Cake Division with Minimal Cuts: Envy-Free Procedures for 3 Persons, 4 Persons, and Beyond[1]

Abstract: The minimal number of parallel cuts required to divide a cake into n pieces is $n-1$. A new 3-person procedure, requiring 2 parallel cuts, is given that produces an envy-free division, whereby each person thinks he or she receives at least a tied-for-largest piece. An extension of this procedure leads to a 4-person division, using 3 parallel cuts, that makes at most one player envious. Finally, a 4-person envy-free procedure is given, but it requires up to 5 parallel cuts, and some pieces may be disconnected. All these procedures improve on extant procedures by using fewer moving knives, making fewer people envious, or using fewer cuts. While the 4-person, 5-cut procedure is complex, endowing people with more information about others' preferences, or allowing them to do things beyond stopping moving knives, may yield simpler procedures for making envy-free divisions with minimal cuts, which are known always to exist.

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