion of technologies and the associated acceleration of embodied TFP growth.

Katz and Murphy [1992] have shown that the experience premium increased dramatically during the 70's and specially in the 80's. If more experienced workers have longer information histories they will be able to cope better with a rapidly changing environment. That is particularly true if the relevant information is idiosyncratic and it is not di±cult to codify. Interestingly, the volatility of individual stock returns increased during the 80's relative to the volatility of aggregate stock returns like the S&P500 or the whole stock market.

The increased in idiosyncratic uncertainty may have implications for the design of the ¯rms and the process of decision making. Management scientist have detected a trend towards less hierarchy and more °exible organizational forms. This move encompasses more autonomy and responsibility being awarded to workers and their performing a wider range of tasks (Caroli [1998]). Therefore, organizational change has led to more decentralization in work organization. With an increase in idiosyncratic uncertainty the information which is idiosyncratic to the client becomes more important. To the extent that information cannot be transmitted perfectly, in the rapidly changing environment it will be optimal to delegate certain decisions to the periphery of the organization. Therefore, the change in the composition of uncertainty may help explain the transformation of organizations.

Finally, to understand the evolution of aggregate TFP growth it is necessary to explain why productivity has been growing so slowly in the service sectors for the last twenty ¯ve years. The main problem with services is that it is di±cult to write down a conceptually accurate production function. Probably, some advance can be made by recognizing the importance of the information in the production of services. As uncertainty increases, information becomes less accurate and the value of some services may decline. This may help explain why the return from introducing improvements in the production of services has been so low since the 1970's.

References

[1] Aghion, P. and P. Howitt (1992), \A Model of Growth Trough Creative Destruction" Econometrica, 60,2 (March), 323-351.

[2] Aghion, P. and P. Howitt (1997), \A Schumpeterian Perspective on Growth and Competition" in Advances in economics and Econometrics: Theory and Applications,Seventh World Congress Vol. II Edited by D. Kreps and K. Wallis p. 279-318.

[3] Barro, R. and X. Sala-i-Mart¶in (1995), Economic Growth, McGraw Hill.

[4] Bartelsman, E. and W. Gray (1996), \The NBER Manufacturing Productivity Database" NBER Technical Paper: 205.

[5] Berman, E., J. Bound, and Z. Griliches (1994), \ Changes in the Demand for Skilled Labor within U.S. Manufacturing: Evidence from the Annual Survey of Manufactures" The Quarterly Journal of Economics Vol. 109 (2). May, 367-97.

[6] Bureau of Economic Analysis (various issues). Survey of Current Business United States Department of Commerce.

[7] Caroli E. (1998) \New Technologies, Organizational Change and the Skill Bias: Going Into the Black Triangle", forthcoming P. Petit and L. Soete eds: Employment and Economic Integration.

[8] Chari, V. and H. Hopenhayn (1991), \Vintage Human Capital, Growth, and the Di®usion of New Technology" Journal of Political Economy, Vol. 99, No. 6. (Dec.), pp. 1142-1165.

[9] Chun, H. (2000), \Can information Technology Explain Deceleration and Acceleration in Productivity Growth" NYU mimeo.

[10] Davies, S. (1979) The Di®usion of Process Innovations Cambridge: Cambridge University Press.

[11] Gordon, R. (1990), The Measurement of Durable Goods Prices Chicago University Press (for NBER).

[12] Gordon, R. (1999), \Has the \New Economy" Rendered the Productivity Slowdown Obsolete?" Northwestern University mimeo.

[13] Gottschalk P. and R. Mo±t (1994), \ The Growth of Earnings Instability in the U.S. Labor Market" Brookings Papers on Economic Activity. Vol. 0 (2). p 217-54.

[14] Greene, A. (1982), \Steelmaking Controls Give Quality, Throughput and Energy E±ciency" Iron Age March 1,p. MP-7.

[15] Greenwood, J. Z. Hercowitz, and P. Krusell (1997), \Long-Run Implications of investment-Speci¯c Technological Change", American Economic Review vol 87 No. 3 (June) pp. 342-362.

[16] Greenwood, J. and B. Jovanovic (1998), \Accounting for Growth", forthcoming in Studies in Income and Wealth: New Directions in Productivity Analysis. C. Hulten, ed, University of Chicago Press (for NBER).

[17] Greenwood, J. and M. Yorukoglu (1997), \1974" Carnegie-Rochester Conference Series on Public Policy. Vol. 46(0). p 49-95. June.

[18] Griliches, Z. (1957), \Hybrid Corn: An Exploration in the Economics of Technological Change" Econometrica Vol. 25(4) p. 501-522.

[19] Helpman, E. and Grossman G. (1991), Innovation and Growth in the Global Economy MIT Press.

[20] Helpman, E. and M. Trajtenberg (1998), \A Time to Sow and a Time to Reap: Growth Based on General Purpose Technologies" in General Purpose Technologies and Economic Growth MIT Press, ed. by E. Helpman.

[21] Hobijn, B. (1999), \Identifying Sources of Growth" mimeo NYU.

[22] Hornstein, A., and P. Krusell (1996), \Can Technology Improvements Cause Productivity Slowdowns?" NBER Macroeconomics Annual, Bernanke B. and J. Rotemberg, eds., Cambridge and London: MIT Press. p 209-59.

[23] Jorgenson, D. (1966), \The Embodiment Hypothesis" Journal of Political Economy, Vol. 74, No. 1. (Feb.), pp. 1-17.

[24] Jorgenson, D. and K. Stiroh (1999), \Information Technology and Growth" American Economic Review, Papers and Proceedings. vol. 89, No. 2, 109-115.

[25] Katz L. and K. Murphy (1992) \Changes in Relative Wages, 1963-1987: Supply and Demand Factors" The Quarterly Journal of Economics, Vol. 107, No. 1. (Feb.), pp. 35-78.

[26] Leahy, J. and T. Whited (1996), \The E®ect of Uncertainty on Investment: Some Stylized Facts" Journal of Money, Credit, and Banking Vol. 28, No. 1, (February) p. 64-82.

[27] McHugh R. and J. Lane (1987), \ The Role of Emdodied Technological Change in the Decline of Labor Productivity " Southern Economic Journal Vol. 53 (4), p. 915-24.

[28] Milgrom P. and J. Roberts (1990), \The Economics of Modern Manufacturing: Technology, Strategy, and Organization" American Economic Review Vol. 80 (3) June, p 511-28.

[29] Morrison, C. (1997) \Assessing the Productivity of Information Technology Equipment in U.S. Manufacturing Industries", Review of Economics and Statistics, 79(3) (August), pp. 471-81.

[30] Mortensen, D. and C. Pissarides (1994), \Job Creation and Job Destruction in the Theory of Unemployment" The Review of Economic Studies Vol. 61 (3). July, p 397-415.

[31] Nelson, R. (1964), \Aggregate Production Functions and Medium-Range Growth Projections" American Economic Review Vol. 54, 5 (September), p. 575-606.

[32] O±ce of Technology Assessment (1984), Computerized Manufacturing Automation. Employment, education and the workplace. Washington D.C.

[33] Piore, M. and C. Sabel (1984), The Second Industrial Divide: Possibilities for Prosperity New York, Basic Books.

[34] Rich, R. and J. Tracy (1999), \Uncertainty and Labor Contract Durations" mimeo Federal Reserve Bank of New York.

[35] Rosenberg, N. (1976), \On Technological Expectations" Economic Journal 86 (September): 523-35.

[36] Solow, R. (1959), \Investment and Technical Change" in Mathematical Methods in the Social Sciences, Stanford.

A Appendix

The appendix is divided in two parts. The first contains the proofs of the claims and propositions made in the text. The second describes the calibration of the model used for the computation of the transition path in section 3.4.

A.1 Proof of Propositions

This appendix contains the proofs of the claims and propositions made in the paper.

Claim 1: Let

$$Y(t) = \begin{cases} e^{\theta t}\left(\int_{v_0}^{t} e^{\lambda v} K(v;t)dv\right)^{\alpha} L(t)^{1-\alpha} & \text{for } t < \bar{t} \\ e^{\theta t}\left(\int_{v_0}^{\bar{t}} e^{\lambda v - \frac{\zeta_o}{\delta_o(t-\bar{t})+1}} K(v;t)dv + \int_{\bar{t}}^{t} e^{\hat{\lambda}(v-\bar{t})+\lambda\bar{t} - \frac{\zeta_n}{\delta_n(t-\bar{t})+1}} K(v;t)dv\right)^{\alpha} L(t)^{1-\alpha} & \text{for } t \geq \bar{t} \end{cases}$$

; then

$$Y(t) \simeq \begin{cases} e^{(\theta+\alpha\lambda)t - \alpha\lambda G_t} K(t)^{\alpha} L(t)^{1-\alpha} & \text{for } t < \bar{t} \\ e^{(\theta+\alpha\bar{\lambda}(t))t - \alpha\tilde{\lambda}(t)G_t} K(t)^{\alpha} L(t)^{1-\alpha} & \text{for } t \geq \bar{t} \end{cases};$$

with both $\bar{\lambda}(t)$ and $\tilde{\lambda}(t)$ smaller than λ for an interval after the arrival of the new GPT

Proof. : Following Nelson (1964), for $t < \bar{t}$;

$$\begin{aligned} \hat{K}(t) &= \int_{v_0}^{t} e^{\lambda v} K(v;t)dv \\ &= e^{\lambda t} \int_{v_0}^{t} e^{\lambda(v-t)} K(v;t)dv \\ &= e^{\lambda t} K(t) \int_{v_0}^{t} e^{\lambda(v-t)} \frac{K(v;t)}{K(t)} dv \\ &\simeq e^{\lambda t} K(t) \int_{v_0}^{t} (1 - \lambda(t-v)) \frac{K(v;t)}{K(t)} dv \\ &= e^{\lambda t} K(t)[1 - \lambda G_t] \\ &\simeq K(t) e^{\lambda(t - G_t)} \end{aligned}$$

For $t \geq \bar{t}$;

$$\hat{K}(t) \approx \int_{v_0}^{\bar{t}} e^{\lambda v - \frac{\zeta_o}{\delta_o(t-\bar{t})+1}} K(v;t)dv + \int_{\bar{t}}^{t} e^{\hat{\lambda}(v-\bar{t})+\lambda\bar{t} - \frac{\zeta_n}{\delta_n(t-\bar{t})+1}} K(v;t)dv$$

$$
\begin{aligned}
&= K(t)\left\{ e^{\hat\lambda_o(t)t}\int_{v_0}^{\tilde v} e^{\left(\lambda-\frac{\zeta_o}{v[\delta_o(t-\tilde v)+1]}\right)(v-t)}\frac{K(v;t)}{K_{<\tilde v}(t)}dv\frac{K_{<\tilde v}(t)}{K(t)}+\right.\\
&\left. e^{\hat\lambda_n(t)t}\int_{\tilde v}^{t} e^{\left(\hat\lambda-(\hat\lambda-\lambda)\frac{\tilde v}{v}-\frac{\zeta_n}{v[\delta_n(t-\tilde v)+1]}\right)(v-t)}\frac{K(v;t)}{K_{\geq\tilde v}(t)}dv\frac{K_{\geq\tilde v}(t)}{K(t)}\right\}\\
&= K(t)\{e^{\hat\lambda_o(t)t}\int_{v_0}^{\tilde v} e^{\xi_o(v;t)(v-t)}\frac{K(v;t)}{K_{<\tilde v}(t)}dv\frac{K_{<\tilde v}(t)}{K(t)}+\\
&e^{\hat\lambda_n(t)t}\int_{\tilde v}^{t} e^{\xi_n(v;t)(v-t)}\frac{K(v;t)}{K_{\geq\tilde v}(t)}dv\frac{K_{\geq\tilde v}(t)}{K(t)}
\end{aligned}
$$

where $K_{<\tilde v}(t) \equiv \int_{v_0}^{\tilde v} K(v;t)$ and $K_{\geq\tilde v}(t) \equiv \int_{\tilde v}^{t} K(v;t)$. For any given distribution of capital, $\hat\lambda_n(t)$ takes values in the interval $[\lambda - \frac{\zeta_n}{\tilde v[\delta_n(t-\tilde v)+1]}, \hat\lambda]$, while $\hat\lambda_o(t) \in [\lambda - \frac{\zeta_o}{v_0[\delta_o(t-\tilde v)+1]}, \lambda]$. At the arrival of the new GPT $\hat\lambda_n(t)$ and $\hat\lambda_o(t)$ are lower than λ for an interval of time (i.e. $\hat\lambda_n(t) < \lambda$, for $t \in [\tilde v, \bar t]$, and in this formulation, $\hat\lambda_o(t) < \lambda$ for $t \in (\tilde v, \infty)$). Since $(\hat\lambda - (\hat\lambda - \lambda)\frac{\tilde v}{v} - \frac{\zeta_i}{v[\delta_i(t-\tilde v)+1]})$ is increasing in v and t, for $i = o, n$, as time passes and more capital is accumulated in the more advanced vintages, $\hat\lambda_i(t)$ will increase. The functions ξ_i is defined as follows: $\xi_n(v;t) \equiv \hat\lambda - (\hat\lambda - \lambda)\frac{\tilde v}{v} - \frac{\zeta_n}{v[\delta_n(t-\tilde v)+1]}$, $\xi_o(v;t) \equiv \hat\lambda - \frac{\zeta_o}{v[\delta_o(t-\tilde v)+1]}$. Note that $\xi_n(v;t) < \lambda$ for all the $v < \tilde v + \frac{\zeta}{(\hat\lambda-\lambda)[\delta(t-\tilde v)+1]}$, and $\xi_o(v;t) < \lambda$ for all v.

It is convenient to introduce the following definitions $sh_{<\tilde v} \equiv \frac{K_{<\tilde v}}{K(t)}$, $sh_{\geq\tilde v} \equiv \frac{K_{\geq\tilde v}}{K(t)}$, $G_{<\tilde v} \equiv \int_{v_0}^{\tilde v}(v-t)\frac{K(v;t)}{K_{<\tilde v}}dv$ and $G_{\geq\tilde v} \equiv \int_{v_0}^{\tilde v}(v-t)\frac{K(v;t)}{K_{\geq\tilde v}}dv$. Using the same approximation as above, the effective capital stock can be expressed as:

$$
\begin{aligned}
\hat K(t) &\simeq K(t)sh_{<\tilde v}e^{\hat\lambda_o(t)t-\check\lambda_o(t)G_{<\tilde v}} + K(t)sh_{\geq\tilde v}e^{\hat\lambda_n(t)t-\check\lambda_n(t)G_{\geq\tilde v}}\\
&\simeq K(t)sh_{<\tilde v}(1+\hat\lambda_o(t)t-\check\lambda_o(t)G_{<\tilde v}) +\\
&\quad K(t)sh_{\geq\tilde v}(1+\hat\lambda_n(t)t-\check\lambda_n(t)G_{\geq\tilde v})\\
&= K(t)\{1+\bar\lambda(t)t-\tilde\lambda(t)G_t\}\\
&\simeq K(t)e^{\bar\lambda(t)t-\tilde\lambda(t)G_t}
\end{aligned}
$$

where for any fixed distribution of capital and for $i = n, o$, $\check\lambda_i(t)$ are increasing functions and $\check\lambda_i(t) < \lambda$ for an interval after the arrival of the new GPT. $\bar\lambda(t)$ and $\tilde\lambda(t)$ are defined in the following way: $\bar\lambda(t) \equiv sh_{<\tilde v}\hat\lambda_o(t) + sh_{\geq\tilde v}\hat\lambda_n(t)$, and $\tilde\lambda(t) \equiv \check\lambda_o(t)\left(\frac{sh_{<\tilde v}G_{<\tilde v}}{G_t}\right) + \check\lambda_n(t)\left(\frac{sh_{\geq\tilde v}G_{\geq\tilde v}}{G_t}\right)$, which implies that both $\bar\lambda(t)$ and $\tilde\lambda(t)$ are smaller than λ for an interval after the arrival of the new GPT.

Plugging back $\hat{K}(t)$ into $Y(t)$ we obtain the desired expression. ∎

Claim 2: The AA and the FF curves are upward sloping in the $(\Delta T^*, f^*)$ space.

Proof. : To show this just need to apply the Implicit Function Theorem to equations (A) and (F).

$$\begin{aligned}
\left.\frac{d\Delta T^*}{df^*}\right|_A &= \frac{-\frac{\partial A}{\partial f^*}}{\frac{\partial A}{\partial \Delta T^*}} \\
&= \frac{-a'(f^*)\sigma^2\left[\frac{e^{\sigma^2\Delta T^*}-1}{\sigma^2}-\frac{e^{(\sigma^2-\rho+\lambda)\Delta T^*}-1}{\sigma^2-\rho+\lambda}\right]}{\pi\lambda\left[e^{-\lambda\Delta T^*}-e^{-\rho\Delta T^*}\right]+a(f^*)\sigma^2 e^{\sigma^2\Delta T^*}\left[1-e^{-(\rho-\lambda)\Delta T^*}\right]} \\
&= \frac{-a'(f^*)\sigma^2\left[\int_0^{\Delta T^*} e^{\sigma^2 t}\left[1-e^{-(\rho-\lambda)t}\right]dt\right]}{>0} \\
&> 0
\end{aligned}$$

where the denominator is positive because $\lambda < \rho$; and the numerator is also positive because $a' < 0$:

$$\begin{aligned}
\left.\frac{d\Delta T^*}{df^*}\right|_F &= \frac{-\frac{\partial F}{\partial f^*}}{\frac{\partial F}{\partial \Delta T^*}} \\
&= \frac{a''(f^*)\frac{e^{(\sigma^2-\rho+\lambda)\Delta T^*}-1}{\sigma^2-\rho+\lambda}+c_m''(f^*)\frac{1-e^{-(\rho-\lambda)\Delta T^*}}{\rho-\lambda}}{-\left[a'(f^*)e^{(\sigma^2-\rho+\lambda)\Delta T^*}+c_m'(f^*)e^{-(\rho-\lambda)\Delta T^*}\right]}
\end{aligned}$$

The numerator in this expression is unambiguously positive because both a'' and c'' are positive. The denominator is also positive if $\frac{-a'(f^*)e^{\sigma^2\Delta T^*}}{c_m'(f^*)} > 1$. Using equation (F), this can be rewritten as

$$\begin{aligned}
1 &> \frac{\frac{e^{-\sigma^2\Delta T^*-\rho+\lambda}-e^{-(\rho-\lambda)\Delta T^*}}{\rho-\lambda-\sigma^2}}{\frac{1-e^{-(\rho-\lambda)\Delta T^*}}{\rho-\lambda}} \\
&= e^{-(\rho-\lambda)\Delta T^*}\frac{\frac{e^{(\rho-\sigma^2-\lambda)\Delta T^*}-1}{\rho-\lambda-\sigma^2}}{\frac{1-e^{-(\rho-\lambda)\Delta T^*}}{\rho-\lambda}} \\
&= \frac{\int_0^{\Delta T^*} e^{-(\rho-\lambda)(\Delta T^*-t)}e^{-\sigma^2 t}dt}{\int_0^{\Delta T^*} e^{-(\rho-\lambda)t}dt}
\end{aligned}$$

and since $\int_0^{\Delta T^*} e^{-(\rho-\lambda)(\Delta T^*-t)}dt = \int_0^{\Delta T^*} e^{-(\rho-\lambda)t}dt$ and $e^{-\sigma^2 t} < 1 \ \forall t > 0$; the condition necessary for the denominator to be positive holds and FF is upward sloping too.■

Proposition 1: There exist two positive numbers, $\hat{c}'_{m\sigma}$ and $-\hat{a}'_{\sigma}$; such that if $c'_m(.)$ is bounded below by $\hat{c}'_{m\sigma}$ and $-a'(.)$ is bounded above by $-\hat{a}'_{\sigma}$ an increase in uncertainty will increase the flexibility and the speed of diffusion. This restriction will be denoted as condition 1.

Proof. : Let's define the functions A and F as follows:

$$
\begin{aligned}
A(\Delta T^*; f^*; \sigma^2; \lambda) &= -\left[\pi - a(f^*)\right] + \left\langle \pi e^{-\lambda \Delta T^*} - a(f^*)e^{\sigma^2 \Delta T^*}\right\rangle + \frac{\lambda\pi}{\rho}\left(1 - e^{-\rho\Delta T^*}\right) + \\
&\quad a(f^*)\sigma^2 \frac{e^{(\sigma^2-\rho+\lambda)\Delta T^*} - 1}{\sigma^2 - \rho + \lambda} + A_c(\rho - \lambda) \\
F(\Delta T^*; f^*; \sigma^2; \lambda) &= -a'(f^*)\frac{e^{(\sigma^2-\rho+\lambda)\Delta T^*} - 1}{\sigma^2 - \rho + \lambda} - c'_m(f^*)\frac{1 - e^{-(\rho-\lambda)\Delta T^*}}{\rho - \lambda}
\end{aligned}
$$

A(.) represents the net marginal benefit from delay and F(.) represents the net marginal value of flexibility. Equations (A) and (F) can be written as:

$$
\begin{aligned}
A(\Delta T^*; f^*; \sigma^2; \lambda) &= 0 \\
F(\Delta T^*; f^*; \sigma^2; \lambda) &= 0
\end{aligned}
$$

For any given tuple $(\Delta T^*; \sigma^2; \lambda)$, $\frac{\partial F}{\partial f^*} < 0$; therefore equation F, implicitly defines a function of f^* in terms of $\Delta T^*; \sigma^2; \lambda$: Let's denote this function as $f^*_F(\Delta T^*; \sigma^2; \lambda)$: Plugging it back into A, we obtain:

$$
A(\Delta T^*; f^*_F(\Delta T^*; \sigma^2; \lambda); \sigma^2; \lambda) = 0
$$

Total differentiation of A leads to:

$$
\begin{aligned}
\frac{d\Delta T^*}{d\sigma^2} &= -\frac{\frac{\partial A}{\partial \sigma^2} + \frac{\partial A}{\partial f^*}\frac{\partial f^*_F}{\partial \sigma^2}}{\frac{\partial A}{\partial \Delta T^*} + \frac{\partial A}{\partial f^*}\frac{\partial f^*_F}{\partial \Delta T^*}} \\
&= \frac{\left.\frac{\partial f^*}{\partial \sigma^2}\right|_A - \left.\frac{\partial f^*}{\partial \sigma^2}\right|_F}{\left.\frac{\partial f^*}{\partial \Delta T^*}\right|_F - \left.\frac{\partial f^*}{\partial \Delta T^*}\right|_A}
\end{aligned}
$$

$$= \frac{MRS^{A}_{\bar{f};\sigma^2} - MRS^{F}_{\bar{f};\sigma^2}}{MRS^{F}_{\bar{f};\Delta\bar{T}} - MRS^{A}_{\bar{f};\Delta\bar{T}}} \quad \text{(A1)}$$

where $MRS^{X}_{y;z}$ denotes the marginal relation of substitution between y and z along equation X. Note that from Claim 2, both $MRS^{F}_{\bar{f};\Delta\bar{T}}$ and $MRS^{A}_{\bar{f};\Delta\bar{T}}$ are positive. In Claim 1.1 I show that $\frac{\partial A}{\partial \sigma^2} < 0$; and $\frac{\partial A}{\partial \bar{f}} > 0$; and therefore $MRS^{A}_{\bar{f};\sigma^2} > 0$. Claim 1 also shows that $MRS^{F}_{\bar{f};\sigma^2} > 0$. As advanced above, this implies that the effect of σ^2 on $\Delta\bar{T}$ is ambiguous.

Claim 1.1: a) $\frac{\partial A}{\partial \sigma^2} < 0$.

b) $\frac{\partial A}{\partial \bar{f}} > 0$.

c) $MRS^{F}_{\bar{f};\sigma^2} > 0$.

Proof. : To show part a), note that the terms in A that involve σ^2 can be written as follows

$$-\sigma^2 a(\bar{f})\left[\int_0^{\Delta\bar{T}} e^{\sigma^2 t}dt - \int_0^{\Delta\bar{T}} e^{(\sigma^2-\rho+\lambda)t}dt\right]$$

Therefore,

$$\begin{aligned}
\frac{\partial A}{\partial \sigma^2} &= \frac{d\left[-\sigma^2 a(\bar{f})\left[\int_0^{\Delta\bar{T}} e^{\sigma^2 t}dt - \int_0^{\Delta\bar{T}} e^{(\sigma^2-\rho+\lambda)t}dt\right]\right]}{d\sigma^2} \\
&= -a(\bar{f})\left[\int_0^{\Delta\bar{T}} e^{\sigma^2 t}dt - \int_0^{\Delta\bar{T}} e^{(\sigma^2-\rho+\lambda)t}dt\right] \\
&\quad -\sigma^2 a(\bar{f})\int_0^{\Delta\bar{T}} te^{\sigma^2 t}\left[1 - e^{-(\rho-\lambda)t}\right]dt \\
&< 0
\end{aligned}$$

□

To show part b), note that the terms in A that involve $\bar{f}$ can be written as

$$-\sigma^2 a(\bar{f})\left[\int_0^{\Delta\bar{T}} e^{\sigma^2 t}dt - \int_0^{\Delta\bar{T}} e^{(\sigma^2-\rho+\lambda)t}dt\right]$$

Therefore,

$$
\begin{aligned}
\frac{\partial A}{\partial \bar{f}} &= \frac{d\left[-\sigma^2 a(\bar{f})\left[\int_0^{\Delta\bar{T}} e^{\sigma^2 t}dt - \int_0^{\Delta\bar{T}} e^{(\sigma^2-\rho+\lambda)t}dt\right]\right]}{d\bar{f}} \\
&= -a'(\bar{f})\sigma^2\left[\int_0^{\Delta\bar{T}} e^{\sigma^2 t}dt - \int_0^{\Delta\bar{T}} e^{(\sigma^2-\rho+\lambda)t}dt\right] \\
&> 0
\end{aligned}
$$

□

To show part c), note that $MRS^F_{\bar{f},\sigma^2} = \frac{-\frac{\partial F}{\partial \sigma^2}}{\frac{\partial F}{\partial \bar{f}}}$; where the denominator is negative because the marginal value of flexibility is diminishing in the level of flexibility (i.e. $-a' > 0$ and $c'_m > 0$). The numerator can be expressed as

$$
\frac{d\left[a'(\bar{f})\int_0^{\Delta\bar{T}} e^{(\sigma^2-\rho+\lambda)t}dt\right]}{d\sigma^2} = a'(\bar{f})\int_0^{\Delta\bar{T}} te^{(\sigma^2-\rho+\lambda)t}dt < 0
$$

,by Leibnitz's rule. Note further that since $a' < 0$; so is this derivative. It follows immediately that $MRS^F_{\bar{f},\sigma^2} > 0$. □

This concludes the proof of claim 1. ■

From expression (A1) it follows that if $MRS^A_{\bar{f},\sigma^2} > MRS^F_{\bar{f},\sigma^2}$ and $MRS^F_{\bar{f},\Delta\bar{T}} < MRS^A_{\bar{f},\Delta\bar{T}}$; $\frac{d\Delta\bar{T}}{d\sigma^2} < 0$. Next I show that this is the case if condition 1 holds.

Claim 1.2: There exists a positive number $-\bar{a}'_{\sigma 1}$ such that if $-\bar{a}'_{\sigma 1} > -a'(.)\ \forall f$; $MRS^A_{\bar{f},\sigma^2} > MRS^F_{\bar{f},\sigma^2}$.

Proof. : By definition, $MRS^A_{\bar{f},\sigma^2} = \frac{-\frac{\partial A}{\partial \sigma^2}}{\frac{\partial A}{\partial \bar{f}}}$ and $MRS^F_{\bar{f},\sigma^2} = \frac{-\frac{\partial F}{\partial \sigma^2}}{\frac{\partial F}{\partial \bar{f}}}$. Therefore $MRS^A_{\bar{f},\sigma^2} > MRS^F_{\bar{f},\sigma^2}$ iff

$$
\frac{a(\bar{f})\frac{d}{d\sigma^2}\left[\sigma^2\int_0^{\Delta\bar{T}} e^{\sigma^2 t}\left[1 - e^{-(\rho-\lambda)t}\right]dt\right]}{-a'(\bar{f})\sigma^2\left[\int_0^{\Delta\bar{T}} e^{\sigma^2 t}(1 - e^{-(\rho-\lambda)t})dt\right]} > \frac{-a'(\bar{f})\frac{d}{d\sigma^2}\left[\int_0^{\Delta\bar{T}} e^{(\sigma^2-\rho-\lambda)t}dt\right]}{a''(\bar{f})\int_0^{\Delta\bar{T}} e^{(\sigma^2-\rho-\lambda)t}dt + c''_m(\bar{f})\int_0^{\Delta\bar{T}} e^{-(\rho-\lambda)t}dt}
$$

Since both denominators are positive, this inequality holds iff the following holds:

$$a(\bar f)\frac{d}{d\sigma^2}\left[\sigma^2\int_0^{\Delta\bar T} e^{\sigma^2 t}\left(1-e^{-(\rho-\lambda)t}\right)dt\right]\left[a''(\bar f)\int_0^{\Delta\bar T} e^{(\sigma^2-\rho-\lambda)t}dt + c''_m(\bar f)\int_0^{\Delta\bar T} e^{-(\rho-\lambda)t}dt\right]$$
$$> a'(\bar f)^2\sigma^2\left[\int_0^{\Delta\bar T} e^{\sigma^2 t}\left(1-e^{-(\rho-\lambda)t}\right)dt\right]\frac{d}{d\sigma^2}\left[\int_0^{\Delta\bar T} e^{(\sigma^2-\rho-\lambda)t}dt\right]$$

And this is equivalent to

$$a'(\bar f)^2 < \frac{a(\bar f)\frac{d}{d\sigma^2}\left[\sigma^2\int_0^{\Delta\bar T} e^{\sigma^2 t}\left(1-e^{-(\rho-\lambda)t}\right)dt\right]\left[a''(\bar f)\int_0^{\Delta\bar T} e^{(\sigma^2-\rho-\lambda)t}dt + c''_m(\bar f)\int_0^{\Delta\bar T} e^{-(\rho-\lambda)t}dt\right]}{\sigma^2\left[\int_0^{\Delta\bar T} e^{\sigma^2 t}\left(1-e^{-(\rho-\lambda)t}\right)dt\right]\frac{d}{d\sigma^2}\left[\int_0^{\Delta\bar T} e^{(\sigma^2-\rho-\lambda)t}dt\right]}$$

A sufficient condition for this inequality to hold is that

$$a'(\bar f)^2 < \mathrm{Min}_{\Delta\bar T;\bar f}\frac{a(\bar f)\frac{d}{d\sigma^2}\left[\sigma^2\int_0^{\Delta\bar T} e^{\sigma^2 t}\left(1-e^{-(\rho-\lambda)t}\right)dt\right]\left[a''(\bar f)\int_0^{\Delta\bar T} e^{(\sigma^2-\rho-\lambda)t}dt+c''_m(\bar f)\int_0^{\Delta\bar T} e^{-(\rho-\lambda)t}dt\right]}{\sigma^2\left[\int_0^{\Delta\bar T} e^{\sigma^2 t}\left(1-e^{-(\rho-\lambda)t}\right)dt\right]\frac{d}{d\sigma^2}\left[\int_0^{\Delta\bar T} e^{(\sigma^2-\rho-\lambda)t}dt\right]} \tag{21}$$

Now to show that if $-a'(\bar f)$ is bounded above by $-\bar a'_{\sigma 1}$ inequality A2 holds I proceed in two steps. First I show that the RHS of A2 is strictly positive and then I show that this together with the upper bound imply the result.

Step 1: The RHS on A2 is strictly positive:

Note that $\forall\ \Delta\bar T > 0$; both the numerator and the denominator are strictly positive. Also, $\forall\ \bar f$; $a(\bar f) > 0$; $c''_m(\bar f) > 0$; and $a''(\bar f) \geq 0$; therefore the RHS of A2 is strictly positive.

Step 2: note that The RHS of A2 can be written as

$$\mathrm{Min}_{\Delta\bar T;\bar f}\frac{\left(a_1+\int_0^{\bar f} a'(x)dx\right)\frac{d}{d\sigma^2}\left[\sigma^2\int_0^{\Delta\bar T} e^{\sigma^2 t}\left(1-e^{-(\rho-\lambda)t}\right)dt\right]\left[a''(\bar f)\int_0^{\Delta\bar T} e^{(\sigma^2-\rho-\lambda)t}dt+c''_m(\bar f)\int_0^{\Delta\bar T} e^{-(\rho-\lambda)t}dt\right]}{\sigma^2\left[\int_0^{\Delta\bar T} e^{\sigma^2 t}\left(1-e^{-(\rho-\lambda)t}\right)dt\right]\frac{d}{d\sigma^2}\left[\int_0^{\Delta\bar T} e^{(\sigma^2-\rho-\lambda)t}dt\right]}$$

, where a_1 is a strictly positive integration constant. It follows that, since the LHS of A2 is strictly increasing in $-a'(\bar f)$ and can be made arbitrarily close to zero by reducing it, and the RHS is strictly positive, that there exists an upper bound $-\bar a'_{\sigma 1}$ such that if $-a'(f) < -\bar a'_{\sigma 1}$; $\forall f$; inequality A2 holds and $MRS^A_{\bar f;\sigma^2} > MRS^F_{\bar f;\sigma^2}$.2

Claim 1.3: There exists a positive number $-\bar{a}'_{\sigma^2}$ such that if $-\bar{a}'_{\sigma^2} > -a'(\cdot)\ \forall f$, $MRS^A_{\bar f;\Delta\bar T} > MRS^F_{\bar f;\Delta\bar T}$.

Proof. : By definition, $MRS^A_{\bar f;\Delta\bar T} = \frac{-\frac{\partial A}{\partial \Delta\bar T}}{\frac{\partial A}{\partial \bar f}}$ and $MRS^F_{\bar f;\Delta\bar T} = \frac{-\frac{\partial F}{\partial \Delta\bar T}}{\frac{\partial F}{\partial \bar f}}$. Therefore $MRS^A_{\bar f;\Delta\bar T} > MRS^F_{\bar f;\Delta\bar T}$ iff

$$\frac{\frac{d}{d\Delta\bar T}\left[\lambda\int_0^{\Delta\bar T}\left(e^{-\lambda t}-e^{-\rho t}\right)dt+\sigma^2\int_0^{\Delta\bar T}e^{\sigma^2 t}\left(1-e^{-(\rho-\lambda)t}\right)dt\right]}{-\sigma^2 a'(\bar f)\int_0^{\Delta\bar T}e^{\sigma^2 t}\left(1-e^{-(\rho-\lambda)t}\right)dt} > \frac{-\frac{d}{d\Delta\bar T}\left[a'(\bar f)\int_0^{\Delta\bar T}e^{(\sigma^2-\rho+\lambda)t}dt+c'_m(\bar f)\int_0^{\Delta\bar T}e^{-(\rho-\lambda)t}dt\right]}{a''(\bar f)\int_0^{\Delta\bar T}e^{(\sigma^2-\rho+\lambda)t}dt+c''_m(\bar f)\int_0^{\Delta\bar T}e^{-(\rho-\lambda)t}dt}$$

Since both denominators are positive, this inequality holds iff the following holds:

$$\frac{d}{d\Delta\bar T}\left[\lambda\int_0^{\Delta\bar T}\left(e^{-\lambda t}-e^{-\rho t}\right)dt+\sigma^2\int_0^{\Delta\bar T}e^{\sigma^2 t}\left(1-e^{-(\rho-\lambda)t}\right)dt\right]$$
$$\left[a''(\bar f)\int_0^{\Delta\bar T}e^{(\sigma^2-\rho+\lambda)t}dt+c''_m(\bar f)\int_0^{\Delta\bar T}e^{-(\rho-\lambda)t}dt\right]$$
$$> \frac{d}{d\Delta\bar T}\left[a'(\bar f)\int_0^{\Delta\bar T}e^{(\sigma^2-\rho+\lambda)t}dt+c'_m(\bar f)\int_0^{\Delta\bar T}e^{-(\rho-\lambda)t}dt\right]\sigma^2 a'(\bar f)\int_0^{\Delta\bar T}e^{\sigma^2 t}\left(1-e^{-(\rho-\lambda)t}\right)dt$$

And this is equivalent to

$$-a'(\bar f) < \frac{\frac{d}{d\Delta\bar T}\left[\lambda\int_0^{\Delta\bar T}\left(e^{-\lambda t}-e^{-\rho t}\right)dt+\sigma^2\int_0^{\Delta\bar T}e^{\sigma^2 t}\left(1-e^{-(\rho-\lambda)t}\right)dt\right]\left[a''(\bar f)\int_0^{\Delta\bar T}e^{(\sigma^2-\rho+\lambda)t}dt+c''_m(\bar f)\int_0^{\Delta\bar T}e^{-(\rho-\lambda)t}dt\right]}{-\frac{d}{d\Delta\bar T}\left[a'(\bar f)\int_0^{\Delta\bar T}e^{(\sigma^2-\rho+\lambda)t}dt+c'_m(\bar f)\int_0^{\Delta\bar T}e^{-(\rho-\lambda)t}dt\right]\sigma^2\int_0^{\Delta\bar T}e^{\sigma^2 t}\left(1-e^{-(\rho-\lambda)t}\right)dt}$$

As before, a sufficient condition from this inequality to hold is that the LHS is smaller than the minimum of the RHS, i.e.

$$-a'(\bar f) < \mathrm{Min}_{\Delta\bar T;\bar f}\frac{\frac{d}{d\Delta\bar T}\left[\lambda\int_0^{\Delta\bar T}\left(e^{-\lambda t}-e^{-\rho t}\right)dt+\sigma^2\int_0^{\Delta\bar T}e^{\sigma^2 t}\left(1-e^{-(\rho-\lambda)t}\right)dt\right]\left[a''(\bar f)\int_0^{\Delta\bar T}e^{(\sigma^2-\rho+\lambda)t}dt+c''_m(\bar f)\int_0^{\Delta\bar T}e^{-(\rho-\lambda)t}dt\right]}{-\frac{d}{d\Delta\bar T}\left[a'(\bar f)\int_0^{\Delta\bar T}e^{(\sigma^2-\rho+\lambda)t}dt+c'_m(\bar f)\int_0^{\Delta\bar T}e^{-(\rho-\lambda)t}dt\right]\sigma^2\int_0^{\Delta\bar T}e^{\sigma^2 t}\left(1-e^{-(\rho-\lambda)t}\right)dt} \tag{A3}$$

$\forall\ \Delta\bar T > 0$ and $\bar f$; the RHS of A3 is strictly positive. Note also that the LHS can be arbitrarily close to zero by bounding $-a'(\bar f)$ above. Therefore, there exists a positive number $-\bar{a}'_{\sigma^2}$ such that if $-a'(\cdot)$ is bounded above by $-\bar{a}'_{\sigma^2}$; inequality A3 holds and $MRS^A_{\bar f;\Delta\bar T} > MRS^F_{\bar f;\Delta\bar T}$. $\square$

By selecting $-\bar{a}'_{\sigma} = \min\{-\bar{a}'_{\sigma 1}; -\bar{a}'_{\sigma 2}\}$, claims 1.2. and 1.3 prove the first part of the proposition, i.e. $\frac{d\triangle\bar{T}}{d\sigma^2} < 0$. It remains to show that condition 1 is sufficient for $\frac{d\bar{f}}{d\sigma^2} > 0$:

For any given tuple $(\bar{f};\sigma^2;\lambda)$, $\frac{\partial A}{\partial \triangle\bar{T}} < 0$; therefore equation A defines an implicit function that maps $(\bar{f};\sigma^2;\lambda)$ into $\triangle\bar{T}$: Let's denote it by $\triangle\bar{T}_A(\bar{f};\sigma^2;\lambda)$: Plugging it back into F, we obtain:

$$F(\triangle\bar{T}_A(\bar{f};\sigma^2;\lambda);\bar{f};\sigma^2;\lambda) = 0$$

Total differentiation of F leads to:

$$\begin{aligned}
\frac{d\bar{f}}{d\sigma^2} &= -\frac{\frac{\partial F}{\partial\sigma^2} + \frac{\partial F}{\partial\triangle\bar{T}}\frac{\partial\triangle\bar{T}_A}{\partial\sigma^2}}{\frac{\partial F}{\partial\bar{f}} + \frac{\partial F}{\partial\triangle\bar{T}}\frac{\partial\triangle\bar{T}_A}{\partial\bar{f}}} \\
&= \frac{\left.\frac{\partial\triangle\bar{T}}{\partial\sigma^2}\right|_F - \left.\frac{\partial\triangle\bar{T}}{\partial\sigma^2}\right|_A}{-\left.\frac{\partial\triangle\bar{T}}{\partial\bar{f}}\right|_F + \left.\frac{\partial\triangle\bar{T}}{\partial\bar{f}}\right|_A} \\
&= \frac{MRS^F_{\triangle\bar{T};\sigma^2} - MRS^A_{\triangle\bar{T};\sigma^2}}{-MRS^F_{\triangle\bar{T};\bar{f}} + MRS^A_{\triangle\bar{T};\bar{f}}}
\end{aligned}$$

By claim 2, both $MRS^F_{\triangle\bar{T};\bar{f}}$ and $MRS^A_{\triangle\bar{T};\bar{f}}$ are positive. Moreover, both $MRS^F_{\triangle\bar{T};\sigma^2}$ and $MRS^A_{\triangle\bar{T};\sigma^2}$ are negative. Therefore, the sign of $\frac{d\bar{f}}{d\sigma^2}$ is ambiguous. However, if $MRS^F_{\triangle\bar{T};\sigma^2} < MRS^A_{\triangle\bar{T};\sigma^2}$ and $MRS^F_{\triangle\bar{T};\bar{f}} > MRS^A_{\triangle\bar{T};\bar{f}}$; $\frac{d\bar{f}}{d\sigma^2} > 0$: This is the case if condition 1 holds.

Claim 1.4: There exist a positive number $\bar{c}'_{m\sigma}$ such that if $c'_m(:) > \bar{c}'_{m\sigma}$ $\forall f$; $MRS^F_{\triangle\bar{T};\sigma^2} < MRS^A_{\triangle\bar{T};\sigma^2}$:

Proof. : By definition, $MRS^F_{\triangle\bar{T};\sigma^2} = \frac{-\frac{\partial F}{\partial\sigma^2}}{\frac{\partial F}{\partial\triangle\bar{T}}}$; and $MRS^A_{\triangle\bar{T};\sigma^2} = \frac{-\frac{\partial A}{\partial\sigma^2}}{\frac{\partial A}{\partial\triangle\bar{T}}}$: Therefore $MRS^F_{\triangle\bar{T};\sigma^2} < MRS^A_{\triangle\bar{T};\sigma^2}$ iff

$$\begin{aligned}
&\frac{a'(\bar{f})\frac{d}{d\sigma^2}\left[\int_0^{\Delta\bar{T}} e^{(\sigma^2-\rho+\lambda)t}dt\right]}{-\left[a'(\bar{f})e^{\sigma^2\Delta\bar{T}} + c'_m(\bar{f})\right]e^{-(\rho-\lambda)\Delta\bar{T}}} \\
< &\frac{a(\bar{f})\frac{d}{d\sigma^2}\left[\sigma^2\int_0^{\Delta\bar{T}} e^{\sigma^2 t}\left[1-e^{-(\rho-\lambda)t}\right]dt\right]}{-\frac{d}{d\Delta\bar{T}}\left[\lambda\int_0^{\Delta\bar{T}}(e^{-\lambda t}-e^{-\rho t})dt + \sigma^2\int_0^{\Delta\bar{T}} e^{\sigma^2 t}(1-e^{-(\rho-\lambda)t})dt\right]}
\end{aligned}$$

Multiplying by (-1)

$$\frac{-a'(f^*)\frac{d}{d\sigma^2}\left[\int_0^{\Delta\tau^*} e^{(\sigma^2-\rho+\lambda)t}dt\right]}{-\left[a'(f^*)e^{\sigma^2\Delta\tau^*}+c'_m(f^*)\right]e^{-(\rho-\lambda)\Delta\tau^*}}$$

$$> \frac{a(f^*)\frac{d}{d\sigma^2}\left[\sigma^2\int_0^{\Delta\tau^*} e^{\sigma^2 t}\left(1-e^{-(\rho-\lambda)t}\right)dt\right]}{\frac{d}{d\Delta\tau^*}\left[\lambda\int_0^{\Delta\tau^*}(e^{-\lambda t}-e^{-\rho t})\,dt+\sigma^2\int_0^{\Delta\tau^*}e^{\sigma^2 t}(1-e^{-(\rho-\lambda)t})\,dt\right]}$$

and this is equivalent to

$$-\left[a'(f^*)e^{\sigma^2\Delta\tau^*}+c'_m(f^*)\right]e^{-(\rho-\lambda)\Delta\tau^*}$$

$$< \frac{-a'(f^*)\frac{d}{d\sigma^2}\left[\int_0^{\Delta\tau^*} e^{(\sigma^2-\rho+\lambda)t}dt\right]\frac{d}{d\Delta\tau^*}\left[\lambda\int_0^{\Delta\tau^*}\left[e^{-\lambda t}-e^{-\rho t}\right]dt+\sigma^2\int_0^{\Delta\tau^*}e^{\sigma^2 t}\left[1-e^{-(\rho-\lambda)t}\right]dt\right]}{a(f^*)\frac{d}{d\sigma^2}\left[\sigma^2\int_0^{\Delta\tau^*}e^{\sigma^2 t}(1-e^{-(\rho-\lambda)t})\,dt\right]}$$

A sufficient condition for this inequality to hold is that

$$-\left[a'(f^*)e^{\sigma^2\Delta\tau^*}+c'_m(f^*)\right]e^{-(\rho-\lambda)\Delta\tau^*} \qquad \text{(A4)}$$

$$< \quad \mathrm{Min}_{\Delta\tau^*,f^*}\frac{-a'(f^*)\frac{d}{d\sigma^2}\left[\int_0^{\Delta\tau^*} e^{(\sigma^2-\rho+\lambda)t}dt\right]\frac{d}{d\Delta\tau^*}\left[\lambda\int_0^{\Delta\tau^*}\left(e^{-\lambda t}-e^{-\rho t}\right)dt+\sigma^2\int_0^{\Delta\tau^*}e^{\sigma^2 t}\left(1-e^{-(\rho-\lambda)t}\right)dt\right]}{a(f^*)\frac{d}{d\sigma^2}\left[\sigma^2\int_0^{\Delta\tau^*}e^{\sigma^2 t}\left(1-e^{-(\rho-\lambda)t}\right)dt\right]}$$

Note that the RHS of A4 is strictly positive for all $\Delta\tau^* > 0$, and all f^*. The LHS of A4 can be arbitrarily close to zero by making the $c'_m(f^*)$ arbitrarily close to $a'(f^*)e^{\sigma^2\Delta\tau^*}$; therefore for any given $a'(f^*)$, there exists a lower bound $\hat{c}'_{m\sigma}$ for $c'_m(\cdot)$ such that if $c'_m(\cdot) > \hat{c}'_{m\sigma}\ \forall f$, inequality A4 holds and $MRS^F_{\triangle\tau,\sigma^2} < MRS^A_{\triangle\tau,\sigma^2}$. $\square$

Claim 1.5: There exists a positive number $-\hat{a}'_{\sigma 4}$ such that if $-\hat{a}'_{\sigma 4} > -a'(\cdot)\ \forall f$, $MRS^F_{\triangle\tau,f^*} > MRS^A_{\triangle\tau,f^*}$.

Proof. : Set $-\hat{a}'_{\sigma 4} = -\hat{a}'_{\sigma 2}$. The result follows immediately from claim 1.3 and from the fact that $MRS^X_{y,z} = \left[MRS^X_{z,y}\right]^{-1}$ $\square$

Claims 1.4. and 1.5 prove the second part of the proposition, i.e. $\frac{df^*}{d\sigma^2} > 0$. This concludes the proof of the proposition. ∎

Claim 3: The net marginal value of flexibility increases with λ, (i.e. $\frac{\partial F}{\partial \lambda} > 0$).

Proof. : The Net Marginal Value of Flexibility can be written as

$$0 = -a'(\hat{T})\int_0^{\Delta \hat{T}} e^{(\sigma^2-\rho+\lambda)t}dt - c'_m(\hat{T})\int_0^{\Delta \hat{T}} e^{-(\rho-\lambda)t}dt \qquad (F)$$

Taking derivatives with respect to λ:

$$\frac{\partial F}{\partial \lambda} = -a'(\hat{T})\int_0^{\Delta \hat{T}} te^{(\sigma^2-\rho+\lambda)t}dt - c'_m(\hat{T})\int_0^{\Delta \hat{T}} te^{-(\rho-\lambda)t}dt$$

Using equation (F) it is very easy to see that the sign of $\frac{\partial F}{\partial \lambda}$ is equal to the sign of the following expression:

$$\text{sign}\left(\frac{\partial F}{\partial \lambda}\right) = \text{sign}\left(\int_0^{\Delta \hat{T}} t\frac{e^{(\sigma^2-\rho+\lambda)t}}{\int_0^{\Delta \hat{T}} e^{(\sigma^2-\rho+\lambda)t}dt}dt - \int_0^{\Delta \hat{T}} t\frac{e^{-(\rho-\lambda)t}}{\int_0^{\Delta \hat{T}} e^{-(\rho-\lambda)t}dt}dt\right)$$

Note that the term in the RHS bracket can be expressed as:

$$\int_0^{\Delta \hat{T}} t\delta_1(t)dt - \int_0^{\Delta \hat{T}} t\delta_2(t)dt$$

where both δ_1 and δ_2 are positive weights that add up to one over the interval $[0; \Delta \hat{T}]$; and δ_1 is increasing with t while δ_2 is decreasing. It is straighforward that since the δ_1 weights put more weight for higher t's the first term must be larger than the second. Hence $\frac{\partial F}{\partial \lambda} > 0$.∎

Proposition 2: There exist two numbers $\bar{c}'_{m\lambda}$ and $\bar{a}'_{\lambda}$ such that if $c'_m(.)$ is bounded below by $\bar{c}'_{m\lambda}$ and $-a'(.)$ is bounded above by $-\bar{a}'_{\lambda}$ an increase in the rate of technological progress raises the flexibility and the speed of diffusion. This restriction will be referred to as Condition 2.

Proof. : The proof of this proposition is identical to the proof of proposition 1 and therefore I have not included it here. However, I will be glad to provide it if requested. ∎

Proposition 3: There exists a lower bound $\bar{sh}_f$ such that if $sh_f(\bar{\sigma}^2) > \bar{sh}_f$; λ_f will be higher than λ_{nf}.

Proof. : Fix any pair of diffusion lags $\Delta \hat{T}_f$ and $\Delta \hat{T}_{nf}$: For any level of n_f; equation L_{nf} defines n_{nf} as a continuously decreasing function of sh_f, at any interior equilibrium.

For any level of n_{nf}, equation L_f defines n_f as a monotonic increasing function of sh_f. Therefore by increasing sh_f, n_f will increase and n_{nf} will fall. If $sh_f = 1$, the value of an inflexible innovation is zero because there is no market for it, and therefore $n_{nf} = 0$. Therefore it exists a lower bound $\bar{sh}_f$ such that if $sh_f(\bar{\sigma}^2) > \bar{sh}_f$, $n_f > n_{nf}$. Since λ is monotonically increasing in n, the first part of the proposition follows immediately. $\blacksquare$

A.2 Calibration

This section describes the parameter values and functional forms used to simulate the model and study the transitional dynamics after an increase in uncertainty. Before describing the calibration a word of caution is appropriate. The purpose of this computational exercise was to illustrate the qualitative evolution of several variables. The parameters and functional forms are almost arbitrary and in no sense pretend to match any actual economy.

Gross profits: $\pi = 6$, $\alpha = 0.3$, $\lambda = 0.01$, $\rho = 0.05$.

The costs of operating the machine take the following form:

$$C(t, d, f) = [a_0 + a_1 f^{-\delta} e^{\gamma d} + c_0 + c_1 f^{\beta}] e^{\lambda t}$$

where $a_0 = 0$, $c_0 = 0$, $a_1 = 0.3$, $c_1 = 0.25$, $\delta = 0.5$, $\beta = 1.2$, $\gamma = 3$.

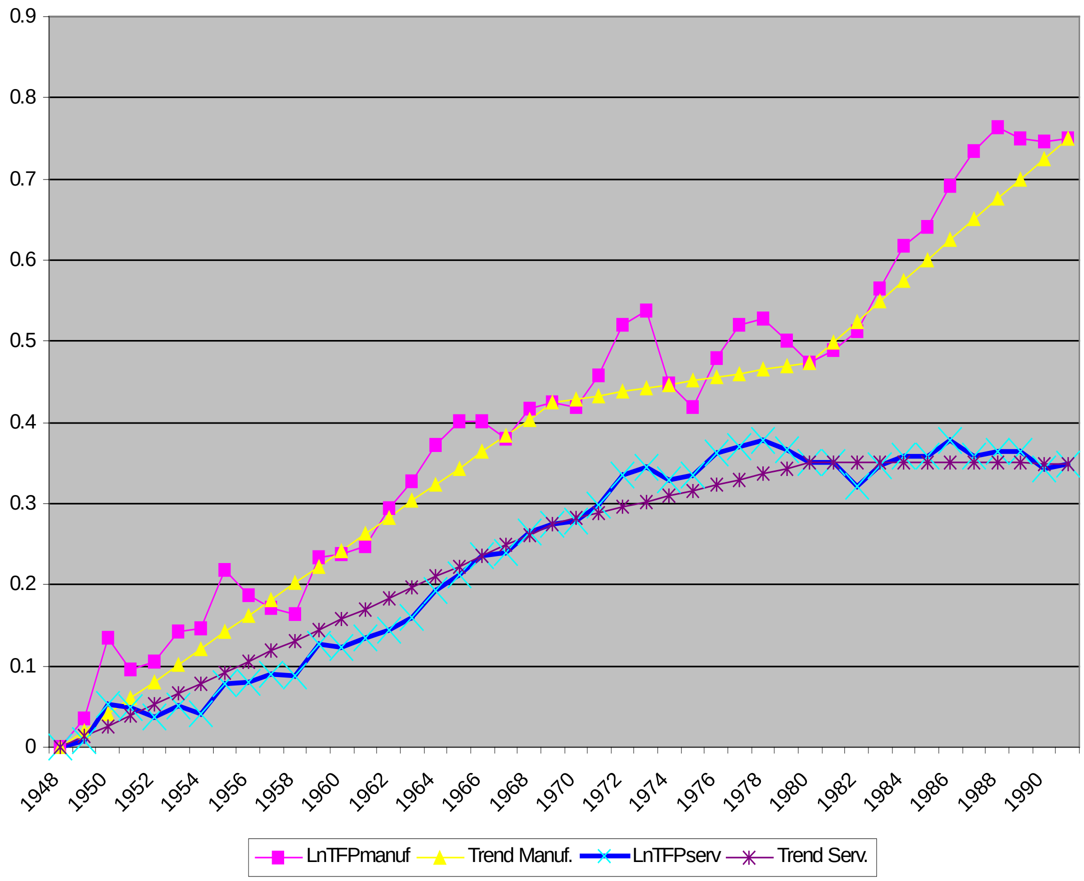
Fixed cost of adoption: $A_c = 2.5$.

Uncertainty: in the low state $\sigma^2 = 0.005776$; in the high state, $\bar{\sigma}^2 = 0.024336$; the marginal cost of producing capital, $c = 0.1$.

The implied adoption lags for the low and high uncertainty steady states are $4\bar{T}_l = 6.9$ and $4\bar{T}_h = 6.4$. The levels of flexibility are $f_l = 0.68$ and $f_h = 0.7563$.

For the economy that cannot change the level of flexibility, this has been set up equal to f_l. The steady state adoption lag in the highly uncertain steady state is equal to 6.3859.

FIGURE 1: Sectorial Heterogeneity in TFP



	Manufactures	Services
48-69	0.0202	0.0131
69-80	0.0045	0.0068
80-91	0.0252	-0.0001

Figure 2

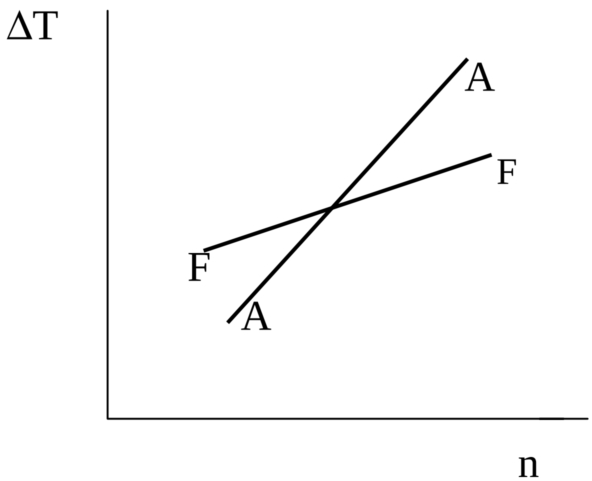


Figure 3: Increase in σ^2

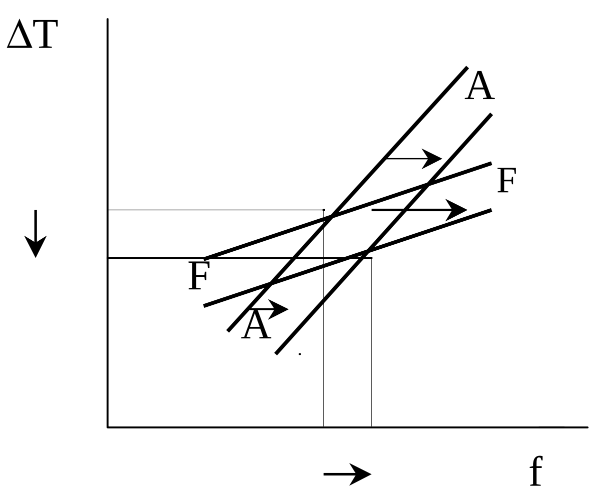


Figure 4: Increase in λ

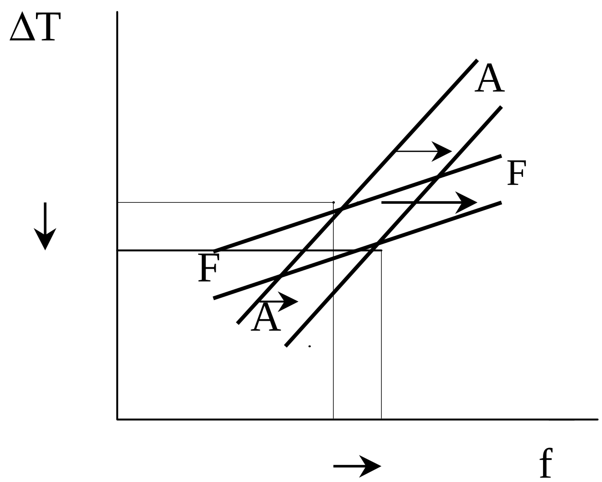


Figure 5: General Equilibrium

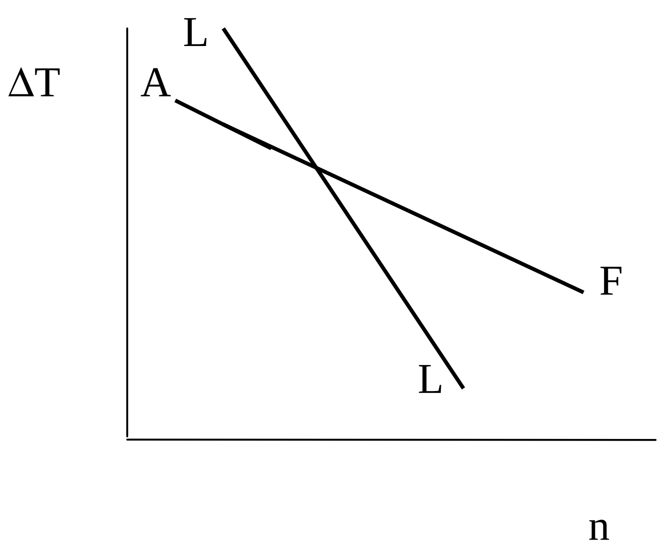


Figure 6. Increase in σ^2

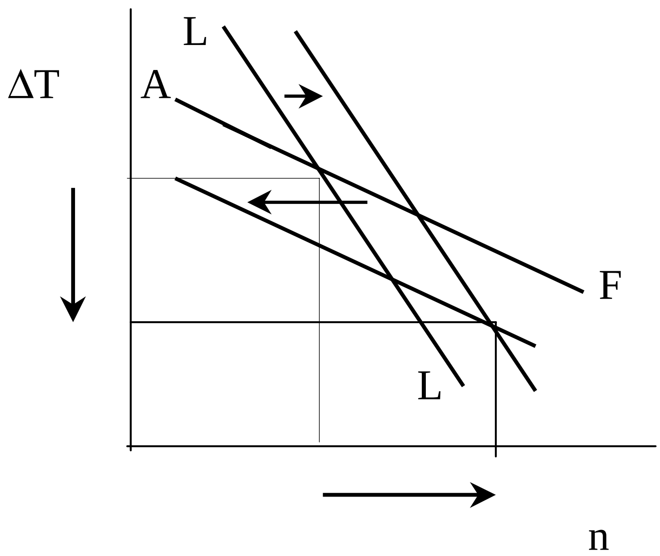


Figure 7: The Effect of ΔT on the Value of Innovations.

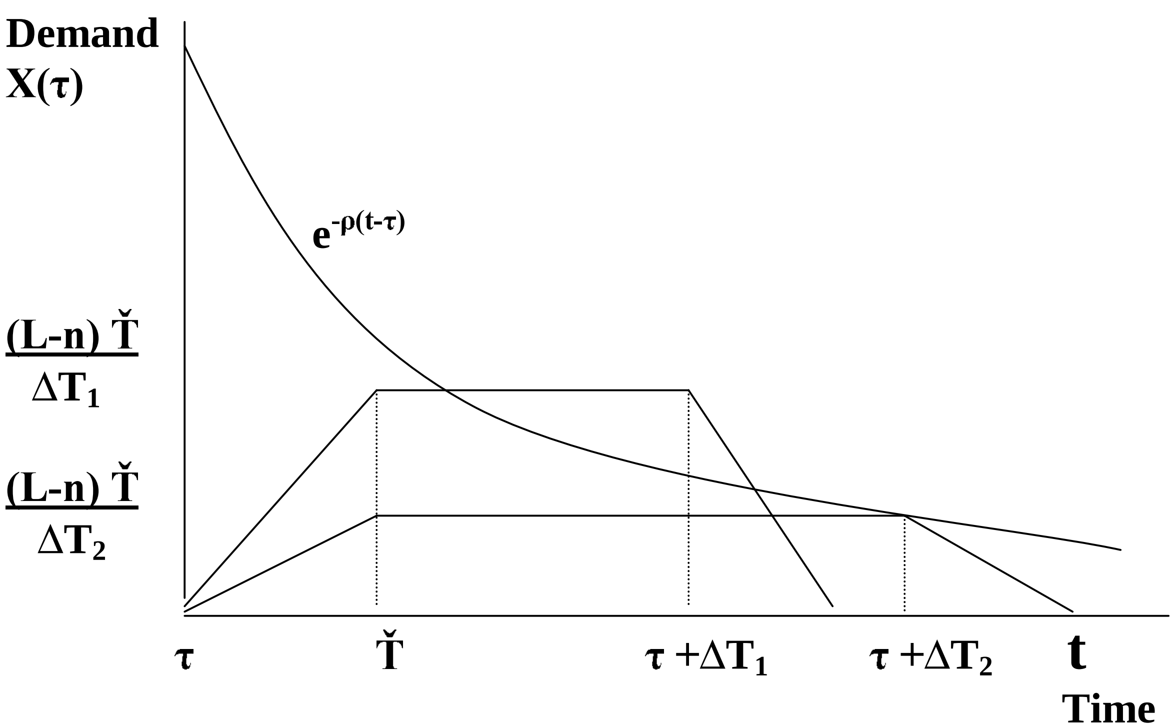


Figure 8
Non-Defense R&D Expenditures as a share of GDP

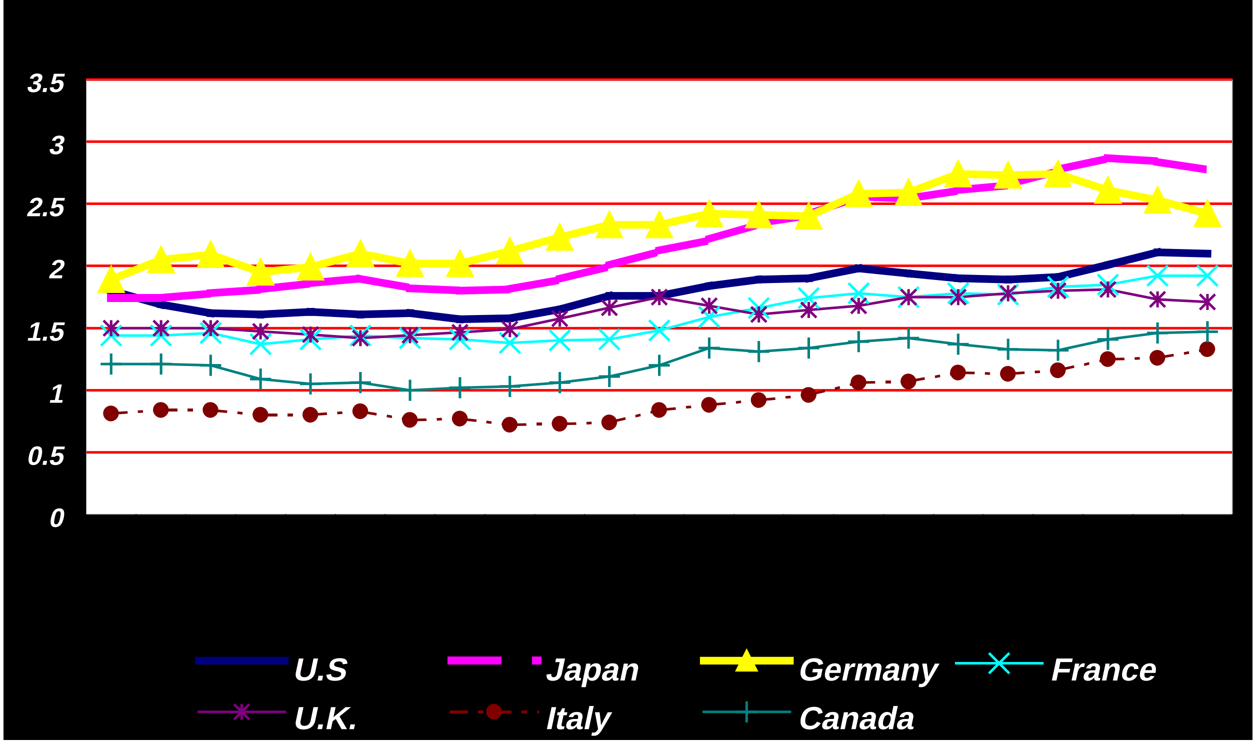


Figure 9: Adoption during the Transition

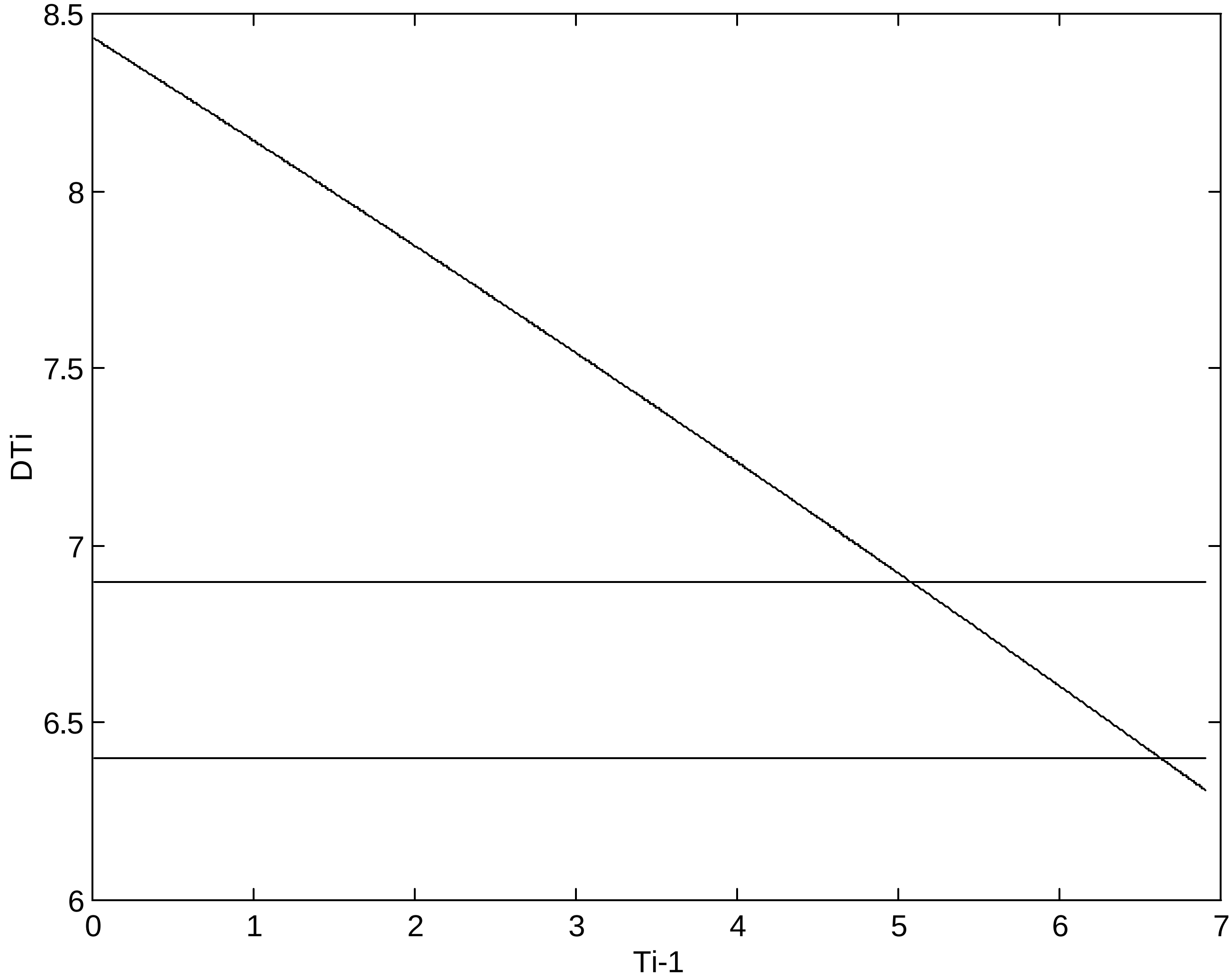


Figure 10: The Evolution of log (TFP)

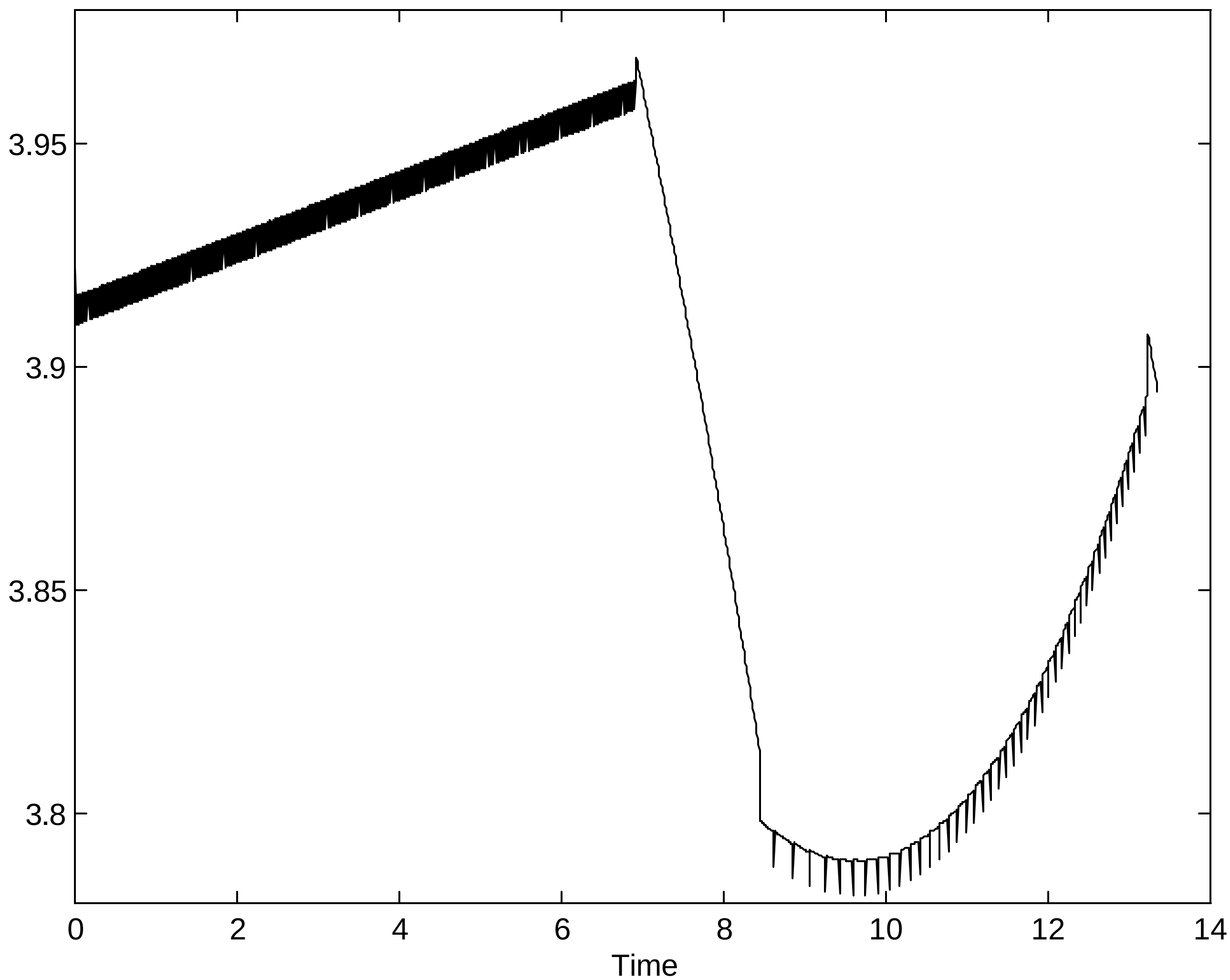
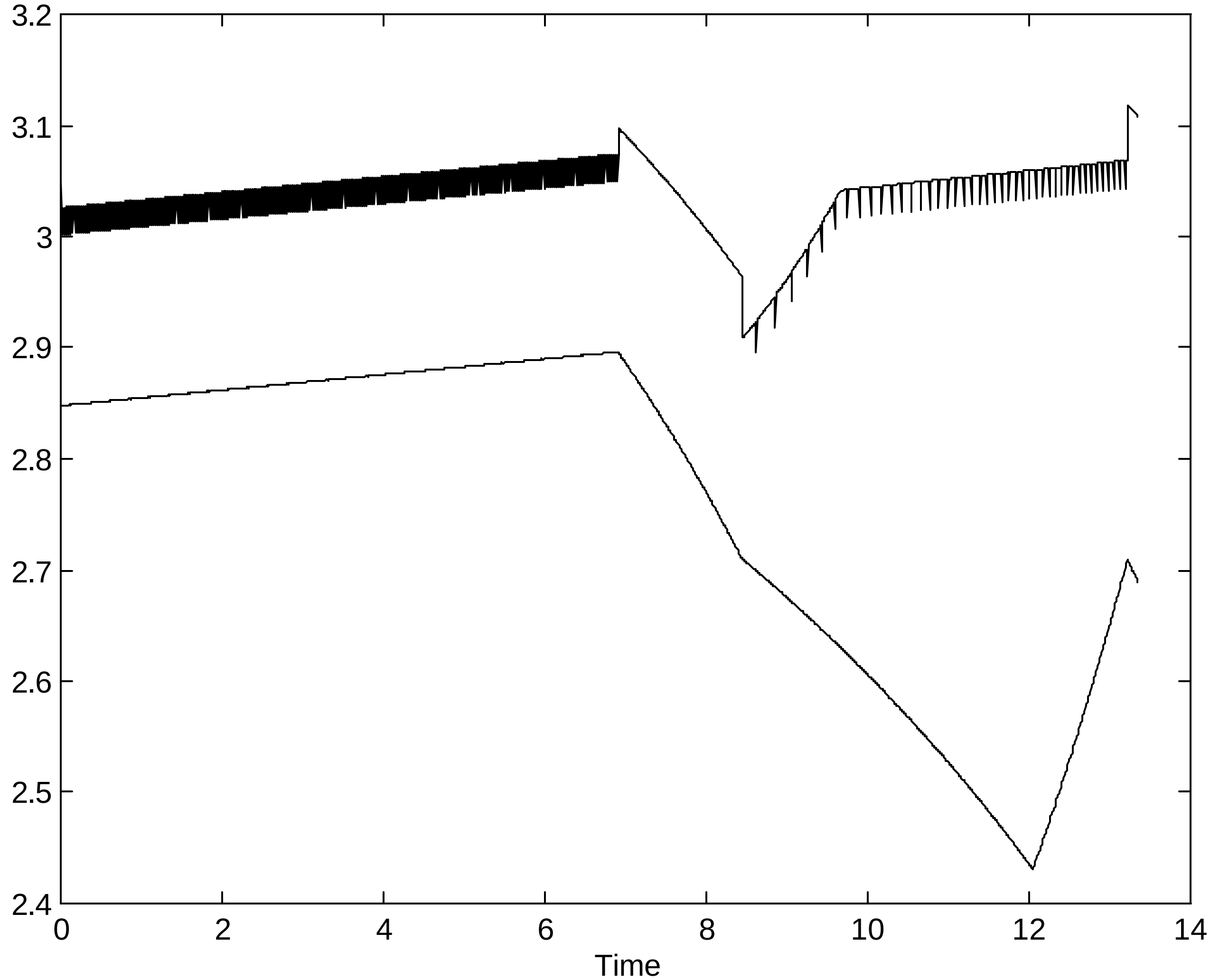


Figure 11: Acceleration of Embodied TFP growth



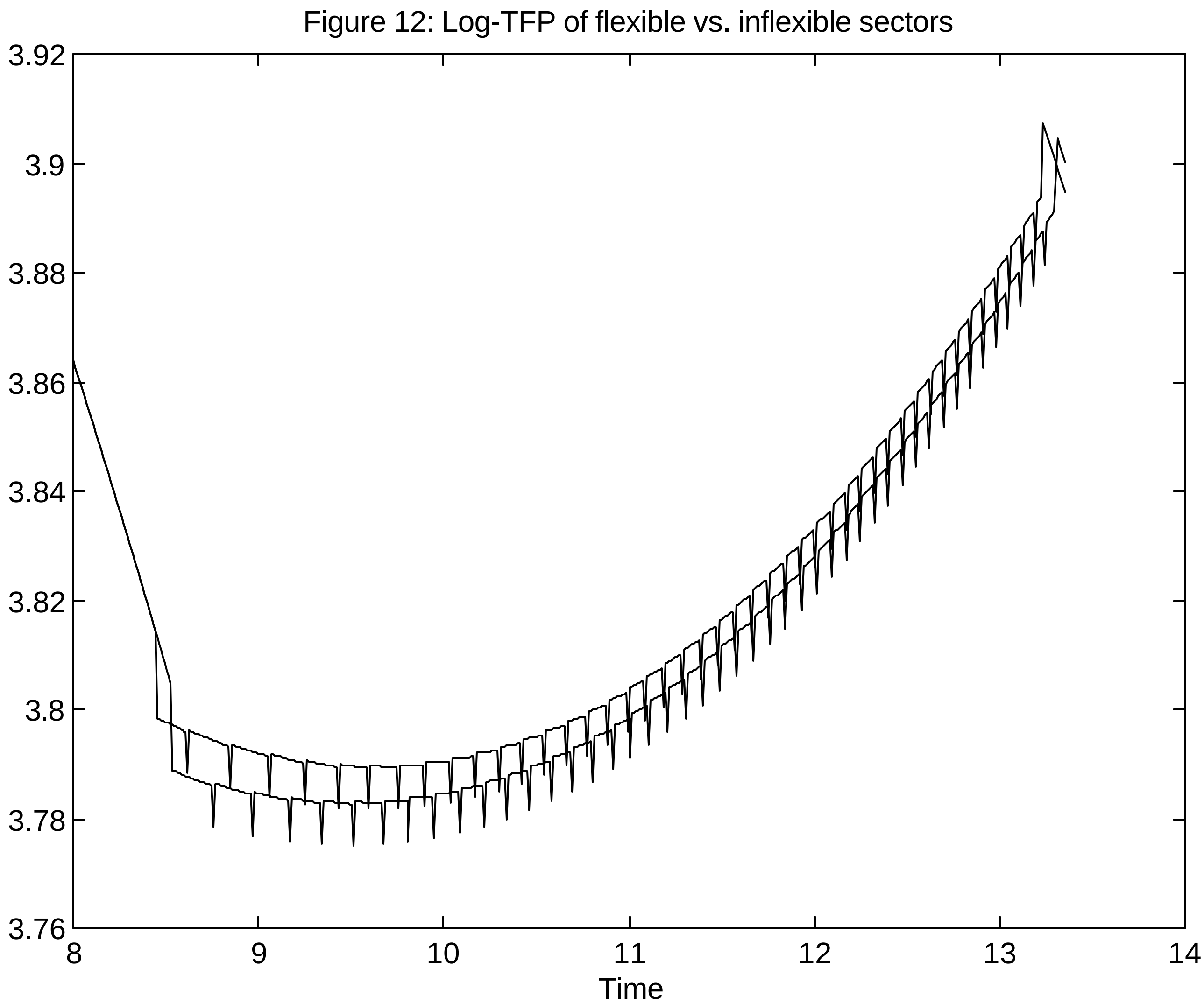


Figure 12: Log-TFP of flexible vs. inflexible sectors
3.92
3.9
3.88
3.86
3.84
3.82
3.8
3.78
3.76
8
9
10
11
12
13
14
Time

Figure 13: Directed Technological Progress

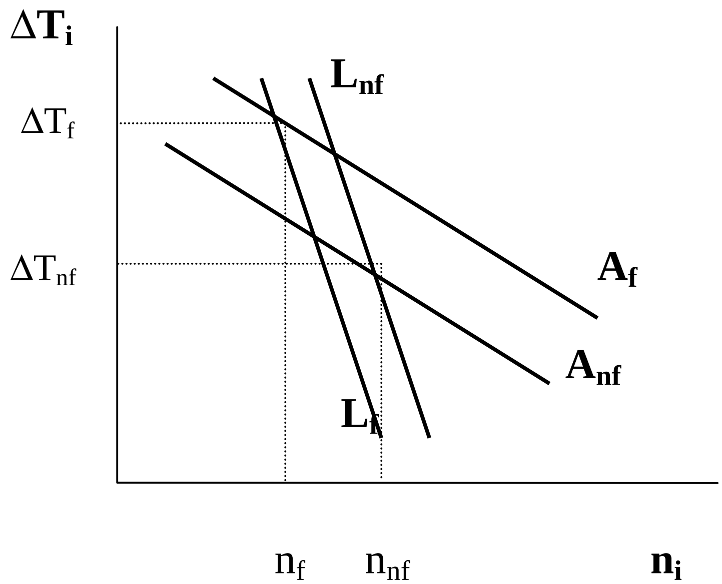


Figure 14: Moving Average of Std. Deviation of First Differences of Real Prices

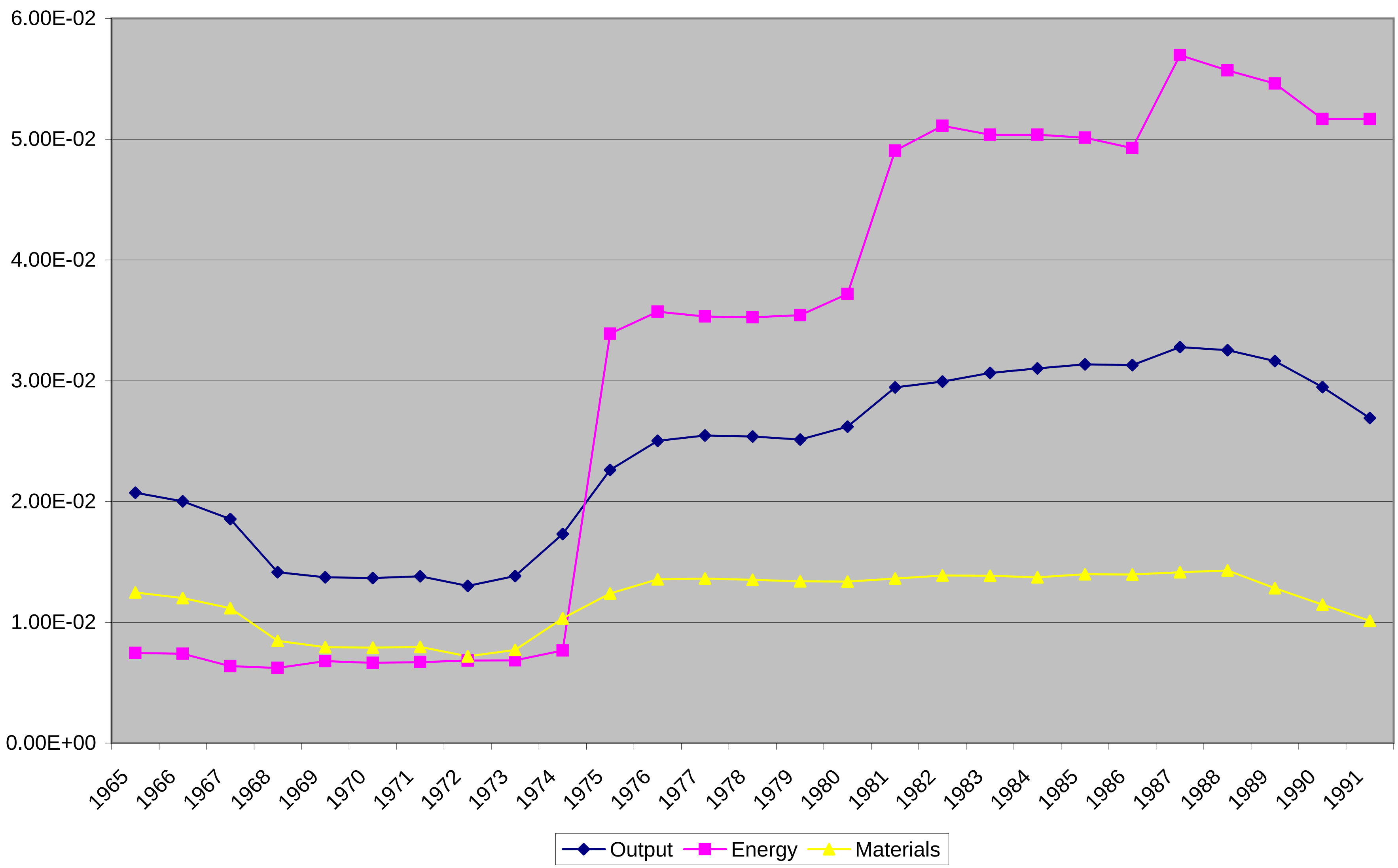


Table 1: Decomposition of TFP growth into Embodied and Disembodied

Basic Equation: $\ln(TFP_{it}) \equiv \ln(Y_{it}) - \alpha \ln(K/L_{it}) = \beta_{0i} + \beta_1 \ln(R_{it}) + \beta_2 t + \beta_3 G_{it}$

Intercept	-4.71 (0.314)	-4.44 (0.324)
R-Capacity Utilization	0.013 (0.003)	0.01 (0.003)
T-Time Trend	0.028 (0.001)	0.029 (0.001)
T74-Time trend Post73	-0.035 (0.003)	-0.031 (0.004)
G-Age of All capital	-0.017 (0.002)	-0.015 (0.002)
G74- Age of All Capital: Post 73		-0.006 (0.002)
Embodied TFP growth	0.017	0.015
Disembodied TFP growth	0.018	0.014
Δ Embodied TFP growth post-73		0.006
Δ Disembodied TFP growth post-73		-0.037

Standard errors in parentheses.
All equations estimated with sector fixed effects.

Table 2: Volatility of Individual Stock Returns

Decade	Mean	>2years	Median	Trend	Annual	Stocks 50's	Stocks 60's	Value Weighted
50's	0.076	0.075	0.073	0.075	0.33	0.076	-	0.059
60's	0.109	0.107	0.097	0.106	0.43	0.085	0.11	0.066
70's	0.147	0.142	0.137	0.142	0.51	0.129	0.14	0.085
80's	0.156	0.156	0.136	0.152	0.5	0.174	0.16	0.089
90's	0.16	0.154	0.136	0.155	0.56	0.19	0.17	0.081

Source: CRSP data set.
The 50's goes from January 1947 to December 1959. The rest go from January of 19x0 to December of 19x9. **Mean** is the average standard deviation of monthly stock returns (including all distributions) during the decade. **>2years** is the average standard deviation of monthly stock returns for those stocks with at least 24 months of observations in the decade. **Median** is the median standard deviation of individual monthly stock returns. **Trend** is the average volatility of the deviations of the monthly stock returns from a linear fit for each stock and decade. **Yearly** is the average standard deviation of yearly stock returns among those stocks with more than two years of monthly stock returns in the decade. **Stocks 50's** and **60's** control for compositional effects by computing **Mean** for only the stocks that are in the data set in the 50's and 60's respectively. **Value Weighted** is the value weighted average standard deviation of monthly stock returns for the decade. The value weights are the share of the individual stock' volume on total volume during the decade.

Table 3: Time Trend in Excess Reallocation Rate

Dependent Variable	Excess Reallocation Rate	Excess Reallocation Rate	Excess Reallocation Rate
Year	0.051 (2.33)	0.068 (3.18)	0.1 (5.72)
Net Employment Growth	0.174 (6.94)	0.11 (4.63)	0.12 (4.9)
Constant	10.52 (1.81)	6.08 (5.1)	1.4 (4.7)
R^2	0.04	0.63	0.65
Sample	420 (n=20, T=21)	420 (n=20, T=21)	420 (n=20, T=21)
Sic-Dummies	YES	YES	YES
Weight (Employment Share)	NO	Fixed	Variable

Note: t-statistics in parenthesis.
Excess job reallocation rate=job creation rate+ job destruction rate- net employment growth from Davis , Haltiwanger and Schuh (1996). **Year** is a time trend. **Sic-Dummies** are the 2-digit sic dummies.

Table 4: Explaining the Increment in Uncertainty

Dependent Variable	**ΔSd7060**	**ΔSd7060**	**ΔSd7060**	**ΔSd7060**
Δop7060	0.05 (2.25)			
Δim7060		0.069 (2.16)		
Δex7060			0.108 (2.24)	
Δcsh7060				0.062 (1.063)
Constant	0.028 (12.36)	0.029 (13.55)	0.028 (12.3)	0.031 (16.5)
R^2	0.01	0.01	0.011	0.0025

Note: t-statistics in parenthesis. All regressions are weighted by share in value added. Sample size is 448 4-digit manufacturing industries.

ΔSd7060 is the increment in average standard deviation of stock returns in the 4-digit manufacturing sector between the 60's and the 70's. **Δim7060** is the increment in the average sectorial import share between the 60's and the 70's. **Δex7060** is the increment in the ratio of exports to shipments between the 60's and the 70's. **Δop7060= Δim7060 + Δex7060. Δcsh7060** is the difference between the share of OCAM in investment in 1977 and the same variable in 1967.

Table 5: Volatility in Prices

	Poly-1			Poly- 3			ΔP		
	Py	Pe	Pm	Py	Pe	Pm	Py	Pe	Pm
50's	.0267	.0128	.0176	.0242	.0123	.0161	.0214	.0083	.0135
60's	.0125	.0068	.0079	.0093	.0043	.0063	.0104	.0067	.0065
70's	.0415	.0375	.0256	.0367	.0371	.0225	.0328	.0426	.0187
80's	.03	.0683	.0167	.0249	.0617	.0145	.0252	.0530	.0109

	ΔP		
	Py	Pe	Pm
Pre-70's	.0183	.0075	.0118
Post-70's	.031	.0489	.0153

All the variables are weighted averages over the 21 2-digit manufacturing sectors of the standard deviations of some price level deflated by the aggregate producer price index of the US economy along a given decade. **Py** is the price of sectorial final output, **Pe** is the sectorial price of energy, **Pm** is the sectorial price of materials. The cells under the heading **Poly-1** contain the average standard deviation of the deviations of the price variable from a decade and sector specific time trend. The cells under the heading **Poly-3** contain the average standard deviation of the deviations of the price variable from a decade and sector specific third order polynomial in time. The cells under the heading **ΔP** contain the average standard deviation of the first difference of the price variable. The 50's go from 1950 to 1959, the 60's go from 1960 to 1969, the 70's go from 1970 to 1979, and the 80's go from 1980 to 1991.

Data source: Jorgenson, http://www.economics.harvard.edu/faculty/jorgenson/data/35klem.html

Table 6: Effect of computers on the TFP Slowdown

Dependent Variable	TFPg 70's	TFPg 70's	TFPg 70's	TFPg 70's	TFPg 70's	TFPg 70's	Δ TFP g	Δ TFP g
Comp. sh. 77 (BEA)	0.446 (16.1)		0.61 (16.35)		0.4 (2.57)			
Comp. sh. 77 (BBG)		0.42 (13.3)		0.93 (12.7)		0.72 (2.2)		
Δ Comp. sh. (BEA)							0.143 (2.73)	
Δ Comp. sh. 87 (BEA)								0.031 (1.1)
Constant	-0.03 (-13.8)	-0.013 (-7.9)	-0.04 (-14.9)	-0.26 (-10)	-0.027 (2.75)	-0.02 (-2.42)	-0.016 (-9.7)	-0.016 (-8)
R^2	0.36	0.284	0.31	-	0.36	0.14	0.016	0.003
Estimation	OLS	OLS	IV comp. sh. 67	IV comp. sh. 67	IV sd60's	IV sd60's	OLS	OLS

Note: t-statistics in parenthesis. Sample size is 448 4-digit manufacturing industries.

TFPg 60's is the average growth rate of TFP between 1958 and 1973. **TFPg 70's** is the average growth rate of TFP between 1973 and 1980. Δ **TFP g** = **TFPg 70's - TFPg 60's. Comp. sh. xx (BEA)** is the share of office computing and accounting machines in total investment in year 19xx. **Comp. sh. 77 (BBG)** is the share of computers in total investment in 1977 as computed by Berman, Bound and Griliches (1994). **Δ Comp. sh. (BEA)= Comp. sh. 77 (BEA) - Comp. sh. 67 (BEA). Δ Comp. sh. 87 (BEA) = [Comp. sh. 92 (BEA)+ Comp. sh. 82 (BEA)]/2- Comp. sh. 77 (BEA).**

Table 7: Did uncertainty cause the slowdown?

Dependent Variable	ΔTFPg70	ΔTFPg70	ΔTFPg70	TFPg	TFPg	ΔTFPg80	ΔTFPg80
Δsdr70	0.061 (1.38)						
Δsdgship70		-0.22 (-9.98)					
ΔsdTFPg70			-0.457 (-12.4)				
Δcsh7767	0.174 (3.14)	0.1 (1.95)	0.1 (2.06)				
Sdgship				0.06 (4.675)			
Sdgtfp					0.116 (5.0)		
Δsdgship80						-0.048 (-1.48)	
ΔsdTFPg80							-0.12 (-2.35)
Δcsh8777						-0.028 (-1.17)	-0.026 (-1.09)
Constant	-0.017 (-7.7)	-0.011 (-7)	-0.011 (-7.35)	0.0013 (0.928)	0.0009 (0.65)	0.007 (4.1)	0.007 (4.4)
R^2	0.027	0.20	0.27	0.016	0.018	0.019	0.016

Note: t-statistics in parenthesis.
All regressions run at the 4-digit manufacturing sector. The sample size for all regressions but columns 4 and 5 is 448, while columns 4 and 5 pool the data for three decades and therefore have 448*3=1344. In all regressions, observations are weighted by the average share of value added.
Δxy represents the increment in the variable x between decade y and the previous one. in period y. Δxyyvv denotes the increment in variable x between 19yy and 19vv. x can be the average growth rate of TFP growth (TFPg), the standard deviation of monthly stock returns (sdr), the standard deviation of the annual growth rate of shipments (sdgship) and the standard deviation of the annual growth rate of TFP (sdtfpg). Csh denotes the share of expenditures in OCAM (office computing and accounting machines) over total investment.

Table 8: Diffusion of Computers

Dependent Variable	Comp sh (BEA)	Comp sh. (BBG)	Comp sh (BEA)	Comp sh (BEA)
Sdl	.38 (19.46)	.35 (14.57)	.1 (5.03)	.116 (3.58)
T67				-.0001 (-.7)
T77				-.0003 (-3.12)
Constant	.0007 (6.36)	-.0003 (-1.87)	.0019 (18.2)	.002 (9.93)
R^2	.297	.19	.285	.29
N	896	896	448x3=1344	448x3=1344
Effects	Random	Random	Fixed	Fixed

Note: t-statistics in parenthesis.
Comp. sh. (BEA) is the share of office computing and accounting machines (OCAM) in total investment in the years 1967 1977. The computer share for 1987 for the BEA is linearly extrapolated from the computer shares in 1982 and 1992 from the same source. **Comp. sh. (BBG)** is the share of computers in total investment in 1977 and 1987 as computed by Berman, Bound and Griliches (1994). **Sdl** is the average standard deviation of monthly stock returns during the previous decade. **Txx** are time specific dummies.