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Going from Theory to Practice: The Mixed Success of Approval Voting

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Abstract

Approval voting (AV) is a voting system in which voters can vote for, or approve of, as many candidates as they like in multicandidate elections. In 1987 and 1988, four scientific and engineering societies, collectively comprising several hundred thousand members, used AV for the first time. Since then, about half a dozen other societies have adopted AV. Usually its adoption was seriously debated, but other times pragmatic or political considerations proved decisive in its selection.

While AV has an ancient pedigree, its recent history is the focus of this paper. Ballot data from some of the societies that adopted AV are used to compare theoretical results with experience, including the nature of voting under AV and the kinds of candidates that are elected. Although the use of AV is generally considered to have been successful in the societies—living up to the rhetoric of its proponents—AV has been a controversial reform.

AV is not currently used in any public elections, despite efforts to institute it, so its success should be judged as mixed. The chief reason for its nonadoption in public elections, and by some societies, seems to be a lack of key “insider” support.

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Going from Theory to Practice: The Mixed Success of Approval Voting¹

Steven J. Brams and Peter C. Fishburn

1. Background

Approval voting (AV) is a voting procedure in which voters can vote for, or approve of, as many candidates as they like in multicandidate elections (i.e., those with more than two candidates). Each candidate approved of receives one vote, and the candidate with the most votes wins.

Beginning in 1987, several scientific and engineering societies adopted AV, including the

- Mathematical Association of America (MAA), with about 32,000 members;
- American Mathematical Society (AMS), with about 30,000 members;
- Institute for Operations Research and Management Sciences (INFORMS), with about 12,000 members;
- American Statistical Association (ASA), with about 15,000 members;
- Institute of Electrical and Electronics Engineers (IEEE), with about 377,000 members.

Smaller societies that use AV include the Society for Judgment and Decision Making, the Social Choice and Welfare Society, the International Joint Conference on Artificial Intelligence, and the European Association for Logic, Language and Information.

Additionally, the Econometric Society has used AV (with certain emendations) to elect fellows since 1980 (Gordon, 1981); likewise, since 1981 the selection of members of the National Academy of Sciences (1981) at the final stage of balloting has been based on AV. Coupled with many colleges and universities that now use AV—from the

¹ We thank Richard F. Potthoff for valuable comments. Brams gratefully acknowledges the support of the C.V. Starr Center for Applied Economics at New York University.

departmental level to the school-wide level—it is no exaggeration to say that several hundred thousand individuals have had direct experience with AV.

Probably the best-known official elected by AV today is the secretary-general of the United Nations (Brams and Fishburn, 1983). AV has also been used in internal elections by the political parties in some states, such as Pennsylvania, where a presidential straw poll using AV was conducted by the Democratic State Committee in 1983 (Nagel, 1984).

Bills to implement AV have been introduced in several state legislatures (see section 2). In 1987, a bill to enact AV in certain statewide elections passed the Senate but not the House in North Dakota. In 1990, Oregon used AV in a statewide advisory referendum on school financing, which presented voters with five different options and allowed them to vote for as many as they wished (Wright, 1990).

In the late 1980s, AV was used in some competitive elections in countries in Eastern Europe and the Soviet Union, where it was effectively “disapproval voting,” because voters were permitted to cross off names on ballots but not to vote for candidates (Shabad, 1987; Keller, 1987, 1988; White, 1989; Federal Election Commission, 1989). But this procedure is logically equivalent to AV: candidates not crossed off are, in effect, approved of, although psychologically there is almost surely a difference between approving and disapproving of candidates.

With this information as background, we trace in section 2 our early involvement, and that of several associates, with AV. After outlining the arguments we and others have made for AV, we discuss in section 3 how AV came to be adopted by the different societies.

In section 4, we report on empirical analyses of ballot data of some professional societies that adopted AV; they help to answer the question of when AV can make a difference in the outcome of an election. In section 5, we investigate the extent to which

AV elects “lowest common denominators.” In section 6, we discuss whether voting is “ideological” under AV.

The confrontation between theory and practice offers some interesting lessons on “selling” new ideas. The rhetoric of AV supporters has been opposed not only by those supporting extant systems like plurality voting (PV)—including incumbents elected under PV—but also by those with competing ideas, particularly proponents of other voting systems like the Borda count and the Hare system of single transferable vote.

We conclude that academics probably are not the best sales people for two reasons: (1) they lack the skills and resources, including time, to market their ideas, even when they are practicable; and (2) they squabble among themselves. Because few if any ideas in the social sciences are certifiably “right” under all circumstances, squabbles may well be grounded in serious intellectual differences. Sometimes, however, they are not.

2. Early History and Rhetoric

In 1976, one of us (Brams) was attracted by the concept of “negative voting” (NV), proposed in a brief essay by Boehm (1976) that was passed on to me by the late Oskar Morgenstern. Under NV, voters can either vote for one candidate or against one candidate, but they cannot do both. Independently, Robert J. Weber had begun working on AV (he was apparently the first to coin the term “approval voting”).

When Brams and Weber met in the summer of 1976 at a workshop at Cornell University under the direction of William F. Lucas, it quickly became apparent that NV and AV are equivalent when there are three candidates. Under both systems, a voter can vote for just one candidate. Under NV, a voter who votes against one candidate has the same effect as a voter who votes for the other two candidates under AV. And voting for all three candidates under AV has the same effect as abstaining under both systems.

When there are four candidates, however, AV enables a voter better to express his or her preferences. While voting against one candidate under NV has the same effect as

voting for the other three candidates under AV, there is no equivalent under NV for voting for two of the four candidates. More generally, everything that a voter can do under NV he or she can do under AV, but not vice versa, so AV affords voters more opportunity to express themselves.

Brams and Weber wrote up their results separately, as did three other analysts who worked independently on AV in the 1970s (discussed in Brams and Fishburn, 1983; see also Weber, 1995). But the *idea* of AV did not spring forth full-blown only about 25 years ago; its provenance is much earlier. Indeed, AV was actually *used*, beginning in the 13th century, in Venice (Lines, 1986) and in papal elections (Colomer and McLean, 1998); it was also used in elections in 19th-century England (Cox, 1987), among other places.

In the summer of 1977, after we met at a conference on Hilton Head Island, SC, under the direction of James S. Coleman, we began a long collaboration, which resulted in one book (Brams and Fishburn, 1983) and many articles on AV and other voting procedures (Brams and Fishburn, 2002).

Our first article (Brams and Fishburn, 1978) was a formal analysis of the properties of AV that included, as an illustration, its application to the 1968 U.S. presidential election, in which there were three significant candidates (Richard M. Nixon, Hubert H. Humphrey, and George Wallace). Our analysis of this election was based on empirical research of Brams's former Yale student, D. Roderick Kiewiet (1979), who showed that Nixon's popular-vote and electoral-vote victory in 1968 would have been much more substantial under AV than it was under PV.²

Even at this early stage AV generated academic controversy (Tullock, 1979; Brams and Fishburn, 1979), which we will say more about later. Nevertheless, we

² For other retrospective studies of elections, including the 1992 presidential election involving Bill Clinton, George Bush, and Ross Perot, see the citations in Brams and Fishburn (2002).

became convinced that AV is a simple and practicable election reform that could ameliorate, if not solve, serious problems in multicandidate elections.

Brams began a “campaign” in 1979 to get it adopted in public elections, beginning with New Hampshire’s first-in-the-nation presidential primaries in February 1980, which had multiple candidates running in both the Democratic and Republican primaries. Although his efforts received both national coverage (e.g., in the *New York Times* and *Los Angeles Times*) and in several New Hampshire newspapers (e.g., the *Manchester Union-Leader* and *Concord Monitor*), he was not successful in getting an AV bill out of committee, despite being a native of New Hampshire (“prodigal son returns”), testifying before Senate and House committees in New Hampshire’s General Court (legislature), and meeting with the governor. Later testimony Brams gave before legislative committees in other states (e.g., New York and Vermont) was similarly unavailing in effecting reform.

Arguments we and others have made for AV proved more persuasive in convincing professional societies to adopt AV. Our rhetoric has remained relatively constant over the years and can be summarized by the following six propositions:

1. *AV gives voters more flexible options.* They can do everything they can under PV—vote for a single favorite—but if they have no strong preference for one candidate, they can express this fact by voting for all candidates they find acceptable. In addition, if a voter’s most-preferred candidate has little chance of winning, then that voter can vote for both a first choice *and* a more viable candidate without worrying about wasting his or her vote on the less popular candidate.

2. *AV helps elect the strongest candidate.* Under PV, the candidate supported by the largest minority often wins, or at least makes the runoff if there is one. Under AV, by contrast, the candidate with the greatest overall support will generally win. In particular, *Condorcet candidates*, who can defeat every other candidate in separate pairwise

contests, almost always win under AV, whereas under PV they often lose because they split the vote with one or more other centrist candidates.

3. *AV will reduce negative campaigning.* AV induces candidates to try to mirror the views of a majority of voters, not just cater to minorities whose votes could give them a slight edge in a crowded plurality contest. AV is therefore likely to cut down on negative campaigning, because candidates will have an incentive to broaden their appeals by reaching out for approval to voters who might have a different first choice. Lambasting such a choice, rather than being more expansive, risks alienating this candidate's supporters, thereby losing their approval.

4. *AV will increase voter turnout.* By being better able to express their preferences, voters are more likely to vote in the first place. Voters who think they might be wasting their votes, or who cannot decide which of several candidates best represents their views, will not have to despair about making a choice.³ By not being forced to make a single—perhaps arbitrary—choice, they will feel that the election system allows them to be more honest, which will make voting more meaningful and encourage greater participation in elections.

5. *AV will give minority candidates their proper due.* Minority candidates will not suffer under AV: their supporters will not be torn away simply because there is another candidate who, though less appealing to them, is generally considered a stronger contender. Because AV allows these supporters to vote for *both* candidates, they will not be tempted to desert the one who is weak in the polls, as under PV. Hence, minority candidates will receive their true level of support under AV, even if they cannot win. This will make election returns a better reflection of the overall acceptability of

³ Perhaps the best recent example of voters who faced this dilemma were supporters of Ralph Nader in the 2000 U.S. presidential election. Although Nader received less than 3 percent of the popular vote in this election, polls show that if his supporters could have voted for a second choice, Al Gore would have been the choice of most. Thereby Gore would have won Florida and its electoral votes, making him rather than George W. Bush the winner.

candidates, relatively undistorted by strategic voting, which is important information often denied to voters today.

6. *AV is eminently practicable.* Unlike more complicated ranking systems, which suffer from a variety of theoretical as well as practical defects, AV is simple for voters to understand and use. Although more votes must be tallied under AV than under PV, AV can readily be implemented on existing voting machines. Because AV does not violate any state constitutions in the United States (or, for that matter, the constitutions of most countries in the world), it requires only an ordinary statute to enact.

Voting systems that involve ranking candidates may appear, at first blush, to be more appealing than AV. One, the Borda count or Borda voting (BV), awards points to candidates according to their ranking. Another, the Hare system of single transferable vote (STV; also called the “alternative vote” or “instant runoff”), progressively eliminates candidates with the fewest first-choice votes and transfers their votes to second choices—and lower choices if necessary—until one candidate emerges with a majority.

Proponents of AV argue that these systems have serious drawbacks. BV fosters “insincere voting”—when, for example, a voter moves a second choice down to last place to minimize that candidate’s threat to his or her top choice—and is also vulnerable to “irrelevant candidates,” who cannot win but can affect the outcome. STV may eliminate a centrist candidate early on and thereby elect one less acceptable to a majority. In addition, STV suffers from “nonmonotonicity,” in which voters, by raising the ranking of a candidate, may actually cause that candidate to lose—just the opposite of what one would want to happen.

PV is also vulnerable to insincere voting, whereby a voter may switch to a second choice if his or her first choice appears to be a long shot, as indicated, for example, by polls. While AV encourages sincere voting—voting for all candidates above the lowest-ranked candidate one considers acceptable—it does not eliminate strategic calculations

altogether. Because approval of a less-preferred candidate can hurt a more-preferred candidate, the voter still faces the decision under AV of where to draw the line between acceptable and nonacceptable candidates.

The pros and cons of AV versus other voting systems have been debated over the last twenty years in numerous publications.⁴ But this is not the subject of this paper, except insofar as the rhetoric has influenced the history of adoptions (and nonadoptions) of AV.⁵ We next discuss the adoption decisions of the first societies to use AV in the late 1980s.

3. The Adoption Decisions in the Societies⁶

Elections are not a burning issue in most scientific societies, with participation rates often considerably below 50 percent of the membership and sometimes closer to about 10 percent. For the candidates, on the other hand, who are often luminaries in their disciplines, outcomes are usually more consequential and sometimes represent, especially if the office is president, recognition of professional achievements over one's career.

It is not surprising, then, that candidates are willing to make subdued versions of what, in political life, would be called campaign statements. In the more rarefied atmosphere of an academic or professional society, these statements, which usually

⁴ For a sampling of this debate, see Arrington and Brenner (1984) and Brams and Fishburn (1984); Niemi (1984, 1985) and Brams and Fishburn (1985); Saari and Van Newenhizen (1988) and Brams, Fishburn, and Merrill (1988); Brams and Fishburn (2001) and Saari (2001a); and Brams and Herschbach (2001a, 2001b) and Richie, Bouricius, and Macklin (2001). Recent popular accounts of the controversy over voting systems by science writers include MacKenzie (2000), Guterman (2002), Klarreich (2002), and Begley (2003).

⁵ Donald G. Saari has been a proponent of BV, most recently in Saari (2001b), but we know of no recent adoptions of BV, though it and a variant have been used in two small Pacific Island countries, beginning about 30 years ago (Reilly, 2002). Proponents of instant runoff voting (IRV), based on STV, recently succeeded in getting it enacted in elections in San Francisco; they formed an organization, the Center for Voting and Democracy (CV&D), which now has a staff of about ten people that includes the authors of Richie, Bouricius, and Macklin (2001) and Hill (2002). As noted in Brams and Herschbach (2001a), IRV supporters have done little serious analysis to back up their claims, although other studies of STV (e.g., Dummett, 1984) have been more probing. On the other hand, CV&D does have human and monetary resources that few academics can claim.

⁶ This and the next two sections are based on Brams and Fishburn (1992a) as well as earlier and later studies that we cite.

accompany a mailed ballot, tend more to emphasize broad goals than specific programs, although candidates often pledge to undertake new initiatives. Most candidates, while listing their past offices and qualifications for the new office, generally do not seek to disparage the opposition.

Genteel as most of these campaigns are, candidates do, nonetheless, try to garner support by highlighting their qualifications, and proposing new approaches or ideas, that differentiate them from their opponents. When AV was first proposed as a reform in the four societies that adopted AV in the late 1980s, no candidates or factions, with one major exception, identified AV as a threat either to their candidacies or points of view.

Of course, after AV's use, there are winners and losers, and some losers, undoubtedly, see themselves as victims of this reform. In one society (The Institute of Management Sciences, or TIMS, before it merged with the Operations Research Society of America, or ORSA, to become INFORMS), this logic worked in reverse: the winner under PV, before AV was adopted, would almost certainly have lost under AV—and this became an argument made for the adoption of AV!

We hasten to add that this argument against PV was not a personal argument directed against the PV winner. Rather, the argument was that another candidate commanded broader support and thereby “deserved” to win.

Next we briefly recount the adoption decisions of the first four societies to use AV:

1. *Mathematical Association of America (MAA)*. In 1985, the president of the MAA, Lynn Arthur Steen, who was familiar with work on AV, asked the Board of Governors of the MAA to consider adoption of AV in its biennial elections for president-elect and other national offices. After “heated but not acrimonious” debate (Steen, 1985), AV was approved by the Board in 1985, passed by the membership in 1986, and used for the first time in the 1987 MAA elections.

Steen earlier had written an article in *Scientific American* (Gardner, 1980) on the mathematics of elections, in which he discussed AV. Before the MAA's consideration of AV, he asked Brams to look into the use of STV by the American Mathematical Society (AMS), the major research society of mathematicians.⁷ Brams (1982) demonstrated via two counterexamples that the "Instructions to Voters" accompanying the 1981 ballot used by the AMS to elect a nominating committee contained an erroneous statement about a property of STV, which led to an exchange with Chandler Davis (1982), who had been a proponent of STV when it was adopted by the AMS several years earlier. The erroneous statement was deleted from future instructions, but AV was not adopted by the AMS until 1992.⁸

Both Steen's knowledge and his position as president of the MAA made him a crucial player in the MAA's adoption of AV. So, also, was Steen's successor as president of the MAA, Leonard Gillman, who was a strong advocate of AV and played an active role in its eventual implementation in the 1987 elections of the Association. For example, he wrote a description of AV for mathematicians, which included results of his own analysis (Gillman, 1987).

2. *The Institute of Management Sciences (TIMS)*, which is now part of INFORMS. The use of AV by TIMS in 1988 was preceded by an experiment in which members were sent a nonbinding AV ballot, along with the regular PV ballot, in the 1985 elections. Although the AV ballot did not count, 85 percent of the members who voted in these elections returned the AV ballot. This permitted Fishburn and Little (1988) to compare the results of voting under the two different systems.

⁷ The MAA is the more teaching-oriented of the two major American mathematical societies at the college-university level.

⁸ It was adopted in part because counting votes by hand under STV proved to be too onerous, and computerizing the counting was not feasible at the time. Even so, AV was adopted only for those offices of the AMS that did not require an amendment to the bylaws, which would have required considerable effort to enact; voting for other offices is still by PV (Daverman, 2002, and Fossum, 2002). Patently, pragmatic considerations played a key role in the AMS's choices.

On the basis of their empirical analysis, which will be discussed later, Fishburn and Little (1988) concluded that AV did a better job of electing Condorcet candidates than did PV. Not only was the experiment “remarkably successful” (Little and Fishburn, 1986), but the results also convinced TIMS Council to adopt AV in 1987, leading to its later adoption by INFORMS when it formed in 1995. In fact, an argument for conducting the experiment in the first place was that management scientists should “practice what we preach” (Jarvis, 1984): before deciding on its usage, TIMS should collect the information necessary to make an informed judgment about the applicability of the theoretical analysis of AV to its own elections.

Both the consideration and adoption of AV by TIMS were certainly helped by the fact that the president of TIMS in 1984-1985, John D. C. Little, was interested in AV and collaborated with Fishburn on the experiment and its analysis. Before undertaking the experiment, inquiries were made of the candidates to ask their permission to participate in it. Because of its research potential, all agreed, prefiguring AV’s eventual adoption.

3. *American Statistical Association (ASA)*. The former chair of the ASA’s Committee on Elections, Richard F. Potthoff, had read about AV and brought it to the attention of his committee. This committee recommended its adoption first in “internal” ASA elections; the ASA Board of Directors approved this recommendation.

After AV’s successful use in 1986 in three elections for Council governors, the election of two editors to serve on the Board, and the election of a Board member to serve on the Executive Committee, the Committee on Elections recommended that AV be used in Association-wide elections, which was approved by the Board (“Amendment to ASA By-Laws,” 1987) and ratified as an amendment in 1987. Unlike the other societies, the ASA has had no Association-wide multicandidate elections since the adoption of AV, though some internal elections and single-winner section elections have had more than two candidates.

4. *Institute of Electrical and Electronics Engineers (IEEE)*. The adoption of AV by the IEEE has a politically charged history (Brams and Nagel, 1991). Beginning in 1984, AV was considered, along with other voting systems, for possible use in multicandidate elections. But not until the 1986 elections—when a petition candidate, Irwin Feerst, ran against two candidates for president-elect who were nominated by the Board of Directors—did the issue of election reform take center stage. The reason is that Feerst, with 35 percent of the vote, defeated one of the two Board-nominated candidates and came within 242 votes (of 52,405 cast) of defeating the other candidate. This result starkly illustrated to the Board how vulnerable their nominees, who together might win a substantial majority in an election, are to a minority candidate if these nominees should split the majority vote more or less evenly.

In 1987 the Board reverted to nominating only one candidate for president-elect, breaking a tradition of nominating two candidates that it had begun in 1982. Feerst was instrumental in bringing the question of how many nominees the Board must nominate to a vote of the entire membership in the 1987 election, in which he did not run and there were no other petition candidates. By a 57-percent majority, members supported a constitutional amendment requiring that the Board nominate at least two candidates, but this fell short of the 2/3's majority needed to amend the IEEE's constitution.

Nevertheless, it was clear that there was strong member support for making IEEE elections more competitive, which renewed interest in AV should the Board return to nominating two candidates and have petition candidates run as well. In 1987, Brams was invited by the then president of the IEEE, Henry L. Bachman, to attend an Executive Council meeting to discuss AV.

Unable to do so, he suggested that Jack H. Nagel of the University of Pennsylvania, who had done extensive research on AV, take his place. Nagel did; he also attended a later meeting of the full Board of Directors, which adopted AV in November 1987. (AV had previously been used in internal IEEE elections, sometimes in modified

form.) With its adoption, the Board voted to nominate at least two candidates for each office.

When the IEEE's adoption of AV was announced at a December 1987 IEEE press conference in New York City that Brams and Nagel attended, Feerst objected strenuously to its use, arguing that it was a deliberate move to undermine his candidacy and the interests of “working engineers,” whom he claimed to represent. When Feerst ran in 1988 for president-elect under AV, he came in fourth in a field of four candidates.

To recapitulate, the paths to adoption of AV in the different societies have been diverse. Only in the MAA did full-scale use of AV begin before it was first tried out in an experiment (TIMS) or in internal elections (ASA and IEEE).

The presidents of the MAA, TIMS, and the IEEE played active roles in AV's adoption in their societies, and each received assistance from an advocate of AV. In the ASA, on the other hand, it was writings on AV that sparked initial interest, which turned into adoption without much controversy.

Controversy was the hallmark of the IEEE deliberations. While the IEEE's adoption of AV was in part a response to a perceived threat to its established leadership, it is important to realize that the IEEE did not view it as its only alternative.

In fact, several other election systems had been considered before AV was selected. For example, a runoff election between the two top contenders, if neither received a majority in the initial balloting under PV, was also seriously considered, but it was viewed as too costly to have a second round of voting and also would have required a constitutional change. Ultimately, a majority of Board members concluded that AV better fit the needs of the organization than any other voting system, and that is why it was adopted.⁹

⁹ By no means do we suggest that AV is a panacea in all elections, especially those involving multiple winners; for such elections, see the AV-related reforms in Brams (1990), Fishburn and Brams (1991), Brams and Fishburn (1992b), and Potthoff and Brams (1998).

This quick overview does not do justice to the serious debates that occurred over the merits of AV, particularly in the MAA and the IEEE. Indeed, although there has been dissent over AV's use in some societies (Kiely, 1991), no society that adopted AV ever rescinded its decision, with one notable exception (the IEEE).¹⁰ Looking at what has AV wrought in them may offer some explanation of why it has been generally, but not universally, accepted.

4. Does Approval Voting Make a Difference?

Clearly, a new voting procedure makes a difference if it leads to the selection of a different winner. The best evidence we have that AV would have elected a different winner is from the 1985 TIMS experiment, in which ballot data for both the PV official elections and the AV nonbinding elections were compared (Fishburn and Little, 1988).

In one of the three 1985 elections, the official PV and actual AV ballot totals are shown in Table I for candidates A, B, and C. Also shown are the AV totals extrapolated

Table I about here

from the 85-percent sample of members who returned their AV nonbinding ballots, which is a very high figure. The extrapolation is a straightforward one: approval votes are added to the actual AV totals for each candidate based on the propensity of the sample respondents who voted for one particular candidate on the PV ballot to vote for each of the other candidates on the AV ballot. This extrapolation is justified by the finding that there are no major differences in voting patterns on the official PV ballot between AV respondents and nonrespondents.

¹⁰ According to the IEEE Executive Director, Daniel J. Senese, AV was abandoned in 2002 because “few of our members were using it and it was felt that it was no longer needed.” Brams responded in an e-mail exchange (June 2, 2002) that since “candidates now can get on the ballot with ‘relative ease’ [according to former IEEE president Henry L. Bachman in the same e-mail exchange] . . . the problem of multiple candidates [in the late 1980s] might actually be exacerbated . . . and come back to haunt you [IEEE] some day.”

Observe that candidate C wins the official PV election by a bare 8 votes (0.4 percent), but B would have won under AV by a substantial 170 votes (6.1 percent). By itself, the fact that C wins more plurality votes and B wins more approval votes does not single out one candidate as the manifestly preferred choice. But on the experimental ballot, voters were asked one piece of additional information: to rank the candidates from best to worst by marking next to their names 1 for their first choice, 2 for their second choice, and so on.

These data can be used to reconstruct who would defeat whom in hypothetical pairwise contests, which is not evident from the PV totals. For example, the fact that C edges out B in presumed first choices, based on the PV totals, does not mean that C would hold his or her lead when the preferences of the 166 A voters are taken into account. In fact, the experimental ballots of these 166 voters show that

- (1) 70 provided rankings in the order ABC;
- (2) 66 provided rankings in the order ACB;
- (3) 3 provided no rankings but approved both A and B;
- (4) 27 made no distinction between B and C by rankings or approval.

In the B-versus-C comparison, it is reasonable to credit (1) and (3) to B (73 votes), (2) to C (66 votes), and (4) to neither candidate. When added to the PV totals, these credits give C (901 votes) exactly one more vote than B (900 votes). However, assuming the 27 voters in (4) split their votes between B and C in the pattern of the 139 voters (70 + 66 + 3) who ranked A first and also expressed a preference between B and C, B would pick up an additional vote (rounded to the nearest vote), resulting in a 914-914 tie.

This extrapolation indicates that there is not a single Condorcet candidate.¹¹ While surprising, the lack of a single Condorcet candidate should not obscure the fact that 170

¹¹ It is worth noting that the usual reason for the nonexistence of a Condorcet candidate is because of a Condorcet paradox, whereby majorities cycle. In this election, however, it is a projected tie that precludes

more voters approved of B rather than C in the extrapolated AV returns, albeit C won the PV contest by 8 votes.

The reason for this discrepancy between the AV and PV results is that whereas C has slightly more *stalwart* supporters (i.e., those who vote only for one candidate) than B, supporters of the third candidate, A, more approve of B than C (36 percent to 23 percent). Furthermore, because more of C's supporters approve of B than B's do of C, B would have won handily under AV.

Is this desirable? In the absence of a Condorcet candidate, Fishburn and Little (1988, pp. 559-560) concluded that

approval voting picks a clear winner on the basis of second choices.

These show that B has a broader acceptance in the electorate than C.

Therefore, the approval process, by eliciting more information from the voters, leads to the election of the candidate with the widest support.

Although it is theoretically possible in close elections that the Condorcet candidate will not be the most approved candidate, it has almost never occurred.¹² But the legitimacy of the AV winner may be questioned on other grounds.

one candidate from defeating the others in pairwise contests. That there is no cycle, and that A in fact would lose to both B and C, is shown by ranking data in Fishburn and Little (1988).

¹² The 1999 election for president of the Social Choice and Welfare Society, which was decided by 2 approval votes among 76 cast, is the only exception we know of: the second-place AV candidate in this election would have defeated the AV winner by 4 votes in a head-to-head contest, based on the hypothetical use of BV, for which voters ranked candidates. Brams and Fishburn (2001) deem this “nail-biting” election essentially a toss-up, whereas Saari (2001a) argues that most positional methods would have chosen the Condorcet candidate (including BC, wherein the Condorcet winner would have defeated the AV winner 60-59); see Laslier (2003a) for more details on voting patterns in this election. Regenwetter and Grofman (1998), using a random-utility model to reconstruct voter preferences in several elections—including some discussed here—show that AV, BV, and Condorcet winners generally coincide. Laslier (2003b) and Laslier and Vander Sraeten (2003) analyze data from a field experiment with AV in the 2002 French presidential election, which involved over 5,000 voters in two French towns, and conclude that AV was easily understood, readily accepted, and provided a more complete picture of the “political space.” Earlier theoretical analyses as well as computer simulations (Brams and Fishburn, 1983; Lijphart and Grofman, 1984; Nurmi, 1987; Merrill, 1988) demonstrate that AV almost always elects a Condorcet winner

5. Does Approval Voting Elect the Lowest Common Denominator?

One fear that has been expressed about the use of AV is that while it may help elect candidates more broadly representative than PV, these candidates could turn out to be rather bland and uninspiring. They may win simply because they offend the fewest voters, not because they excite the passions of many.

It is difficult to say whether, in principle, a compromise candidate is a better or worse social choice than a more extreme candidate who is the darling of some voters but the bane of others. In practice, fortunately, this dichotomous choice seems rarely to arise, as the data from the AV elections of the four societies demonstrate. Specifically, the winners under AV were candidates who were generally popular among *all* voters, however many candidates they voted for in the different elections. Thus, a divergence between forceful minority candidates, approved of by few, and “wishy-washy” majority candidates, approved of by many, is probably an infrequent event.

There are, however, examples of elections in which the winner was not strong among all classes of voters. Consider the 1987 MAA election shown in Table 2 (Brams,

Table 2 about here

1988b), wherein the votes received by the five candidates in this election are broken down by the votes each of the candidates received from voters’ casting exactly one vote (1-voters), voters’ casting exactly two votes (2-voters), and so on. Excluded from these totals are 9 voters who voted for all the candidates, whose undifferentiated support obviously has no effect on the outcome.

In this election, 3,081 of the 3,924 voters (79 percent) were 1-voters, while the remaining 843 voters cast 1,956 votes, or an average of 2.3 votes each. Thus, the multiple

if there is one. If there is not one, as in the 1985 TIMS election experiment, then proponents of AV argue that AV provides a compelling way to break either a cycle or a tie.

voters cast 39 percent of the votes, though they constituted only 21 percent of the electorate.

Did the multiple voters make a difference? It would appear not, because the winner (A) received 28 percent more votes from 1-voters than the 1-voters' runner-up (D) did, just edged out B among 2-voters, but lost to several candidates among 3-voters and among 4-voters. A's victory, then, is largely attributable to the substantial margin received from 1-voters, not from the presumably more lukewarm support received from multiple voters.

Define a candidate who wins among all classes of voters—those who cast few votes (narrow voters) and those who cast many votes (wide voters)—as *AV-dominant*. In the MAA election, we assume narrow voters are those who cast 1 or 2 votes, and wide voters are those who cast 3 or 4 votes.

It turns out that A is not AV-dominant, because he or she wins among narrow but not among wide voters. Does this vitiate A's winning status? In winning so decisively among 1-voters, whose preference intensities would seem to be greatest, it would be hard to argue that A is any kind of lowest common denominator. It should be noted, however, that some of the 37 voters who voted for four of the five candidates probably also had intense preferences—but against the one candidate they chose to leave off their approved lists.

In 12 of the 16 multicandidate AV elections analyzed in the four societies, the winners were AV-dominant. In the four elections in which there was not an AV-dominant winner, the pattern is similar to that in the 1987 MAA election shown in Table 2: the winner won by virtue of receiving greater support among narrow voters than among wide voters. These AV-nondominant winners, therefore, do not fit the mold of lowest common denominators—the choice of many wide voters but few narrow voters—but rather the opposite, which reinforces, not undermines, their legitimacy as winners.

The fact that the winners in three-quarters of the elections were AV-dominant is perhaps not surprising, because one would expect such candidates would do better than losers across different types of voters. A little reflection, however, shows that this need not be the case. Paradoxically, a candidate may lose among every possible class of voters—that is, be *AV-dominated*—and still be the AV winner. For example, A might be the victor over C among narrow voters, and B might be the victor over C among wide voters. But C could emerge as the AV winner if A did badly among wide voters, B did badly among narrow voters, but C was a close second among both types.

No winners in the 16 elections were AV-dominated. As already noted, even the support of the four AV-nondominant winners appeared to be more intense and heartfelt (i.e., from narrow voters) than that of the losers, so AV does not appear to elect lowest common denominators.

5. Is Voting Ideological?

Consider again the 1987 MAA election. As can be calculated from Table 2, 2-voters gave the candidates 22-26 percent of all their votes, 3-voters 10-16 percent, and 4-voters 2-5 percent. Venn diagrams (not shown here) indicate the shared support among the 10 subsets of two candidates, 10 subsets of three candidates, 5 subsets of four candidates, and 1 of all five candidates. Examination of the *sources* of this support, as shown in the Venn diagrams, does not reveal any particular pairs, triples, or quadruples that received unusually great support, indicating that there was not obvious coalitional voting.

On the contrary, multiple votes are spread about as one would expect according to the null hypothesis that votes are distributed in proportion to the candidates' totals. In the case of A, for example, there were 82 shared votes with just B, 91 with just C, 80 with just D, and 23 with just E, which is roughly in accord with the candidates' overall totals.

Indeed, every one of the 32 subsets in this election—including the 2.6 percent who abstained—got at least 3 votes.

The story is very different for the 1988 IEEE election shown in Table 3 (Brams

Table 3 about here

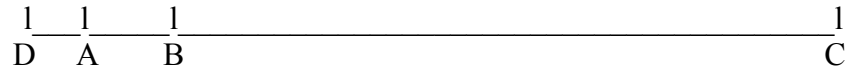
and Nagel, 1991), wherein the approval vote totals are shown for all 16 subsets of the four candidates in this race. Consider first the 3-voters, and note that nearly everyone in this category voted for ABD—5,605 voters, to be precise. By contrast, only 148, 143, and 89 voters, respectively, supported the other 3-subsets of ABC, ACD, and BCD that contain C.

Evidently, the numerous supporters of ABD voted against C by voting for everybody except C. This essentially negative kind of voting against C can also be seen in voting for the six 2-subsets. The three 2-subsets that do not include C (AB, AD, and BD) had an average of 4,027 voters each, whereas the three that included C (AC, BC, and CD) had an average of only 897 voters each.

In addition to the predominant clustering of support around A, B, and D, there are some subtle differences in the sharing of support. For each pair of candidates, Brams and Nagel (1991) computed an index of shared support by taking the ratio of ballots approving both candidates by 2-voters and 3-voters to total ballots, excluding abstentions and votes for all four candidates. By this measure, A and D have the most affinity, with 22.9 percent shared support. They are followed by A and B, with 17.2 percent; and then by B and D, with 13.9 percent. Although A, B, and D share much less support with C, B at 3.1 percent shares slightly more with C than do A (1.8 percent) and D (1.5 percent).

From these results, one might infer an underlying dimension on which D and C occupy opposite extremes, whereas A and B are located at intermediate positions. A is

somewhat closer than B to D, but both B and A are much closer to D than to C, as shown in the following hypothetical continuum:



This representation corresponds to certain facts about the candidates. D and A were both Board nominees, whereas C was a vociferous critic of IEEE officers, Board, and staff. B, though like C a petition candidate, was in other ways close to the IEEE establishment, having previously served on the Board. As for the slight distinction between D and A, judging from the candidates' biographies and statements it may reflect D's emphasis on technical research, which perhaps made him seem most distant from C, who sought to champion the working engineer.

Of the 54,204 ballots analyzed in this election, only 3,323 (6.1 percent) are "inconsistent" with the assumption that voters' preferences are based on the foregoing DABC ordering of candidates. *Inconsistent ballots* include approval of two nonadjacent candidates without including the adjacent candidate(s) between them, notably DC (608), AC (659), DAC (143), and DBC (89). Accounting for more than half the inconsistencies is the relatively minor inconsistency—in terms of perceived differences—represented by the pattern DB (1,824). Of the multiple voters, 17,435 (84.0 percent) cast ballots consistent with the hypothetical ordering.

Thus, candidates with obvious affinities tended disproportionately to share approval from multiple voters. In this sense voting was ideological: it reflected a pattern consistent with an underlying ordering of the candidates. Only in this election, however, was such a pattern found; far more typically, voting in the societies is nonideological, which is consistent with the null hypothesis alluded to earlier. But if AV is used in public elections, their more political character could well lead to the kind of ideological cleavages observed in the IEEE election.

It is important to note, however, that nonideological voting may mirror regularities not evident in the AV data themselves. As a case in point, the winner in the 1987 MAA election (Table 2) was a woman, and this pattern was repeated in the next MAA election in 1989. We have not analyzed data from the latter election, but the 1987 winner's victory, as shown earlier, cannot be impeached on grounds that she won mostly because of lukewarm support from wide voters. Nonetheless, as the only women in each of the two races, it may be the case that they were helped by their uniqueness: by some they were perceived as the single best choice; by others they were seen as broadly acceptable.

6. Summary and Conclusions

AV has proved to be a practical and viable election reform in the four scientific and engineering societies that used it for the first time in 1987 and 1988. While AV supporters played a role in its adoption in three of the four societies (TIMS, MAA, and IEEE), none of its proponents was even aware of its consideration in the fourth society (ASA) until its adoption was imminent.

In all these societies, AV's adoption rested principally on the arguments—summarized earlier—that it is preferable to PV in multicandidate races. In the IEEE, a petition candidate's near-win with vocal but only minority support certainly gave urgency to these arguments, accelerating AV's adoption after the Board's attempt to limit the number of Board-nominated candidates to one person met with the membership's disapprobation. Only in the case of the AMS's 1992 adoption of AV did practical considerations give it an edge over STV, and then only in some elections that were relatively easy to change.

The empirical analyses of election returns from the different societies indicate that AV may make a difference. So far it seems not to have elected candidates who can be characterized as lowest common denominators but instead candidates who either enjoyed support among all classes of voters, or who did particularly well among narrow voters

whose support is presumed to be more intense. Although voting seems generally nonideological in most society elections, a clear ordering of positions was identified in the IEEE election, and voting tended to be only for adjacent candidates in this ordering.

Condorcet candidates almost always win under AV, with the only known exception being the 1999 Social Choice and Welfare election, which was a near-tie under both AV (the official procedure) and BV (the hypothetical procedure). If there is no single Condorcet candidate, as was illustrated in the 1985 TIMS election experiment, then AV provides a way of determining which candidate receives the most support from all voters, not just those who rank this person first.

Not all societies that have been approached about adopting AV, including three that Brams belongs to—the American Political Science Association (APSA), the International Studies Association (ISA), and the Public Choice Society (PCS)—have been amenable to election reform, much less the adoption of AV. Significantly, these societies are dominated, or heavily populated by, academic political scientists; none holds competitive elections unless a petition candidate challenges the official slate (this has never happened in the ISA or PCS; in the APSA, the last challenges occurred more than 25 years ago).

Among the lessons we draw from our experience is that the adoption of AV, and probably any election reform, requires key support from within an organization. We never received this kind of support from politicians or political parties in our attempts to get AV adopted in public elections. By contrast, the society adoptions would not have occurred without influential members of each society favoring reform, sometimes for practical or political reasons. Of course, they also needed to make their cases with arguments based on democratic principles; we like to believe that both the rhetoric of AV supporters as well as their analyses helped in this regard.

Table 1
PV and AV Vote Totals in 1985 TIMS Election

Candidates	Official PV	Actual AV	Extrapolated AV
A	166	417	486
B	827	1,038	1,224
C	835	908	1,054
Total	1,828	2,363	2,764
No. of Voters	1,828	1,567	1,828

Table 2
AV Vote Totals in 1987 MAA Election

Candidates	1-Voters	2-Voters	3-Voters	4-Voters	Total
A	848	276	122	21	1,267
B	618	275	127	32	1,052
C	652	264	134	34	1,084
D	660	273	118	31	1,082
E	303	132	87	30	552
Total	3,081	1,220	588	148	5,037
No. of Voters	3,081	610	196	37	3,924

Table 3
Numbers of Voters Who Voted for 16 Different Subsets in 1988
IEEE Election and AV Totals

Subsets

None = 1,100					
A = 10,738	B = 6,561	C = 7,626	D = 8,521		
AB = 3,578	AC = 659	AD = 6,679	BC = 1,425	BD = 1,824	CD = 608
ABC = 148	ABD = 5,605	ACD = 143	BCD = 89		
All = 523					

Totals

A = 28,073	B = 19,753	C = 11,221	D = 23,992
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