Adverse Selection and Self-fulfilling Business Cycles*

Jess Benhabib† Feng Dong‡ Pengfei Wang§

This Version: May 2016

First Version: June 2014

Abstract

We introduce a simple adverse selection problem arising in credit markets into a standard textbook real business cycle model. There is a continuum of households and a continuum of anonymous producers who use intermediate goods to produce the final goods. These producers do not have the resources to make up-front payments to purchase inputs and have to finance their working capital by borrowing from competitive financial intermediates. Lending to these producers, however, is risky: honest borrowers will always pay their debt back, but dishonest borrowers will always default. This gives rise to an adverse selection problem for financial intermediaries. In a continuous-time real business cycle setting we show that such adverse selection generates multiple steady states and both local and global indeterminacy, and can give rise to boom and bust cycles driven by sunspots under calibrated parameterization. Introducing reputational effects eliminates defaults and results in a unique but still indeterminate steady state. Finally we generalize the model to firms with heterogeneous and stochastic productivity, and show that indeterminacies and sunspots persist.

Keywords: Adverse Selection, Local Indeterminacy, Global Dynamics, Sunspots.

JEL codes: E44, G01, G20.

*We are indebted to Lars Peter Hansen, Alessandro Lizzeri, Jianjun Miao, Venky Venkateswaran, Yi Wen and Tao Zha for very enlightening comments.

†New York University. Email: benhabib@nyu.edu

‡Shanghai Jiao Tong University. Email: fengdong@sjtu.edu.cn

§Hong Kong University of Science and Technology. Email: pfwang@ust.hk

1 Introduction

The seminal work of Wilson (1980) shows that in a static model, adverse selection can generate multiple equilibria because of asymmetric information about product quality. The aim of this paper is to analyze how adverse selection in credit markets can give rise to lending externalities that generate multiple steady states and a continuum of equilibria in an otherwise standard dynamic general equilibrium model of business cycles.

To make this point, we introduce a simple type of adverse selection arising in credit markets into a standard textbook real business cycle model. The model features a continuum of households and a continuum of anonymous producers. These producers use intermediate goods to produce the final goods. They do not have the resources to make the up-front payments to purchase intermediate inputs. Therefore, to finance their working capital, they must borrow from competitive financial intermediaries. Lending to these producers however is risky, as some borrowers may default. We assume that there are two types of borrowers (producers). In our baseline model, the honest borrowers will always pay back their loans, while the dishonest borrowers will always default. The financial intermediaries do not know which borrower is honest and which is not. This gives rise to adverse selection: for any given interest rate, the dishonest borrowers have a stronger incentive to borrow. In such an environment, an increase in lending from some optimistic financial intermediaries encourages more honest producers to borrow. The increased quality of borrowers reduces the default risk, which in turn stimulates other financial intermediaries to lend. The resulting decline in the interest rate brings down the production cost for all producers/borrowers. This stimulates an expansion in output, further expands the credit supply from the households, and generates more future lending. In other words, a lending externality exists both intratemporally and intertemporally.

In a dynamic setting market forces and competition can mitigate adverse selection through reputational effects absent from our baseline model in Section 2. We therefore examine, in Section 3, whether indeterminacy survives under reputational effects. We follow Kehoe and Levine (1993) and assume that a borrower who defaults may, with some probability, lose reputation, and is then excluded from the credit market forever.

In our baseline model in Section 2, we study the local dynamics of our model to show that this lending externality not only generates two steady state equilibria with low and high average default rates, but also gives rise to a continuum of equilibria around one of the steady states under calibrated parameterizations. We then move on to characterize the global dynamics of our model economy. The additional insight from the global dynamics analysis is that even in the absence of local indeterminacy we may still have global indeterminacy, with boom and bust cycles in output under rational expectations. In the model with reputational considerations, we show that the steady state equilibrium is unique, and no default occurs in equilibrium. Nevertheless, perhaps surprisingly, indeterminacy in the form of a continuum of equilibria persists.

Adverse selection in the credit market seems to be a realistic feature, both in poor and rich countries.[1] Our model has several implications that are supported by empirical evidence. First, a large literature has documented that credit risk is countercyclical and has far-reaching macroeconomic consequences. For instance, Gilchrist and Zakrajšek (2012) find that a shock to credit risk leads to significant declines in consumption, investment, and output. Pintus, Wen and Xing (2015) show that interest rates faced by US firms move countercyclically and lead the business cycle. These facts are consistent with our model's predictions. Second, our model delivers a countercyclical markup, an important empirical regularity well documented in the literature. Because of information asymmetry, dishonest borrowers enjoy an information rent. However, when the average quality of borrowers increases due to higher lending, this information rent is diluted. So the measured markup declines, which is critical to sustaining indeterminacy by bringing about higher real wages, a positive labor supply response, and a higher output that dominates the income effect on leisure. Third, our extended model in Section 4 can explain the well-known procyclical variation in productivity. The procyclicality of average quality in the credit market implies that resources are reallocated towards producers with lower credit risk when aggregate output increases. The improved resource allocation then raises productivity endogenously. The procyclical endogenous TFP immediately implies that increases in inputs will lead to a more than proportional increase in total aggregate output, mimicking aggregate increasing returns. This effective increasing returns to scale arises only at the aggregate level. It is also consistent with the results of Basu and Fernald (1997), who find slightly decreasing returns to scale for typical two-digit industries in the US, but strong increasing returns to scale at the aggregate level.

Related Literature Our paper is closely related to several branches of literature in macroeconomics. First, our paper builds on a large strand of literature on the possibility of indeterminacy in RBC models. Benhabib and Farmer (1994) point out that increasing returns to scale

[1] See Sufi (2007) for evidence of syndicated loans in the US and Karlan and Zinman (2009) for evidence from field experiments in South Africa.

can generate indeterminacy in an RBC model. The degree of increasing returns to scale in production required to generate indeterminacy, however, is considered be too large (See Basu and Fernald (1995, 1997)). Subsequent work in the literature has introduced additional features to the Benhabib-Farmer model that reduce the degree of increasing returns required for indeterminacy. In an important contribution, Wen (1998) adds variable capacity utilization and shows that indeterminacy can arise with a magnitude of increasing returns similar to that in the data. Gali (1994) and Jaimovich (2007) explore the possibility of indeterminacy via countercyclical markups due to output composition and firm entry respectively. The literature has also shown that models with indeterminacy can replicate many of the standard business cycle moments as the standard RBC model (see Farmer and Guo (1994)). Furthermore, indeterminacy models may outperform the standard RBC model in many other dimensions. For instance, Benhabib and Wen (2004), Wang and Wen (2008), and Benhabib and Wang (2014) show that models with indeterminacy can explain the hump-shaped output dynamics and the relative volatility of labor and output, which are challenges for the standard RBC models. Our paper complements this strand of literature by adding adverse selection as an additional source of macroeconomic indeterminacy. The adverse selection approach also provides a micro-foundation for increasing returns to scale at the aggregate level. Indeed, once we specify a Pareto distribution for firm productivity, our model in Section 4 is isomorphic to those that have a representative-firm economy with increasing returns. It therefore inherits the ability to reproduce the business cycle features mentioned above without having to rely on increasing returns.[2]

Second, our paper is closely related to a burgeoning literature that study the macroeconomic consequences of adverse selection. Kurlat (2013) builds a dynamic general equilibrium model with adverse selection in the second-hand market for capital assets. Kurlat (2013) shows that the degree of adverse selection varies countercyclically. Since adverse selection reduces the efficiency of resource allocation, a negative shock that lowers aggregate output will exacerbate adverse selection and worsen resource allocation efficiency. So the impact of the initial shocks on aggregate output is propagated through time. Like Kurlat (2013), Bigio (2014) develops an RBC model with adverse selection in the capital market. As firms must sell their existing capital to finance investment and employment, adverse selection distorts both capital and labor markets. Bigio (2014) shows that the adverse selection shock widens a dispersion of capital quality, exacerbates the distortion, and creates a recession with a quantitative pattern similar to

[2]Liu and Wang (2014) provide an alternative mechanism to generate increasing returns via financial constraints.

that observed during the Great Recession of 2008. Our model generates similar predictions to Kurlat (2013) and Bigio (2014). First, adverse selection is also countercyclical in our model, so the propagation of fundamental shocks via adverse selection, as highlighted by Kurlat (2013) is also present in our model. Second, in our model, adverse selection in the credit markets naturally creates the distortions to both capital and labor inputs. Introducing stochastic and heterogeneous productivities into our extended model in Section 4 aggravates adverse selection, and makes the economy more vulnerable to self-fulfilling expectation-driven fluctuations. While Kurlat (2013) and Bigio (2014) emphasize the role of adverse selection in propagating business cycles shocks, our paper complements their work by showing that adverse selection generates multiple steady states and indeterminacy, and hence can be a source of large business cycle fluctuations driven by self-fulfilling expectations.[3] It is worth noting that, all of the above papers focuses on local dynamics via log linearization. As underscored by Brunnermeier and Sannikov (2014) and He and Krithnamurthy (2012), analyzing the local dynamics may miss insights about economic fluctuations and crises that come from studying the global dynamics. To this end, we use a continuous-time setup to characterize both the local and global dynamics in the presence of information asymmetries. Indeed, global dynamics analysis in our model shows that large economic crises can be triggered by confidence shocks occurring in the credit market, arguably an important feature of the recent 2008 financial crisis.

Finally, our extended model in Section 3 with reputation effects is also related to that of Chari, Shourideh and Zeltin-Jones (2014), who build a model of a secondary loan market with adverse selection, and show how reputation effects can generate persistent adverse selection. Multiple equilibria also arise in their model as in the classic signaling model by Spence (1973). In contrast, multiple equilibria in our reputational model take form of indeterminacy, and are generated by a different mechanism, that of endogenously countercyclical markups that mimics aggregate increasing returns.

The rest of the paper is organized as follows. Section 2 describes the baseline model, characterizes the conditions for local indeterminacy, and then proceeds to the analysis of global dynamics. Section 3 incorporates reputation effects into the baseline model and shows that indeterminacy may still arise, even without defaults in equilibrium. In Section 4 we introduce a continuous distribution of heterogeneous and stochastic firm productivities, and show that

[3]Many other papers have also addressed adverse selection in a dynamic environment. Examples include Williamson and Wright (1994), Eisfeldt (2004), House (2006), Guerrieri, Shimer, and Wright (2010), Chiu and Koeppl (2012), Daley and Green (2012), Chang (2014), Camargo and Lester (2014), and Guerrieri and Shimer (2014).

adverse selection in that model can induce endogenous TFP, amplification, aggregate increasing returns to scale and a continuum of equilibria. Section 5 concludes.

2 The Baseline Model

Time is continuous and proceeds from zero to infinity. There is an infinitely-lived representative household and a continuum of final goods producers. The final goods producers purchase intermediate goods as input to produce the final good, which is then sold to households for consumption and investment. The intermediate goods are produced by capital and labor in a competitive market. We assume no distortion in the production of intermediate goods. Final goods firms do not have resources to make up-front payments to purchase intermediate goods before production takes place and revenues from sales are realized. They must therefore borrow from competitive financial intermediaries (lenders) to finance their working capital. Lending to these final goods producers is risky, as they may default. We assume that there are two types of producers (borrowers): honest borrowers who have the ability to produce and will always pay back the loan after the production, and dishonest borrowers who will fully default on their loan. The lenders do not have information about the borrower types. They make loans to firms by with the adverse selection problem in mind. We begin by assuming that all trade is anonymous so we exclude the possibility of reputation effects. We relax these strong assumptions in Section 3, where we introduce reputation effects.

2.1 Setup

Households The representative household has a lifetime utility function

$$\int_0^\infty e^{-\rho t}\left(\log\left(C_t\right)-\psi\frac{N_t^{1+\gamma}}{1+\gamma}\right)dt \tag{1}$$

where $\rho > 0$ is the subjective discount factor, C_t is the consumption, N_t is the hours worked, $\psi > 0$ is the utility weight for labor, and $\gamma \geq 0$ is the inverse Frisch elasticity of labor supply. The household faces the following budget constraint

$$C_t + I_t \leq R_t u_t K_t + W_t N_t + \Pi_t, \tag{2}$$

where R_t, W_t and Π_t denote respectively the rental price, wage and total profits from all the firms and financial intermediaries. As in Wen (1998) we introduce an endogenous capacity utilization rate u_t. As is standard in the literature, the depreciation rate of capital increases with the capacity utilization rate according to

$$\delta(u_t) = \delta^0 \frac{u_t^{1+\theta}}{1+\theta}, \tag{3}$$

where $\delta^0 > 0$ is a constant and $\theta > 0$.[4] Finally, the law of motion for capital is governed by

$$\dot{K}_t = -\delta(u_t)K_t + I_t. \tag{4}$$

The households choose a path of consumption X_t, C_t, N_t, u_t, and K_t to maximize their utility function (1), taking R_t, W_t and Π_t as given. The first-order conditions are

$$\frac{1}{C_t}W_t = \psi N_t^{\gamma}, \tag{5}$$

$$\frac{\dot{C}_t}{C_t} = u_t R_t - \delta\left(u_t\right) - \rho, \tag{6}$$

and

$$R_t = \delta^0 u_t^{\theta}. \tag{7}$$

The left-hand side of equation (5) is the marginal utility of consumption obtained from an additional unit of work, and the right-hand side is the marginal disutility of a unit of work. equation (6) is the usual Euler equation. Finally, a one-percent increase in the utilization rate raises the total rent by $R_t K_t$ but also increases total depreciation by $\delta_0 u_t^{\theta} K_t$, so equation (7) states that the marginal benefit is equal to the marginal cost of utilization. Finally the transversality condition is given by $\lim_{t\to\infty} e^{-\rho t}\frac{1}{C_t}K_t = 0$.

Final goods producers There is a unit measure of final goods producers indexed by $i \in [0,1]$. A fraction π of them are dishonest and a fraction $1-\pi$ are honest. Each one of the honest producers is endowed with an indivisible project as in Stiglitz and Weiss (1981), which transforms Φ units of intermediate goods to Φ units of final goods. Let P_t be the price of the intermediate goods input. Each project then requires ΦP_t of working capital. The dishonest producers, however, can claim to be honest and borrow $P_t\Phi$ and then default and keep (for simplicity all) the borrowed funds. They enjoy profits of $P_t\Phi$ by doing so. Anticipating this adverse selection problem, the final intermediates will therefore charge all borrowers a gross

[4] Dong, Wang, and Wen (2015) develop a search-based theory to offer a micro-foundation for the convex depreciation function.

interest rate $R_{ft} > 1$. Hence the profit from borrowing and producing for a honest producer is given by

$$\Pi_t^h = (1 - R_{ft}P_t)\,\Phi. \tag{8}$$

Denote by s_t the measure of honest producers who invest in their projects:

$$s_t = \begin{cases} 1-\pi & \text{if } R_{ft} < \frac{1}{P_t} \\ \in [0, 1-\pi) & \text{if } R_{ft} = \frac{1}{P_t} \\ 0 & \text{if } R_{ft} > \frac{1}{P_t} \end{cases}. \tag{9}$$

The total demand for intermediate goods is hence given by

$$X_t = s_t\Phi. \tag{10}$$

Since each firm also produces Φ units of the final goods, the total quantity of the final goods produced is:

$$Y_t = s_t\Phi = X_t \tag{11}$$

Intermediate goods The intermediate goods is produced by capital and labor with the technology

$$X_t = A\tilde{K}_t^{\alpha}N_t^{1-\alpha}, \tag{12}$$

where $\tilde{K}_t = u_tK_t$ is total capital supply from the households. In a competitive market the profit of producers is $\Pi_t^x = P_tA\tilde{K}_t^{\alpha}N_t^{1-\alpha} - W_tN_t - R_t\tilde{K}_t$. The first-order conditions are

$$R_t = P_t\alpha\frac{X_t}{\tilde{K}_t} = P_t\alpha\frac{X_t}{u_tK_t}, \tag{13}$$

$$W_t = P_t(1-\alpha)\frac{X_t}{N_t}. \tag{14}$$

Under competition profits are zero, so $\Pi_t^x = 0$, and $W_tN_t + R_tu_tK_t = P_tX_t$.

Financial Intermediaries The financial intermediaries are also operated under competition. Anticipating a fraction Θ_t of the loan will be paid back, the interest rate is then given by

$$R_{ft} = \frac{1}{\Theta_t}. \tag{15}$$

So the financial intermediaries earn zero profit. The honest producers altogether borrow X_tP_t of working capital and the dishonest producers altogether borrow $\pi\Phi P_t$ as working capital. Since only the honest producers pay back their loan, the average payback rate is

$$\Theta_t = \frac{X_tP_t}{\pi\Phi P_t + X_tP_t} = \frac{X_t}{\pi\Phi + X_t}. \tag{16}$$

2.2 Equilibrium

We focus on an interior solution so $R_{ft} = \frac{1}{P_t}$.[5] In equilibrium, the total profit is simply $\pi P_t \Phi$. Hence the total budget constraint becomes

$$C_t + I_t = P_t X_t + \pi P_t \Phi. \tag{17}$$

Since $P_t = \frac{1}{R_{ft}} = \Theta_t = \frac{X_t}{\pi\Phi + X_t}$, the above equation can be further reduced to

$$C_t + I_t = P_t X_t + \pi P_t \Phi = X_t = Y_t. \tag{18}$$

We then obtain the resource constraint,

$$C_t + \dot{K}_t = Y_t - \delta(u_t) K_t. \tag{19}$$

The inverse of markup, using equation (18), is therefore is given by:

$$\phi_t \equiv 1 - \frac{\Pi_t}{Y_t} = 1 - \frac{\pi P_t \Phi}{X_t} = \Theta_t = P_t,$$

as $\phi_t = \Theta_t$, it then also represents the average quality of the borrowers in the credit market. Finally, the rental price of capital is given by

$$R_t = \phi_t \cdot \frac{\alpha Y_t}{u_t K_t}. \tag{20}$$

Likewise, the wage rate is given by

$$W_t = \phi_t \cdot \frac{(1-\alpha) Y_t}{N_t}. \tag{21}$$

Equations (5), (6) and (7) then become

$$\psi N_t^{\gamma} = \left(\frac{1}{C_t}\right)(1-\alpha)\phi_t \frac{Y_t}{N_t}, \tag{22}$$

$$\frac{\dot{C}_t}{C_t} = \alpha \phi_t \frac{Y_t}{K_t} - \delta(u_t) - \rho, \tag{23}$$

$$\alpha \phi_t \frac{Y_t}{u_t K_t} = \delta^0 u_t^{\theta} = (1+\theta) \frac{\delta(u_t)}{u_t}. \tag{24}$$

Then we have

$$u_t = \left(\frac{\alpha \phi_t Y_t}{\delta^0 K_t}\right)^{\frac{1}{1+\theta}}, \tag{25}$$

[5] We assume, without loss of generality, that Φ is big enough, so $\Phi > A K_t^{\alpha} N_t^{1-\alpha}$. We can also assume that there is free entry and an infinite measure of potential honest producers as potential entrants. The free entry condition then implies $R_{ft} = \frac{1}{P_t}$.

and thus

$$\frac{\dot{C}_t}{C_t} = \left(\frac{\theta}{1+\theta}\right) \alpha \phi_t \frac{Y_t}{K_t} - \rho. \tag{26}$$

Equation (16) then becomes

$$\phi_t = \frac{Y_t}{\pi \Phi + Y_t} \tag{27}$$

Finally the aggregate production function becomes

$$Y_t = A \left(u_t K_t\right)^{\alpha} N_t^{1-\alpha}. \tag{28}$$

In short, the equilibrium can be characterized by equations (22), (23), (24), (28), (19) and (27). These six equations fully determine the dynamics of the six variables C_t, K_t, Y_t, u_t, N_t and ϕ_t.

Equation (27) implies that ϕ_t increases with aggregate output. Note that $\frac{1}{\phi_t} = \frac{Y_t}{R_t u_t K_t + W_t N_t}$ is the aggregate markup. Therefore the endogenous markup in our model is countercyclical, which is consistent with the empirical regularity well documented in the literature.[6] The credit spread is given by $R_{ft} - 1 = \pi \Phi / Y_t$, moving in a countercyclical fashion as in the data.

The countercyclical markup has important implications. For example, it can make hours and the real wage move in the same direction. To see this, suppose N_t increases, so that output increases. Then according to equation (27), the marginal cost ϕ_t increases as well, which in turn raises the real wage in equation (21). If the markup is a constant, then the real wage would be proportional to the marginal product of labor and would fall when hours increase. Note also that when $\pi = 0$, *i.e.*, there is no adverse selection in the credit markets, equation (27) implies that $\phi_t = 1$, and our model simply collapses into a standard real business cycle model. The markup is $1/\phi_t > 1$ if and only if dishonest firms obtain rent due to information asymmetry.

2.3 Steady State

We first study the steady state of the model. We use Z to denote the steady state of variable Z_t. To solve the steady state, we first express all other variables in terms of ϕ and then we solve ϕ as a fixed-point problem. Combining equations (23) and (24) yields

$$\delta^0 u^{\theta+1} - \frac{\delta^0 u^{\theta+1}}{1+\theta} = \rho,$$

or $u = \left(\frac{1}{\delta^0} \frac{\rho}{\theta} (1+\theta)\right)^{\frac{1}{1+\theta}}$. Note that u only depends on δ^0, ρ and θ. Therefore, without loss of generality, we can set $\delta^0 = \frac{\rho}{\theta}(1+\theta)$ so that $u = 1$ at the steady state. The steady state

[6] See, e.g., Bils (1987) and Rotemberg and Woodford (1999).

depreciation rate then is $\delta(u) = \rho/\theta$. Given ϕ, we have

$$
\begin{aligned}
k_y &= \frac{K}{Y} = \frac{\alpha\phi}{\rho + \rho/\theta} = \frac{\alpha\phi\theta}{\rho(1+\theta)}, && (29)\\
c_y &= 1 - \delta k_y = 1 - \frac{\alpha\phi}{1+\theta}, && (30)\\
N &= \left(\frac{(1-\alpha)\phi}{1 - \frac{\alpha\phi}{1+\theta}}\frac{1}{\psi}\right)^{\frac{1}{1+\gamma}}, && (31)\\
Y &= A^{\frac{1}{1-\alpha}}\left(\frac{\alpha\phi\theta}{\rho(1+\theta)}\right)^{\frac{\alpha}{1-\alpha}}\left(\frac{(1-\alpha)\phi}{1 - \frac{\alpha\phi}{1+\theta}}\frac{1}{\psi}\right)^{\frac{1}{1+\gamma}} \equiv Y(\phi). && (32)
\end{aligned}
$$

Then we can use equation (27) to pin down ϕ from

$$\bar{\Phi} \equiv \pi\Phi = \left(\frac{1-\phi}{\phi}\right)\cdot Y(\phi) \equiv \Psi(\phi), \tag{33}$$

where the left-hand side is the total debt obligation of the dishonest borrowers, and the right hand-side is the maximum amount of bad loans that the credit market can tolerate under adverse selection, given that the average credit quality is ϕ. The total loss from these dishonest borrowers equals $\pi\Phi = \pi\Phi P R_f$ is exactly compensated from interest gain from the honest borrowers, $\left(\frac{1-\phi}{\phi}\right)\cdot Y(\phi) = (R_f - 1)Y(\phi)$, if equation (33) holds. When $\frac{\alpha}{1-\alpha} + \frac{1}{1+\gamma} > 1$, $\Psi(\phi)$ is a non-monotonic function of ϕ since $\Psi(0) = 0$ and $\Psi(1) = 0$. On the one hand, if the average credit quality is 0, the total supply of credit will be zero, and hence no lending will possible. On the other hand, if the average quality is one, *i.e.*, $\phi = 1$, then by definition no bad loan will be made. So given $\bar{\Phi}$, there may exist two steady state values of ϕ. Denote $\Psi_{\max} = \max_{0\le\phi\le1}\Psi(\phi)$, and $\phi^* = \arg\max_{0\le\phi\le1}\Psi(\phi)$. Then we have the following lemma regarding the possibility of multiple steady state equilibria.

Lemma 1 *When $0 < \bar{\Phi} < \Psi_{\max}$, there exists two steady states ϕ that solve $\bar{\Phi} = \Psi(\phi)$.*

It is well known that adverse selection can generate multiple equilibria in a static model (see, e.g., Wilson (1980)). So it is not surprising that our model has multiple steady state equilibria. A credit expansion by financial intermediaries invites more honest firms to borrow and produce. The increased quality of borrowers reduces the default risk, which then stimulates more lending from other financial intermediaries. In turn, the interest rate charged by financial intermediaries decreases, bringing down the production cost. This triggers an output expansion, and further encourages credit supply from the households, and thus generates more future lending. In a nutshell, lending externality exists both intratemporally and intertemporally. We will show that this type of lending externality generates a new type of multiplicity, which shares some similarities with the indeterminacy literature following Benhabib and Farmer (1994).

2.4 Local Dynamics

A number of studies have explored the role of endogenous markup in generating local indeterminacy and endogenous fluctuations (see e.g., Jaimovich (2006) and Benhabib and Wang (2013)). Following the standard practice, we study the local dynamics around the steady state.

Note that at the steady state ϕ and $\bar{\Phi}$ are linked by $\bar{\Phi} = \Psi(\phi)$, so we can parameterize the steady state either by $\bar{\Phi}$ or ϕ. We will use ϕ as it is more convenient for the study of local dynamics. Denote by $\hat{x}_t = \log X_t - \log X$ the percent deviation from its steady state. First, we log-linearize equation (27) to obtain

$$\hat{\phi}_t = (1 - \phi)\hat{y}_t \equiv \tau \hat{y}_t, \tag{34}$$

which states that the percent deviation of the marginal cost is proportional to output. Log-linearizing equations (28) and (24) yields

$$\hat{y}_t = \frac{\alpha\theta\hat{k}_t + (1+\theta)(1-\alpha)\hat{n}_t}{1+\theta-(1+\tau)\alpha} \equiv a\hat{k}_t + b\hat{n}_t, \tag{35}$$

where $a \equiv \frac{\alpha\theta}{1+\theta-(1+\tau)\alpha}$ and $b \equiv \frac{(1+\theta)(1-\alpha)}{1+\theta-(1+\tau)\alpha}$. We assume that $1+\theta-(1+\tau)\alpha > 0$, or equivalently $\tau < \frac{1+\theta}{\alpha} - 1$, to make $a > 0$ and $b > 0$. In general these restrictions are easily satisfied (see section 2.5). We can also substitute out $\hat{n}_t$ after log-linearizing equation (22) to express $\hat{y}_t$ as

$$\hat{y}_t = \frac{a(1+\gamma)}{1+\gamma-b(1+\tau)}\hat{k}_t - \frac{b}{1+\gamma-b(1+\tau)}\hat{c}_t \equiv \lambda_1\hat{k}_t + \lambda_2\hat{c}_t. \tag{36}$$

It is worth mentioning that $a+b = \frac{1+\theta-\alpha}{1+\theta-(1+\tau)\alpha} = 1$ if $\tau = 0$. Recall that $\tau = 0$ corresponds to the case without adverse selection. Thus endogenous capacity utilization alone does not generate an increasing returns to scale effect at the aggregate level. However, $a+b = \frac{1+\theta-\alpha}{1+\theta-(1+\tau)\alpha} > 1$ if $\tau > 0$. That is, through general equilibrium effects, adverse selection combined with endogenous capacity utilization mimics increasing returns to scale, even though production has constant returns to scale. Furthermore, if $\tau > \theta$, then $b > 1$. The model can then explain the procyclical movements in labor productivity $\hat{y}_t - \hat{n}_t$ without resorting to exogenous TFP shocks.

The effective increasing returns in production can generate locally indeterminate steady states as in Benhabib and Farmer (1994). If increasing capital can increase the marginal product of capital, given a fixed discount rate, the relative price of capital must fall and the relative price of consumption must rise so that the total return including capital gains or losses equals the discount rate. The increase in the relative price of consumption boosts consumption at the expense of investment, so capital drifts back towards the steady state instead of progressively exploding. The steady state then becomes a sink rather than a saddle, and therefore becomes indeterminate. The mechanism responsible for the increase in the marginal product of capital however is the increase in the supply of labor in response to higher wages that offset diminishing returns to capital in production. In standard contexts this is not possible if leisure is a normal good. In our adverse selection context however the countercyclical markups, associated with lower non-repayment rates and higher intermediate goods prices that increase with output levels, allow wages to rise sufficiently. The resulting higher labor supply can then mimic increasing returns, as the marginal product of capital rises with capital.[7]

This mechanism can be seen directly from equation (35): a one-percent increase in capital directly increases output and the marginal product of labor by a percent and, from equation (34), reduces the markup by $a\tau$ percent. Thanks to its higher marginal productivity, the labor supply also increases. A one-percent increase in labor supply then increases output by b percent. The exact increase in labor supply depends on the Frisch elasticity γ. This explains why the equilibrium output elasticity with respect to capital, λ_1, depends on parameters a and b and through them on γ and τ. On the household side, since both leisure and consumption are normal goods, an increase in consumption has a wealth effect on labor supply. The effect of a change in labor supply on output induced by a change in consumption, as seen from equation (36) obtained after substituting for labor in equation (35), works through the marginal cost channel, and also depends on τ. Again since both a and b increase with τ, output elasticities with respect to capital and consumption are increasing functions of τ. In other words, the presence of adverse selection makes equilibrium output more sensitive to changes in capital and to changes in autonomous consumption, and creates an amplification mechanism for business fluctuations.

Formally, using equation (36) and the log-linearized equations (19) and (23), we can then characterize the local dynamics as follows:

$$\begin{bmatrix} \dot{k}_t \\ \dot{c}_t \end{bmatrix} = J \cdot \begin{bmatrix} \hat{k}_t \\ \hat{c}_t \end{bmatrix}, \tag{37}$$

[7]The same mechanism for local indeterminacy can also operate in models of collateral constraints that also give countercyclical markups as in Benhabib and Wang (2013).

where

$$J \equiv \delta \begin{bmatrix} \left(\frac{1+\theta}{\alpha\phi}\right)\lambda_1 - (1+\tau)\,\lambda_1 & \left(\frac{1+\theta}{\alpha\phi}\right)(\lambda_2 - 1) + 1 - (1+\tau)\lambda_2 \\ \theta\,[(1+\tau)\lambda_1 - 1] & \theta(1+\tau)\lambda_2 \end{bmatrix}, \tag{38}$$

and $\lambda_1 \equiv \frac{a(1+\gamma)}{1+\gamma-b(1+\tau)}$, $\lambda_2 \equiv -\frac{b}{1+\gamma-b(1+\tau)}$, and $\delta = \rho/\theta$ is the steady state depreciation rate. The local dynamics around the steady state is determined by the roots of J. The model economy exhibits local indeterminacy if both roots of J are negative. Note that the sum of the roots equals the trace of J, and the product of the roots equals the determinant of J. Thus the sign of the roots of J can be observed from the sign of its trace and determinant. The following lemma specifies the sign for the trace and determinant condition for local indeterminacy.

Lemma 2 *Denote $\tau_{\min} \equiv \frac{(1+\theta)(1+\gamma)}{(1+\theta)(1-\alpha)+\alpha(1+\gamma)} - 1$ and $\tau_{\max} \equiv 1 - \phi^*$, then Trace$(J) < 0$ if and only if $\tau > \tau_{\min}$, and Det$(J) > 0$ if and only if $\tau_{\min} < \tau < \tau_{\max}$.*

According to Lemma 2, our baseline model will be indeterminate if and only if $\tau_{\min} < \tau < \tau_{\max}$. In this case, Trace$(J) < 0$ and Det$(J) > 0$ jointly imply that both roots of J are negative. We summarize this result in the following proposition.

Proposition 1 *The model exhibits local indeterminacy around a particular steady state if and only if*

$$\tau_{\min} < \tau < \tau_{\max}. \tag{39}$$

Equivalently, indeterminacy emerges if and only if $\phi \in (\phi_{\min}, \phi_{\max})$, where $\phi_{\min} \equiv 1 - \tau_{\max} = \phi^*$, and $\phi_{\max} \equiv 1 - \tau_{\min}$.

To understand the intuition behind Proposition 1, first note that if $\tau > \tau_{\min}$, we have

$$1 + \gamma - b(1+\tau) < 1 + \gamma - \frac{(1+\theta)(1-\alpha)}{1+\theta-(1+\tau_{\min})\alpha}(1+\tau_{\min}) = 0. \tag{40}$$

Then the equilibrium elasticity of output with respect to consumption λ_2 becomes positive, namely, an autonomous change in consumption will lead to an increase in output. Since capital is predetermined, labor must increase by equation (35). To induce an increase in labor, the real wage must increase enough to overcome the income effect, which is only possible if the increase in markup is large enough. In other words, τ in equation (34) must be large enough.

We have used the mapping between τ and steady state output to characterize the indeterminacy condition in terms of the model's deep parameter values. Notice that $\tau_{\max} = 1 - \phi^*$, where $\phi^* \equiv \arg\max_{0 \leq \phi \leq 1} \Psi(\phi)$. Since $1 - \bar{\phi}_L > 1 - \phi^* = \tau_{\max}$, the local dynamics around the steady state associated with $\phi = \bar{\phi}_L$ are determinate according to Proposition 1. Indeterminacy is only possible in the neighborhood of the steady state associated with $\phi = \bar{\phi}_H$. The following corollary formally characterizes the indeterminacy condition in terms of $\bar{\Phi}$.

Corollary 1 *Denote* $\bar{\Phi} = \pi\Phi$.

1. *If* $\bar{\Phi} \in (0, \Psi(\phi_{\max}))$, *then both steady states are saddles.*
2. *If* $\bar{\Phi} \in (\Psi(\phi_{\max}), \Psi_{\max})$, *then the local dynamics around the steady state* $\phi = \bar{\phi}_H$ *exhibits indeterminacy while the local dynamics around the steady state* $\phi = \bar{\phi}_L$ *is a saddle.*

As suggested by Lemma 1, we focus on the nontrivial region in which $\bar{\Phi} < \Psi_{\max}$. When $\Psi(\phi_{\max}) < \bar{\Phi} < \Psi_{\max}$, we have $\phi_{\min} = \phi^* < \bar{\phi}_H < \phi_{\max}$, and $\bar{\phi}_L < \phi_{\min}$. As a result, according to Proposition 1, the steady state $\bar{\phi}_H$ exhibits indeterminacy. For the steady state $\phi = \bar{\phi}_L$, by Lemma 2, we can conclude that the determinant of J is negative. So the two roots of J must have opposite signs and this implies a saddle. But if $0 < \bar{\Phi} < \Psi(\phi_{\max})$, we have $\bar{\phi}_H > \phi_{\max}$ and $\bar{\phi}_L < \phi_{\min}$. In this case, the determinants of J at both steady states are negative. So both steady states are saddles.

We summarize these different scenarios in Figure 1. The inverted U curve illustrates the relationship between ϕ and $\bar{\Phi}$ specified in equation (33). In Figure 1, ϕ is on the horizontal axis and $\bar{\Phi}$ is on the vertical axis. For a given $\bar{\Phi}$, the two steady states $\bar{\phi}_L$ and $\bar{\phi}_H$ can be located from the intersection of the inverted U curve and a horizontal line through point $(0, \bar{\Phi})$. The two vertical lines passing points $(\phi_{\min}, 0)$ and $(\phi_{\max}, 0)$ divide the diagram into three regions. In the left and right regions, the determinant of the Jacobian matrix J is negative, implying that one of the roots is positive and the other is negative. So if a steady state ϕ falls into either of these two regions, it is a saddle. In the middle region, $\text{Det}(J) > 0$ and $\text{Trace}(J) < 0$, and thus both roots are negative. So if the steady state ϕ falls into the middle region it is a sink which supports multiple self-fulfilling expectation-driven equilibria, or indeterminacy in its neighborhood.

Since $\bar{\Phi} = \pi\Phi$, we can reinterpret the above corollary in terms of π, the proportion of dishonest firms. For simplicity, assume Φ is large enough such that $\Phi > \Psi_{\max}$. Denote $\pi_L \equiv \Psi(\phi_{\max})/\Phi$ and $\pi_H \equiv \Psi(\phi_{\min})/\Phi = \Psi_{\max}/\Phi$, and thus $0 < \pi_L < \pi_H < 1$. Then we know that (i) if $\pi \in (0, \pi_L]$, both steady states are saddles, (ii) if $\pi \in (\pi_L, \pi_H)$, the steady state with $\phi = \bar{\phi}_L$ is a saddle while the steady state with $\phi = \bar{\phi}_H$ is a sink, and (iii) if $\pi \in [\pi_H, 1]$, then there exist no non-degenerate steady state equilibria. As indicated in Lemma 1, the third case

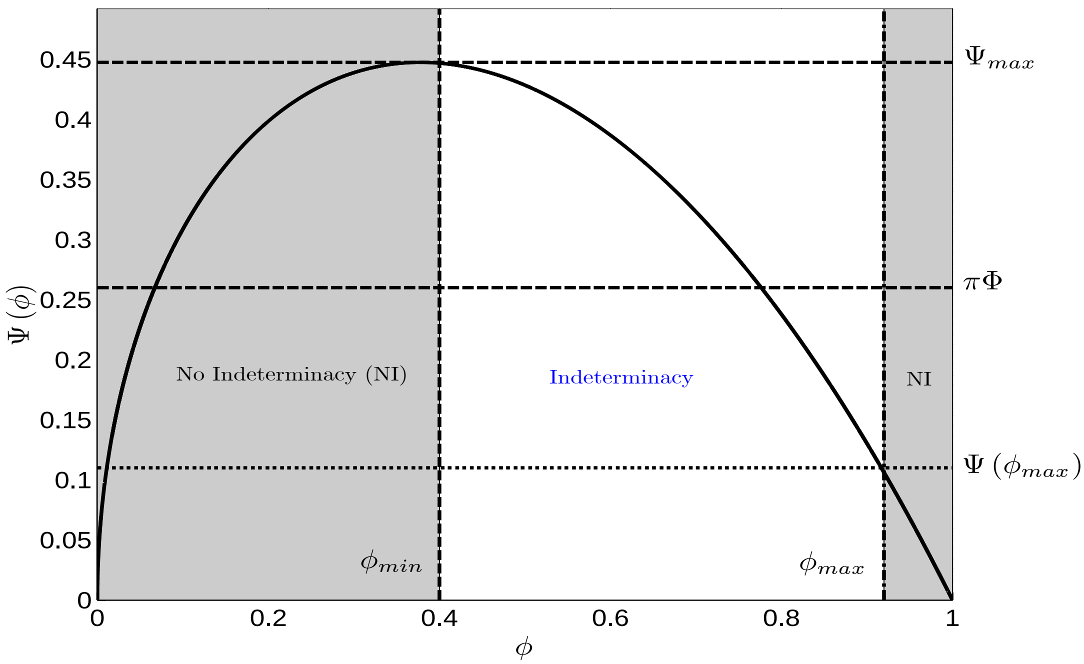

Figure 1: **Multiple Steady States and the Indeterminacy Region**

is the least interesting, and thus we focus on the scenarios in which $\pi < \pi_H$. Then the model is indeterminate if the adverse selection problem is severe enough, *i.e.*, $\pi > \pi_L$. We summarize the above argument in the following corollary.

Corollary 2 *The likelihood of indeterminacy increases with π, the proportion of dishonest firms.*

Arguably, adverse selection is more severe in developing countries. Our study then also suggests that developing countries are more likely to be subject to self-fulfilling expectation-driven fluctuations and hence exhibit higher economic volatility, which is in line with the empirical regularity emphasized by Ramey and Ramey (1995) and Easterly, Islam, and Stiglitz (2000).

2.5 Empirical Possibility of Indeterminacy

We have proved that our model with adverse selection can generate self-fulfilling equilibria in theory. We now examine the empirical plausibility of self-fulfilling equilibria under calibrated parameter values. The frequency is a quarter. We set $\rho = 0.01$, implying an annual risk-free interest rate of 4%. We set $\theta = 0.3$ so the depreciation rate at steady state is 0.033 and the annualized investment-to-capital ratio is 12% (see Cooper and Haltiwanger (2006)). We set $\alpha = 0.33$ as in the standard RBC model. We assume that labor supply is elastic, and thus set $\gamma = 0$. We normalize the aggregate productivity $A = 1$. We set $\psi = 1.75$ so that $N = \frac{1}{3}$ in the "good" steady state. We set $\overline{\Phi} = \pi\Phi = 0.13$ so that $\phi = \bar{\phi}_H = 0.9$, which is consistent with average profit rate in the data. The associated $\bar{\phi}_L = 0.011$. If we further set $\pi = 0.1$, *i.e.*, the proportion of dishonest borrowers is around 10%, then $\Phi = 1.3$.[8] Consequently, based on our calibration and the indeterminacy condition (39), we conclude that our baseline model does generate self-fulfilling equilibria.

Parameter	Value	Description
ρ	0.01	Discount factor
θ	0.3	Utilization elasticity of depreciation
δ	0.033	Depreciation rate
α	0.33	Capital income share
γ	0	Inverse Frisch elasticity of labor supply
ψ	1.75	Coefficient of labor disutility
π	0.1	Proportion of firms that produce lemons
Φ	1.3	Maximum firm capacity

Table 1: Calibration

Our calibration uses a delinquency rate of approximately 10%, which of in the same magnitude as in the Great Recession. but is higher than the average delinquency rate in the data (the average is 3.73% from period 1985 to 2013). Delinquency rates do vary over time, however. For example commercial residential mortgages had high delinquency rates during 2009-2013, which spread panic to financial markets through mortgage-backed securities and other derivatives. Nevertheless we will show in Section 3, where we introduce reputation effects, that indeterminacy will arise even if there is no default in equilibrium.

2.6 Global Dynamics

So far we have characterized the steady states and the local dynamics around these steady states. We showed that for some parameters, the equilibrium around one of the steady states is locally determinate. In this section, we analyze the global dynamics and then show that

[8]As shown in equation (33), only the product $\pi\Phi$ matters for ϕ.

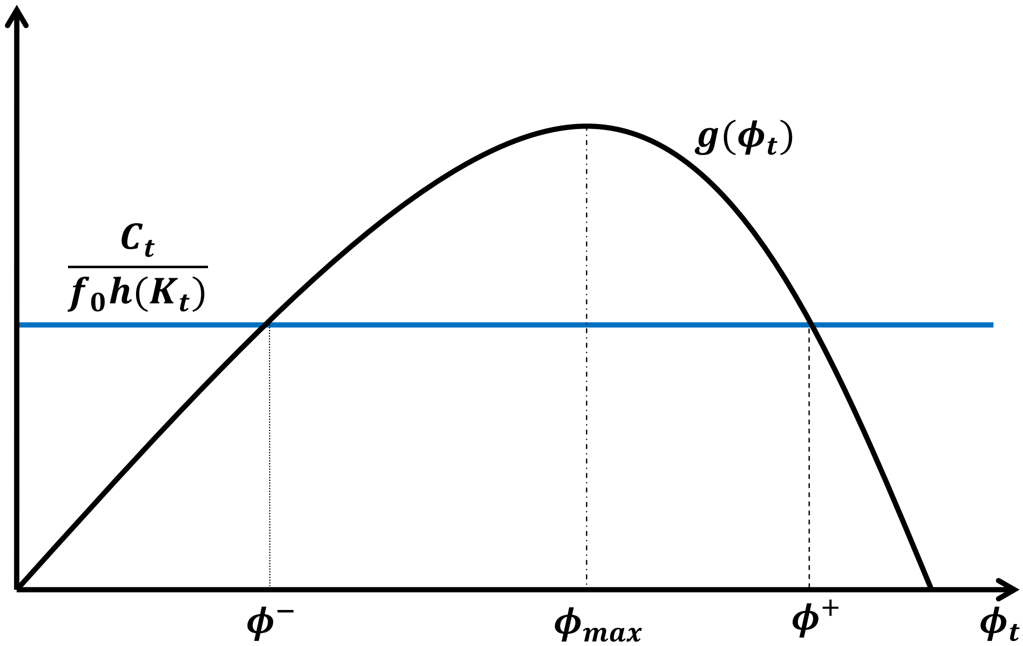

Figure 2: **Illustration of** ϕ_t

global indeterminacy always exists in our model, even in the case where both steady states are saddles and locally determinate.[9]

It is worth noting that it is impossible for us to obtain a two-dimensional autonomous dynamical system that is only related to (C_t, K_t). This is because we do not analytically formulate ϕ_t in terms of (C_t, K_t). One possible solution is to characterize a three-dimensional dynamical system on (C_t, K_t, ϕ_t). The main concern, however, is it will be difficult, if not impossible, for us to completely characterize the economic properties of the high-dimensional dynamical system. Fortunately, we can still reduce the dynamical system to a two-dimensional one, but in terms of (ϕ_t, K_t), as shown in the following proposition.

Proposition 2 *The autonomous dynamical system on* (ϕ_t, K_t) *is given by*

$$\left(1-\alpha+\frac{\alpha(1+\gamma)}{1+\theta}\right)\left(\frac{\phi_{\max}-\phi_t}{1-\phi_t}\right)\frac{\dot{\phi}_t}{\phi_t}+\left(\frac{\alpha\theta(1+\gamma)}{1+\theta}\right)\frac{\dot{K}_t}{K_t} = (1-\alpha)\left(\frac{\alpha\theta}{1+\theta}\phi_t\frac{Y(\phi_t)}{K_t}-\rho\right) \tag{41}$$

$$\dot{K}_t = \left(1-\frac{\alpha\phi_t}{1+\theta}\right)Y(\phi_t)-C(\phi_t,K_t) \tag{42}$$

[9]For an early growth model with countercyclical markups, multiple steady states and global indeterminacies see Gali (1996).

with $Y_t = Y(\phi_t) = \frac{\pi\Phi\phi_t}{1-\phi_t}$, $\phi_{\max} \equiv 1 - \tau_{\min}$, $\tau_{\min}$ *defined in Lemma 2, and*

$$C_t = C(\phi_t, K_t) = f_0 \cdot g(\phi_t) \cdot h(K_t) \tag{43}$$

where $f_0 = A^{\frac{1+\gamma}{1-\alpha}} \left(\frac{\alpha}{\delta^0}\right)^{\frac{\alpha(1+\gamma)}{(1+\theta)(1-\alpha)}} \left(\frac{1-\alpha}{\psi}\right)$, $h(K_t) = K_t^{\frac{\alpha\theta(1+\gamma)}{(1+\theta)(1-\alpha)}}$, *and*

$$g(\phi_t) = \left[\phi_t^{1-\alpha+\frac{\alpha(1+\gamma)}{1+\theta}} Y(\phi_t)^{1-\alpha-\left(1-\frac{\alpha}{1+\theta}\right)(1+\gamma)}\right]^{\frac{1}{1-\alpha}}. \tag{44}$$

As shown in equation (43), we can formulate C_t as a function function of ϕ_t and K_t. In turn, We have the following corollary regarding the relationship between equilibrium ϕ_t and C_t.

Corollary 3 *For any* $K_t > 0$ *and* $C_t < f_0 \cdot h(K_t) \cdot g(\phi_{\max})$, *there exist two possible* ϕ_t *values, denoted by* $\phi_t = \phi^+\left(\frac{C_t}{f_0 h(K_t)}\right) > \phi_{\max}$ *and* $\phi_t = \phi^-\left(\frac{C_t}{f_0 h(K_t)}\right) < \phi_{\max}$, *that yield the same level of consumption defined by (43).*

We illustrate these two possible equilibria ϕ_t in Figure 2. The function $g(\phi_t)$ has an inverted U shape. It attains the maximum at $\phi_{\max}$. Notice that $g(0) < C_t/[f_0 \cdot h(K_t)] < g(\phi_{\max})$, and then by the intermediate value theorem, there exist an ϕ_t^- such that $0 < \phi_t^- < \phi_{\max}$ and $g(\phi_t^-) = C_t/[f_0 \cdot h(K_t)]$. Since $g'(\phi) > 0$ for $0 < \phi < \phi_{\max}$, ϕ_t^- must be unique. Similarly, $g(1) < C_t/[f_0 \cdot h(K_t)] < g(\phi_{\max})$ and $g'(\phi) < 0$ for $\phi_{\max} < \phi < 1$, so there exists a unique ϕ_t^+ such that $\phi_{\max} < \phi_t^+ < 1$ and $g(\phi_t^+) = C_t/[f_0 \cdot h(K_t)]$.

As discussed by Lemma 1, the dynamical system on (ϕ_t, K_t) have two steady states. Motivated by Corollary 1, we consider two cases. In the first case, one of the steady states is a sink and the other is a saddle. In the second case, both steady states are saddles.

2.6.1 Global Dynamics with Local Indeterminacy

We first consider the case in which one steady state is a sink. As illustrated in Figure 1, π (the proportion of dishonest firms) is high and both steady state ϕ values are smaller than $\phi_{\max}$ in this case. As noted before, there is local indeterminacy around the upper steady state but local determinacy around the lower steady state. However, globally the local steady state is also indeterminate as Figure 3 shows.

In Figure 3, the thick red line is the $\dot{K}_t = 0$ locus and the thick blue line is the $\dot{\phi}_t = 0$ locus. The small circles indicate the initial conditions of trajectories. These two loci intersect twice at upper and lower steady states, respectively. For a given K_t, there is a unique level of ϕ_t such

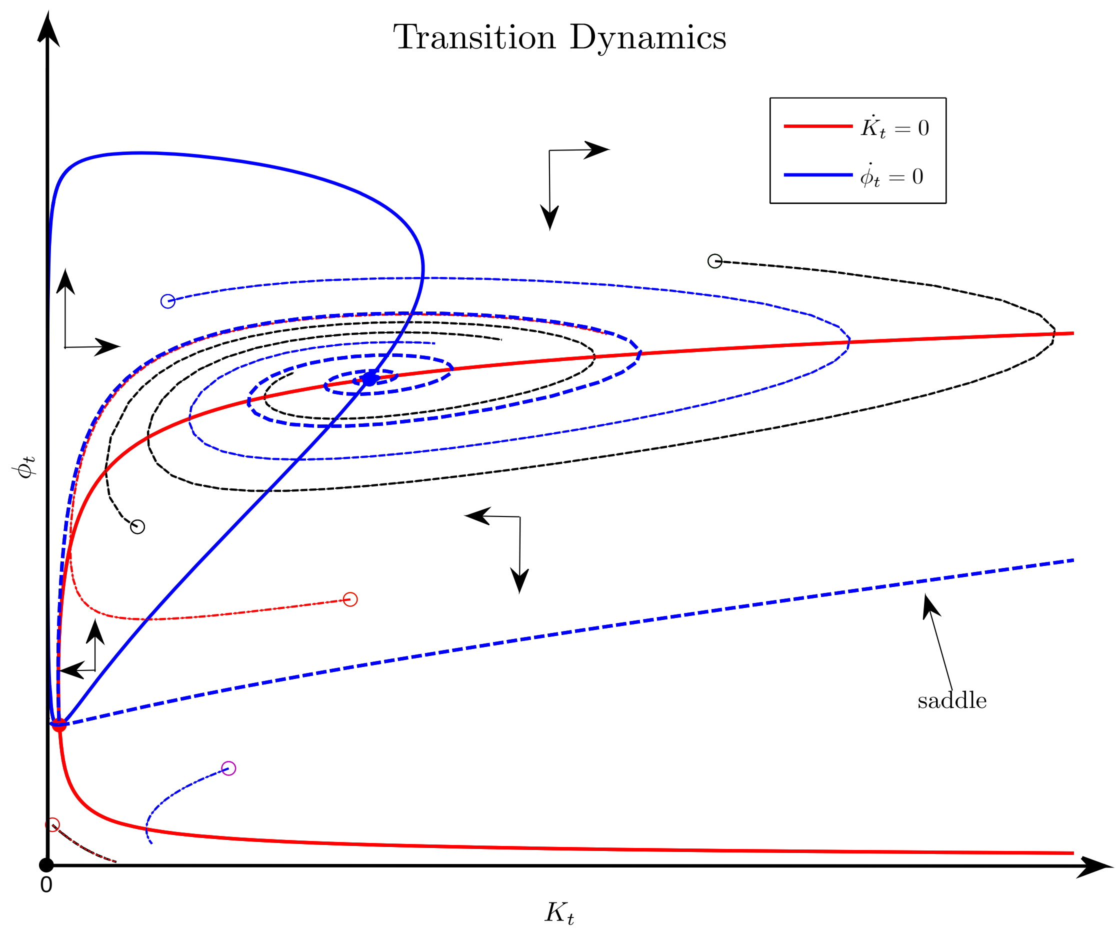

Figure 3: **Global Dynamics with One Saddle: A High π (We set $\pi = 0.2923$. All the other parameter values are from Table 1.)**

that the economy converges to the lower steady state. The function giving the unique level of ϕ_t as K_t and converging to the lower steady state is the saddle path in Figure 3, a dashed blue line. If the initial ϕ_t is below this saddle path, the economy will eventually converge to the horizontal axis with $\phi_t = 0$ and some positive capital.[10] By equation (43), this implies zero consumption and the transversality condition for households will be violated, so paths starting below this saddle path are ruled out. However, for a given K_t in the neighborhood of the lower steady state, a path starting above the saddle path cannot be ruled out. Figure 3 shows that a trajectory that starting above the saddle path takes the economy initially down and to the left, but then then turns right and up. The economy then circles around the upper steady state and eventually converges to it. As both the differential equations and the households' transversality conditions are satisfied, such a path is indeed an equilibrium path. As Figure 3 indicates, almost every initial ϕ_t that is above the saddle path associated with the lower steady state will eventually converge to the upper steady state. It is clear that during the convergence, the economy exhibits oscillations in K_t and ϕ_t. Since output is $Y_t = \pi\Phi\phi_t/(1-\phi_t)$, it also exhibits boom and bust cycles. Such transition dynamics toward the upper steady state therefore implies a rich propagation mechanism for exogenous shocks. For example, if a transitory exogenous shock moves the economy away from the upper steady state, then the economy will display persistent oscillation in output before it returns to the upper steady state.[11]

Figure 3 shows that for a given initial capital stock K_0, there are many (infinite) deterministic equilibria defined by the initial value of ϕ_0 that converges to the upper steady state smoothly. However, there are at least two other types of equilibria with jumps in ϕ_t and hence discontinuity in output. We delay discussing such equilibria in the case where both steady states are saddles to the next section. The stark contrast between the local dynamics and the global dynamics is better illustrated in that context.[12]

[10]When $\phi_t = 0$, both the capital utilization rate and the depreciation rate is zero.

[11]The global dynamics depicted in the case of a local saddle and a sink may be analyzed via the two-parameter Bogdanov-Takens (BT) bifurcation, which occurs at parameter values for the tangency point $\Psi(\phi_{\max}) = \pi\Phi$, or the BT point. By varying the parameters away from the BT point it is possible to analytically characterize the dynamics for various parameter regions yielding either zero and two steady states, and the qualitative dynamics and phase diagram in the region encompassing both steady states, including the saddle connection between the steady states, as depicted in Figure 3. See in particular Kuznetsov, 1998, p. 322. However not all parameter combinations may be economically admissible, so for Figure 3 we pick parameters in the economically admissible range. The qualitative dynamics, steady states and the saddle connection will remain as we perturb parameters.

[12]A large literature on local indeterminacy has already constructed stochastic equilibria by randomizing over the deterministic equilibria (with random jumps). So it may come as no surprise for some readers that there exist equilibria with jumps in ϕ_t when one of the steady states is locally indeterminate.

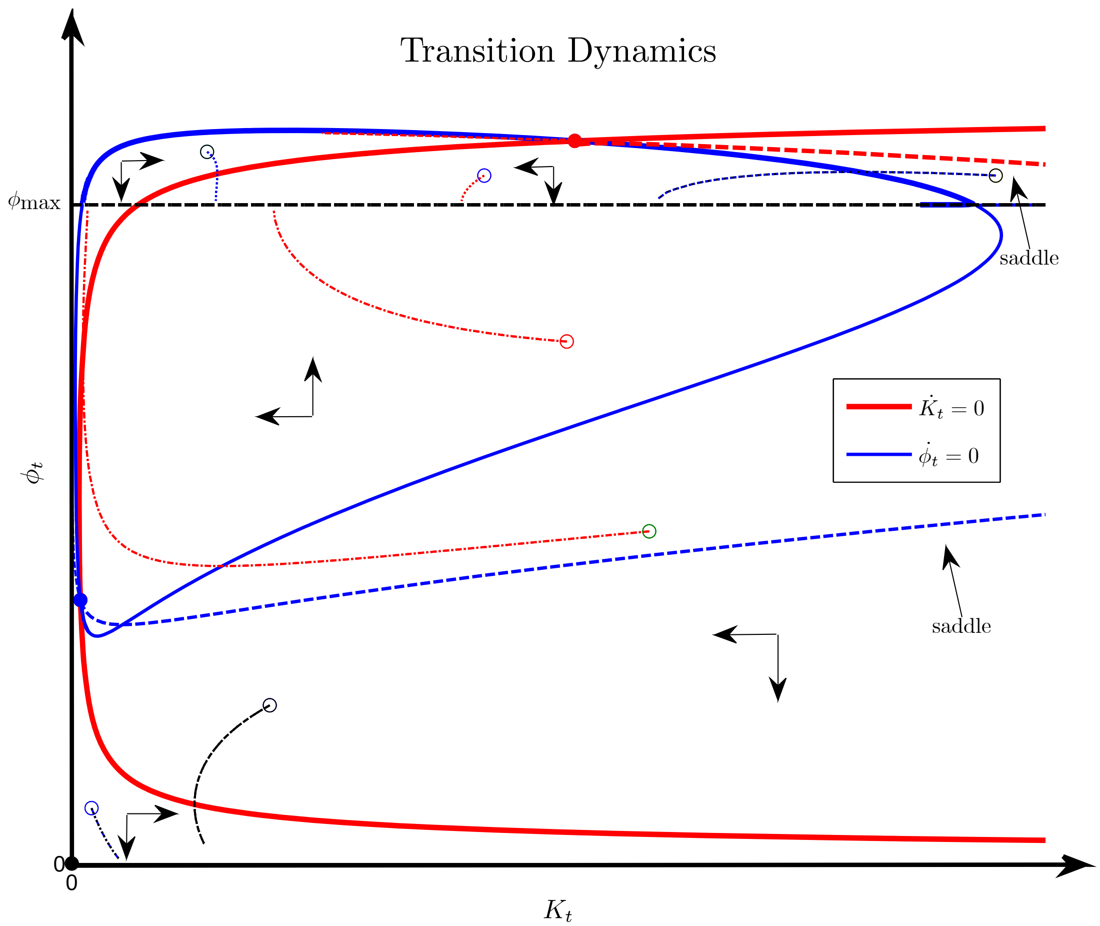

Figure 4: **Global Dynamics with One Saddle: Relatively High π (We set $\alpha = 0.62$ and $\Phi = 22$. All the other parameter values are from Table 1.)**

2.6.2 Global Dynamics with Two Saddles

In this section we study the global dynamics in the case where both π is low such that steady states are saddles, where $\bar{\phi}_H > \phi_{\max}$ and $\bar{\phi}_L < \phi_{\max}$. We set $\pi = 0.0615$ for the following numerical analysis, including in Figures 5 and 6. All the other parameter values are from Table 1.[13] Figure 4 graphs the two saddle paths associated with these two steady states. This then implies that both steady states are globally indeterminate: for any given K_t, the economy can be on either saddle path. So globally there is still indeterminacy even around each of the steady states. Furthermore, we can create very complicated equilibrium paths if we allow ϕ_t to jump. We can construct two types of jumps to illustrate the point. The first type of jump in ϕ_t are deterministic and fully anticipated. Utility maximization then requires consumption to change continuously. That is consumption does not jump when ϕ_t jumps. Notice that $\phi_t = \phi_t^+$ and $\phi_t = \phi_t^-$ yield the same consumption level for a given capital K_t. The economy can always jump from $\phi_t = \phi_t^+ > \phi_{\max}$ to $\phi_t = \phi_t^- < \phi_{\max}$ and back without changing the value of consumption, on a deterministic cycle.

Figure 5 graphs one such possible equilibrium path for each of consumption, investment, output and interest spread once we allow ϕ_t to jump. The economy starts from point $K = 6.2783$ and $\phi = 0.9717 > \phi_{\max}$ and so $C = 0.8723$. With $K = 6.2783$, there exists another $\phi = 0.8249 < \phi_{\max}$ that yields $C = 0.8723$. The economy then follows the trajectory according to equations (41) and (42). It takes around 4.41 years for the model economy to reach $K = 11.1719$, $\phi = 0.9270$ and $C = 0.9307$. We then let ϕ jump down to a level that allows consumption to remain at 0.9307 upon the jump. By construction, this leads to $\phi = 0.8241 < \phi_{\max}$ after the jump. We then let the economy follow the trajectory dictated by equations (41) and (42) again by another 8.02 years to reach $K = 6.2783$, $\phi = 0.8249$ and hence $C = 0.8723$. Notice that the consumption level has returned to its initial level. We then let ϕ jump from $\phi = 0.8249$ to $\phi = 0.9717$. Again by construction, consumption does not change immediately. We repeat this process and obtain the deterministic cycles in consumption, investment, output and credit spread in Figure 5. The adverse selection problem is modest when $\phi_t > \phi_{\max}$, but it becomes much worse when $\phi_t < \phi_{\max}$. So when ϕ_t jumps down, there is a collapse in output. Households can insure their consumption by disinvesting capital after ϕ_t jumps down. In general, there are infinite ways to construct these deterministic cycles, as

[13] To better illustrate the global dynamics with two saddles in Figure 4, we set α from 0.33 to 0.62, and Φ from 1.3 to 22. All the other parameter values are from Table 1. The numerical analysis in this section, however, uses standard parameterization in Table 1, only changing the value of π from 0.1 to 0.0615.

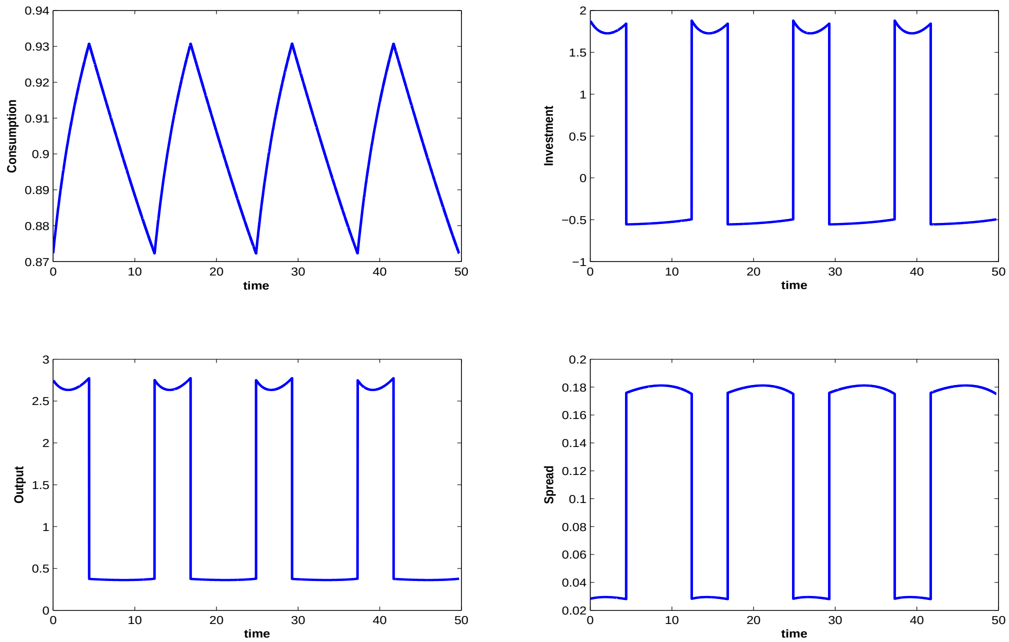

Figure 5: **Deterministic Cycles**

pointed out by Christiano and Harrison (1999).[14] Around the upper steady state, equilibrium ϕ_t can take many (possibly infinite values). So the equilibrium around the upper steady-state is still indeterminate, although it is a saddle.

Sunspot Equilibria Finally we can also construct a stochastic sunspot equilibrium by allowing ϕ_t to jump randomly. More specifically, we introduce sunspot variables z_t, which take two values, 1 and 0. We assume that in a short time interval dt, there is probability λdt that the sunspot variable will change from 1 to 0 and probability ωdt that will change from 0 to 1. We construct the equilibrium ϕ_t as a function of K_t and sunspot z_t, *i.e.*, $\phi_t = \phi(K_t, z_t)$, such that $\phi(K_t, 1) > \phi(K_t, 0)$. So the equilibrium ϕ_t will jump with an anticipated probability when z_t changes its value. When $z_t = 1$, economic confidence is high so adverse selection is modest. But when $z_t = 0$, economic confidence is low, and adverse selection becomes severe. We use the change of z_t from 1 to 0 to trigger an economic crisis, and from 0 to 1 to stop the crisis as

[14]These two ϕ_t that yield the same level of consumption correspond to two different branches in the differential equations defined by C_t and K_t. As pointed out by Christiano and Harrison (1999) a model with two branches can display rich global dynamics, regardless of the local determinacy. For example, we can construct an equilibrium with regime switches along these branches. The jumps for ϕ_t in the differential equations defined by ϕ_t and K_t correspond to the switch of branches in the dynamics defined for C_t and K_t.

economic confidence is restored. We set $\lambda = 0.01$ and $\omega = 0.025$ as an example, which means that the economy will remain in the normal, non-crisis mode with probability 0.7143. Since jumps in ϕ_t are now stochastic, consumption is exposed to a jump risk. Therefore equation (41) must be modified to take this risk into account. Denote $\phi_{1t} = \phi(K_t, 1)$ and $\phi_{0t} = \phi(K_t, 0)$. We then have

$$\begin{aligned} & \left(1-\alpha+\frac{\alpha\left(1+\gamma\right)}{1+\theta}\right)\left(\frac{\phi_{\max}-\phi_{1t}}{1-\phi_{1t}}\right)\frac{\dot{\phi}_{1t}}{\phi_{1t}}+\left(\frac{\alpha\theta\left(1+\gamma\right)}{1+\theta}\right)\frac{\dot{K}_t}{K_t} \\ = \; & (1-\alpha)\left[\frac{\alpha\theta}{1+\theta}\phi_{1t}\frac{Y_{1t}}{K_t}-\rho+\lambda\left(\frac{g(\phi_{1t})}{g(\phi_{0t})}-1\right)\right], \end{aligned}$$

for normal non-crisis times. Here the last term $\frac{g(\phi_{1t})}{g(\phi_{0t})} - 1$ reflects the percentage change in consumption due to the jump from ϕ_{1t} to ϕ_{0t} and $Y_{1t} = \pi\Phi\phi_{1t}/\left(1-\phi_{1t}\right)$ is aggregate output when $\phi_t = \phi_{1t}$. Similarly we have

$$\begin{aligned} & \left(1-\alpha+\frac{\alpha\left(1+\gamma\right)}{1+\theta}\right)\left(\frac{\phi_{\max}-\phi_{0t}}{1-\phi_{0t}}\right)\frac{\dot{\phi}_{0t}}{\phi_{0t}}+\left(\frac{\alpha\theta\left(1+\gamma\right)}{1+\theta}\right)\frac{\dot{K}_t}{K_t} \\ = \; & (1-\alpha)\left[\frac{\alpha\theta}{1+\theta}\phi_{0t}\frac{Y_{0t}}{K_t}-\rho+\omega\left(\frac{g(\phi_{0t})}{g(\phi_{1t})}-1\right)\right], \end{aligned}$$

in crisis times when $z_t = 0$.

It is evident that if $\lambda = \omega = 0$, then $\phi_{1t} = \phi(K_t, 1)$ and $\phi_{0t} = \phi(K_t, 0)$ are functions defining the saddle paths toward the upper and lower steady states, respectively. By continuity, these two functions exist for small λ and ω. We solve these two functions using the collocation method discussed in Miranda and Fackler (2002). More specifically we employ a 15-degree Chebychev polynomial of K to approximate these two functions. Once we obtain $\phi_{1t} = \phi(K_t, 1)$ and $\phi_{0t} = \phi(K_t, 0)$ as functions of capital K_t, we can then use equation (41) to simulate the dynamic path of capital. Figure 6 shows a possible dynamic path for this the economy.

We assume that the economy is initially in the normal non-crisis mode with $z_t = 1$ for a sufficiently long period. So capital, consumption, output, and investment do not change. The parameter values we choose yield $K = 10.5427$. Due to precautionary savings, this level of capital is higher than the deterministic upper steady state level of capital, as households have an incentive to save to insure against the stochastic crash in output. The economy stays at this level of capital for 2.5 years, and then a crisis emerges, triggered by a drop in z_t from 1 to 0. The spread (the bottom-right panel of Figure 6) immediately jumps up as the adverse selection problem in the credit market deteriorates sharply. As a result, production and output collapse (the bottom-left panel). Since the time of this collapse in output is unpredictable *ex*

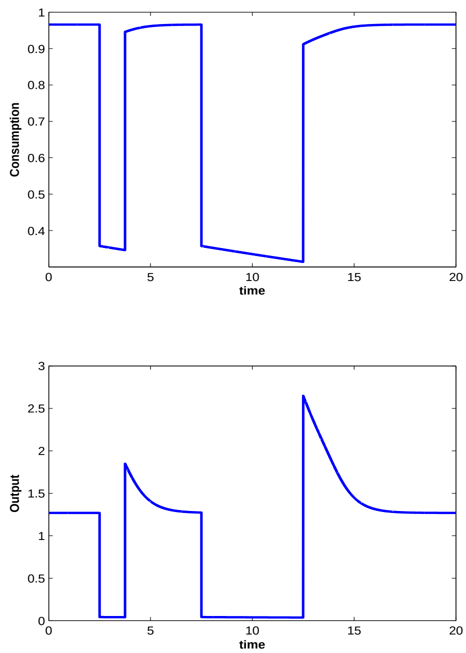

Figure 6: **Stochastic Switches between Branches**

ante, consumption drops immediately (the top-left panel). Investment (the top-right panel) falls for two reasons: one is to partially offset the fall in output to finance consumption, and the other is due to the decline in the effective return as a result of severe adverse selection in the credit market. The economy stays in crisis mode for about a year and then confidence is restored and the recession is over. Interestingly output and investment both over-shoot when the recession is over, and the longer the economy stays in recession, the larger this overshooting. The longer recession, the smaller is the amount of capital left. So the return to investment is very high, and the households opt to work hard and invest more to enjoy this high return from investment. Figure 6 shows several large boom and bust cycles due to stochastic jumps in the sunspot variables. This shows that there are rich multiple-equilibria in our benchmark model regardless of the model parameters.

3 Reputation

We now study the sensitivity of our indeterminacy results to reputation effects under adverse selection. If firms are not anonymous in the market, they may refrain from defaulting and instead may want to build their reputation. Lenders may also refrain from lending to firms with a bad credit history. Arguably, these market forces can alleviate the asymmetric information problem. So we examine whether the indeterminacy results obtained in our baseline model survives if such reputational effects are taken into account.

We follow Kehoe and Levine (1993) closely in modeling reputation. Firms are infinitely-lived, and can choose to default at any time. Firms that default may, with some probability, acquire a bad reputation and are excluded from the credit market forever. In equilibrium, the fear of loosing all future profits from production discourages firms from defaulting. We will show that self-fulfilling equilibria still exist even if there are no defaults in equilibrium.

To keep the model analytically tractable, we assume that all firms are owned by a representative entrepreneur. The entrepreneur's utility function is given by

$$U(C_{et}) = \int_0^\infty e^{-\rho_e t} \log(C_{et}) dt, \tag{45}$$

where C_{et} is the entrepreneur's consumption and ρ_e her discount factor. For tractability, we assume $\rho_e << \rho$ so that the entrepreneur does not accumulate capital. The entrepreneur's consumption equals the firm's profits,

$$C_{et} = \int_0^1 \Pi_t(i) di \equiv \Pi_t, \tag{46}$$

where $\Pi_t(i)$ denotes the profit of firm i.

Since the only cost of default is the loss of future production opportunities, the price must exceed the marginal cost (also the average cost) of production to be profitable. If the price exceeds the marginal cost, each firm will then have an incentive to produce an infinite amount. To overcome this problem, we assume that the production projects of firms are indivisible, as in the benchmark model, and that they produce to meet the orders they receive. A production project produces a flow of final goods Φ from intermediate goods. Each unit of the final good requires one unit of the intermediate good for its production. The project will be carried out only if the firms receive a purchase order. Denote the total demand for the final good by Y_t. Then a fraction $\eta_t = Y_t/\Phi$ of firms will receive a purchase order. Again we assume that firms need to borrow to finance their working capital. Denote the intermediate good price by P_t, so they need to borrow $P_t\Phi$ to produce Φ.

To illustrate the reputation problem, let us consider a short time interval from t to $t + dt$. We use V_{1t} (V_{0t}) to denote the value of a firm that receives an order (no orders). We can then formulate V_{1t} recursively as

$$V_{1t} = (1-\phi_t)\Phi dt + e^{-\rho_e dt}\left(\frac{C_{e,t}}{C_{e,t+dt}}\right)(\eta_{t+dt}V_{1t+dt} + (1-\eta_{t+dt})V_{0t+dt}), \tag{47}$$

where $\phi_t = P_t$ is the unit production cost. If $\phi_t < 1$, then the firm receives a positive profit from production. The second term on the right-hand side is the continuation value of the firms. Since firms are owned by the entrepreneur, the future value is discounted by the marginal utility of the entrepreneur. Since there is no default in equilibrium, the gross interest rate for a working capital loan is $R_{ft} = 1$.

The firms can also choose to default on their working capital, and obtain instantaneous gain of $\Phi\phi_t$. However, default comes with the risk of acquiring a bad reputation. Upon default, a firm acquires a bad reputation in the short time interval between t and $t+dt$ with probability λdt. In that case, the firm will be excluded from production forever. The payoff for defaulting is hence

$$V_t^d = \Phi dt + e^{-\rho_e dt}(1-\lambda dt)E_t\left(\frac{C_{e,t}}{C_{e,t+dt}}\right)(\eta_{t+dt}V_{1t+dt} + (1-\eta_{t+dt})V_{0t+dt}). \tag{48}$$

The value of a firm that does not receive any order is given by

$$V_{0t} = e^{-\rho_e dt}E_t\left(\frac{C_{e,t}}{C_{e,t+dt}}\right)(\eta_{t+dt}V_{1t+dt} + (1-\eta_{t+dt})V_{0t+dt}). \tag{49}$$

Define $V_t = \eta_t V_{1t} + (1-\eta_t)V_{0t}$ as the expected value of the firm. The firm has no incentive to produce lemons if and only if $V_{1t} \geq V_t^d$, or

$$\Phi dt \leq (1-\phi_t)\Phi dt + \lambda dt e^{-\rho_e dt}\left(\frac{C_{e,t}}{C_{e,t+dt}}\right)V_{t+dt}. \tag{50}$$

In the limit $dt \to 0$, the incentive compatibility condition becomes $\phi_t\Phi \leq \lambda V_t$.[15] Then the expected value of the firm is given by the present discounted value of all future profits as

$$V_t = \int_0^\infty e^{-\rho_e s}\frac{C_{et}}{C_{es}}\Pi_s ds. \tag{51}$$

For simplicity, we assume Φ is big enough such that $\eta_t = Y_t/\Phi < 1$ always holds. The average profit is then obtained as $\Pi_t = (1-\phi_t)Y_t$. Then using $C_{ej} = \Pi_j$ and integrating the right hand side of equation 51, we have

$$V_t = \frac{(1-\phi_t)Y_t}{\rho_e}. \tag{52}$$

[15] Under the incentive compatibility condition we can consider one-step deviations since V_{1t},and V_{0t} are then optimal value functions.

The households' budget constraint changes to

$$C_t + I_t \leq R_t u_t K_t + W_t N_t = \phi_t Y_t. \tag{53}$$

Then the incentive constraint (50) becomes

$$\phi_t \Phi \leq \lambda \frac{(1-\phi_t)Y_t}{\rho_e}. \tag{54}$$

From the household budget constraint (53), we know that household utility increases with ϕ_t and thus the incentive constraint (54) must be binding. Then equation (54) can be simplified as

$$\phi_t = \frac{Y_t}{\pi\Phi + Y_t} < 1, \tag{55}$$

where now $\pi \equiv \frac{\rho_e}{\lambda}$. Similar to the baseline model, here firms also receive an information rent. However, the rent in the baseline is derived from hidden information while the rent here arises from hidden action. As indicated in equation (55), ϕ_t is procyclical and hence the markup is countercyclical. When output is high, the total profit from production is high. Therefore the value of a good reputation is high and the opportunity cost of defaulting also increases. This then alleviates the moral hazard problem since high output dilutes information rent.

The cost minimization problem again yields the factor prices given by equation (20) and (21). Since households do not own firms, their budget constraint is modified as

$$C_t + \dot{K}_t = \phi_t Y_t - \delta(u_t) K_t. \tag{56}$$

The equilibrium system of equations is the same as in the baseline model except that equation (19) is replaced by equation (56). The steady state can be computed similarly. The steady state output is given by

$$Y = A^{\frac{1}{1-\alpha}} \left[\frac{\alpha\phi\theta}{\rho(1+\theta)}\right]^{\frac{\alpha}{1-\alpha}} \left[\left(\frac{1-\alpha}{1-\frac{\alpha}{1+\theta}}\right) \cdot \frac{1}{\psi}\right]^{\frac{1}{1+\gamma}} \equiv Y(\phi), \tag{57}$$

and ϕ can be solved from

$$\bar{\Phi} \equiv \pi\Phi \equiv \Psi(\phi) = \left(\frac{1-\phi}{\phi}\right) \cdot Y(\phi). \tag{58}$$

Unlike in the baseline model, here the steady state equilibrium is unique as $Y(\phi)$ is monotonic.[16] We summarize the result in the following lemma.

[16] Note that compared to equation (32), ϕ is missing from the numerator of the second bracket in equation (57).

Lemma 3 *If $\alpha < \frac{1}{2}$, a consistently standard calibrated value of α, then the steady state equilibrium is unique for any $\bar{\Phi} > 0$.*

We can now study the possibility of self-fulfilling equilibria around the steady state. Since ϕ and $\bar{\Phi}$ form a one-to-one mapping, we will treat ϕ as a free parameter in characterizing the indeterminacy condition. We can then use equation (58) to back out the corresponding value of $\bar{\Phi}$. The following proposition specifies the condition under which self-fulfilling equilibria arises.

Proposition 3 *Let $\tau = 1 - \phi$. Then indeterminacy emerges if and only if*

$$\tau_{\min} < \tau < \min\left\{\frac{1+\theta}{\alpha} - 1, \tau_H\right\} \equiv \tau_{\max},$$

where $\tau_{\min} \equiv \frac{(1+\theta)(1+\gamma)}{(1+\theta)(1-\alpha)+\alpha(1+\gamma)} - 1$, and τ_H is the positive solution to $A_1\tau^2 - A_2\tau - A_3 = 0$, where

$$\begin{aligned} A_1 &\equiv s\left(1+\theta\right)\left(2+\alpha+\alpha\gamma\right) \\ A_2 &\equiv \left(1+\theta\right)\left(1+\alpha\gamma\right) - s\left[\left(1+\theta\right)\left(1-\alpha\right)\left(1-\gamma\right) + \left(1+\gamma\right)\alpha\right] \\ A_3 &\equiv \left(1+\theta\right)\left(1-\alpha\right)\left[s + \left(1-s\right)\gamma\right]. \end{aligned}$$

Indeterminacy implies that the model exhibits multiple expectation-driven equilibria around the steady state. The steady state equilibrium is now unique however, which suggests that the continuum of equilibria implied by indeterminacy cannot be obtained in static models studied the earlier literature. So far, the condition to sustain indeterminacy is given in terms of ϕ and τ. The following corollary specifies the underlying condition in terms of ρ_e, λ and Φ.

Corollary 4 *Indeterminacy emerges if and only if $\frac{\Psi(1-\tau_{\min})}{\Phi} < \frac{\rho_e}{\lambda} < \frac{\Psi(1-\tau_{\max})}{\Phi}$.*

Given the other parameters, a decrease in ρ_e or an increase in λ increases the steady state ϕ. According to the above lemma, it makes indeterminacy less likely. The intuition is straightforward. A large λ means the opportunity cost of defaulting increases, as the firm becomes more likely to be excluded from future production. This alleviates the moral hazard problem, which is the source of indeterminacy. Similarly, a decrease of ρ_e means that the entrepreneurs become more patient. So the future profit flow from production is more valuable to them, which again increases the opportunity cost of producing lemons and thus alleviates the moral hazard problem.

4 Adverse Selection with Heterogeneous Productivity

Liu and Wang (2014) show that credit constraints can generate aggregate increasing returns to scale. We now explore the possibility of increasing returns to scale by modifying our model in Section 2 . The households' problems as in the benchmark model and thus the first order conditions are still equations (5), (6) and (7).

We now assume that the risk of lending to final good firms is continuous. We index the final goods firms with $j \in [0,1]$. Again each final goods firm has one production project, which requires Φ units of the intermediate goods. The loan is risky as the final goods firms' production may not be successful. More specifically, we assume that final goods firm j's output is governed by

$$y_{jt} = \begin{cases} a_{jt}x_{jt}, & \text{with probability } q_{jt} \\ 0, & \text{with probability } 1-q_{jt} \end{cases}, \tag{59}$$

where x_{jt} is the intermediate input for firm j and a_{jt} the firm's productivity. We assume q_{jt} is *i.i.d.* and drawn from a common distribution function $F(q)$ and $a_{jt} = a_{\min} q_{jt}^{-\tau}$. So a higher productivity a_{jt} is associated with a lower probability of success q_{jt}. Notice that expected productivity is given by $q_{jt}a_{jt} = a_{\min} q_{jt}^{1-\tau}$. We assume however that $\tau < 1$, *i.e.*, a firm with a higher success probability enjoys a higher expected productivity. Denote by P_t the price of intermediate goods. Then the total borrowing is given by $P_t x_{jt}$. Denote by R_{ft} the gross interest rate. Then final goods firm $j's$ profit maximization problem becomes

$$\max_{x_{jt} \in \{0,\Phi\}} q_{jt} \left(a_{jt}x_{jt} - R_{ft}P_t x_{jt} \right), \tag{60}$$

Note that due to limited liability, the final goods firm pays back the working capital loan only if the project is successful. This implies that, given R_{ft} and P_t, the demand for x_{jt} is simply given by

$$x_{jt} = \begin{cases} \Phi & \text{if } a_{jt} > R_{ft}P_t \equiv a_t^* \\ 0 & \text{otherwise} \end{cases}, \tag{61}$$

or equivalently,

$$a_{\min} q_{jt}^{-\tau} > a_t^*,\ q_{jt} < \left(\frac{a_t^*}{a_{\min}} \right)^{-\frac{1}{\tau}} = q_t^* = \left(\frac{R_{ft}P_t}{a_{\min}} \right)^{-\frac{1}{\tau}}. \tag{62}$$

This establishes that only firms with risky production opportunities will enter the credit markets, which highlights the adverse selection problem in the financial market. Firms with $q_{jt} > q_t^*$ are driven out of the financial market, despite their higher social expected productivity. Since financial intermediaries are assumed to be fully competitive, we have

$$R_{ft}P_t\Phi \int_0^{q_t^*} q dF(q) = P_t\Phi \int_0^{q_t^*} dF(q), \tag{63}$$

where the left-hand side is the actual repayment from the final goods firms, and the right-hand side is the actual lending. Then the interest rate is given by

$$R_{ft} = \frac{1}{\int_0^{q_t^*} q dF(q) / \int_0^{q_t^*} dF(q)} = \frac{1}{E\left(q | q \leq q_t^*\right)} > 1, \tag{64}$$

where the denominator is the average success rate. The above equation says that the interest rate decreases with the average success rate.

The total production of final goods is

$$Y_t = \int_0^1 q_j a_{jt} x_{jt} dF(q) = \Phi \int_0^{q_t^*} a_{\min} q^{1-\tau} dF(q). \tag{65}$$

where the second equality follows equation (61). The total production of intermediate goods is

$$X_t = \Phi \int_0^{q_t^*} dF(q). \tag{66}$$

Finally the intermediate goods are produced according to $X_t = A_t \left(u_t K_t\right)^{\alpha} N_t^{1-\alpha}$, where $u_t K_t$ is the capital rented from the households. Combining equations (65) and (66) then yields

$$Y_t = \Gamma(q_t^*) A_t \left(u_t K_t\right)^{\alpha} N_t^{1-\alpha}, \tag{67}$$

where $\Gamma(q_t^*) = \left(\int_0^{q_t^*} a_{\min} q^{1-\tau} dF(q)\right) / \int_0^{q_t^*} dF(q)$ depends on the threshold q_t^* and the distribution. The above equation then says that the measured TFP is obtained as

$$TFP_t = \frac{Y_t}{(u_t K_t)^{\alpha} N_t^{1-\alpha}} = \Gamma(q_t^*) A_t. \tag{68}$$

Since $\Gamma'(q_t^*) = \frac{a_{\min} f(q_t^*) \int_0^{q_t^*} \left(q_t^{*1-\tau} - q^{1-\tau}\right) dF(q)}{\left(\int_0^{q_t^*} dF(q)\right)^2} > 0$, the endogenous TFP increases with the threshold q_t^*. This is very intuitive: as the threshold increases, more firms with high productivity enter the credit market, making resource allocation more efficient. Equation (65) implies that q_t^* increases with Y_t, so we get the following lemma.

Lemma 4 *TFP is endogenous and increase in Y, i.e.,* $\frac{\partial \Gamma(q_t^*)}{\partial Y_t} > 0$.

We have therefore established that the endogenous TFP is procyclical. Notice that the procyclicality of endogenous TFP holds generally for continuous distributions. So without loss of generality, we now assume $F(q) = q^{\eta}$ for tractability. In turn, firm-level measured productivity $\frac{1}{q}$ follows a Pareto distribution with the shape parameter of η, which is consistent with the findings of a large literature (see, e.g., Melitz (2003) and references therein). Under the assumption of a power distribution, combining equations (65) and (67) yields the aggregate output

$$Y_t = \left(\frac{\eta}{\eta - \tau + 1}\right) a_{\min} \Phi^{-\frac{1-\tau}{\eta}} \left(A_t u_t^{\alpha} K_t^{\alpha} N_t^{1-\alpha}\right)^{1+\frac{1-\tau}{\eta}} . \tag{69}$$

The intuition is as follows. Here a lending externality kicks in because of adverse selection in the credit markets. Suppose that the total lending from financial intermediaries increases. This creates downward pressure on interest rate R_{ft}, which increases the cutoff q_t^* according to the definition in equation (62). Firms with a higher q have a smaller risk of default. A rise in the cutoff q_t^* therefore reduces the average default rate. If the rise is big enough, it can in turn stimulate more lending from the financial intermediaries. Since firms with higher q are also more productive on average, the increased efficiency in reallocating credit implies that resources are better allocated across firms. Notice that the aggregate output again exhibits increasing returns to scale. Equation (69) reveals that the degree of increasing returns to scale clearly depends on the adverse selection problem and decreases with τ and η. When $\eta = \infty$, the firms' product quality is homogeneous. Hence there is no asymmetric information or adverse selection. If $\tau = 1$, firms are equally productive in the sense their expected productivity is the same. It therefore does not matter how credits are allocated among firms. Given $\tau < 1$, a smaller η implies that firms are more heterogenous, creating a larger asymmetric information problem. Similarly, given η, a smaller τ implies that the productivity of firms deteriorates faster with respect to their default risk, making the adverse selection more damaging to resource allocation. We formally state this result in the following proposition.

Proposition 4 *The reduced-form aggregate production in our model exhibits increasing returns to scale if and only if there exists adverse selection, i.e.,* $\tau < 1$ *and* $\eta < \infty$.

In an important contribution, Basu and Fernald (1997) document that increasing returns to scale exist in aggregate production but not at the micro level. In a recent paper, Liu and Wang (2014) show how credit constraints can generate endogenous variation in TFP, and hence aggregate increasing returns. In their model, the less productive firms are driven out of production. Different from Liu and Wang (2014), firms in our model do not suffer from credit constraints; the more productive firms in our model are driven out of production due to adverse selection.

As in the benchmark model, both the credit spread, $R_{ft} - 1$, and the expected default risk, $1 - E\left(q | q \leq q_t^*\right)$, are countercyclical. These predictions are consistent with the empirical regularities by Gilchrist and Zakrajšek (2012) and many others.

4.1 Indeterminacy

It is straightforward to show that $W_t = \phi \frac{(1-\alpha)Y_t}{N_t}$ and $R_t = \phi \frac{\alpha Y_t}{u_t K_t}$ respectively. Here $\phi = \frac{\eta+1-\tau}{\eta+1}$ and is constant instead of procyclical. Together with equations (5), (6), (7), (69), and (19), we can determine the seven variables, C_t, Y_t, N_t, u_t, K_t, W_t and R_t. The steady state can be obtained as in the baseline model. We can express the other variables in terms of the steady state ϕ. Since ϕ is unique, unlike in the baseline model, the steady state here is unique. We assume that Φ is large enough so that an interior solution to q^* is always guaranteed. The following proposition summarizes the conditions for indeterminacy in this extended model.

Proposition 5 *Given the power distribution, i.e., $F(q) = (q/q_{\max})^{\eta}$, (or equivalently, firm productivity conforms to a Pareto distribution), the steady state is unique. Moreover, the model is indeterminate if and only if*

$$\sigma_{\min} < \sigma < \sigma_{\max} \tag{70}$$

where $\sigma \equiv \frac{1-\tau}{\eta}$, $\sigma_{\min} \equiv \left(\frac{1}{\frac{1-\alpha}{1+\gamma} + \frac{\alpha}{1+\theta}} \right) - 1$ and $\sigma_{\max} \equiv \frac{1}{\alpha} - 1$.

To better understand the proposition, we first consider how output responds to a fundamental shock, such as a change in A, the true TFP. Holding factor inputs constant, we have

$$1 + \tilde{\sigma} \equiv \frac{d \log Y_t}{d \log A} = (1+\sigma) \left[\frac{1+\theta}{1+\theta - \alpha(1+\sigma)} \right] > 1, \tag{71}$$

The above equations show that adverse selection and variable capacity utilization can amplify the impact of a TFP shock on output. Let us define $1+\tilde{\sigma}$ as the multiplier of adverse selection. Note that the necessary condition $\sigma > \sigma_{\min}$ can be written as

$$(1+\tilde{\sigma})(1-\alpha) - 1 > \gamma. \tag{72}$$

The model will be indeterminate if the multiplier effect of adverse selection is sufficiently large. The restriction $\sigma < \sigma_{\max}$ is typically automatically satisfied. The restriction $\sigma < \frac{1}{\alpha} - 1$ simply requires that $\alpha(1+\sigma) < 1$, which is the condition needed to rule out explosive growth in the model.

Whether the model is indeterminate or not, equation (71) implies that the response of output to TFP shocks will be amplified. In addition, by Proposition 4, the economy is more likely to be indeterminate if η is smaller. Our results are hence in the same spirit as those of Kurlat (2013) and Bigio (2014), showing that a dispersion in quality will strengthen the amplification effect of adverse selection.

Empirical Possibility of Indeterminacy To empirically evaluate the possibility of indeterminacy, we set the same values for ρ, θ, δ, α and γ as in Table 1.[17] We also have new parameters in this extended model, (τ, η). We use two moments to pin them down and set τ and η to match the steady state markup $\frac{\eta+1-\tau}{\eta+1} = 0.9$. Basu and Fernald (1997) estimate aggregate increasing returns to scale in manufacturing to approximately 1.1. So we set $\sigma = 0.1$. This leads to $\tau = 0.55$ and $\eta = 4.5$. We have $\sigma_{\min} = 0.083$ and $\sigma_{\max} \equiv 2$, which meet the indeterminacy conditions. Hence, with these parameters the model exhibits self-fulfilling equilibria.

5 Conclusion

We have shown that in realistically calibrated dynamic general equilibrium models, adverse selection in credit markets can generate a continuum of equilibria in the form of indeterminacy, either through endogenous markups or endogenous TFP. Adverse selection can therefore potentially explain high output volatility and boom and bust cycles in the absence of fundamental shocks. For example, an RBC model with a negative TFP shock cannot fully explain the increase in labor productivity during the Great Recession (see Ohanian (2010)). Yet this feature of the Great Recession is consistent with the prediction of our baseline model in Section 2, and is driven by pessimistic beliefs about aggregate output. The pessimistic beliefs reduce aggregate demand and increase markups, leading to a lower real wage and a lower labor supply. Labor productivity however rises due to decreasing returns to labor.

To keep our analysis simple, we abstracted from some important features of the credit markets, for example, runs on various financial intermediaries that may amplify the initial adverse selection problem, as for example in the subprime mortgages during the Great Recession. Future research may examine the effects of adverse selection among financial intermediaries.

[17] $q_{\max}$ and Φ do not affect the indeterminacy condition, so we do not need to specify their values.

Appendix

A Proofs

Proof of Lemma 1: The proof is straightforward. First, from the explicit form of $Y(\phi)$, we can easily prove that $\Psi(\phi) \equiv \left(\frac{1-\phi}{\phi}\right) \cdot Y(\phi)$ strictly increases with ϕ when $\phi \in (0, \phi^*)$ but strictly decreases with ϕ when $\phi \in (\phi^*, 1)$. Second, since $\Psi(0) < \bar{\Phi} < \Psi^* = \Psi(\phi^*)$, there exists a unique solution between zero and ϕ^*, denoted by $\bar{\phi}_L$, that solves $\Psi(\phi) = \bar{\Phi}$. Likewise, there also exists a unique solution between ϕ^* and 1, denoted by $\bar{\phi}_H$, that solves $\Psi(\phi) = \bar{\Phi}$.

Proof of Lemma 2: Denote by φ_1 and φ_2 the eigenvalues of matrix J so that we have $\varphi_1 + \varphi_2 =$Trace(J) and $\varphi_1\varphi_2 =$Det(J). Then the model is indeterminate if the trace of J is negative and the determinant is positive. The trace and the determinant of J are

$$\begin{aligned}
\frac{\text{Trace}\,(J)}{\delta} &= \left(\frac{1+\theta}{\alpha\phi}\right)\lambda_1 - (1+\tau)\,\lambda_1 + \theta\,(1+\tau)\,\lambda_2, \\
\frac{\text{Det}\,(J)}{\delta^2\theta} &= \left[(1+\tau)\,\lambda_1 - 1 + \lambda_2\right]\left(\frac{1+\theta}{\alpha\phi} - 1\right) - \tau\lambda_2,
\end{aligned}$$

respectively, where

$$\lambda_1 = \frac{a(1+\gamma)}{1+\gamma - b(1+\tau)}, \text{ and } \lambda_2 = -\frac{b}{1+\gamma-b(1+\tau)},$$

as defined in equation (36).

Substituting out λ_1 and λ_2 we obtain

$$\begin{aligned}
\frac{\text{Trace}\,(J)}{\delta} &= \left[\frac{1}{\gamma+1-(1+\tau)b}\right] \cdot \left[\left(\frac{1+\theta}{\alpha\phi} - 1 - \tau\right)a(1+\gamma) - \theta(1+\tau)b\right] \\
&= \left[\left(\frac{\theta}{\phi}\right)\left(\frac{\alpha\,(1+\gamma) + (1+\theta)\,(1-\alpha)}{1+\theta-(1+\tau)\alpha}\right)\right] \cdot \left[\frac{\frac{(1+\gamma)(1+\theta)}{\alpha(1+\gamma)+(1+\theta)(1-\alpha)} - \phi\,(1+\tau)}{\gamma+1-(1+\tau)b}\right] \\
&= \left[\left(\frac{\theta}{\phi}\right)\left(\frac{\alpha\,(1+\gamma) + (1+\theta)\,(1-\alpha)}{1+\theta-(1+\tau)\alpha}\right)\right] \cdot \left[\frac{\frac{(1+\gamma)(1+\theta)}{\alpha(1+\gamma)+(1+\theta)(1-\alpha)} - 1 + \tau^2}{\gamma+1-(1+\tau)b}\right]
\end{aligned}$$

Notice that $\gamma + 1 - (1+\tau)b < 0$ is equivalent to

$$\tau > \tau_{\min} \equiv \frac{(1+\gamma)\,(1+\theta)}{\alpha(1+\gamma)+(1+\theta)(1-\alpha)} - 1.$$

Since $\tau_{\min} > 0$, we know that

$$\frac{(1+\gamma)\,(1+\theta)}{\alpha(1+\gamma)+(1+\theta)(1-\alpha)} - 1 + \tau^2 > 0.$$

Therefore Trace$(J) < 0$ if and only if $\tau > \tau_{\min}$. It remains for us to determine the condition under which Det$(J) > 0$. Note that Det(J) can be rewritten as

$$
\begin{aligned}
\frac{\operatorname{Det}(J)}{\delta^2\theta} &= \left[\frac{1}{\gamma+1-(1+\tau)b}\right]\cdot\left[\left(\frac{1+\theta}{\alpha\phi}-1\right)((1+\gamma)\left[a(1+\tau)-1\right]+\tau b)+\tau b\right] \\
&= \frac{1+\theta}{(1+\tau)b-(\gamma+1)}\left\{(1+\gamma)(1-\alpha)-\left[\frac{(1-\alpha)(1+\theta)}{(1+\theta-\alpha\phi)}+(1+\gamma)\alpha\right]\tau\right\}.
\end{aligned}
$$

If $\tau < \tau_{\min}$, then we immediately have Det$(J) < 0$. Thus to guarantee that Det$(J) > 0$, we must have $\tau > \tau_{\min}$, which then implies that $(1+\tau)b-(\gamma+1) > 0$. As a result, given that $\tau > \tau_{\min}$, Det$(J) > 0$ if and only if

$$(1+\gamma)(1-\alpha)-\left[\frac{(1-\alpha)(1+\theta)}{1+\theta-\alpha\phi}+(1+\gamma)\alpha\right]\tau > 0,$$

which can be further simplified as

$$\tau < \frac{(1+\gamma)(1-\alpha)}{\frac{(1-\alpha)(1+\theta)}{1+\theta-\alpha\phi}+(1+\gamma)\alpha}.$$

Since $\phi = 1-\tau$, the above inequality can be reformulated as

$$\Delta(\tau) \equiv \alpha^2\tau^2+\left[\alpha\theta+\frac{(1-\alpha)(1+\theta)}{(1+\gamma)}\right]\tau-(1-\alpha)(1+\theta-\alpha) < 0.$$

Denote $\xi \equiv \alpha\theta+\frac{(1-\alpha)(1+\theta)}{(1+\gamma)}$. Then det$(J) > 0$ if and only if $\tau > \tau_{\min}$ and

$$\tau < \tau_{\max} \equiv \frac{-\xi+\sqrt{\xi^2+4\alpha^2(1-\alpha)(1+\theta-\alpha)}}{2\alpha^2}.$$

It remains for us to prove that $\tau_H = 1-\phi^*$, where $\phi^* = \arg\max_{0\le\phi\le1}\Psi(\phi)$. The first-order condition of $\log\Psi(\phi)$ suggests

$$\left(\frac{1}{1+\gamma}+\frac{2\alpha-1}{1-\alpha}\right)\left(\frac{1}{\phi}\right)+\left(\frac{1}{1+\gamma}\right)\left(\frac{\alpha}{1+\theta}\right)\left(\frac{1}{1-\frac{\alpha\phi}{1+\theta}}\right)-\frac{1}{1-\phi}=0,$$

which is equivalent to

$$\Gamma(\phi) \equiv \alpha^2\phi^2-\left[\frac{(1-\alpha)(1+\theta)}{1+\gamma}+\alpha\theta+2\alpha^2\right]\phi+\left[\frac{(1-\alpha)(1+\theta)}{1+\gamma}+(2\alpha-1)(1+\theta)\right]=0.$$

Besides, we can easily verify that, for $\phi \in (0,1)$, it always holds that

$$\frac{d^2}{d\phi^2}(\log\Psi(\phi)) < 0.$$

Since $\tau \equiv 1-\phi$, we know that $\Delta(1-\phi) = \Gamma(\phi)$. Denote by ϕ_1 and ϕ_2 the solutions to $\Gamma(\phi) = 0$. Note that $\phi_1+\phi_2 > 0$, $\phi_1\cdot\phi_2 > 0$, and $\Gamma(0) > 0$, $\Gamma(1) > 0$. Therefore we know that $0 < \phi_1 < 1 < \phi_2$. Consequently we conclude that

$$\phi^* = \phi_1 = 1-\tau_{\max} \in (0,1).$$

Proof of Proposition 1: Notice that, by definition, $\tau_{\max} = 1 - \phi_{\min}$. Therefore we have $\phi_{\min} = \phi^*$. Then by Lemma 2 we know that

1. If $\phi < \phi_{\min}$, then Trace$(J) < 0$, and Det$(J) < 0$.
2. If $\phi \in (\phi_{\min}, \phi_{\max})$, then Trace$(J) < 0$, and Det$(J) > 0$.
3. If $\phi > \phi_{\max}$, then Trace$(J) > 0$, and Det$(J) < 0$.

Proof of Corollary 1: First, when adverse selection is severe enough, *i.e.*, $\bar{\Phi} = \pi\Phi \geq \Psi_{\max}$, the economy collapses. The only equilibrium is the trivial case with $\phi = 0$. Given that $\bar{\Phi} < \Psi_{\max}$, Lemma 1 implies that there are two solutions, which are denoted by $\left(\bar{\phi}_H, \bar{\phi}_L\right)$. It always holds that $\bar{\phi}_L < \phi^* < \bar{\phi}_H$. Then Lemma 2 immediately suggests that the steady state $\bar{\phi}_L$ is always a saddle. Since $\Psi(\phi)$ decreases with ϕ when $\phi > \phi^*$, as shown in Proposition 1, indeterminacy emerges if and only if $\phi \in (\phi^*, \phi_{\max})$. Therefore the local dynamics around the steady state $\phi = \bar{\phi}_H$ exhibits indeterminacy if and only if $\Psi(\phi_{\max}) < \bar{\Phi} < \Psi_{\max}$.

Proof of Corollary 2: Holding Φ constant, $\bar{\Phi}$ increases with π, the proportion of firms producing lemon products. As is proved in Corollary 1, given $\bar{\Phi} < \Psi_{\max}$, indeterminacy emerges if and only if $\bar{\Phi} > \Psi(\phi_{\max})$. Therefore the likelihood of indeterminacy increases with π.

Proof of Proposition 2: As shown in Section 2, the dynamical system on (C_t, K_t) is given by

$$\frac{\dot{C}_t}{C_t} = \left(\frac{\theta}{1+\theta}\right)\alpha\phi_t\frac{Y_t}{K_t} - \rho, \tag{A.1}$$

$$\dot{K}_t = Y_t - \left(\delta^0\frac{u_t^{1+\theta}}{1+\theta}\right)K_t - C_t. \tag{A.2}$$

where

$$u_t^{1+\theta} = \frac{\alpha}{\delta^0}\frac{\phi_t Y_t}{K_t}, \tag{A.3}$$

$$Y_t = Y(\phi_t) \equiv \left(\frac{\phi_t}{1-\phi_t}\right)\pi\Phi, \tag{A.4}$$

and

$$\delta(u_t) \equiv \delta^0\frac{u_t^{1+\theta}}{1+\theta},$$

in which $\delta^0 = \frac{\rho}{\theta}(1+\theta)$ so that $u=1$ at the steady state.

First, equation (A.3) implies

$$u_t = \left(\frac{\alpha\phi_t Y_t}{\delta^0 K_t}\right)^{\frac{1}{1+\theta}},$$

and thus we have

$$N_t^{1-\alpha} = \frac{Y_t}{A u_t^{\alpha} K_t^{\alpha}} = \frac{Y_t^{1-\frac{\alpha}{1+\theta}} \phi_t^{-\frac{\alpha}{1+\theta}} K_t^{-\frac{\alpha\theta}{1+\theta}}}{A\left(\frac{\alpha}{\delta^0}\right)^{\frac{\alpha}{1+\theta}}}. \tag{A.5}$$

Substituting equation (A.5) into (5) yields

$$\left[\frac{Y_t^{1-\frac{\alpha}{1+\theta}} \phi_t^{-\frac{\alpha}{1+\theta}} K_t^{-\frac{\alpha\theta}{1+\theta}}}{A\left(\frac{\alpha}{\delta^0}\right)^{\frac{\alpha}{1+\theta}}}\right]^{1+\gamma} = \left[\left(\frac{1}{C_t}\right)\left(\frac{1-\alpha}{\psi}\right)\phi_t Y_t\right]^{1-\alpha},$$

which can be further simplified as

$$\frac{Y_t^{\left(1-\frac{\alpha}{1+\theta}\right)(1+\gamma)} \phi_t^{-\frac{\alpha(1+\gamma)}{1+\theta}} K_t^{-\frac{\alpha\theta(1+\gamma)}{1+\theta}}}{A^{1+\gamma}\left(\frac{\alpha}{\delta^0}\right)^{\frac{\alpha(1+\gamma)}{1+\theta}}} = C_t^{-(1-\alpha)}\left(\frac{1-\alpha}{\psi}\right)^{(1-\alpha)} \phi_t^{1-\alpha} Y_t^{1-\alpha},$$

or equivalently,

$$C_t^{1-\alpha} = A^{1+\gamma}\left(\frac{\alpha}{\delta^0}\right)^{\frac{\alpha(1+\gamma)}{1+\theta}}\left(\frac{1-\alpha}{\psi}\right)^{(1-\alpha)} \phi_t^{1-\alpha+\frac{\alpha(1+\gamma)}{1+\theta}} Y_t^{1-\alpha-\left(1-\frac{\alpha}{1+\theta}\right)(1+\gamma)} K_t^{\frac{\alpha\theta(1+\gamma)}{1+\theta}}. \tag{A.6}$$

Substituting equation (A.4) into (A.6) yields

$$C_t = C(\phi_t, K_t) = f_0 \cdot g(\phi_t) \cdot h(K_t), \tag{A.7}$$

where $f_0 = A^{\frac{1+\gamma}{1-\alpha}}\left(\frac{\alpha}{\delta^0}\right)^{\frac{\alpha(1+\gamma)}{(1+\theta)(1-\alpha)}}\left(\frac{1-\alpha}{\psi}\right)$, $h(K_t) = K_t^{\frac{\alpha\theta(1+\gamma)}{(1+\theta)(1-\alpha)}}$, and

$$g(\phi_t) = \left[\phi_t^{1-\alpha+\frac{\alpha(1+\gamma)}{1+\theta}} Y(\phi_t)^{1-\alpha-\left(1-\frac{\alpha}{1+\theta}\right)(1+\gamma)}\right]^{\frac{1}{1-\alpha}}.$$

In turn, differentiating both sides of equation (A.7) yields

$$C_t^{1-\alpha} = A^{1+\gamma}\left(\frac{\alpha}{\delta^0}\right)^{\frac{\alpha(1+\gamma)}{1+\theta}}\left(\frac{1-\alpha}{\psi}\right)^{(1-\alpha)} \phi_t^{1-\alpha+\frac{\alpha(1+\gamma)}{1+\theta}} Y_t^{1-\alpha-\left(1-\frac{\alpha}{1+\theta}\right)(1+\gamma)} K_t^{\frac{\alpha\theta(1+\gamma)}{1+\theta}},$$

which immediately implies

$$
\begin{aligned}
(1-\alpha)\frac{\dot{C}_t}{C_t} &= \left(1-\alpha+\frac{\alpha(1+\gamma)}{1+\theta}\right)\frac{\dot{\phi}_t}{\phi_t}+\left(1-\alpha-\left(1-\frac{\alpha}{1+\theta}\right)(1+\gamma)\right)\frac{\dot{Y}_t}{Y_t}+\left(\frac{\alpha\theta(1+\gamma)}{1+\theta}\right)\frac{\dot{K}_t}{K_t} \\
&= \left(1-\alpha+\frac{\alpha(1+\gamma)}{1+\theta}+\left(1-\alpha-\left(1-\frac{\alpha}{1+\theta}\right)(1+\gamma)\right)\frac{Y'(\phi_t)\phi_t}{Y(\phi_t)}\right)\frac{\dot{\phi}_t}{\phi_t}+\left(\frac{\alpha\theta(1+\gamma)}{1+\theta}\right)\frac{\dot{K}_t}{K_t} \\
&= \left(1-\alpha+\frac{\alpha(1+\gamma)}{1+\theta}-\left(\left(1-\frac{\alpha}{1+\theta}\right)(1+\gamma)-(1-\alpha)\right)\left(\frac{1}{1-\phi_t}\right)\right)\frac{\dot{\phi}_t}{\phi_t}+\left(\frac{\alpha\theta(1+\gamma)}{1+\theta}\right)\frac{\dot{K}_t}{K_t} \\
&= \left(1-\alpha+\frac{\alpha(1+\gamma)}{1+\theta}\right)\left(\frac{\phi_{\max}-\phi_t}{1-\phi_t}\right)\frac{\dot{\phi}_t}{\phi_t}+\left(\frac{\alpha\theta(1+\gamma)}{1+\theta}\right)\frac{\dot{K}_t}{K_t} \qquad \text{(A.8)}
\end{aligned}
$$

Additionally, we have

$$
u_t=\left(\frac{\alpha}{\delta^0}\frac{\phi_t Y(\phi_t)}{K_t}\right)^{\frac{1}{1+\theta}} \equiv u(K_t,\phi_t). \qquad \text{(A.9)}
$$

In the end, substituting equation (A.7) and (A.9) into (A.1) and (A.2) yields

$$
\begin{aligned}
\left(1-\alpha+\frac{\alpha(1+\gamma)}{1+\theta}\right)\left(\frac{\phi_{\max}-\phi_t}{1-\phi_t}\right)\frac{\dot{\phi}_t}{\phi_t}+\left(\frac{\alpha\theta(1+\gamma)}{1+\theta}\right)\frac{\dot{K}_t}{K_t} &= (1-\alpha)\left(\frac{\alpha\theta}{1+\theta}\phi_t\frac{Y(\phi_t)}{K_t}-\rho\right), \\
\dot{K}_t &= \left(1-\frac{\alpha\phi_t}{1+\theta}\right)Y(\phi_t)-C(\phi_t,K_t),
\end{aligned}
$$

the desired autonomous dynamical system in Proposition 2.

Proof of Corollary 3: We can easily verify that $g(0)=g(1)=0$, $g''(\phi)<0$, and $g'(\phi_{\max})=0$, where $\phi_{\max}=1-\tau_{\min}$, and $\tau_{\min}$ is defined in Lemma 2. Therefore we have $\phi_{\max}=\arg\max g(\phi)$. It then follows from equation (43) that C_t is a hump-shaped function of ϕ_t for a given level of K_t. Then we immediately obtain the results in Lemma 3.

Proof of Lemma 3: Notice that $\Psi(\phi)=\left(\frac{1-\phi}{\phi}\right)\cdot Y(\phi)\propto(1-\phi)\phi^{\frac{2\alpha-1}{1-\alpha}}$. When $\alpha<\frac{1}{2}$, we know that $(1-\phi)\phi^{\frac{2\alpha-1}{1-\alpha}}$ is decreasing in ϕ. It is easy to check that $\lim_{\phi\to 0}\Psi(\phi)=\infty$ and $\lim_{\phi\to 1}\Psi(\phi)=0$. Hence equation (58) uniquely pins down the steady state ϕ for any $\bar{\Phi}>0$.

Proof of Proposition 3: The dynamical system with reputation is given by

$$\begin{aligned}
\psi N_t^{\gamma} &= \frac{1}{C_t}(1-\alpha)\phi_t \frac{Y_t}{N_t}, \\
\frac{\dot{C}_t}{C_t} &= \alpha\phi_t \frac{Y_t}{K_t} - \delta(u_t) - \rho, \\
\alpha\phi_t \frac{Y_t}{u_t K_t} &= \delta^0 u_t^{\theta}, \\
C_t + \dot{K}_t + C_t^e &= Y_t - \delta\left(u_t\right) K_t, \\
Y_t &= A\left(u_t K_t\right)^{\alpha} N_t^{1-\alpha}, \\
\phi_t &= \frac{Y_t}{\pi\Phi + Y_t}, \\
C_t^e &= \left(1-\phi_t\right) Y_t,
\end{aligned}$$

where $\pi \equiv \frac{\rho_e}{\lambda}$. Denote $s \equiv 1 - \frac{\alpha}{1+\theta}$. Then some of the key ratios in the steady state can be obtained as

$$\begin{aligned}
k_y &= \frac{K}{Y} = \frac{\alpha\phi\theta}{\rho\left(1+\theta\right)}, \\
c_y &= \frac{C}{Y} = s\phi = \left(1 - \frac{\alpha}{1+\theta}\right)\phi, \\
N &= \left[\frac{\left(1-\alpha\right)\phi}{c_y} \cdot \frac{1}{\psi}\right]^{\frac{1}{1+\gamma}} = \left[\left(\frac{1-\alpha}{1-\frac{\alpha}{1+\theta}}\right) \cdot \frac{1}{\psi}\right]^{\frac{1}{1+\gamma}}, \\
Y &= A^{\frac{1}{1-\alpha}} \left(k_y\right)^{\frac{\alpha}{1-\alpha}} N = A^{\frac{1}{1-\alpha}} \left[\frac{\alpha\phi\theta}{\rho\left(1+\theta\right)}\right]^{\frac{\alpha}{1-\alpha}} \left[\left(\frac{1-\alpha}{1-\frac{\alpha}{1+\theta}}\right) \cdot \frac{1}{\psi}\right]^{\frac{1}{1+\gamma}}. \qquad \text{(A.10)}
\end{aligned}$$

We can use equation (58) to solve for the steady state ϕ and use equation (A.10) to obtain the steady state Y. Consumption and capital can then be computed from $C = c_y Y$ and $K = k_y Y$, respectively. The log-linearization of the system of equilibrium equations is given by:

$$\begin{aligned}
0 &= \hat{\phi}_t + \hat{y}_t - \left(1+\gamma\right)\hat{n}_t - \hat{c}_t, \\
\dot{c}_t &= \rho\left(\hat{\phi}_t + \hat{y}_t - \hat{k}_t\right), \\
\hat{y}_t &= \alpha\left(\hat{u}_t + \hat{k}_t\right) + (1-\alpha)\hat{n}_t, \\
\hat{u}_t &= \frac{1}{1+\theta}(\hat{\phi}_t + \hat{y}_t - \hat{k}_t), \\
\dot{k}_t &= \left(\frac{s\phi}{k_y}\right)\left(\hat{\phi}_t + \hat{y}_t - \hat{k}_t\right) - \left(\frac{c_y}{k_y}\right)\left(\hat{c}_t - \hat{k}_t\right), \\
\hat{\phi}_t &= \left(1-\phi\right)\hat{y}_t \equiv \tau\hat{y}_t.
\end{aligned}$$

As in the baseline model, we can substitute $\hat{u}_t$ and $\hat{\phi}_t$ to obtain a reduced form of output in terms of capital and labor as

$$\hat{y}_t = \frac{\alpha\theta\hat{k}_t + (1+\theta)(1-\alpha)\hat{n}_t}{1+\theta-(1+\tau)\alpha} \equiv a\hat{k}_t + b\hat{n}_t,$$

where $a \equiv \frac{\alpha\theta}{1+\theta-(1+\tau)\alpha}$ and $b \equiv \frac{(1+\theta)(1-\alpha)}{1+\theta-(1+\tau)\alpha}$. We assume $\tau < \frac{1+\theta}{\alpha} - 1$, which is a reasonable restriction under standard calibrations, so that $a > 0$ and $b > 0$. Finally $\hat{n}_t$ can be expressed as a function of $\hat{y}_t$ and $\hat{c}_t$, and thus we have

$$\hat{y}_t = \frac{a(1+\gamma)}{1+\gamma-b(1+\tau)}\hat{k}_t - \frac{b}{1+\gamma-b(1+\tau)}\hat{c}_t \equiv \lambda_1\hat{k}_t + \lambda_2\hat{c}_t,$$

where $\lambda_1 \equiv \frac{a(1+\gamma)}{1+\gamma-b(1+\tau)}$ and $\lambda_2 \equiv -\frac{b}{1+\gamma-b(1+\tau)}$. Consequently the local dynamics is characterized by the following differential equations:

$$\begin{aligned}
\begin{bmatrix} \dot{k}_t \\ \dot{c}_t \end{bmatrix} &= \delta \begin{bmatrix} \left(\frac{1+\theta}{\alpha\phi}\right) s\phi\left(1+\tau\right)\lambda_1 & \left(\frac{1+\theta}{\alpha\phi}\right)\left[s\phi\left(1+\tau\right)\lambda_2 - \left(1-s\phi\right)\right] \\ \theta\left[(1+\tau)\lambda_1 - 1\right] & \theta(1+\tau)\lambda_2 \end{bmatrix} \begin{bmatrix} \hat{k}_t \\ \hat{c}_t \end{bmatrix}, \\
&\equiv J \begin{bmatrix} \hat{k}_t \\ \hat{c}_t \end{bmatrix},
\end{aligned}$$

where $s \equiv 1-\frac{\alpha}{1+\theta}$, $c_y = s\phi$, $\delta = \rho/\theta$. The local dynamics around the steady state is determined by the roots of J. Notice that the trace and the determinant of J are

$$\begin{aligned}
\frac{\text{Trace}\,(J)}{\delta} &= \left(\frac{1+\theta}{\alpha}\right) s\left(1+\tau\right)\lambda_1 + \theta(1+\tau)\lambda_2 < 0, \\
\frac{\text{Det}\,(J)}{\delta^2\theta\left(\frac{1+\theta}{\alpha\phi}\right)} &= s\phi\left(1+\tau\right)\lambda_2 + \left(1-s\phi\right)\left(1+\tau\right)\lambda_1 - \left(1-s\phi\right) > 0.
\end{aligned}$$

Similar to the analysis of the indeterminacy for our baseline model, here Trace$(J) < 0$ if and only if $\tau > \tau_{\min} \equiv \frac{(1+\theta)(1+\gamma)}{(1+\theta)(1-\alpha)+\alpha(1+\gamma)} - 1$. Given that $\tau > \tau_{\min}$, some algebraic manipulation shows that Det$(J) > 0$ if and only if $\tau < \frac{1+\theta}{\alpha} - 1$, and

$$A_1\tau^2 - A_2\tau - A_3 < 0,$$

where

$$\begin{aligned}
A_1 &\equiv s\left(1+\theta\right)\left(2+\alpha+\alpha\gamma\right) > 0 \\
A_2 &\equiv \left(1+\theta\right)\left(1+\alpha\gamma\right) - s\left[\left(1+\theta\right)\left(1-\alpha\right)\left(1-\gamma\right) + \left(1+\gamma\right)\alpha\right] \\
A_3 &\equiv \left(1+\theta\right)\left(1-\alpha\right)\left[s + + \left(1-s\right)\gamma\right] > 0.
\end{aligned}$$

Therefore $A_1\tau^2 - A_2\tau - A_3 < 0$ if and only if $\tau < \tau_H$, where τ_H is the positive solution to $A_1\tau^2 - A_2\tau - A_3 = 0$.

Proof of Corollary 4: Combining Lemma 3 and Proposition 2 immediately yields the desired result.

Proof of Lemma 4: First, using the Implicit Function Theorem, equation (67) suggests that $\frac{\partial q^*}{\partial Y} > 0$. Second, since $TFP = \Gamma(q^*)A$, it is obvious that $\frac{\partial TFP}{\partial q^*} > 0$. Then using the chain rule gives $\frac{\partial TFP}{\partial Y} = \left(\frac{\partial TFP}{\partial q^*}\right)\left(\frac{\partial q^*}{\partial Y}\right) > 0$.

Proof of Proposition 4: We immediately reach the proposition by observing equation (69).

Proof of Proposition 5: First, given the power distribution, *i.e.*, $F(q) = (q/q_{\max})^\eta$, we can analytically obtain the dynamical system, and then easily verify the uniqueness of the steady state. It remains for us to pin down the indeterminacy region. To establish the conditions for indeterminacy, we first log-linearize the equilibrium equations. Substituting $\hat{u}_t$ from the log-linearized equation (24), we obtain

$$\hat{y}_t = a\hat{k}_t + b\hat{n}_t,$$

where $a = \frac{\theta\alpha(1+\sigma)}{1+\theta-\alpha(1+\sigma)}$ and $b = \frac{(1+\theta)(1-\alpha)(1+\sigma)}{1+\theta-\alpha(1+\sigma)}$. Finally, expressing $\hat{n}_t$ from the log-linearized equation (22), we obtain

$$\hat{y}_t = \lambda_1\hat{k}_t + \lambda_2\hat{c}_t,$$

where $\lambda_1 \equiv \frac{a(1+\gamma)}{1+\gamma-b}$ and $\lambda_2 \equiv -\frac{a}{1+\gamma-b}$. We hence obtain a two-dimensional system of differential equations

$$\begin{bmatrix} \dot{k}_t \\ \dot{c}_t \end{bmatrix} = \delta \begin{bmatrix} \left(\frac{1+\theta}{\alpha\phi} - 1\right)\lambda_1 & \left(\frac{1+\theta}{\alpha\phi}\right)(\lambda_2 - 1) + 1 - \lambda_2 \\ \theta(\lambda_1 - 1) & \theta\lambda_2 \end{bmatrix} \begin{bmatrix} \hat{k}_t \\ \hat{c}_t \end{bmatrix} \equiv J \begin{bmatrix} \hat{k}_t \\ \hat{c}_t \end{bmatrix}$$

where $\delta = \rho/\theta$. The local dynamics around the steady state is determined by the roots of J. The trace and the determinant of J are

$$\begin{aligned} \frac{\mathrm{Trace}\,(J)}{\delta} &= \left(\frac{1+\theta}{\alpha\phi} - 1\right)\lambda_1 + \theta\lambda_2 = \frac{\left(\frac{1+\theta}{\alpha\phi} - 1\right)(1+\gamma)\,a - \theta b}{1+\gamma-b}, \\ \frac{\det(J)}{\delta^2\theta} &= \left(\frac{1+\theta}{\alpha\phi} - 1\right)(\lambda_1 - 1 + \lambda_2) = \left(\frac{1+\theta}{\alpha\phi} - 1\right)\left[\frac{(1+\gamma)(a-1)}{1+\gamma-b}\right]. \end{aligned}$$

Indeterminacy arises if $\mathrm{Trace}(J) < 0$ and $\det(J) > 0$. Under the assumption $a < 1$, or $\alpha(1+\sigma) < 1$, $\mathrm{Det}(J) > 0$ is equivalent to $1+\gamma-b$, or $\sigma > \sigma_{\min} \equiv \left(\frac{1}{\frac{1-\alpha}{1+\gamma} + \frac{\alpha}{1+\theta}}\right) - 1$. Then

Trace$(J) < 0$ requires $\left(\frac{1+\theta}{\alpha\phi} - 1\right)(1+\gamma)\, a > \theta b$. Rearranging terms yields the requirement, $\frac{(1+\sigma)\eta}{1+\eta} < \left(\frac{1}{\frac{1-\alpha}{1+\gamma} + \frac{\alpha}{1+\theta}}\right)$. Recall that $\sigma = \frac{1-\chi}{\chi+\eta}$, or $\frac{(1+\sigma)\eta}{1+\eta} = \frac{\eta}{\chi+\eta}$, so that this requirement is automatically satisfied.

References

Allen, Franklin. "Reputation and Product Quality." *The RAND Journal of Economics* (1984): 311-327.

Akerlof, George A. "The Market for "Lemons": Quality Uncertainty and the Market Mechanism." *The Quarterly Journal of Economics* (1970): 488-500.

Basu, Susanto, and **John G. Fernald.** "Are Apparent Productive Spillovers A Figment of Specification Error?." *Journal of Monetary Economics* 36, no. 1 (1995): 165-188.

Basu, Susanto, and **John G. Fernald.** "Returns to Scale in US Production: Estimates and Implications." *Journal of Political Economy* 105, no. 2 (1997): 249-283.

Benhabib, Jess, and **Roger EA Farmer.** "Indeterminacy and Increasing Returns." *Journal of Economic Theory* 63, no. 1 (1994): 19-41.

Benhabib, Jess, and **Pengfei Wang.** "Financial Constraints, Endogenous Markups, and Self-fulfilling Equilibria." *Journal of Monetary Economics* 60, no. 7 (2013): 789-805.

Benhabib, Jess, and **Yi Wen.** "Indeterminacy, Aggregate Demand, and the Real Business Cycle." *Journal of Monetary Economics* 51, no. 3 (2004): 503-530.

Bernanke, Ben, and **Mark Gertler.** "Agency Costs, Net worth, and Business Fluctuations." *The American Economic Review (1989)*: 14-31.

Bigio, Saki. "Endogenous Liquidity and the Business Cycle." *The American Economic Review*, forthcoming (2014).

Bils, Mark. "The Cyclical Behavior of Marginal Cost and Price." *The American Economic Review* (1987): 838-855.

Broda, Christian, and **David E. Weinstein.** "Product Creation and Destruction: Evidence and Price Implications." *The American Economic Review*, 100(3): 691-723 (2010).

Brunnermeier, Markus K., and **Yuliy Sannikov.** "A Macroeconomic Model with a Financial Sector." *The American Economic Review* 104.2 (2014): 379-421.

Camargo, Braz, and **Benjamin Lester.** "Trading Dynamics in Decentralized Markets with Adverse Selection." *Journal of Economic Theory*, 153 (2014) 534–568.

Chang, Briana. "Adverse Selection and Liquidity Distortion in Decentralized Markets." Working Paper, University of Wisconsin, Madison (2014).

Chari, V. V., Ali Shourideh, and **Ariel Zeltin-Jones.** "Reputation and Persistence of Adverse Selection in Secondary Loan Markets.", *The American Economic Review*, forthcoming (2014).

Christiano, Lawrence J., and **Sharon G. Harrison.** "Chaos, Sunspots and Automatic Stabilizers." *Journal of Monetary Economics* 44.1 (1999): 3-31.

Chiu, Jonathan, and **Thorsten V. Koeppl.** "Trading Dynamics with Adverse Selection and Search: Market freeze, Intervention and Recovery." No. 2011-30. Bank of Canada Working Paper, 2011.

Cooper, Russell, and **Thomas W. Ross.** "Monopoly Provision of Product Quality with Uninformed Buyers." *International Journal of Industrial Organization* 3, no. 4 (1985): 439-449.

Cooper, Russell W., and **John C. Haltiwanger.** "On the Nature of Capital Adjustment Costs." *The Review of Economic Studies* 73, no. 3 (2006): 611-633.

Daley, Brendan, and **Brett Green.** "Waiting for News in the Market for Lemons." *Econometrica* 80, no. 4 (2012): 1433-1504.

Dong, Feng, Pengfei Wang, and **Yi Wen.** "Credit Search and Credit Cycles." *Economic Theory* (2015): 1-25.

Eisfeldt, Andrea L. "Endogenous Liquidity in Asset Markets." *The Journal of Finance* 59, no. 1 (2004): 1-30.

Easterly, William, Roumeen Islam, and **Joseph E. Stiglitz.** "Shaken and Stirred: Explaining Growth Volatility." In Annual World Bank Conference on Development Economics, vol. 191, p. 211. 2001.

Farmer, Roger EA, and **Jang-Ting Guo.** "Real Business Cycles and the Animal Spirits Hypothesis." *Journal of Economic Theory* 63.1 (1994): 42-72.

Gali, Jordi. "Monopolistic Competition, Business Cycles, and The Composition of Aggregate Demand." *Journal of Economic Theory* 63, no. 1 (1994): 73-96.

Gali, Jordi. "Multiple Equilibria in a Growth Model with Monopolistic Competition." *Economic Theory* 8.2 (1996): 251-266.

Gilchrist, Simon, and **Egon Zakrajšek**. "Credit Spreads and Business Cycle Fluctuations." *The American Economic Review* 102, no. 4 (2012): 1692-1720.

Guerrieri, Veronica, Robert Shimer, and **Randall Wright.** "Adverse Selection in Competitive Search Equilibrium." *Econometrica* 78, no. 6 (2010): 1823-1862.

Guerrieri, Veronica, and **Robert Shimer.** 2014. "Dynamic Adverse Selection: A Theory of Illiquidity, Fire Sales, and Flight to Quality." *The American Economic Review*, 104(7): 1875-1908.

He, Zhiguo, and **Arvind Krishnamurthy.** "A model of capital and crises." *The Review of Economic Studies* 79.2 (2012): 735-777.

House, Christopher L. "Adverse Selection and the Financial Accelerator." *Journal of Monetary Economics* 53, no. 6 (2006): 1117-1134.

Jaimovich, Nir. "Firm Dynamics and Markup Variations: Implications for Sunspot Equilibria and Endogenous Economic Fluctuations." *Journal of Economic Theory* 137.1 (2007): 300-325.

Karlan, Dean, and **Jonathan Zinman.** "Expanding Credit Access: Using Randomized Supply Decisions to Estimate the Impacts." *Review of Financial studies* (2009): hhp092.

Kehoe, Timothy J., and **David K. Levine.** "Debt-constrained Asset Markets." *The Review of Economic Studies* 60.4 (1993): 865-888.

Kiyotaki, Nobuhiro, and **John Moore.** "Credit Cycles." *The Journal of Political Economy* 105, no. 2 (1997): 211-248.

Klein, Benjamin, and **Keith B. Leffler.** "The Role of Market Forces in Assuring Contractual Performance." *The Journal of Political Economy* (1981): 615-641.

Kurlat, Pablo. "Lemons Markets and the Transmission of Aggregate Shocks." *The American Economic Review* 103, no. 4 (2013): 1463-1489.

Kuznetsov, Yuri A., *Elements of Applied Bifurcation Theory*, Second Edition, New York: Springer-Verlag, 1998, http://wwwf.imperial.ac.uk/˜dturaev/kuznetsov.pdf

Liu, Zheng, and **Pengfei Wang.** "Credit Constraints and Self-Fulfilling Business Cycles." *American Economic Journal: Macroeconomics* 6, no. 1 (2014): 32-69.

Melitz, Marc J. "The Impact of Trade on Intra-industry Reallocations and Aggregate Industry Productivity." *Econometrica* 71.6 (2003): 1695-1725.

Miranda Mario, J., and **Paul L. Fackler.** "Applied Computational Economics and Finance." (2002).

Ohanian, Lee E. "The Economic Crisis from A Neoclassical Perspective." *The Journal of Economic Perspectives* 24, no. 4 (2010): 45-66.

Pintus, Patrick, Yi Wen, and **Xiaochuan Xing.** "Interest Rate Dynamics, Variable-Rate Loan Contracts, and the Business Cycle." FRB St. Louis Working Paper 2015-32 (2015).

Priest, George L. "A Theory of the Consumer Product Warranty." *Yale Law Journal* (1981): 1297-1352.

Ramey, Garey, and **Valerie A. Ramey.** "Cross-Country Evidence on the Link Between Volatility and Growth." *The American Economic Review* 85.5 (1995): 1138-1151.

Rotemberg, Julio J., and **Michael Woodford.** "The Cyclical Behavior of Prices and Costs." *Handbook of Macroeconomics* 1 (1999): 1051-1135.

Shapiro, Carl. "Consumer Information, Product Quality, and Seller Reputation." *The Bell Journal of Economics* (1982): 20-35.

Spence, Michael. "Job Market Signaling." *The Quarterly Journal of Economics* (1973): 355-374.

Sufi, Amir. "Information Asymmetry and Financing Arrangements: Evidence from Syndicated Loans." *The Journal of Finance* 62.2 (2007): 629-668.

Stiglitz, Joseph E., and **Andrew Weiss.** "Credit Rationing in Markets with Imperfect Information." *The American Economic Review* (1981): 393-410.

Wang, Pengfei, and **Yi Wen.** "Imperfect Competition and Indeterminacy of Aggregate Output." *Journal of Economic Theory* 143, no. 1 (2008): 519-540.

Wen, Yi. "Capacity Utilization under Increasing Returns to Scale." *Journal of Economic Theory* 81, no. 1 (1998): 7-36.

Williamson, Stephen, and **Randall Wright.** "Barter and Monetary Exchange under Private Information." *The American Economic Review* (1994): 104-123.

Wilson, Charles. "The Nature of Equilibrium in Markets with Adverse Selection." *The Bell Journal of Economics* (1980): 108-130.