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Abstract

Auction theory has emphasized the importance of private information to the profits of bidders. However, the theory has failed to consider the question of whether or not bidders will be able to keep their information private. We show that in a variety of contexts bidders will reveal all their information, even if this information revelation is (ex ante) detrimental to them. Similarly, a seller may reveal all her information even when this revelation lowers revenues. We also show that bidders may be harmed by private information.

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1 Introduction

An auction with interdependent values involves the sale of a good whose (expected) value to each bidder depends upon public information as well as information privately held by the bidders and the seller. For instance, the value of a painting purportedly by Hyppolite will depend on each party's estimation that the artwork is authentic. Though the idiosyncratic information the various agents possess might initially be private, much of it may be *verifiable* and nothing prevents the agents from revealing such information if

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they choose to do so. Indeed, it is well known that in a symmetric auction, when the agents' signals are affiliated[1] *i*) if the seller can publicly commit to a revelation policy she will maximize ex ante revenue by committing to *always* reveal her information (Milgrom and Weber (1982*a*)) and *ii*) even if the seller cannot make a such a commitment, she will always reveal her information in a perfect Bayesian equilibrium (Milgrom (1987)). In contrast, there has been little investigation into the revelation behavior of the buyers. Perhaps this paucity stems from the belief that "it is more important to a bidder that his information be private than that it be precise." (McAfee and McMillan (1987)).

However, even if it is true that bidders profit from the privacy of their information, it does not follow that they will be able to refrain from revealing it. Suppose that signals are affiliated. Even if, say, bidder 1 favors an ex ante policy of never revealing his signal, ex post he may well prefer to conceal highly positive signals, but reveal very negative signals. This is because a negative signal has the potential to depress the bids of the other players, both because their valuations of the object have fallen and because they expect the other players to lower their bids. Thus, absent the possibility of commitment, in many cases bidder 1 will in fact reveal dismal information. But if the other players know that bidder 1 is acting thus, he will be "forced" to reveal moderately poor information as well, since this information becomes dismal relative to the possibilities the other players entertain if no disclosure is made. The argument can be reapplied iteratively, so that the bidder ends up revealing even positive signals.

Though this type of *unraveling* argument is familiar in other contexts, the fact that bidders will often deleteriously reveal their information may have escaped attention because they will not necessarily do so in the simplest models of common value auctions. However, these models are misleading in this regard. Indeed, we will argue that they are discontinuous in the sense that slight modeling changes can lead from a situation of no information revelation to one of almost complete revelation. Private information will also be revealed in very different contexts, including some pure private value auctions.

The literature on auction theory has emphasized the benefits of private information to buyers. Typical comments include "A bidder without special

[1] Roughly speaking, high values of one agent's estimates make high values of the other agents' estimates more likely.

private information ... can never earn a positive expected payoff" (Milgrom (1981)) and "the winning bidder's surplus is due to her private information" (Klemperer (1999)). However, this literature is at best incomplete since it has not considered whether or not bidders will be able to keep their information private, at least when it is verifiable. Moreover, in many important auctions much information is indeed verifiable. For instance, telecommunications firms bidding on licenses often hire consultants to help them estimate the value of these licenses. The consultants' reports can easily be made public. Similarly, geological reports about oil tracts to be auctioned off can be disseminated.

Furthermore, there is more than just a lacuna in the theory: a bidder may actually be harmed by private information. This finding is in sharp contrast with the received theory, as illustrated by Milgrom and Weber's (1982b) finding that a "bidder's profits rise when he gathers extra information" (absent the possibility of information revelation).

2 No Revelation

We begin with a standard pure common value model in which it is an equilibrium for the players not to reveal any information. We then modify the game slightly to obtain a game where full information revelation is the unique outcome.

There are 2 risk-neutral bidders. The value of the good to both bidders is given by $V = v(X_1, X_2)$, where X_i is player i's private signal. Consider, say, a simple first-price sealed-bid auction. Under weak conditions the game has a positive value to each player. Suppose the signals are verifiable and alter the game by adding a preliminary stage in which either bidder can reveal her signal. A bidder that chooses to disclose her information earns zero in the ensuing auction, regardless of the revelation policy of the other bidders (Milgrom and Weber (1982b)). Thus, neither player has an incentive to divulge any realization of a signal, favorable or unfavorable. This conclusion is quite misleading, however. It depends upon the fact that a player with no private information always earns zero. This fact itself is driven by at least two aspects of the model.

i) **No Private Component.** As the name suggests, in a pure common value auction there is no private component to the bidders' valuations. With respect to mineral rights, Milgrom and Weber (1982a, p. 1093) argue that this simplification is appropriate since "To a first approximation, the values of these mineral rights to the various bidders can be regarded as equal." However, while this first approximation is harmless for the usual analytical purposes it is deceptive when considering the disclosure of information.

ii) **Imprecision.** Each player's information is either precisely revealed or not revealed at all. Players must reveal their information in a verifiable manner, which limits the strategies available to them. Nonetheless, the implicit assumption that players must end up disclosing all or none of their information is unduly restrictive. Consider an auction for an oil tract. A bidder investigating the worth of the tract might first receive a preliminary estimate indicating whether the value of the tract is say, low, medium, or high and then a more accurate indication within the initial category. Since the latter information dominates the former, it might seem without loss of generality to ignore the first signal. However, explicit recognition of the two signals allows the bidder to reveal only the initial estimate, which, as we shall see, is important.

Modifying either of these aspects can have dramatic consequences. In the next section we illustrate this claim with an example that we will reconsider in greater detail in Section 4.1.

2.1 Full Revelation: An Example

Consider a good worth $z_1 + w$ to player 1 and w to player 2. The private component z_1 is common knowledge, but only player 1 is informed of the signal w, which is drawn from the uniform distribution on $[0, 1]$. The good is sold via a first-price sealed-bid auction in which player 1 wins the good if his bid is at least as large as player 2's bid. First suppose that $z_1 = 0$.[2] The auction has a unique equilibrium in which player 1 bids $\frac{1}{2}w$ and player 2 bids $b \in \left[0, \frac{1}{2}\right]$ with cumulative distribution $2b$. Player 1 receives an ex ante payoff of $\frac{1}{6}$. Furthermore, given any realization of $w > 0$, player 1 earns a strictly positive (expected) payoff. Now give player 1 the opportunity to disclose his signal w. If he does so, both players bid w in the ensuing auction, yielding 1 a payoff of 0. Thus player 1 has no incentive to disclose any realized signal and there is an equilibrium in which player 1 refrains from ever making such a disclosure. Note that a policy of disclosing all his signals would earn player 1 an ex ante payoff of 0.

Now suppose that z_1 is arbitrarily small but strictly positive. The first-

[2]Engelbrecht-Wiggans et al (1983) solve the model for this $z_1 = 0$ case.

price auction has an equilibrium which approximates the $z_1 = 0$ equilibrium, namely, player 1 bids max $\left[z_1, \frac{1}{2}w\right]$ and player 2 bids $b \in \left[z_1, \frac{1}{2}\right]$ with cumulative distribution approximately $2b$.[3] Player 1 receives an ex ante payoff of approximately $\frac{1}{6}$. Again give player 1 the opportunity to disclose his signal w. If he does so, both players again bid w in the ensuing auction's (undominated) equilibrium, yielding player 1 a payoff of $z_1 \approx 0$. A policy of disclosing all signals again harms player 1, earning him an ex ante payoff of approximately 0. Thus far, our analysis of the case $z_1 \approx 0$ mirrors our analysis of the $z_1 = 0$ case. There is, however, a crucial difference.

Suppose that player 1 has received a signal $w \leq z_1$. If 1 discloses this signal, in the ensuing auction player 2 bids w instead of randomizing between $z_1 \geq w$ and higher bids. Hence, player 1 will in fact disclose any signal $w \leq z_1$, thereby earning z_1 instead of strictly less. By continuity, there exists a $\underline{w} > z_1$ such that player 1 will also disclose all $w < \underline{w}$. But then if player 1 does not disclose a signal w, player 2 knows that $w \geq \underline{w}$, and in the ensuing equilibrium 2 randomizes among bids $\underline{w}$ and above. Player 1 benefits from disclosing $\underline{w}$ and all signals $w < \underline{w}'$ for some $\underline{w}' > \underline{w}$. The argument can be reapplied, leading to the conclusion that in any equilibrium, player 1 essentially reveals all his information despite the fact that this is ex ante detrimental to him.

3 A General Framework

In this section we develop a general framework for analyzing information revelation in auctions. The reader who is only interested in the applications can skip this section and proceed directly to Section 4.

In a fairly general auction setting, there are n players each of whom receives a verifiable private signal X_i drawn from the joint distribution F. A good whose value to player i is $v_i(X_1, X_2, \ldots, X_n)$ is to be auctioned off. At the interim stage in which i has seen a signal x_i, but before the auction takes place, i has an expected equilibrium payoff which we can write as $\hat{u}_i(x_i, F)$ (if the auction has multiple equilibria, assume that some selection has been made).

Now suppose that each player is given the option of disclosing her signal before playing the auction. Since the signal is verifiable, its disclosure must be truthful. We have the following game:

[3] A precise description is given in Section 4.1.

1. Nature chooses $(x_1, x_2, ..., x_n)$ from F; player i is informed only of x_i.
2. Each player i reports $t_i \in \{x_i, \emptyset\}$.
3. The good is auctioned off.

In effect, the disclosure option in stage 2 changes the joint distribution from which the signals are drawn for the auction in stage 3. In the overall equilibrium of this new game, following the reports the bidders play to an equilibrium of the auction using an updated conditional joint distribution function. Let $r_i : \mathbb{X}_i \to \{\mathbb{X}_i, \emptyset\}$ be a reporting strategy for player i. Given a (presumed) reporting strategy combination r and the (actual) reports t, let $F(\cdot \mid t, r)$ be the joint distribution of x conditional on t and r. In the auction of stage 3, a player i with signal x_i gets a payoff of $\hat{u}_i(x_i, F(\cdot \mid t, r))$, which is simply i's equilibrium payoff in a standard setting where the types are drawn from the distribution $F(\cdot \mid t, r)$.

Now consider stage 2 where player i has seen his own signal x_i, but before the reports of the other players are made public. If the other players follow r, while player i reports t_i, then i has an expected payoff of

$$E_{x_{-i}} \hat{u}_i(x_i, F(\cdot \mid (t_i, r_{-i}(x_{-i})), r)), \tag{1}$$

which is derived by taking an expectation over x_{-i} given x_i, and where $(t_i, r_{-i}(x_{-i})) \equiv (r_1(x_1), ..., t_i, ..., r_n(x_n))$. For instance, suppose $n = 3$ and that the conditional distribution function has an associated density function. Then, player 1 has an expected payoff of

$$\int \int \hat{u}_1(x_1, F(\cdot \mid (t_1, r_2(z_2), r_3(z_3)), r)) f(z_2, z_3 \mid x_1) dz_2 dz_3.$$

A perfect Bayesian equilibrium of this game is a reporting strategy combination r^* in which $t_i = r_i^*(x_i)$ maximizes (1) for all i and all $x_i \in \mathbb{X}_i$.

As a preliminary to the general result of the next section define

$$u_i(x_i, t_i, r) \equiv E_{x_{-i}} \hat{u}_i(x_i, F(\cdot \mid (t_i, r_{-i}(x_{-i})), r)). \tag{2}$$

Then a perfect Bayesian equilibrium is an r^* such that:

$$\begin{aligned} u_i(x_i, r_i^*(x_i), r^*) &\geq u_i(x_i, x_i, r^*) \\ u_i(x_i, r_i^*(x_i), r^*) &\geq u_i(x_i, \emptyset, r^*) \\ \forall i \forall x_i &\in \mathbb{X}_i \end{aligned}$$

3.1 The Disclosure Game

We now derive a general unravelling result which enables us to avoid duplicating unravelling arguments in our various auction applications.

We first define a generic n-person game in which each player i receives a private verifiable signal X_i drawn from a compact set $\mathbb{X}_i \subseteq \mathbf{R}$, and is given the option of (truthfully) disclosing it. A *disclosure game* is the following three stage game:

1. Nature chooses $(x_1, x_2, ..., x_n)$ according to the distribution F; player i is informed only of x_i.
2. Each player i chooses a report $t_i \in \{x_i, \emptyset\}$.
3. Each player i receives a payoff $u_i(x_i, t_i, r)$, where $r_i : \mathbb{X}_i \to \{\mathbb{X}_i, \emptyset\}$ such that $\forall x_i \in \mathbb{X}_i \; r_i(x_i) \in \{x_i, \emptyset\}$.

We can think of r_i as a reporting strategy for player i.

A *disclosure game equilibrium* is an r^* such that

$$\begin{aligned} u_i(x_i, r_i^*(x_i), r^*) &\geq u_i(x_i, x_i, r^*) \\ u_i(x_i, r_i^*(x_i), r^*) &\geq u_i(x_i, \emptyset, r^*) \\ \forall i \forall x_i &\in \mathbb{X}_i \end{aligned}$$

Thus, a disclosure game equilibrium is a reporting strategy combination such that each type of each player maximizes by following the reporting strategy.[4] When u is an auction payoff, as in (2) of the previous section, a disclosure game equilibrium gives a perfect Bayesian equilibrium of the auction preceded by the possibility of disclosure, and vice-versa.

Given a joint distribution F, let F_i be the marginal distribution of X_i. We will (abusively) use F and F_i to denote probability measures as well. Thus, $F_i(X_i > x_i) \equiv 1 - F_i(x_i)$, while $F_i\{X_i = x_i\} = \Pr_{F_i}(X_i = x_i)$. As usual, $F(\cdot \mid t, r)$ denotes the joint distribution conditional on t and r. Correspondingly, $F_i(\cdot \mid t_i, r_i)$ is the marginal distribution conditional on i's report and reporting strategy. Observe that since x_i is verifiable, $F_i(X_i = x_i \mid x_i, r_i) \equiv \Pr_{F_i(\cdot \mid x_i, r_i)}(X_i = x_i) = 1$ regardless of r_i.

[4] Note that a disclosure game equilibrium is not equivalent to a Nash equilibrium of the disclosure game.

Given a strategy profile r, we say that player i's signal is *almost surely known* if either $r_i(x_i) = x_i$ for almost all $x_i \in X_i$, or $F_i(X_i = x_i \mid \emptyset, r_i) = 1$ for some $x_i \in X_i$. That is, the set of undisclosed signals with positive measure is at most a singleton.

We now give a sufficient condition for player i to essentially disclose all her information. Theorem 1 says that if player i always wants to disclose a signal at the bottom of the support of her signals, then she will essentially reveal (almost) all her signals.

Theorem 1 *Suppose that for all $x_i \in \mathbb{X}_i$ and reporting strategies r, $u_i(x_i, x_i, r) > u_i(x_i, \emptyset, r)$, whenever $\min\{Support\ F_i(\cdot \mid \emptyset, r_i)\} = x_i$ and $F_i(X_i > x_i \mid \emptyset, r_i) > 0$. Suppose further that $u_i(x_i, x_i, r)$ and $u_i(x_i, \emptyset, r)$ are continuous functions of x_i. Then player i's signal is almost surely known in any disclosure game equilibrium.*

Proof. All proofs are in the appendix ■

The above theorem is a general result about unravelling. In contrast to most results in the literature about unravelling (for instance, Grossman (1981) and Milgrom and Roberts (1986)), it covers the case of many informed parties. The previous result which is most similar to Theorem 1 is in Okuno-Fujiwara et al. (1990), where several informed parties play a revelation stage, and then a game amongst themselves. Both our result and theirs show that strategic considerations do not alter the standard result that full revelation obtains. Our result, however, does not assume that the signals are independent – a particularly poor assumption in an interdependent value auction – or that they are drawn from a finite space.

4 Applications

Though Theorem 1 is straightforward, applying it to auctions is a bit tricky as auctions with disclosure possibilities are inherently asymmetric, and it is often difficult to provide closed-form equilibrium characterizations in such games. For the most part we will consider variants of the following simple two-player pure common value first-price sealed bid auction.[5]

[5]This model was introduced by Wilson (1967) and studied later by Wilson (1975), Weverburgh (1979) and Engelbrecht-Wiggans et al. (1983).

1. Player 1 receives a verifiable signal $W \in [w_m, w_M]$ drawn from the atomless distribution function $F(W)$.

2. Player 1 submits a bid $b_1(w)$ and player 2 submits a bid b_2. Player 1 wins the good if and only if $b_1 \geq b_2$. The payoff to player i is $w - b_i$ if he wins the object and 0 otherwise.

Note that player 1, and only player 1, is perfectly informed of the value of the object. Henceforth we refer to this game as the *one-sided common value game*. Engelbrecht-Wiggans et al (1983) show that this game has a unique equilibrium in which player 1 chooses

$$b_1(w, F) = E_F[W \mid W \leq w] \tag{3}$$

and player 2 chooses $b_2(F)$ from $[0, E_F[W]]$ according to the distribution H defined by

$$H(b_2(F)) = \Pr[b_1(w, F) \leq b_2(F)]. \tag{4}$$

Given w, equilibrium payoffs are

$$\begin{array}{l} \hat{u}_1(w, F) = F(w)(w - b_1(w, F)) \\ \hat{u}_2(F) = 0 \end{array} \tag{5}$$

We note that expression (5) remains valid even if F is not atomless.

We now give player 1 the opportunity to disclose his information. Since the signal is verifiable, the disclosure must be truthful. The players engage in the following game:

1. Player 1 receives a signal $w \in [w_m, w_M]$ according to the distribution function $F(w)$.

2. Player 1 chooses whether or not to disclose his signal w.

3. Each player submits a bid. Payoffs are:

$$u(b_1, b_2) = \begin{cases} (w - b_1, 0) & \text{if } b_1 \geq b_2 \\ (0, w - b_2) & \text{if } b_1 < b_2 \end{cases}$$

We will refer to this game as the *one-sided common value disclosure game*. Player 1 never has an incentive to reveal his information in this game, since if he does both players bid w, resulting in a payoff of 0 to him. Thus there is a perfect Bayesian equilibrium in which no disclosure takes place, so that the addition of the revelation stage 2 is irrelevant. However, as discussed in Section 2, this pure common value model is misleadingly restrictive in at least two respects. The next two subsections address these respects.

4.1 Private Component

In the one-sided common value disclosure game, player 1 always ends up with a profit of zero when he discloses his signal. Crucial to this result is the (extreme) assumption that the value of the good is the same to both players. In this section we show that a continuous departure from this assumption can have a discontinuous impact upon equilibrium behavior. Specifically, we add a (possibly small) private component to 1's valuation of the good. We assume that this private component is common knowledge so that no new informational considerations are introduced. The one-sided common value game is usually considered to be a reasonable model of the auction of an oil tract in which player 1 has a neighboring tract, and thus superior information. The added private component can be thought of as an independent benefit player 1 would obtain from owning adjacent land, say from reduced clean-up costs.

In the modified game, when player 1 wins the good with a bid of $b_1 \geq b_2$ his payoff is $z_1 + w - b_1$; when player 2 wins the good with a bid of $b_2 > b_1$ her payoff is (still) $w - b_2$. As before, player 1's only private information is w, which is drawn from the distribution F; the parameter $z_1 > 0$ is common knowledge. When z_1 is small this game "approximates" the one-sided common value game; in particular, revealing w yields player 1 about zero. Nonetheless, adding this component has drastic consequences when player 1 is given the option to disclose his signal. Consider the following game.

1. Player 1 receives a signal $w \in [w_m, w_M]$ according to the distribution function F.

2. Player 1 chooses whether or not to disclose his signal w.

3. Each player submits a bid. Payoffs are:

$$u(b_1, b_2) = \begin{cases} (z_1 + w - b_1, 0) & \text{if } b_1 \geq b_2 \\ (0, w - b_2) & \text{if } b_1 < b_2 \end{cases}$$

We assume that players use undominated strategies.

Recall that player 1's signal is *almost surely known* if *i*) player 1 discloses almost all signals in stage 2, or *ii*) the set of undisclosed signals with positive measure is at most a singleton.

Proposition 1 *In any perfect Bayesian equilibrium of the above game, player 1's signal is almost surely known.*

The proposition is proved in the appendix by defining the appropriate disclosure game and applying Theorem 1. In essence, player 1 always strictly wants to disclose a signal at the bottom of the support of his signals and an unravelling ensues. One subtlety is worth observing. When Player 1 discloses a signal at the bottom of the support, player 2's bids are lowered in the first order stochastic domination sense. However, when 1 reveals signals close to the bottom, while 2's bids are "mostly" lowered, they are not lowered in the same first order sense. In order to conclude that player 1 wants to disclose these bids as well (and hence obtain full unravelling) it is important that player 1 *strictly* prefers revealing the bottom signal – hence the role of z_1.

Note the discontinuity. For all $z_1 > 0$, player 1's signal is almost surely known in any equilibrium and as z_1 tends to 0, so do player 1's profits. On the other hand, when $z_1 = 0$ there is an equilibrium in which player 1 conceals all signals and earns a positive profit.

Let us return to the specific example of Section 2.1, where $w \sim U[0,1]$ and $0 < z_1 < \frac{1}{2}$. In the standard sealed-bid auction in which player 1 is not given the disclosure option, the equilibrium[6] is that player 1 bids

$$\max\left[z_1, \frac{1}{2}w\right],$$

while player 2 bids $b \in \left[z_1, \frac{1}{2}\right]$ with cumulative distribution

$$\frac{2z_1}{2z_1+1} + \frac{2}{2z_1+1}b.$$

Player 1's ex ante payoff is

$$\frac{1}{6}\frac{-8z_1^3 + 12z_1^2 + 6z_1 + 1}{2z_1+1}.$$

[6]We believe this is the unique equilibrium of the sealed bid auction, though we have not proved this. As z_1 approaches 0, this equilibrium converges to player 1 bidding $\frac{1}{2}w$ and player 2 bidding $b \in \left[0, \frac{1}{2}\right]$ with cumulative distribution $2b$, which is the equilibrium of the auction in which $z_1 = 0$.

On the other hand, when given the possibility to reveal his signal, player 1 does so, yielding him a payoff of z_1.[7] Note that

$$\frac{1}{6}\frac{-8z_1^3+12z_1^2+6z_1+1}{2z_1+1}>z_1$$

In particular, when $z_1=0$ the left hand side is $\frac{1}{6}$. This is consistent with the general belief in the literature that a player is harmed by relinquishing his private information. Nonetheless, he relinquishes it.

Since $z_1>0$ it is common knowledge that the good is worth more to player 1 than player 2. We now show that the fact that *some* revelation must take place does not depend upon this feature.

Suppose that z_1 is an unverifiable random variable which may be positive or negative. Specifically, consider the game

1. Player 1 receives the signals z_1 and w, where w has support $[0,1]$ and $z_1=\begin{cases} a>0 & \text{with probability } p\in(0,1) \\ -a & \text{with probability } 1-p \end{cases}$
2. Player 1 chooses whether or not to disclose his signal w.
3. A good worth z_1+w to player 1 and w to player 2 is sold via a sealed-bid first-price auction. If $b_1=b_2$ player 1 decides who gets the good.

We are uncertain whether or not full revelation is necessary in this game (though our belief is that it is not). In any case, as we now show, there must be a positive measure of revelation.

Suppose on the contrary that $r(w)=\emptyset$ for almost all $w\in W$.

i) We first establish that in the auction equilibrium following a report of $\emptyset$, $p(b_2=0)\neq 1$. Suppose on the contrary that $p(b_2=0)=1$. Player 1's best response to this is to always bid 0. But, given 1's response, player 2 is better off with a strategy of $p(b_2=\varepsilon)=1$ for small enough ε.

ii) Thus, following a report of $\emptyset$, $p(b_2=0)<1$. Following a report of w, in any undominated equilibrium, player 2 bids at most w. For small enough w, player 1 should disclose when $z_1=a$, so that $r(w)=w$ for a positive measure of $w\in W$.

[7] We note that even if it turns out that the sealed-bid auction has other equilibria, player 1's signal is still revealed. The proof of Proposition 1 given in the appendix is not tied to any specific equilibrium.

4.2 Imprecision

In the previous section we saw that a player will reveal his information if there is some private component to the good's value. In this section, we show that absent a private component to the valuation, a player still fully reveals his information if a fraction of the information remains private. This is true even if the fraction of private information is arbitrarily small.

In the one-sided common value game, suppose that player 1 first receives an estimate of the good's value. Specifically, 1 receives a verifiable signal telling him in which one of n equal intervals the value of the object lies. After choosing whether or not to disclose this information, he receives another signal telling him the exact value. The bidders then play a first-price sealed-bid auction as before. For convenience we assume that $w_m = 0$ and $w_M = 1$, and that F has an associated density function f which is strictly positive on $[0, 1]$. Formally, we have:

1. Player 1 receives a signal $x \in \{0, 1, \ldots, n-1\}$ with probability $F\left(\frac{x+1}{n}\right) - F\left(\frac{x}{n}\right)$.

2. Player 1 chooses whether or not to reveal x.

3. Player 1 receives a signal $w \in \left[\frac{x}{n}, \frac{x+1}{n}\right]$ according to F.

4. Each player submits a bid. Payoffs are:

$$u(b_1, b_2) = \begin{cases} (w - b_1, 0) & \text{if } b_1 \geq b_2 \\ (0, w - b_2) & \text{if } b_1 < b_2 \end{cases}$$

For large n, this game approximates the one-sided common value disclosure game. However, equilibrium behavior is quite different.

Proposition 2 *In any perfect Bayesian equilibrium of this game, player* 1*'s signal is almost surely known.*

An implication of Proposition 2 is that as n approaches infinity, the disclosure game yields player 1 an arbitrarily small profit. In contrast, if player 1 is somehow prevented from divulging his signal he earns a positive profit which is independent of n.

4.3 Pure Private Values

One reason for believing that a player benefits from having private information in an interdependent value auction is that winner's curse fears depress rivals' bids. In this section we analyze a pure private values setting in which there are no such fears. We find that while the players reveal their information, they are indeed not harmed by this revelation.

Consider a pure private values second-price auction where, in addition to information about his own valuation, each player has private information about the other player's valuation. Specifically, $v_1 = z_1 + x_2$ and $v_2 = z_2 + x_1$, where player i observes (z_i, x_i). Following the disclosure decisions, the players engage in a second-price sealed-bid auction, where it is a dominant strategy for each player to bid his (expected) value.

We define a disclosure game in which each player receives his expected payoff from a second-price auction. For ease of exposition (in the appendix), we set $z_1 = z_2 = 0$.

1. Nature chooses $x_i \in [x_m, x_M]$ according to the strictly increasing atomless distribution function F_i, for $i = 1, 2$; player i is informed only of x_i.

2. Player i chooses whether or not to disclose x_i.

3. Each player submits a bid. Payoffs are:

$$u(b_1, b_2) = \begin{cases} (x_2 - b_2, 0) & \text{if } b_1 \geq b_2 \\ (0, x_1 - b_1) & \text{if } b_1 < b_2 \end{cases}$$

Proposition 3 *In any perfect Bayesian equilibrium of this game, the players' signals are almost surely known. Furthermore, each player is better off almost fully disclosing his signals than not disclosing any signals (regardless of the disclosure policy of the other player).*

While the players are not harmed by the disclosure of their information in this pure private values game, it is not always true that with pure private values, full disclosure does not harm a player. For instance, consider a first-price sealed-bid auction of a good worth x_i to player $i = 1, 2$, where the the x_i' are i.i.d $U[0, 1]$. Player 1's payoff to this game is $\frac{1}{6}$. On the other hand, if his signal is always revealed he has a payoff of $\frac{1}{8}$ (Vickrey (1961) analyzes the game in which player 1's signal is known). We note, however, that in this game player 1 is not "forced" to reveal his signal. That is, there is an equilibrium of the appropriate disclosure game in which no disclosure is made.[8]

4.4 Seller Revelation

Recall that when the seller receives a signal in a symmetric affiliated signals auction, then *i)* she maximizes ex ante revenues by committing to reveal the signal and *ii)* even if she cannot make a such a commitment, she reveals the signal in the appropriate disclosure game. At this point the reader may suspect that, despite appearances, these two statements are essentially unrelated. The following asymmetric example confirms this suspicion.

There are two bidders who receive unverifiable private signals x_1 and x_2 and a seller who receives a verifiable private signal s; all signals are drawn from distributions with support $[0, 1]$. The valuations of the bidders are,

$$\begin{aligned} v_1(x_1, x_2, s) &= x_1 + \alpha(x_2 + s) \\ v_2(x_1, x_2, s) &= x_2, \end{aligned}$$

where $\alpha \in \left(0, \frac{1}{2}\right)$. Suppose the good is sold using a second-price sealed-bid auction. Krishna (2002) shows[9] that if S is *never* disclosed the equilibrium price is

$$P^N = \min\left\{\frac{1}{1-\alpha}X_1 + \frac{\alpha}{1-\alpha}E[S], X_2\right\},$$

whereas when S is disclosed the equilibrium price is

$$P^S = \min\left\{\frac{1}{1-\alpha}X_1 + \frac{\alpha}{1-\alpha}S, X_2\right\}.$$

As Krishna observes, $E\left[P^S\right] < E\left[P^N\right]$ so that here full disclosure is detrimental to the seller. Nevertheless, as we now show, absent commitment possibilities the seller still fully discloses.

Consider the following game:

[8] This follows from the fact that player 1 is ex post harmed by the revelation of any signal $x_i > 0$.

[9] Krisha (2002) makes further distributional assumptions on the game, but these are not necessary.

1. Bidder i receives a signal x_i, and the seller receives a signal s. The signals are independently drawn from distributions with support $[0, 1]$.

2. The seller chooses whether or not to disclose his signal s.

3. Each bidder submits a bid. Payoffs to the bidders are

$$u_b(b_1, b_2, s) = \begin{cases} (x_1 + \alpha(x_2 + s) - b_2, 0) & \text{if } b_1 \geq b_2 \\ (0, x_2 - b_1) & \text{if } b_1 < b_2 \end{cases}$$

while the payoff to the seller is

$$u_s(b_1, b_2, s) = \min(b_1, b_2)$$

Proposition 4 *In any perfect Bayesian equilibrium of this game, the seller's signal is almost surely known.*

5 Inducing Disclosure

In a pure common value auction, a player with no private information always earns zero profit; full disclosure is not inevitable, since it is never (strictly) beneficial for a player to reveal his signal. Nonetheless, we now show that the seller can "force" full revelation by providing an arbitrarily small payment. We do this in a *general* interdependent value setting, where the good may or may not have a common value.

Consider a good worth $v_1(x_1, x_2)$ to player 1 and $v_2(x_1, x_2)$ to player 2, where each v_i is increasing in both arguments. The good is sold using a second price auction. The seller offers to pay $\varepsilon > 0$ to any player that reveals his information. Formally, we have

1. Nature chooses the signal x_i according to the distribution F_i. Player i is informed of x_i.

2. Player i chooses whether or not to disclose his signal. Specifically, i chooses $t_i \in \{x_i, \emptyset\}$, where $t_i = \emptyset$ indicates that i makes no disclosure.

3. Each player submits a bid. Payoffs are:

$$\begin{aligned} u(b_1, b_2) &= \begin{cases} (v_1(x_1, x_2) - b_2 + \varepsilon(t_1), \varepsilon(t_2)) & \text{if } b_1 \geq b_2 \\ (\varepsilon(t_1), v_2(x_1, x_2) - b_1 + \varepsilon(t_2)) & \text{if } b_1 < b_2 \end{cases} \\ \text{where } \varepsilon(t_i) &= \begin{cases} \varepsilon & \text{if } t_i = x_i \\ 0 & \text{if } t_i = \emptyset \end{cases} \end{aligned}$$

Proposition 5 *In any undominated perfect Bayesian equilibrium of the above game, both players' signals are almost surely known.*

As an example, suppose that $v_1(x_1, x_2) = v_2(x_1, x_2) = v(x_1, x_2)$. If the seller does not offer a payment ($\varepsilon = 0$), there is a perfect Bayesian equilibrium in which no signals are disclosed. The symmetric equilibrium strategies in the auction phase call for i to bid $v(x_i, x_i)$. For any realization of signals, the seller's profit is $\min_{i=\{1,2\}} v(x_i, x_i)$. On the other hand, if the seller offers $\varepsilon > 0$, (essentially) all signals are known. The buyers bid $v(x_1, x_2)$ and the seller's profit is $v(x_1, x_2) - \varepsilon \approx v(x_1, x_2) > \min v(x_i, x_i)$ for small ε. Thus, in this pure common value case, offering a payment of ε results in a "virtually optimal" auction (the seller extracts virtually all the surplus). Furthermore, offering a small payment can be used to design virtually optimal auctions in many common value settings, of which the one-sided common value game of Engelbrecht-Wiggans et al is one example.

6 Harmful Information

We now reconsider the imprecision model of section 4.2 to show that private information may be harmful to a bidder.

There is a single good worth $w \sim U[0, 1]$ to two players. Player 1 has two types of signals potentially available to him, x which indicates in which one of n equal intervals w lies and w itself. He must decide whether to receive the signal x alone, or to receive x and w. In either case, player 2 is informed of player 1's decision.

First suppose that player 1 does not have the option of disclosing his information. Theorem 4 of Milgrom and Weber (1982b) shows that player 1 prefers receiving both x and w to receiving x alone: more information is better.

Now suppose that player 1 has the option of disclosing the signal x upon receiving it. Will he prefer to receive only the estimate x, or x and then the precise signal w? We have already seen that if he receives the two signals, then he will disclose his signal x. On the other hand if he receives only x he never has an incentive to divulge it (since both players then bid $E(w \mid x)$ yielding player 1 zero). Thus, when player 1 receives only one signal there is an equilibrium in which no information is revealed. This non-disclosure equilibrium yields player 1 a higher payoff than the disclosure equilibrium, so that he prefers *less* information to more. In particular, as the number of intervals $n \to \infty$, player 1's equilibrium payoff approaches $\frac{1}{6}$ when he does not disclose whereas his payoff approaches 0 when he always discloses, as he does when he receives two signals.

Milgrom and Weber (1982*b*) also argue that a bidder would rather gather information on the value of an item overtly than covertly. Their intuition is that overt information gathering induces a fear of the winner's curse, which causes the other players to bid timidly. Hence they would expect a specialist to loudly proclaim his presence at an auction. Our intuition is quite different. The other players will not fear the winner's curse as they know that the specialist will disclose his information (unless, possibly, it is highly favorable). Our specialist would prefer to send an anonymous proxy to do his bidding.

In the present context, suppose that both players know that player 1 knows x. With no disclosure possibilities, if player 1 is to receive the signal w as well, he wants player 2 to be aware of this. With disclosure possibilities, he prefers that player 2 be unaware that he has the extra information.[10]

7 Multiplicity

We have emphasized games in which the disclosure behavior is essentially unique. We now consider multiplicity.

In the imprecision game of Section 4.2, a bidder receives two signals pertaining to a good's value – an initial estimate x, which he has the option of disclosing, followed by an exact indication w. In any equilibrium the bidder almost surely reveals his estimate.

We now modify the sequential nature of the signals by supposing that player 1 receives both the estimate and the exact signal before he has the opportunity to report the estimate, or the exact signal. With this change, the game has many perfect Bayesian equilibria because player 2 can "punish" player 1 with his beliefs. For instance, for all $0 \leq j < n-1$ there is an equilibrium in which player 1 always discloses x_1^k for $k = 0, ..., j$ and never discloses for $k = j+1, ..., n-1$; off the equilibrium path player 2 believes that $w = \frac{k+1}{n}$ if player 1 discloses x_1^k for $k = j+1, ..., n$. Essentially, player 2 keeps player 1 from disclosing by threatening to believe that only the highest

[10] In Section 7.1 we consider a model in which player 2 is uncertain as to whether or not player 1 is informed.

type within an interval discloses.[11]

Consider again the one-sided common value disclosure game of section 4. Suppose that w is drawn uniformly from $[0,1]$. There is a perfect Bayesian equilibrium in which player 1 never discloses his signal. Given a realization w', he earns $\frac{1}{2}w'^2$. A disclosure of w' would have earned player 1 zero. Although disclosing his signal is never beneficial to 1, there is a perfect Bayesian equilibrium of this game in which player 1 always discloses his signal, thereby earning 0. This equilibrium is supported by an out-of-equilibrium belief of player 2 that $w = 1$ if no disclosure is made. Thus, the discontinuities we noted in Sections 4.1 and 4.2 were failures of lower hemicontinuity.

This last example suggests that the conditions necessary for full disclosure to obtain in some equilibrium, as opposed to all equilibria, are quite weak. In an auction with affiliated signals it is typically bad for player i if the other players think that he has received a high signal, since they then bid high. Thus, suppose the signals are drawn from compact intervals and that each player's payoffs are minimized when the other players believe that he has received his highest signal. Then there is a Perfect Bayesian equilibrium in which all players fully disclose, and silence by a player is interpreted to mean that he has received his highest signal. In disclosure game terms, we have the following theorem.

Theorem 2 *Suppose that for each i there is a signal $\bar{x}_i \in X_i$ such that for all $x_i \in X_i$ and reporting strategies r, $u_i(x_i, x_i, r) \geq u_i(x_i, \emptyset, r)$ whenever $F_i(X_i = \bar{x}_i \mid \emptyset, r_i) = 1$. Then there exists a disclosure game equilibrium in which all signals are disclosed.*

7.1 Partial Revelation

If an agent always wants to disclose a signal that is at the bottom of the support of his signals, then he will fully reveal his information in equilibrium; that is the content of Theorem 1. The required condition is fairly strong, but so is the conclusion. If we are only interested in knowing when *some*

[11] There seems to be no good reason for player 2 to have such a belief, but this game does not have enough structure for refinements such as sequential equilibrium to be of any use. We note that if player 2 adopts the neutral posture of viewing out of equilibrium moves as being mistakes that could just easily have been made by any player, then full disclosure is the unique outcome.

revelation must take place, then it is sufficient that some agent wants to disclose starting from a situation of no disclosure.

We now consider an example with only partial revelation.

Consider a good worth x_1 to player 1 and x_2 to player 2. The x_i's are independently drawn from $U[0,1]$. Player 1 observes x_1 and with probability $\frac{1}{2}$ observes x_2 as well. He has the option of disclosing x_2. Player 2 makes no observation. Following player 1's disclosure decision a second-price sealed-bid auction takes place. Note that for the first time it is not common knowledge that an informed agent is informed. This extension of the model is of independent importance.

We first establish that player 1 must disclose some (positive measure of) signals whenever $x_1 > 0$. If he does not, player 2 will bid $\frac{1}{2}$ in the auction – her expected value – and player 1 would then prefer to disclose any $x_2 < \frac{1}{2}$ whenever $x_1 > \frac{1}{2}$. On the other hand, player 1 will not disclose all his signals. If he did, following no revelation 2 would bid $\frac{1}{2}$ on the presumption that 1 had not observed x_2, and so high types of player 1 would prefer to conceal all $x_2 > \frac{1}{2}$.

The game has many partial revelation equilibria. For instance, in one perfect Bayesian equilibrium, player 1 reveals all $x_2 \leq 2 - \sqrt{2}$ and conceals higher signals.

8 Conclusion

There seems to be a consensus in auction theory that bidders derive their profits from private information. In this paper we argue that the analysis of this issue has been incomplete. The literature has considered the question of the value of information to a bidder under the implicit assumption that she will be able to keep her information private. However, this ability needs to be demonstrated. In fact, as we have shown, in a variety of contexts the bidders' information will be revealed. This revelation may be complete, or partial. In any case, there is little justification for the presumption that no information will be revealed. At the same time, the well-known result that a seller will reveal her information is unrelated to the ex ante profitability of this revelation.

We offer the following as a real world example of information disclosure by bidders. In 1993 and 1994, one of us worked with a stock brokerage firm in Uruguay. Before every auction of Government bonds, the largest participants in the market (most notably 3 banks, and 3 stockbrokers) hired the services of economics consulting firms which, among other things, assessed the "quality" or "value" of the bonds. Every market participant knew who the economic advisors of each firm was and any information that could be shared by bidders prior to the auction was of a verifiable nature. Invariably, in the hours that preceded the submission of the bids, the market participants would call each other up and share the information that their consultants had communicated. Information was never shared regarding the actual prices to be submitted and, to the best of our knowledge, collusion was not an issue in this market.

We interpret this situation as evidence in favor of our model. At the same time, others can doubtlessly list situations where verifiable private information remains private. The question for the theorist, then, is why?

9 Appendix

In this section we provide proofs of the theorems and propositions.

Proof of Theorem 1 . Suppose the conditions of the theorem are met and let r^* be a disclosure game equilibrium. Let $\bar{y}_i$ be defined by

$$\bar{y}_i = \min \left\{ Support F_i \left(\cdot \mid \emptyset, r_i \right) \right\}.$$

Thus, almost every signal below $\bar{y}_i$ is revealed. Two cases must be considered.

Case 1: $F_i \left\{ \bar{y}_i < X_i : r_i^* \left(X_i \right) = X_i \right\} = F_i \left\{ \bar{y}_i < X_i \right\}$. That is, almost every signal above $\bar{y}_i$ is also revealed.

a) $F_i \left\{ X_i = \bar{y}_i \right\} = 0$. In this case, $r_i^* \left(x_i \right) = x_i$ for almost all $x_i \in X_i$.

b) $F_i \left\{ X_i = \bar{y}_i \right\} > 0$. In this case, almost every signal of player i is revealed, with the possible exception of $\bar{y}_i$. If $\bar{y}_i$ is revealed, almost every signal is in fact revealed. If $\bar{y}_i$ is not revealed, $F_i \left(X_i = \bar{y}_i \mid \emptyset, r_{-i} \right) = 1$ since $\bar{y}_i$ is the only positive measure non-disclosing event.

Case 2: $F_i \left\{ \bar{y}_i < X_i : r_i^* \left(X_i \right) = X_i \right\} < F_i \left\{ \bar{y}_i < X_i \right\}$. Then min{Support $F_i \left(\cdot \mid \emptyset, r_i \right)\} = \bar{y}_i$ and $F_i \left(X_i > \bar{y}_i \mid \emptyset, r_{-i} \right) > 0$ so that, by assumption,

$$u_i \left(\bar{y}_i, \bar{y}_i, r^* \right) > u_i \left(\bar{y}_i, \emptyset, r^* \right). \tag{6}$$

Therefore $r_i^* \left(\bar{y}_i \right) = \bar{y}_i$.

a) There exists a $\widehat{y} \in \mathbb{X}_i$, $\widehat{y} > \bar{y}_i$ such that $\left(\bar{y}_i, \widehat{y} \right) \cap \mathbb{X}_i = \emptyset$

Then, $F_i \left\{ X_i < \widehat{y} : r_i^* \left(X_i \right) = X_i \right\} = F_i \left\{ X_i < \widehat{y} \right\}$ and $\bar{y}_i < \widehat{y}$, contradicting the definition of $\bar{y}_i$.

b): There is no $\widehat{y} \in \mathbb{X}_i$, $\widehat{y} > \bar{y}_i$ such that $(\bar{y}_i, \widehat{y}) \cap \mathbb{X}_i = \emptyset$. Since $\mathbb{X}_i$ is compact, for ε small enough $\bar{y}_i + \varepsilon \in \mathbb{X}_i$ and by continuity $u_i(\bar{y}_i + \varepsilon, \bar{y}_i + \varepsilon, r^*) > u_i(\bar{y}_i + \varepsilon, \emptyset, r^*)$. Thus, for small enough ε, $r_i^*(\bar{y}_i + \varepsilon) = \bar{y}_i + \varepsilon$, again contradicting the definition of $\bar{y}_i$. ■

Proof of Proposition 1. We apply Theorem 1. In order to do this, we must define the appropriate disclosure game. Let $\hat{u}_i(w, z_1, F)$ be player i's equilibrium payoff in the first-price sealed-bid auction once player 1 has seen w. If the auction has several equilibria then we choose an equilibrium in undominated strategies. If there are several such equilibria then some selection is made. The disclosure game is:

1. Player 1 receives a signal $w \in [w_m, w_M]$ according to the distribution function F.
2. Player 1 chooses $t \in \{w, \emptyset\}$.
3. Player 1 receives

$$u_1(w, t, r) = \hat{u}_1(w, z_1, F(\cdot \mid t, r))$$

   If the auction in which 1's signal is drawn from $F(\cdot \mid t, r)$ has no equilibrium, we set $\hat{u}_1(w, z_1, F(\cdot \mid t, r)) = 0$.

First consider the first-price sealed-bid auction. Since the object is worth less to player 2 than player 1, and player 2 has no private information, 2 earns 0 in any equilibrium (see Theorem 2, in Engelbrecht-Wiggans et al. (1983)). That is, $E_G \hat{u}_2(w, z_1, G) = 0$ for any distribution function G over w.

In the unique undominated equilibrium of the sealed-bid auction where w is common knowledge, player 1 wins the good for w. Thus, $u_1(w, w, r) = z_1$ for all w, which is continuous.

Now consider the sealed-bid auction where the signals are drawn from $F(\cdot \mid \emptyset, r)$ with $\min\{\text{Support } F(\cdot \mid \emptyset, r)\} = \underline{w}$ and $F(W > \underline{w} \mid \emptyset, r) > 0$. Given the signal $\underline{w}$, if player 1 bids b_1 he earns

$$p(b_1)[z_1 + \underline{w} - b_1],$$

where $p(b_1)$ is the probability that a bid of b_1 wins the object. We now show that $p(b_1)[z_1 + \underline{w} - b_1] < z_1$.

Clearly, $p(b_1)[z_1 + \underline{w} - b_1] \geq z_1$ only if $b_1 \leq \underline{w}$. If $b_1 = \underline{w}$ then it must be that $p(\underline{w}) = 1$. Therefore all of 2's bids are at most $\underline{w}$ and $b_1(w) \leq \underline{w}$ for all w. But this cannot be an equilibrium, since 2 could earn a positive profit with a bid of $\underline{w} + \varepsilon$, for small enough ε.

Therefore, all of 1's winning bids must be strictly below $\underline{w}$. But this cannot be the case either since then player 2 could earn a positive profit with a bid of $\underline{w} - \varepsilon$, for small enough ε.

Hence, $p(b_1)[z_1 + \underline{w} - b_1] < z_1$ so that $u_1(\underline{w}, \emptyset, r) = \hat{u}_1(\underline{w}, z_1, F(\cdot \mid \emptyset, r)) < z_1$ whenever $\min\{\text{Support } F(\cdot \mid \emptyset, r)\} = \underline{w}$ and $F(W > \underline{w} \mid \emptyset, r) > 0$. Also $u_1(w, \emptyset, r)$ is clearly continuous in w.

The conditions of Theorem 1 are met, establishing the proposition. ■

Proof of Proposition 2. We apply Theorem 1. In order to do this, we first define the appropriate disclosure game.

1. Player 1 receives a signal $x \in \{0, 1, \ldots, n-1\}$ with probability $F\left(\frac{x+1}{n}\right) - F\left(\frac{x}{n}\right)$.

2. Player 1 chooses $t \in \{x, \emptyset\}$.

3. Player 1 receives

   $$u_1(x, t, r) = \int_{\frac{x}{n}}^{\frac{x+1}{n}} F(w \mid t, r)(w - b_1(w, F(\cdot \mid t, r))) \frac{f(w)}{F\left(\frac{x+1}{n}\right) - F\left(\frac{x}{n}\right)} dw$$

   In particular, if x is revealed, letting $G^x(w) = \frac{F(w) - F\left(\frac{x}{n}\right)}{F\left(\frac{x+1}{n}\right) - F\left(\frac{x}{n}\right)}$ and g^x its density, we have

   $$u_1(x, x, r) = \int_{\frac{x}{n}}^{\frac{x+1}{n}} G^x(w)(w - b_1(w, G^x)) g^x(w) dw,$$

Suppose that $\min\{\text{Support } F(\cdot \mid \emptyset, r)\} = x$ and $F(X > x \mid \emptyset, r) > 0$. Then $F(\cdot \mid \emptyset, r)$ is the posterior of F conditional on

$$w \in \left[\frac{x}{n}, \frac{x+1}{n}\right] \cup \left[\frac{k_1}{n}, \frac{k_1+1}{n}\right] \cup \ldots \cup \left[\frac{k_t}{n}, \frac{k_t+1}{n}\right] \equiv K,$$

for some $x < k_1 < k_2, < \cdots < k_t$. We have that

$$f(w \mid \emptyset, r) = \begin{cases} \frac{f(w)}{F(K.)} & w \in K. \\ 0 & \text{otherwise} \end{cases}$$

and for $w \in \left[\frac{x}{n}, \frac{x+1}{n}\right]$

$$F(w \mid \emptyset, r) = \frac{F(w) - F\left(\frac{x}{n}\right)}{F(K)}.$$

We first note that $b_1(w, F(\cdot \mid \emptyset, r)) = b_1(w, F(\cdot \mid x, r))$ for all $w \in \left[\frac{x}{n}, \frac{x+1}{n}\right]$. This is so because for all $w \in \left[\frac{x}{n}, \frac{x+1}{n}\right]$,

$$\begin{aligned} b_1(w, F(\cdot \mid x, r)) &= E_{F(\cdot|x,r)}[W \mid W \leq w] = \int_{\frac{x}{n}}^{w} W \frac{f(W \mid x, r)}{F(w \mid x, r)} dW \\ &= \int_{\frac{x}{n}}^{w} W \frac{\frac{f(w)}{F\left(\frac{x+1}{n}\right) - F\left(\frac{x}{n}\right)}}{\frac{F(w) - F\left(\frac{x}{n}\right)}{F\left(\frac{x+1}{n}\right) - F\left(\frac{x}{n}\right)}} dW = \int_{\frac{x}{n}}^{w} W \frac{f(w)}{F(w) - F\left(\frac{x}{n}\right)} dW \\ &= \int_{\frac{x}{n}}^{w} W \frac{\frac{f(w)}{F(K)}}{\frac{F(w) - F\left(\frac{x}{n}\right)}{F(K)}} dW = E_{F(\cdot|\emptyset,r)}[W \mid W \leq w] \\ &= b_1(w, F(\cdot \mid \emptyset, r)) \end{aligned}$$

We have:

$$\begin{aligned} u_1(x, \emptyset, r) &= \int_{\frac{x}{n}}^{\frac{x+1}{n}} F(w \mid \emptyset, r)(w - b_1(w, F(\cdot \mid \emptyset, r))) \frac{f(w)}{F\left(\frac{x+1}{n}\right) - F\left(\frac{x}{n}\right)} dw \\ &= \int_{\frac{x}{n}}^{\frac{x+1}{n}} F(w \mid \emptyset, r)(w - b_1(w, F(\cdot \mid x, r))) \frac{f(w)}{F\left(\frac{x+1}{n}\right) - F\left(\frac{x}{n}\right)} dw \\ &< \int_{\frac{x}{n}}^{\frac{x+1}{n}} F(w \mid x, r)(w - b_1(w, F(\cdot \mid x, r))) \frac{f(w)}{F\left(\frac{x+1}{n}\right) - F\left(\frac{x}{n}\right)} dw \\ &= u_1(x, x, r) \end{aligned}$$

where the inequality follows from the fact that $F(w \mid \emptyset, r) < F(w \mid x, r)$ for all $w > \frac{x}{n}$.

To complete the proof, we note that continuity is trivially satisfied. ■

Proof of Proposition 3. We first note that in the unique undominated equilibrium of a second-price auction, each player bids the conditional expected value of the object.

Let r^* be a reporting equilibrium strategy and suppose that, say, player 1 does not reveal a positive measure of signals. Define

$$\underline{x}_i = \min \text{Support} F_i(\cdot \mid \emptyset, r_i^*).$$

By continuity, player 1 weakly prefers not to reveal $\underline{x}_1$. If 1 discloses $\underline{x}_1$ then 2 bids $\underline{x}_1$, whereas if 1 does not disclose $\underline{x}_1$, 2 bids $E(x_1 \mid \emptyset, r_1^*) > \underline{x}_1$. Since 1 does not benefit from revealing $\underline{x}_1$, he must win the object with probability zero. This implies that for almost every revelation of player 2,player 1 bids weakly less than $\underline{x}_1$, so that $r_2^*(x_2) = \emptyset$ for almost all $x_2 > x_1$. Hence, $\underline{x}_1 \geq \underline{x}_2$. Symmetric reasoning establishes that $\underline{x}_1 = \underline{x}_2$. But then, $i)$ there is a positive probability that player 2 does not disclose and $ii)$ $E(x_2 \mid \emptyset, r_2^*) > \underline{x}_2 = \underline{x}_1$, so that 1 wins the object with positive probability if he discloses $\underline{x}_1$; a contradiction.

We now show that the ex ante payoff to 1 from full disclosure is larger than the payoff from not revealing, regardless of 2's revelation strategy. Let $E_2 \equiv E(X_2 \mid \emptyset)$, $NR \equiv r_2^{-1}(\emptyset)$ and let R be the set of types of 2 that reveals, that is, $R \equiv [x_m, x_M] - r_2^{-1}(\emptyset)$. We have that the payoff to 1 of always revealing is

$$F_2(R) \int_0^1 \int_R \max\{x_2 - x_1, 0\} \frac{f_2(x_2)}{F_2(R)} dx_2 dF_1 + F_2(NR) \int_0^1 \max\{E_2 - x_1, 0\} dF_1 \tag{7}$$

and the payoff to 1 of never revealing is

$$F_2(R) \int_R \max\{x_2 - E_1, 0\} \frac{f_2(x_2)}{F_2(R)} dx_2 + F_2(NR) \max\{E_2 - E_1, 0\} \tag{8}$$

Note that

$$\int_R \max\{x_2 - x_1, 0\} \frac{f_2(x_2)}{F_2(R)} dx_2$$

in (7) is a convex function of x_1, so that the expected value with respect to x_1 is higher than

$$\int_R \max\{x_2 - E_1, 0\} \frac{f_2(x_2)}{F_2(R)} dx_2$$

by Jensen's inequality. Therefore, the first term in (7) is larger than the first term in (8). Similarly, since $\max\{E_2 - x_1, 0\}$ is a convex function of x_1, its expectation is larger than $\max\{E_2 - E_1, 0\}$, and so the second term in (7) is larger than the second term in (8). ■

Proof of Proposition 4. We apply Theorem 1. In order to do this, we first define the appropriate disclosure game.

1. The seller receives a signal $\check{s}$ from the distribution $F(\check{s})$ for $\check{s} \in [0,1]$.

2. The seller reports $t \in \{\check{s}, \emptyset\}$

3. The seller receives

$$u(\check{s}, t, r) = E \min \left\{ \frac{1}{1-\alpha} X_1 + \frac{\alpha}{1-\alpha} E\left[1 - \check{S} \mid t, r\right], X_2 \right\}$$

Suppose $\min\{\text{Support } F(\cdot \mid \emptyset, r)\} = \check{s}$ and $F_i\left(\check{S} > \check{s} \mid \emptyset, r\right) > 0$, so that $E\left[1 - \check{S} \mid \check{s}, r\right] > E\left[1 - \check{S} \mid \emptyset, r\right]$. For large enough X_2 and small enough X_1

$$\begin{aligned} \frac{1}{1-\alpha} X_1 + \frac{\alpha}{1-\alpha} E\left[1 - \check{S} \mid \emptyset, r\right] &< \frac{1}{1-\alpha} X_1 + \frac{\alpha}{1-\alpha} E\left[1 - \check{S} \mid \check{s}, r\right] < X_2 \\ &\Rightarrow u(\check{s}, \check{s}, r) > u(\check{s}, \emptyset, r) \end{aligned}$$

Theorem 1 implies the proposition. ■

Proof of Proposition 5. If player 1's signal, x, is known to player 2 then 2 bids $v_2(x, x_2)$. Suppose that player 2 knows that $x_1 \geq x$. Then 2 bids at least $v_2(x, x_2)$ since for any realization of x, player 2 is glad to win the good with a bid of $v_2(x, x_2)$. Therefore, absent the payment of ε, revealing x is at least as good as not revealing it, and with the payment, revealing x is strictly better. Similarly for player 2's signal. Theorem 1 implies the result. ■

Proof of Theorem 2. For all i, let r_i^* be such that $r_i^*(x_i) = x_i$ for all $x_i \in X_i$, and set $F_i(X_i = \bar{x}_i \mid \emptyset, r_i^*) = 1$. The hypothesis of the theorem implies that $u_i(x_i, x_i, r^*) \geq u_i(x_i, \emptyset, r^*)$ for all x_i, so that r^* is a disclosure game equilibrium. ■

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