Adverse Selection and Self-fulfilling Business Cycles

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Abstract

This paper introduces a simple adverse selection problem arising in credit markets into a standard textbook continuous-time real business cycle model. Such adverse selection generates multiple steady states and both local and global indeterminacy, and can give rise to equilibria with probabilistic jumps in credit, consumption, investment and employment driven by Markov sunspots under calibrated parameterizations and fully rational expectations. Introducing reputational effects eliminates defaults and results in a unique but still indeterminate steady state. Finally we generalize the model to firms with heterogeneous and stochastic productivity, and show that indeterminacies and sunspots persist.

Keywords: Adverse Selection, Local Indeterminacy, Global Dynamics, Sunspots.

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1. Introduction

The seminal work of Wilson (1980) shows that in a static model, adverse selection can generate multiple equilibria because of asymmetric information about product quality. This paper analyzes how adverse selection in credit markets can create lending externalities that generate multiple steady states and a continuum of equilibria in an otherwise standard dynamic general equilibrium model of business cycles. To this end, we embed a simple type of adverse selection arising in credit markets into a standard textbook real business cycle (RBC) model. To understand the main intuition of our model, imagine an economy in which a continuum of anonymous final goods producers (borrowers) must borrow from competitive financial intermediaries to purchase intermediate goods to produce. The intermediate goods in turn are produced by competitive intermediate firms using capital and labor. There are two types of borrowers: a fixed measure of dishonest borrowers and an endogenous measure of honest borrowers. Only the honest borrowers know how to produce and will always pay back their loans while the dishonest borrowers will simply run away with the borrowed funds. Adverse selection problems arise as dishonest borrowers always borrow and default, regardless of the interest rate. In this environment, adverse selection is naturally countercyclical as higher aggregate demand for final output encourages more honest producers to borrow and produce.

Multiple equilibria emerge as countercyclical adverse selection creates positive feedback between expected and actual aggregate output and make expectations self-fulfilling. Optimistic expectations of higher aggregate output cause financial intermediaries believe that there exists a greater number honest borrowers in the credit market. Then the perceived default risk decreases and each financial intermediary is willing to charge a lower interest rate. This in turn will attract more honest borrowers to enter production and confirm beliefs of lower default risk. A greater number of honest borrowers (producers) create stronger competition in intermediate goods and drives up wage and rental rates of capital. Finally higher wages induce a higher labor supply, production increases, and the expectation of higher aggregate output is fulfilled. Similarly, pessimistic expectations for aggregate output will lead to a lower aggregate output and a higher default risk. This explains the occurrence of multiple steady state equilibria in our model.

In RBC models, capital accumulation gives rise to a new type of multiple equilibria, beyond those in the static adverse selection model. Namely, there exist a continuum of sunspot equilibria around one of the steady states under a calibrated parametrization of our baseline model. A continuum of sunspot equilibria still exists even if the steady state equilibrium is unique in an extended model with reputation, as shown in section 3.1.

The intuition behind local indeterminacy and the existence of sunspot equilibria is as follows. Procyclical wages under countercyclical adverse selection allow for both capital and its marginal product to increase, as the higher supply of labor in response to higher wages can offset diminishing returns to capital. In equilibrium a higher marginal product of capital must imply a fall in the shadow price of capital since the negative capital gain in the shadow price plus the marginal product of capital must add up to the fixed discount rate. In other words in response to the fall in the shadow price of capital (equal to the marginal utility of consumption), consumption must rise and investment must fall. This reverses the initial increase in capital, which then drifts back towards its steady state value instead of becoming explosive. The steady state then becomes a sink instead of a saddle, giving rise to indeterminacy, consistent with a continuum of equilibria parametrized by initial self-fulfilling expectations of the marginal product of capital. This opens up the possibility of sunspot randomizations over local equilibria.[1]

Furthermore, trajectories diverging away from "locally determinate" steady states cannot be ruled out as equilibria on the grounds that they violate transversality or feasibility conditions, as they can converge to the other steady state and satisfy all requirements of rational expectations equilibria. Thus a model could have "local determinacies" when in fact it exhibits global indeterminacies, as demonstrated in section 2.5. The additional insight from the global dynamics analysis is that global indeterminacy with significant boom and bust cycles in output may exist under rational expectations. Our adverse selection model can exhibit jumps across equilibria, so that credit, consumption, investment and employment can suddenly collapse with some probability, driven by a Markov sunspot or a confidence crisis. Our model is constructed such that agents expect such probabilistic jumps and thus build them into their optimal decisions. Jump probabilities can then capture occasional confidence and credit crises, or boom and bust cycles, demonstrated in section 2.5.

To further highlight the difference between multiplicity of equilibria in our model and those in static asymmetric information models (Wilson (1980)), we conduct two additional robustness analyses in section 3. First, in a dynamic setting where producers who borrow

[1] As Brunnermeier and Sannikov (2014) and He and Krishnamurthy (2012) have cautioned, analyzing the local dynamics may not yield the same insights about economic fluctuations and crises that analyzing global dynamics does. Thus we use a continuous-time setup to characterize both the local and global dynamics in the presence of information asymmetry to show that large economic crises can be triggered by confidence shocks in the credit market, arguably an important feature of the recent 2008 financial crisis.

are not completely anonymous, market forces and competition can mitigate adverse selection through reputational effects absent from our baseline model in section 2. Therefore section 3.1 first explores the role of reputation to examine whether multiplicity of equilibria survives under reputation effects. As in Kehoe and Levine (1993), a borrower who defaults is assumed to lose reputation with some probability, and is then excluded from the credit market forever. Such considerations, under certain conditions, can eliminate default and yield a unique steady state. However, the unique steady state can still be indeterminate under adverse selection and capital accumulation. Second, section 3.2 extends our model to incorporate a continuum of types of producers facing different risks in production. Adverse selection arises as the riskier firms have stronger incentives to borrow under limited liability. For given inputs at any moment in time, the static equilibrium can be shown to be unique and hence the steady state is also unique.[2] Again, the unique steady state can still be indeterminate. These two extensions further contrast the multiplicity of equilibria in our model to those arising in static asymmetric information models. They show that the dynamic nature of our model is crucial: multiple equilibria would be impossible without dynamic capital accumulation in these settings.

Our emphasis on multiple equilibria due to adverse selection in a business cycle model also differentiates our contribution from the recent business cycle studies such as Eisfeldt (2004), Kurlat (2013) and Bigio (2015) that emphasize the role of adverse selection in amplifying and propagating business cycle shocks.[3] Similar to our paper, Eisfeldt (2004), Kurlat (2013) and Bigio (2015) all have the feature that the procyclicality of average quality in the credit market implies that resources are reallocated towards producers with lower credit risk when aggregate output increases. Our paper complements their work by showing that adverse selection generates multiple steady states and indeterminacy, and hence can be a source of large business cycle fluctuations driven by self-fulfilling expectations.[4] Eisfeldt (2004) does not model labor inputs.[5] Kurlat (2013) assumes an inelastic labor supply and hand-to-mouth

[2] We intentionally focus on the interior solution by ruling out the uninteresting market collapse equilibrium.

[3] Obviously, our model builds on a large strand of literature on the possibility of indeterminacy in RBC models. The literature is too large to review here. For an recent example of multiple equilibria due to credit market friction without adverse selection, see Benhabib and Wang (2014) and the references therein.

[4] Many other papers have also addressed adverse selection in a dynamic environment. Examples include Camargo and Lester (2014), Chang (2017), Chiu and Koeppl (2016), Daley and Green (2012), Guerrieri, Shimer, and Wright (2010), Guerrieri and Shimer (2014), House (2006), and Williamson and Wright (1994).

[5] Malherbe (2014) has a model that builds on Eisfeldt (2004) and introduces self-fulfilling liquidity dry-ups. In particular, he builds a three-period model of liquidity with adverse selection in financial markets. Then cash holdings by some agents with high-quality assets, who do not participate in asset trading, generate a negative externality on others, and thus reduce market liquidity, which in turn strengthens the incentive to hold cash.

workers. In these setups. adverse selection does not change the labor input and thus dynamic indeterminacy is not possible. Optimistic beliefs or expectations of higher output cannot be self-fulfilling, as output is pre-determined by the capital stock. In Bigio (2015) labor supply is endogenous, but it depends only on real wages (due to his assumption that workers do not save), which is similar to the case of GHH preferences by Greenwood, Hercowitz and Huffman (1988). As demonstrated by Jaimovich (2008), dynamic indeterminacy is not possible when there are no income effects on the supply of labor, even under increasing returns and externalities. In our paper, with adverse selection and countercyclical default risk, labor supply can increase sufficiently to raise the marginal product of capital under the standard household preferences, which is crucial for indeterminacy.

Finally, our extended model in section 3.1 with reputational effects is also related to that of Chari, Shourideh and Zeltin-Jones (2014), who model a secondary loan market with adverse selection and show how reputational effects can generate persistent adverse selection. Multiple equilibria also arise in their model due to signalling, as in Spence's (1973) classic signaling model. In contrast, multiple equilibria in our reputational model take the form of indeterminacy. They are generated instead by endogenously countercyclical markups that mimic aggregate increasing returns.

Our model has several implications that are supported by empirical evidence. First, a large literature has documented that credit risk is countercyclical and has far-reaching macroeconomic consequences. For instance, Gilchrist and Zakrajšek (2012) find that shocks to credit risks lead to significant declines in consumption, investment, and output. Meanwhile, the recent empirical work by Krishnamurthy and Muir (2017) suggests that credit expansions are associated with lower average borrower quality. This is consistent with our model featuring time varying risk premia and default premia leading to credit expansions involving lower quality borrowers. Additionally, Pintus, Wen and Xing (2015) show that interest rates faced by US firms move countercyclically and lead the business cycle. Second, the measured markup in our model is simply the inverse of the price of intermediate goods, and therefore it is countercyclical, an important empirical regularity well documented in the literature. Because of information asymmetry, dishonest borrowers enjoy an information rent. However, when the average quality of borrowers increases due to higher lending, this information rent is diluted relative to aggregate output. Hence the measured markup declines, which is critical to sustaining indeterminacy by bringing about higher real wages, a positive labor supply response, and a higher output that dominates the income effect on leisure. Third, our extended model in section 3.2 can explain the well-known procyclical variation in productivity. The procyclicality of average quality in the credit market implies that resources are reallocated towards producers with lower credit risk when aggregate output increases. The improved resource allocation then raises productivity endogenously. The procyclical endogenous TFP immediately implies that increases in inputs will lead to a more than proportional increase in total aggregate output, mimicking aggregate increasing returns. This effective increasing returns to scale arises only at the aggregate level. It is also consistent with the results of Basu and Fernald (1997), who find slightly decreasing returns to scale for typical two-digit industries in the US, but strong increasing returns to scale at the aggregate level. Adverse selection in credit markets then becomes realistic, in rich as well as developing countries.[6]

The rest of the paper is organized as follows. Section 2 describes the baseline model, characterizes the conditions for local indeterminacy, and ends by analyzing global dynamics. Section 3 undertakes robustness analysis. First, section 3.1 incorporates reputational effects into the baseline model and shows that indeterminacy may still arise in equilibrium even without defaults. Second, section 3.2 introduces a continuous distribution of heterogeneous and stochastic firm productivity, and shows that adverse selection in that model can induce endogenous TFP, amplification, aggregate increasing returns to scale and a continuum of equilibria. Section 4 concludes. The Appendix details the proofs of all propositions, lemmas and corollaries.

2. The Baseline Model

Time is continuous and proceeds from zero to infinity. There is an infinitely-lived representative household and a continuum of final goods producers. The final goods producers purchase intermediate goods as input to produce the final good, which is then sold to households for consumption and investment. The intermediate goods are produced with capital and labor in a competitive market. No distortion exists in the production of intermediate goods. Final goods firms do not have resources to make up-front payments to purchase intermediate goods before production takes place and revenues from sales are realized. They must therefore borrow from competitive financial intermediaries (lenders) to finance their working capital. Lending to these final goods producers is risky, however, as they may default. There are two types of producers (borrowers): honest borrowers who have the ability to produce

[6] See Sufi (2007) for evidence of syndicated loans in the US, and Karlan and Zinman (2009) for evidence from field experiments in South Africa.

and will always pay back the loan after production, and dishonest borrowers who will always default on their loan. The lenders do not know which borrower is which. They make loans to all firms, fully aware of the adverse selection problem. We begin by assuming that all trades are anonymous by excluding the possibility of reputational effects. This strong assumption will be relaxed in section 3.1, which introduces reputational effects.

2.1. Setup

Households The representative household has a lifetime utility function

$$\int_0^\infty e^{-\rho t}\left(\log\left(C_t\right) - \psi \frac{N_t^{1+\gamma}}{1+\gamma}\right) dt \tag{1}$$

where $\rho > 0$ is the subjective discount factor, C_t is consumption, N_t is hours worked, $\psi > 0$ is the utility weight for labor, and $\gamma \geq 0$ is the inverse Frisch elasticity of labor supply. The household faces the following budget constraint:

$$C_t + I_t \leq R_t u_t K_t + W_t N_t + \Pi_t, \tag{2}$$

where R_t, W_t and Π_t denote respectively the rental price, wage and the profits of all firms and financial intermediaries. As in Wen (1998), an endogenous capacity utilization rate u_t is introduced. As is standard in the literature, the depreciation rate of capital is given by $\delta(u_t) = \delta^0 \frac{u_t^{1+\theta}}{1+\theta}$, where $\delta^0 > 0$ is a constant and $\theta > 0$.[7] Finally, the law of motion for capital is governed by $\dot{K}_t = -\delta(u_t)K_t + I_t$.

The households choose a path of consumption X_t, C_t, N_t, u_t, and K_t to maximize their utility function (1), taking R_t, W_t and Π_t as given. The first-order conditions are given by

$$\frac{1}{C_t} W_t = \psi N_t^\gamma, \tag{3}$$

$$\frac{\dot{C}_t}{C_t} = u_t R_t - \delta\left(u_t\right) - \rho, \tag{4}$$

$$R_t = \delta^0 u_t^\theta. \tag{5}$$

The left-hand side of equation (3) is the marginal utility of consumption obtained from an additional unit of work, and the right-hand side is the marginal disutility of a unit of work. Equation (4) is the usual Euler equation. Finally, a one-percent increase in the utilization

[7] Dong, Wang, and Wen (2015) develop a search-based theory to offer a microfoundation for the convex depreciation function.

rate raises the total rent by $R_t K_t$ but also increases total depreciation by $\delta_0 u_t^\theta K_t$. Equation (5) thus states that the marginal benefit is equal to the marginal cost of utilization. Finally the transversality condition is given by $\lim_{t\to\infty} e^{-\rho t}\frac{1}{C_t}K_t = 0$.

Final goods producers There is a continuum of final goods producers of measure $\pi + S < \infty$, indexed by $i \in [0, 1]$. A measure π are dishonest and a measure S are honest. Each honest producer is endowed with an indivisible project, as in Stiglitz and Weiss (1981), which transforms one unit of intermediate good to one unit of final good. Let P_t be the price of the intermediate goods input. Each project then requires P_t of working capital. The dishonest producers, however, can claim to be honest and borrow P_t and then default and keep, for simplicity, all of the borrowed funds. They thus enjoy a profit of P_t. Anticipating this adverse selection problem, the final intermediaries will charge all borrowers a gross interest rate $R_{ft} > 1$. Hence the profit from borrowing and producing for a honest producer is given by $\Pi_t^h = 1 - R_{ft}P_t$.

Denote by s_t the measure of honest producers who produce goods:

$$s_t = \begin{cases} S & \text{if } R_{ft} < \frac{1}{P_t} \\ \in [0, S) & \text{if } R_{ft} = \frac{1}{P_t} \\ 0 & \text{if } R_{ft} > \frac{1}{P_t} \end{cases} . \tag{6}$$

The total demand for intermediate goods is $X_t = s_t$. Since each firm also produces one unit of the final goods, the total quantity of final goods produced is $Y_t = s_t = X_t$.

Intermediate goods The intermediate goods are produced with capital and labor using the technology

$$X_t = A\tilde{K}_t^\alpha N_t^{1-\alpha}, \tag{7}$$

where $\tilde{K}_t = u_t K_t$ is total capital supply from the households. In a competitive market the profit of producers is given by $P_t A\tilde{K}_t^\alpha N_t^{1-\alpha} - W_t N_t - R_t\tilde{K}_t$. The first-order conditions are

$$R_t = P_t\alpha\frac{X_t}{\tilde{K}_t} = P_t\alpha\frac{X_t}{u_t K_t}, \tag{8}$$

$$W_t = P_t(1-\alpha)\frac{X_t}{N_t}. \tag{9}$$

Under competition profits are zero, so $W_t N_t + R_t u_t K_t = P_t X_t$.

Financial Intermediaries The financial intermediaries must compete for business. Anticipating that only a fraction Ω_t of the loans will be paid back, the interest rate is then given by

$$R_{ft} = \frac{1}{\Omega_t}. \tag{10}$$

Hence the financial intermediaries make zero profits.

2.2. Equilibrium

Only interior solution is considered so $R_{ft} = \frac{1}{P_t}$.[8] Since each honest producer needs one unit of intermediate goods, if X_t units of intermediate goods are produced, there must be X_t honest borrowers in the credit market. The price of intermediate goods is P_t, so each of must borrow P_t working capital to produce. Hence they borrow a total $X_t P_t$ of working capital, while the dishonest producers altogether borrow πP_t of working capital. Since only the honest producers pay back their loan the average payback rate is

$$\Omega_t = \frac{X_t P_t}{\pi P_t + X_t P_t} = \frac{X_t}{\pi + X_t}. \tag{11}$$

In equilibrium, the total profit is simply πP_t. Hence the total budget constraint in equation (2) becomes

$$C_t + I_t = P_t X_t + \pi P_t. \tag{12}$$

where the zero profit condition for the intermediate goods is used to substitute for $W_t N_t + R_t u_t K_t$ by $P_t X_t$. Since $P_t = \frac{1}{R_{ft}} = \Omega_t = \frac{X_t}{\pi + X_t}$, the above equation can be further reduced to

$$C_t + I_t = P_t X_t + \pi P_t = X_t = Y_t. \tag{13}$$

The resource constraint is then given by

$$C_t + \dot{K}_t = Y_t - \delta(u_t) K_t. \tag{14}$$

Equation (13) yields

$$\phi_t \equiv 1 - \frac{\Pi_t}{Y_t} = 1 - \frac{\pi P_t}{X_t} = \Omega_t = P_t. \tag{15}$$

As $\phi_t = \Omega_t$, ϕ_t also represents the average quality of borrowers in the credit market, namely the fraction of the loans will be paid back. Equation (11) then becomes

$$\phi_t = \frac{Y_t}{\pi + Y_t}. \tag{16}$$

Finally the aggregate production function becomes

$$Y_t = A \left(u_t K_t\right)^{\alpha} N_t^{1-\alpha}. \tag{17}$$

[8] We assume that there is free entry and that an infinite measure of potential honest producers exist as potential entrants. The free entry condition then implies $R_{ft} = \frac{1}{P_t}$.

Proposition 1 *The competitive equilibrium is characterized by equations (3), (4), (5), (8), (9), (17), (14) and (16). These eight equations fully determine the dynamics of the eight variables $C_t, K_t, Y_t, u_t, N_t, w_t, R_t$ and ϕ_t.*

Equation (16) implies ϕ_t increases with aggregate output. By assumption, only honest borrowers produce. Since each of them can only produce a maximum of one unit, an increase in aggregate output is possible only if their number increases. As the number of dishonest borrower is fixed, the average borrower quality ϕ_t must increase with aggregate output. Note that $\frac{1}{\phi_t} = \frac{Y_t}{R_t u_t K_t + W_t N_t}$ is the aggregate markup. Therefore the endogenous markup in our model is countercyclical, which is consistent with the empirical regularity well documented in the literature.[9] The credit spread is given by $R_{ft} - 1 = \pi / Y_t$ and moves in a countercyclical fashion as in the data.

The countercyclical markup has important implications. For example, it can make the number of hours worked and the real wage move in the same direction. To see this, suppose that the real wage increases but the markup is a constant. Higher wages would lead intermediate firms to reduce their labor inputs, so the number of hours worked and the real wage move in the opposite directions. Aggregate production, given by (17) would then decrease. But if ϕ_t increases as well, increases in the price of intermediate goods P_t (recall $P_t = \phi_t$) , if strong enough, can induce the intermediate goods producers to increase their labor inputs, despite the increase of labor cost according to equation (9). Note also that when $\pi = 0$, *i.e.*, when there is no adverse selection in the credit markets, equation (16) implies that $\phi_t = 1$, and our model simply reduces to a standard RBC model. The markup is $1/\phi_t > 1$ if and only if dishonest firms obtain rents due to the information asymmetries.

2.3. Steady State

This section studies the steady state of the model. We use Z to denote the steady state of variable Z_t. It turns out that it suffices to solve for steady-state ϕ as other variables can all other variables can be expressed in in terms of ϕ. It is easy to obtain $u = \left(\frac{\rho(1+\theta)}{\delta^0 \theta}\right)^{\frac{1}{1+\theta}}$ in the steady state. Note that u only depends on δ^0, ρ and θ. Therefore, without loss of generality, one can set $\delta^0 = \frac{\rho(1+\theta)}{\theta}$ so that $u = 1$ at the steady state. The steady state depreciation rate is then $\delta = \rho/\theta$. To solve for steady-state ϕ, equation (16) can be rearranged as

$$\pi = \left(\frac{1-\phi}{\phi}\right) \cdot Y(\phi) \equiv \Psi(\phi), \tag{18}$$

[9] See, e.g., Bils (1987) and Rotemberg and Woodford (1999).

where the aggregate output is given by[10]

$$Y(\phi) = A^{\frac{1}{1-\alpha}} \left(\frac{\alpha\phi\theta}{\rho(1+\theta)} \right)^{\frac{\alpha}{1-\alpha}} \left(\frac{(1-\alpha)\phi}{1-\frac{\alpha\phi}{1+\theta}} \frac{1}{\psi} \right)^{\frac{1}{1+\gamma}}. \tag{19}$$

Notice that the total loss from lending to dishonest borrowers is $\pi \cdot P$, and the total gain of lending to honest borrowers is $(R_f - 1)s \cdot P = \left(\frac{1-\phi}{\phi}\right) \cdot Y(\phi) \cdot P$. Equation (18) simply implies that competition must drive the total loss equal to the total profit, so the net profit of financial intermediaries is zero in the equilibrium. The total profit from lending to honest borrowers depends on the product of the interest rate difference $R_f - 1$ and the total amount of loans made. A decrease in ϕ increases the interest rate difference $R_f - 1$, but it decreases the total amount of loans made to honest borrowers. So $\Psi(\phi)$ is generally non-monotonic. When $\frac{\alpha}{1-\alpha} + \frac{1}{1+\gamma} > 1$, $\Psi(\phi)$ becomes a typical Laffer curve in ϕ since $\Psi(0) = 0$ and $\Psi(1) = 0$. On the one hand, if the average credit quality is 0, there are no honest borrowers in the credit market. As a result, the profit from lending to honest borrowers must be zero. On the other hand, if the average quality ϕ is one, and therefore $R_f = 1$ due to competition, then R_f is equal to the financial intermediaries' funding cost, and they make no profit from lending to honest borrowers. So given π, there may exist two steady state values of ϕ.[11]

Denote by $\phi^*$the steady-state ϕ that maximizes the value of Laffer curve, where π^* is the maximum value of the Laffer curve so that $\phi^* \equiv \arg\max_{0\le\phi\le1} \Psi(\phi)$ and $\pi^* \equiv \max_{0\le\phi\le1} \Psi(\phi)$. The following lemma establishes the possibility of multiple steady state equilibria.

Lemma 2 *When $0 < \pi < \pi^*$, there exists two steady states ϕ that solve $\pi = \Psi(\phi)$.*

Adverse selection can generate multiple equilibria in a static model (see, e.g., Wilson (1980)). Therefore it is not surprising that our model has multiple steady state equilibria. A belief of a lower default risk leads the financial intermediaries to reduce the interest rate,

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[10] We can show that $Y = A^{\frac{1}{1-\alpha}} (k_y)^{\frac{\alpha}{1-\alpha}} N$, where $k_y = \frac{K}{Y} = \frac{\alpha\phi\theta}{\rho(1+\theta)}$, and $N = \left(\frac{(1-\alpha)\phi}{1-\frac{\alpha\phi}{1+\theta}} \frac{1}{\psi} \right)^{\frac{1}{1+\gamma}}$. Then we obtain equation (19).

[11] We thank the anonymous referee very much for providing a static framework to intuitively illustrate the multiplicity of equilibria for our model in equation (18). Denote s as the number of honest borrowers, then zero profit of financial intermediary implies $\pi = (R_f - 1) \cdot s$. Note that price per input equals to marginal cost, i.e., $P(s) = \chi'(s)$, where $\chi(s)$ denotes the cost function. Then $R_f = \frac{1}{P(s)} = \frac{1}{\chi'(s)}$, which defines s as an implicit function of R_f. In turn,

$$\pi = (R_f - 1) \cdot s(R_f).$$

If the cost $\chi(s)$ is convex, then $\chi'(s)$ increases in s, and thus $s(R_f)$ decreases with R_f. Consequently, the right hand of the above equation is a typical Laffer curve in R_f. Since $R_f = \frac{1}{\phi}$, and $s = Y(\phi)$, we can then rewrite the above equation as equation (18).

which in turn invites more honest firms to borrow and produce. The increased quality of borrowers then reduces the default risk, which stimulates more lending from other financial intermediaries. The decreases in the interest rate charged by financial intermediaries brings down the production cost. This triggers an output expansion, which further encourages savings from the households, and thus generates more future lending. We will show that, with capital accumulation, our model in fact generates a continuum of equilibria.

2.4. Local Dynamics

This section first explores the possibility of a continuum of equilibria under local indeterminacy around the steady state, and shows that the mechanism generating indeterminacy in our model is similar to the that in Benhabib and Farmer (1994). Adverse selection in effect generates a type of increasing returns, which is known to give rise to locally indeterminate steady states. The global dynamics will be explored in the next section.

Note that at the steady state, ϕ and π are linked by $\pi = \Psi(\phi)$, so the steady state can be parameterized either by π or ϕ. We will use ϕ as it is more convenient for the study of local dynamics. Denote by $\hat{x}_t = \log X_t - \log X$ the percentage deviation from the steady state. To see the effective increasing returns to scale, log-linearize equations (17) and (5) to obtain the linearized output $\hat{y}_t$ as

$$\hat{y}_t = a\hat{k}_t + b\hat{n}_t, \tag{20}$$

where $a \equiv \frac{\alpha\theta}{1+\theta-(1+\tau)\alpha}$ and $b \equiv \frac{(1+\theta)(1-\alpha)}{1+\theta-(1+\tau)\alpha}$ are the output elasticities with respective to capital and labor, respectively, and $\tau \equiv 1-\phi$. We assume that $1+\theta-(1+\tau)\alpha > 0$, or equivalently $\tau < \frac{1+\theta}{\alpha} - 1$, to make $a > 0$ and $b > 0$. In general these restrictions are easily satisfied as shown by the brief calibration at the end of this section. Note in particular that $a+b = \frac{1+\theta-\alpha}{1+\theta-(1+\tau)\alpha} = 1$ if $\phi = 1$. Thus endogenous capacity utilization alone does not generate an increasing returns to scale effect at the aggregate level without adverse selection. However, $a+b = \frac{1+\theta-\alpha}{1+\theta-(1+\tau)\alpha} > 1$ if $\phi < 1$. That is, through general equilibrium effects, the combination of adverse selection as well as endogenous capacity utilization mimics increasing returns to scale, even though production has constant returns to scale. Furthermore, if $\phi < 1-\theta$, then $b > 1$. The model can then explain the procyclical movements in labor productivity $\hat{y}_t - \hat{n}_t$ without resorting to exogenous TFP shocks.

The effective increasing returns in production can generate locally indeterminate steady states as in Benhabib and Farmer (1994) if increasing capital can increase marginal product of capital. This is possible if the increase in the supply of labor in response to higher wages can offset diminishing returns to capital in production. To see this, substitute out labor after log-linearizing equation (3) to express $\hat{y}_t$ as

$$\hat{y}_t = \frac{a(1+\gamma)}{1+\gamma-b(1+\tau)}\hat{k}_t - \frac{b}{1+\gamma-b(1+\tau)}\hat{c}_t \equiv \lambda_1\hat{k}_t + \lambda_2\hat{c}_t. \tag{21}$$

The equilibrium output elasticity with respect to capital, λ_1, depends on parameters a and b, the Frisch elasticity γ, and the degree of adverse selection measured by τ. A one-percent increase in capital directly increases output and the marginal product of labor by a percent, and it increases the intermediate good price by $a\tau$ percent since $\hat{p}_t = \tau\hat{y}_t$. Both forces lead to a higher marginal productivity of labor, so labor supply also increases. The exact increase in labor supply depends on the Frisch elasticity γ while the increases in output depend on b. The effect of a change in labor supply on output induced by a change in consumption works through the price of intermediate good, and also depends on τ and the Frisch elasticity γ. Again since both a and b increase with τ, output elasticities with respect to capital and consumption are increasing functions of τ. In other words, the presence of adverse selection makes equilibrium output more sensitive to changes in capital and to changes in autonomous consumption, and creates an amplification mechanism for business fluctuations.

Denote $\tau_{\min} \equiv \frac{(1+\theta)(1+\gamma)}{(1+\theta)(1-\alpha)+\alpha(1+\gamma)} - 1$. It is easy to see that if $\tau > \tau_{\min}$ or $\phi < 1 - \tau_{\min} \equiv \phi_{\max}$, then the equilibrium elasticity of output with respect to consumption λ_2 becomes positive, namely, an autonomous change in consumption leads to an increase in output. Since capital is predetermined, labor must increase. To induce an increase in labor, the real wage must increase enough to overcome the income effect, which is only possible if $\hat{p}_t$, the increase in intermediate good price, is large enough. As $\hat{p}_t = \tau\hat{y}_t$, τ must be large enough.

It turns out that the condition that $\phi < \phi_{\max}$ is crucial for the existence of a continuum of equilibria around the steady state. Suppose that the agent increases her investment due to a higher expected future marginal product of capital. Given a fixed discount rate, the relative price of capital must fall and the relative price of consumption must rise, so that the total return including capital gains or losses equals the discount rate. The increase in the relative price of consumption boosts consumption at the expense of investment, so capital drifts back towards the steady state instead of exploding. The steady state then becomes a sink rather than a saddle, and therefore becomes indeterminate. When the condition $\phi < \phi_{\max}$ holds, the increase in consumption induces an increase instead of a decrease in labor, and hence raises the marginal product of capital, which justifies the agent's initial belief of a higher expected future marginal product of capital.

However, if ϕ is too small, λ_2 becomes too large. In this case, the increase in the relative consumption price stimulates consumption and generates too much additional output. This boosts investment and makes capital explode, and can eventually violate the transversality condition. This suggests that there exists a lower bound for the steady state value of ϕ such that the steady state is indeterminate. Interestingly this lower bound is simply ϕ^*, which maximizes the value of Laffer curve. Formally, the following proposition lays down the indeterminacy condition.

Proposition 3 *The model exhibits local indeterminacy around a steady state if and only if $\phi \in (\phi^*, \phi_{\max})$.*

Denote $\pi^{**} = \Psi(\phi_{\max})$. Note that $\pi^{**} < \pi^*$ by the definition $\pi^* \equiv \max_{0 \leq \phi \leq 1} \Psi(\phi)$. We can use the relationship $\pi = \Psi(\phi)$ in the steady state to specify the indeterminacy region for the parameter π and obtain the following corollary.

Corollary 4 *1. If $\pi \in (0, \pi^{**})$, then both steady states are saddles.*

2. If $\pi \in (\pi^{**}, \pi^*)$, then the local dynamics around the steady state with $\phi > \phi^*$ exhibits indeterminacy while the local dynamics around the steady state with $\phi < \phi^*$ is a saddle.

These different scenarios are summarized in Figure 1. The inverted U curve illustrates the relationship between ϕ and π specified in equation (18). In Figure 1, ϕ is on the horizontal axis and π is on the vertical axis. For a given π, the two steady states $\bar{\phi}_L$ and $\bar{\phi}_H$ can be located from the intersection of the inverted U curve and a horizontal line through point $(0, \pi)$. The two vertical lines passing through points $(\phi^*, 0)$ and $(\phi_{\max}, 0)$ divide the diagram into three regions. In the left and right regions, the determinant of the Jacobian matrix J is negative, implying that one of the roots is positive and the other is negative. Therefore if a steady state ϕ falls into either of these two regions, it is a saddle. In the middle region, $\text{Det}(J) > 0$ and $\text{Trace}(J) < 0$, and thus both roots are negative. Therefore if the steady state ϕ falls into the middle region it is a sink, which supports multiple self-fulfilling expectation-driven equilibria, or indeterminacy, in its neighborhood.

Our model is mainly focused on the theoretical possibility of self-fulfilling equilibria, but as an illustration, we briefly illustrate its empirical plausibility under a reasonable parameterization. Let $\rho = 0.01$, implying an annual risk-free interest rate of 4%, $\theta = 0.3$ so that the depreciation rate at steady state is 0.033 and the annualized investment-to-capital ratio is 12% (see Cooper and Haltiwanger (2006)). Aggregate productivity is normalized to $A = 1$. Let labor supply be elastic so that $\gamma = 0$ and let $\alpha = 0.33$ as in the standard RBC model. Let $\psi = 1.75$ which implies that $N = \frac{1}{3}$ in the high ϕ steady state and $\pi = 0.13$ so that $\phi = \bar{\phi}_H = 0.9$, implying a 10% default rate. The associated $\bar{\phi}_L = 0.011$. It is easy to check that under these parameter values, the steady state where $\phi = 0.9$ is indeterminate. Table 1 summarizes the calibrations.

2.5. Global Dynamics

This section moves from the characterization of the local dynamics around steady states to the global dynamics. It shows that global indeterminacy always exists in our model, even in cases where both steady states are saddles and locally determinate.[12]

The terminology "local determinacy" of a steady state is typically used to describe the case where, given an initial condition on the state variable (K_0 in our case) in any small neighborhood of that steady state, there exists a unique equilibrium trajectory that stays in that neighborhood and converges to that steady state. Local indeterminacy on the other hand implies that there are multiple, typically a continuum of of such equilibria that stay in any neighborhood of the steady state. Such results are obtained by analyzing the linearized dynamics around the steady state. But a local analysis is insufficient. For example, even if a steady state is locally determinate, the paths that diverge from it may also be equilibria, for example if the divergent paths in fact converge to another steady state and satisfy transversality conditions for agent optimization. Thus there can be local determinacy around steady states, but globally there may exist a continuum of equilibria that start from an initial condition K_0 in the neighborhood of a "locally determinate" steady state, implying global indeterminacy, as discussed in section 2.5.1 and illustrated in Figure 3 below.

The dynamical system in (ϕ_t, K_t), described in section 2.2 and Proposition 1, has two steady states (see Lemma 2). While we cannot obtain a two-dimensional autonomous dynamical system in (C_t, K_t) the analysis of the dynamical system can still be reduced to two-dimensions, but in terms of (ϕ_t, K_t), as shown in the following Proposition. The dynamics of C_t can then be characterized in terms of (ϕ_t, K_t) , as shown below in equation (24) .

[12] See Gali (1996) for an early growth model with countercyclical markups, multiple steady states and global indeterminacy.

Proposition 5 *The autonomous dynamical system in (ϕ_t, K_t) is given by*

$$\left(1-\alpha+\frac{\alpha(1+\gamma)}{1+\theta}\right)\left(\frac{\phi_{\max}-\phi_t}{1-\phi_t}\right)\frac{\dot{\phi}_t}{\phi_t}+\left(\frac{\alpha\theta(1+\gamma)}{1+\theta}\right)\frac{\dot{K}_t}{K_t} = (1-\alpha)\left(\frac{\alpha\theta}{1+\theta}\phi_t\frac{Y(\phi_t)}{K_t}-\rho\right) \quad (22)$$

$$\dot{K}_t = \left(1-\frac{\alpha\phi_t}{1+\theta}\right)Y(\phi_t)-C(\phi_t,K_t) \quad (23)$$

with $Y(\phi_t)=\frac{\pi\phi_t}{1-\phi_t}$, $\phi_{\max}=1-\tau_{\min}$, and

$$C_t = C(\phi_t, K_t) = f_0\cdot g(\phi_t)\cdot h(K_t), \quad (24)$$

where f_0, $g(\phi_t)$, and $h(K_t)$ are defined in the Appendix.

The relationship between equilibrium ϕ_t and C_t is given in the following corollary, which we will then use to describe the global dynamics in (C_t, K_t).

Corollary 6 *For any $K_t > 0$ and $C_t < f_0\cdot h(K_t)\cdot g(\phi_{\max})$, there exist two possible ϕ_t values, denoted by $\phi_t=\phi^+\left(\frac{C_t}{f_0 h(K_t)}\right) > \phi_{\max}$ and $\phi_t=\phi^-\left(\frac{C_t}{f_0 h(K_t)}\right) < \phi_{\max}$, that yield the same level of consumption defined by (24).*

Notice that $Y_t=\frac{\pi\phi_t}{1-\phi_t}$. So Corollary 6 basically says that there are two possible equilibrium levels of output for given consumption and capital. The intuition is similar to the existence of two steady states summarized in Lemma 2. If the agents in the model believe that the default risk is lower, the financial intermediary is willing to lower interest rate, which attracts more honest borrowers and in fact lowers the default risk. The reduction in the interest rate charged by the financial intermediaries increases the real wage, which induces higher labor supply for a given consumption level, as implied by equation (3). This leads to a higher equilibrium level of output. Conversely, if the agent expects a higher default rate, the equilibrium default rate will indeed be higher and output will be lower in equilibrium. The two possible equilibria ϕ_t are illustrated in Figure 2. The function $g(\phi_t)$ has an inverted U shape. It attains the maximum at $\phi_{\max}$. For $C_t < f_0\cdot g(\phi_{\max})\cdot h(K_t)$, there exists two steady state equilibria, ϕ_t^- and ϕ_t^+. If $0<\phi_t^-<\phi_t^+<\phi_{\max}$, the former corresponds to a saddle and the latter to a sink. If $0<\phi_t^-<\phi_{\max}<\phi_t^+$, then both steady states are saddles. They are discussed below and illustrated in Figures 3 and 4.

2.5.1. Global Dynamics with Local Indeterminacy

Consider first the case in which one steady state is a sink. In Figure 1, when π (the proportion of dishonest firms) is high both steady state ϕ values are smaller than $\phi_{\max}$. There is local indeterminacy around the upper steady state but local determinacy around the lower steady state. However, the locally determinate steady state can be globally indeterminate. This can be seen from the dynamics illustrated in Figure 3. The solid red line is the $\dot{K}_t = 0$ locus and the solid blue line is the $\dot{\phi}_t = 0$ locus. These two loci intersect twice at the upper and lower steady states. The small circles indicate the initial conditions of trajectories. Given an initial value of K_0, which could be in the neighborhood of the lower steady state, we could choose an initial value ϕ_0^- and its corresponding consumption level C_0 such that the trajectory $\{K_t, C_t\}_{t=0}^{\infty}$ converges to the lower steady state along a saddle path. However there also exists a continuum of initial values of ϕ_0 and their corresponding consumption levels such that $\{K_t, C_t\}_{t=0}^{\infty}$ converges to the higher steady state, the sink. In either case, the resulting trajectories of capital and consumption are rational expectations equilibria, driven by self-fulfilling expectation about $\{K_t, C_t\}_{t=0}^{\infty}$, and the default rates along them. The trajectories that diverge away from the steady state that is a saddle, or "locally determinate," do not explode or become infeasible and violate transversality conditions. Instead they converge to the other steady that is a sink and satisfy all requirements of rational expectations equilibria under perfect foresight.

As Figure 3 indicates, almost every trajectory from an initial ϕ_0 that is above the saddle path associated with the lower steady state eventually converges to the upper steady state. It is clear that during the convergence, the economy exhibits oscillations in K_t and ϕ_t. Since output is $Y_t = \pi\phi_t/(1-\phi_t)$, it also exhibits boom and bust cycles. Such transition dynamics toward the upper steady state therefore implies a rich propagation mechanism for exogenous shocks. For example, if a transitory exogenous shock can move the economy away from the upper steady state, then the economy will display persistent oscillations in output before returning to the upper steady state.[13]

Figure 3 shows that for a given initial capital stock K_0, there are infinitely many deterministic equilibria defined by the initial value of ϕ_0 that smoothly converge to the upper steady state. However, there are at least two other types of equilibria with jumps in ϕ_t and hence discontinuity in output. We now turn to equilibria when both steady states are

[13]The global dynamics depicted in the case of a local saddle and a sink may be analyzed via the two-parameter Bogdanov-Takens (BT) bifurcation, which occurs at parameter values for the tangency point $\Psi(\phi_{\max}) = \pi$, or the BT point. By varying the parameters away from the BT point it is possible to analytically characterize the dynamics for various parameter regions yielding either zero and two steady states, and the qualitative dynamics and phase diagram in the region encompassing both steady states, including the saddle connection between the steady states, as depicted in Figure 3 (see in particular Kuznetsov, 1998, p. 322). However, not all parameter combinations are economically admissible. For Figure 3 we picked parameters in the economically admissible range. The qualitative dynamics, steady states and the saddle connection remain the same as we perturb parameters.

saddles to the next section. The stark contrast between the local dynamics and the global dynamics is better illustrated in that context.[14]

2.5.2. *Global Dynamics with Two Saddles*

In this section we study the global dynamics when π is low such that both steady states are saddles, where $\bar{\phi}_H > \phi_{\max}$ and $\bar{\phi}_L < \phi^*$, as is clear from Figure 1. Figure 4 graphs the two saddle paths associated with these two steady states. This then implies that both steady states can be globally indeterminate: for a given K_0, the economy can be placed on either saddle path by the appropriate choice of ϕ_0. Therefore globally there is still indeterminacy, though there is a unique saddle path associated with each of the two steady states.

Now there can be very complicated equilibrium paths if ϕ_t is allowed to jump. We can construct two types of jumps to illustrate this point. The first type of jumps in ϕ_t are deterministic and fully anticipated. Utility maximization then requires consumption to change continuously. That is, consumption does not jump when ϕ_t jumps. Notice that $\phi_t = \phi_t^+$ and $\phi_t = \phi_t^-$ yield the same consumption level for a given capital K_t. The economy can always jump across saddle paths from $\phi_t = \phi_t^+ > \phi_{\max}$ to $\phi_t = \phi_t^- < \phi^*$ and back without changing the value of consumption on a deterministic cycle.

The numerical analysis in this section only uses standard parameterization in Table 1, only changing the value of π from 0.1 to 0.0615.[15] Figure 5 graphs one such possible equilibrium path for each of consumption, investment, output and interest spread once we allow ϕ_t to jump. Initially, the economy is at point $K = 6.2783$ and $\phi = 0.9717 > \phi_{\max}$ and so $C = 0.8723$. With $K = 6.2783$, there exists another $\phi = 0.8249 < \phi_{\max}$ that yields $C = 0.8723$. The economy then follows the trajectory according to equations (22) and (23). It takes around 4.41 years for the model economy to reach $K = 11.1719$, $\phi = 0.9270$ and $C = 0.9307$. We then let ϕ drop to a level that allows consumption to remain at 0.9307 upon the jump. By construction, this leads to $\phi = 0.8241 < \phi_{\max}$ after the drop. and the economy follows the trajectory dictated by equations (22) and (23) again for another 8.02 years to reach $K = 6.2783$, $\phi = 0.8249$ and hence $C = 0.8723$. Notice that the consumption level has returned to its initial level. We then let ϕ jump from $\phi = 0.8249$ to $\phi = 0.9717$. Again by construction, consumption does not change immediately. This process can be repeated to

[14] A large literature on local indeterminacy has already constructed stochastic equilibria by randomizing over deterministic equilibria (with random jumps). Therefore it may come as no surprise to some readers that there exist equilibria with jumps in ϕ_t when one of the steady states is locally indeterminate.

[15] To graphically better illustrate the global dynamics with two saddles in Figures 4 however, we set $\alpha = 0.62$.

obtain the deterministic cycles in consumption, investment, output and the credit spread in Figure 5. The adverse selection problem is mild when $\phi_t > \phi_{\max}$, but it becomes much worse when $\phi_t < \phi_{\max}$. Thus when ϕ_t jumps down, there is an output collapse. Households can ensure their consumption by disinvesting capital after ϕ_t drops. In general, there are infinite ways to construct these deterministic cycles, as pointed out by Christiano and Harrison (1999).[16] Around the upper steady state, equilibrium ϕ_t can take many (possibly infinite) values. Hence the equilibrium around the upper steady-state is still indeterminate, albeit a saddle.

Sunspot Equilibria Finally there can also be stochastic sunspot equilibria if ϕ_t jumps randomly. More specifically, we introduce sunspot variables z_t, which take two values, 1 and 0, and assume that in a short time interval dt there is probability λdt that the sunspot variable will change from 1 to 0 and probability ωdt that it will change from 0 to 1. The equilibrium ϕ_t is constructed as a function of K_t and sunspot z_t, *i.e.*, $\phi_t = \phi(K_t, z_t)$, such that $\phi(K_t, 1) > \phi(K_t, 0)$. Thus the equilibrium ϕ_t will jump with an anticipated probability when z_t changes its value. When $z_t = 1$, economic confidence is high so adverse selection is mild. But when $z_t = 0$, economic confidence is low, and adverse selection becomes severe. The change in z_t from 1 to 0 triggers an economic crisis, and the change from 0 to 1 ends the crisis as economic confidence is restored. Set $\lambda = 0.01$ and $\omega = 0.025$ as an example, which means that the economy will remain in the normal, non-crisis state with probability 0.7143. Since jumps in ϕ_t are now stochastic, consumption is exposed to a jump risk. Therefore equation (22) must be modified to take this risk into account. Denote $\phi_{1t} = \phi(K_t, 1)$ and $\phi_{0t} = \phi(K_t, 0)$. Then

$$\left(1-\alpha+\frac{\alpha\left(1+\gamma\right)}{1+\theta}\right)\left(\frac{\phi_{\max}-\phi_{1t}}{1-\phi_{1t}}\right)\frac{\dot{\phi}_{1t}}{\phi_{1t}}+\left(\frac{\alpha\theta\left(1+\gamma\right)}{1+\theta}\right)\frac{\dot{K}_t}{K_t}=\left(1-\alpha\right)\left[\frac{\alpha\theta}{1+\theta}\phi_{1t}\frac{Y_{1t}}{K_t}-\rho+\lambda\left(\frac{g(\phi_{1t})}{g(\phi_{0t})}-1\right)\right]$$

for normal, non-crisis times. Here the last term $\frac{g(\phi_{1t})}{g(\phi_{0t})} - 1$ reflects the percentage change in consumption due to the jump from ϕ_{1t} to ϕ_{0t} and $Y_{1t} = \pi\phi_{1t}/\left(1-\phi_{1t}\right)$ is aggregate output

[16] These two ϕ_t which yield the same level of consumption correspond to two different branches in the differential equations defined by C_t and K_t. As pointed out by Christiano and Harrison (1999), a model with two branches can display rich global dynamics, regardless of the local determinacy. For example, we can construct an equilibrium with regime switches between these branches. The jumps for ϕ_t in the differential equations defined by ϕ_t and K_t correspond to the switching of branches in the dynamics defined for C_t and K_t.

when $\phi_t = \phi_{1t}$. Similarly

$$\left(1-\alpha+\frac{\alpha\,(1+\gamma)}{1+\theta}\right)\left(\frac{\phi_{\max}-\phi_{0t}}{1-\phi_{0t}}\right)\frac{\dot{\phi}_{0t}}{\phi_{0t}}+\left(\frac{\alpha\theta\,(1+\gamma)}{1+\theta}\right)\frac{\dot{K}_t}{K_t}=(1-\alpha)\left[\frac{\alpha\theta}{1+\theta}\phi_{0t}\frac{Y_{0t}}{K_t}-\rho+\omega\left(\frac{g(\phi_{0t})}{g(\phi_{1t})}-1\right)\right]$$

in crisis times when $z_t = 0$.

It is evident that if $\lambda = \omega = 0$, then $\phi_{1t} = \phi(K_t, 1)$ and $\phi_{0t} = \phi(K_t, 0)$ are functions defining the saddle paths toward the upper and lower steady states, respectively. By continuity, these two functions exist for small λ and ω. We solve these two functions using the collocation method discussed in Miranda and Fackler (2004). Specifically, a 15-degree Chebychev polynomial of K is used to approximate these two functions. Once $\phi_{1t} = \phi(K_t, 1)$ and $\phi_{0t} = \phi(K_t, 0)$ are obtained, equation (22) can be used to simulate the dynamic path of capital. Figure 6 shows a possible dynamic path for this economy.

Let the economy be initially in the normal, non-crisis state with $z_t = 1$ for a sufficiently long period. Hence capital, consumption, output, and investment do not change. The parameter values in Table 1 yield $K = 10.5427$. Due to precautionary savings, this level of capital is higher than the deterministic upper-steady-state level of capital, as households have an incentive to save to insure against a stochastic crash in output. The economy stays at this level of capital for 2.5 years, and then a crisis emerges, triggered by a drop in z_t from 1 to 0. The spread (the bottom-right panel of Figure 6) immediately jumps up as the adverse selection problem in the credit market deteriorates sharply. As a result, production and output collapse (the bottom-left panel). Since the timing of this collapse in output is unpredictable *ex ante*, consumption drops immediately (the top-left panel). Investment (the top-right panel) falls for two reasons: one is to partially offset the fall in output to finance consumption, and the other is due to the decline in the effective return as a result of severe adverse selection in the credit market. The economy stays in crisis mode for about one year before confidence is restored and the recession is over. Interestingly output and investment both overshoot when the recession is over, and the longer the economy stays in recession, the greater the overshoot. The longer the recession, the smaller the amount of capital remaining. The return to investment therefore is very high, and the households opt to work hard and invest more to enjoy this high return to investment. Figure 6 shows several large boom and bust cycles due to stochastic jumps in the sunspot variables. Thus there are rich multiple-equilibria in our benchmark model regardless of the model parameters.

3. Robustness Analysis

This section considers two variants of our baseline model to highlight the difference between the dynamic indeterminacy in our model and the multiple equilibria in the static models.

In the first model, reputation can alleviate the adverse selection problem in dynamic setting, and eliminate defaults. Reputation is an important consideration in adverse selection models as borrowers may not be anonymous, and excluded from markets once detected as dishonest. In this case the steady state is unique, yet there still exists a continuum of equilibria around the steady state.

In the second model, the probability of default follows a continuous distribution instead of a binary distribution. As in the case of reputation, the steady state will be shown to be unique, but again there exists a continuum of equilibria around the steady state. Multiplicity of equilibria seems to be a robust feature under these extensions of our model.

3.1. Reputation

If firms were anonymous in the market, they may default all the time without a care for their reputation. But they are not and lenders may refrain from lending to firms with a bad credit history. Arguably, these market forces can alleviate the asymmetric information problem. Therefore it is important to examine whether the indeterminacy results obtained in our baseline model can survive if such reputational effects are taken into account.

We follow Kehoe and Levine (1993) closely in modeling reputation. A continuum of firms with measure S that are infinitely-lived and can choose to default at any time. Firms that default, with some probability, acquire a bad reputation and may be excluded from the credit market forever. In equilibrium, the fear of that happening discourages firms from defaulting. It turns out that self-fulfilling equilibria still exist even if there are no defaults in equilibrium.

To keep the model analytically tractable, all firms are assumed to be owned by a representative entrepreneur. The entrepreneur's utility is $U(C_{e,t}) = \int_0^\infty e^{-\rho_e t}\log(C_{e,t})dt$, where $C_{e,t}$ is the entrepreneur's consumption and ρ_e her discount factor. For tractability, ρ_e is assumed to be much smaller than ρ so that the entrepreneur does not accumulate capital. The entrepreneur's consumption equals the firm's profits, $C_{e,t} = \int_0^S \Pi_t(i)di \equiv \Pi_t$, where $\Pi_t(i)$ denotes the profit of firm i.

Since the only cost of defaulting is the loss of future production opportunities, the price must exceed the marginal cost (also the average cost) of production to be profitable. If the price exceeds the marginal cost, each firm will then have an incentive to produce an infinite amount. To overcome this problem, we assume that the production projects of firms are indivisible, as in the benchmark model, and that they produce to meet the orders they receive. A production project produces a flow of 1 unit of final goods from intermediate goods. Each unit of the final good requires one unit of the intermediate good for its production. The project is carried out only if the firms receive a purchase order. Denote the total demand for the final good by Y_t. Then a fraction $\eta_t = Y_t/S$ of firms will receive a purchase order. S is assumed to be sufficiently large so $\eta_t < 1$ holds always. Again assume that firms must borrow to finance their working capital. Denote the price of the intermediate good by P_t, so they must borrow P_t to produce 1 unit.

To illustrate the reputation problem, let us consider a short time interval from t to $t+dt$. Let V_{1t} (V_{0t}) to denote the value of a firm that receives an order (no orders). On the one hand, V_{1t} can be formulated recursively as

$$V_{1t} = (1-\phi_t)dt + e^{-\rho_e dt}\left(\frac{C_{e,t}}{C_{e,t+dt}}\right)\left(\eta_{t+dt}V_{1t+dt} + (1-\eta_{t+dt})V_{0t+dt}\right), \tag{25}$$

where $\phi_t = P_t$ is the unit production cost. If $\phi_t < 1$, then the firm makes a positive profit from production. The second term on the right-hand side is the continuation value of the firms. Since firms are owned by the entrepreneur, the future value is discounted by the ratio of marginal utilities of the entrepreneur. Since there is no default in equilibrium, the gross interest rate for a working capital loan is $R_{ft} = 1$. On the other hand, V_{0t} is given by

$$V_{0t} = e^{-\rho_e dt}E_t\left(\frac{C_{e,t}}{C_{e,t+dt}}\right)\left(\eta_{t+dt}V_{1t+dt} + (1-\eta_{t+dt})V_{0t+dt}\right). \tag{26}$$

The firms can also choose to default on their working capital and obtain an instantaneous gain of 1. However, default comes with the risk of acquiring a bad reputation. Upon default, a firm acquires a bad reputation in the short time interval between t and $t+dt$ with probability λdt. The firm will then be excluded from production forever. The payoff for defaulting is hence given by

$$V_t^d = 1\cdot dt + e^{-\rho_e dt}(1-\lambda dt)E_t\left(\frac{C_{e,t}}{C_{e,t+dt}}\right)\left(\eta_{t+dt}V_{1t+dt} + (1-\eta_{t+dt})V_{0t+dt}\right). \tag{27}$$

Define $V_t = \eta_t V_{1t} + (1-\eta_t)V_{0t}$ as the expected value of the firm. The firm has no incentive to default if and only if $V_{1t} \geq V_t^d$, or

$$dt \leq (1-\phi_t)dt + \lambda dt e^{-\rho_e dt}\left(\frac{C_{e,t}}{C_{e,t+dt}}\right)V_{t+dt}. \tag{28}$$

In the limit $dt \to 0$, the incentive compatibility condition becomes $\phi_t \leq \lambda V_t$.[17] Then the expected value of the firm is given by

$$V_t = \int_0^\infty e^{-\rho_e j} \frac{C_{e,t}}{C_{e,t+j}} \frac{\Pi_{t+j}}{S} dj. \tag{29}$$

The average profit is then obtained as $\Pi_t = (1 - \phi_t)Y_t$. Then using $C_{e,t+j} = \Pi_{t+j}$ and integrating the right-hand side of (29) yields $V_t = \frac{(1-\phi_t)Y_t}{\rho_e}\frac{1}{S}$.

The households' budget constraint becomes $C_t + I_t \leq R_t u_t K_t + W_t N_t = \phi_t Y_t$. Then the incentive constraint (28) becomes

$$\phi_t \leq \lambda \frac{(1-\phi_t)Y_t}{\rho_e} \frac{1}{S}. \tag{30}$$

Final goods firms compete for intermediate goods, so marginal cost $\phi_t = P_t$ rises with P_t, and from the budget constraint, household utility increases with ϕ_t so the incentive constraint (30) must be binding. Then equation (30) can be simplified to

$$\phi_t = \frac{Y_t}{\pi + Y_t} < 1, \tag{31}$$

by defining $\pi \equiv \rho_e S/\lambda$, which is exactly equation (16). Similar to the baseline model, here firms also receive an information rent. However, the rent in the baseline is derived from hidden information while the rent here arises from hidden action. As indicated by equation (31), ϕ_t is procyclical and hence the markup is countercyclical. When output is high, the total profit from production is high. Therefore the value of a good reputation is high and the opportunity cost of defaulting also increases. This then alleviates the moral hazard problem since high output dilutes the information rent relative to aggregate output.

Similar to equation (18), ϕ can be solved from

$$\pi = \Psi_R(\phi) \equiv \left(\frac{1-\phi}{\phi}\right) \cdot Y(\phi), \tag{32}$$

where $Y(\phi) \equiv A^{\frac{1}{1-\alpha}} \left(\frac{\alpha\phi\theta}{\rho(1+\theta)}\right)^{\frac{\alpha}{1-\alpha}} \left(\frac{1-\alpha}{1-\frac{\alpha}{1+\theta}} \frac{1}{\psi}\right)^{\frac{1}{1+\gamma}}$. Unlike in the baseline model, here the steady-state equilibrium is unique as $\Psi_R(\phi)$ is monotonic when $\alpha < 0.5$, which is consistent with standard calibration. We summarize the result in the following lemma.

Lemma 7 *If $\alpha < \frac{1}{2}$, then the steady state equilibrium is unique for any $\pi > 0$.*

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[17] Under the incentive compatibility condition we can consider one-step deviations since V_{1t}, and V_{0t} are then optimal value functions.

The similarity between equations (16) and (31) suggests that the similar intuition applies: the steady state is indeterminate when the steady state ϕ falls in a range between lower bound and upper bound. The following proposition formally specifies the condition under which self-fulfilling equilibria arise.

Proposition 8 *Indeterminacy emerges if and only if* $\phi_R^* < \phi < \phi_{\max}$*, where the exact expression of* ϕ_R^* *is given in the Appendix and* $\phi_{\max}$ *is the same as in Proposition 3.*

Corollary 9 *Indeterminacy emerges if and only if* $\Psi_R(\phi_{\max}) < \pi < \Psi_R(\phi_R^*)$.

Given the other parameters, a decrease in ρ_e or an increase in λ raises the steady state ϕ. According to Lemma 7, this makes indeterminacy less likely. The intuition is straightforward. A large λ means the opportunity cost of defaulting increases, so the chance of the firm's being excluded from future production increases. This alleviates the moral hazard problem, which is the source of indeterminacy. Similarly, a decrease in ρ_e means that the entrepreneurs become more patient. The future profit flow from production becomes more valuable to them, which again increases the opportunity cost of defaulting and thus alleviates the moral hazard problem.

3.2. Adverse Selection with Heterogeneous Productivity

We now extend our baseline model by introducing productivity heterogeneity. The households' problems are the same as in the benchmark model and thus the first-order conditions are still described by equations (3), (4) and (5).

The risk of lending to final goods firms, of measure S, is now assumed to be continuous. The final goods firms are indexed by $j \in [0,1]$. Again each final goods firm has one production project, which requires 1 unit of the intermediate good. The loan is risky as production may not be successful. More specifically, final goods firm j's output is governed by

$$y_{jt} = \begin{cases} a_{jt}x_{jt}, & \text{with probability } q_{jt} \\ 0, & \text{with probability } 1 - q_{jt} \end{cases}, \tag{33}$$

where x_{jt} is the intermediate input for firm j, a_{jt} the firm's productivity, and q_{jt} is *i.i.d.* and drawn from a common distribution function $F(q)$ with $a_{jt} = a_{\min} q_{jt}^{-\zeta}$. So a higher productivity a_{jt} is associated with a lower probability of success q_{jt}. Notice that expected productivity is given by $q_{jt}a_{jt} = a_{\min} q_{jt}^{1-\zeta}$. However, $\zeta < 1$, *i.e.*, a firm with a higher success probability enjoys a higher expected productivity. Denote by P_t the price of intermediate goods. Then the total borrowing is given by $P_t x_{jt}$. Denote by R_{ft} the gross interest rate. Then final goods firm $j's$ profit maximization problem becomes $\max_{x_{jt}\in\{0,1\}} q_{jt}\left(a_{jt}x_{jt} - R_{ft}P_t x_{jt}\right)$.

Note that due to limited liability, the final goods firm pays back the working capital loan only if the project is successful. This implies that, given R_{ft} and P_t, the demand for x_{jt} is simply given by

$$x_{jt} = \begin{cases} 1 & \text{if } a_{jt} > R_{ft}P_t \equiv a_t^* \\ 0 & \text{otherwise} \end{cases}. \tag{34}$$

or equivalently, $q_{jt} < q_t^* = \left(\frac{a_t^*}{a_{\min}}\right)^{-\frac{1}{\zeta}} = \left(\frac{R_{ft}P_t}{a_{\min}}\right)^{-\frac{1}{\zeta}}$. This establishes that only firms with risky production opportunities will enter the credit markets, which highlights the adverse selection problem in the financial market. Firms with $q_{jt} > q_t^*$ are driven out of the financial market, despite their higher social expected productivity. Since financial intermediaries are assumed to be fully competitive, the interest rate is given by $R_{ft} = \frac{1}{E(q|q\leq q_t^*)} > 1$, where the denominator is the average success rate.

The total production of final goods is

$$Y_t = \int_0^S q_j a_{jt} x_{jt} dj = S\int_0^{q_t^*} a_{\min} q^{1-\zeta} dF(q). \tag{35}$$

where the second equality follows from equation (34). The total production of intermediate goods is

$$X_t = S\int_0^{q_t^*} dF(q). \tag{36}$$

Again S is assumed to be large enough so that an interior solution to q^* is always guaranteed.

Finally the intermediate goods are produced according to $X_t = A_t\left(u_t K_t\right)^\alpha N_t^{1-\alpha}$, where $u_t K_t$ is the capital borrowed from the households. Combining equations (35) and (36) then yields

$$Y_t = \Gamma(q_t^*) A_t \left(u_t K_t\right)^\alpha N_t^{1-\alpha}, \tag{37}$$

where $\Gamma(q_t^*) \equiv \left(\int_0^{q_t^*} a_{\min} q^{1-\zeta} dF(q)\right) / \int_0^{q_t^*} dF(q)$ depends on the threshold q_t^* and the distribution. The above equation then suggests that the measured TFP is

$$TFP_t \equiv \frac{Y_t}{(u_t K_t)^\alpha N_t^{1-\alpha}} = \Gamma(q_t^*) A_t. \tag{38}$$

Since $\Gamma'(q_t^*) > 0$, the endogenous TFP increases with the threshold q_t^*. This is intuitive: as the threshold increases, more firms with high productivity enter the credit market, making resource allocation more efficient. Equation (35) implies that q_t^* increases with Y_t, and hence the credit spread $R_{ft} - 1$ decreases with Y_t. Finally the aggregate default rate is simply equal to $\int_0^{q_t^*} (1-q)\, dF(q) / \int_0^{q_t^*} dF(q) = 1 - \frac{1}{R_{ft}}$, which also decreases with Y_t. These results are summarized in the following lemma.

Lemma 10 *The default rate and credit spread both decrease with Y, while TFP is endogenous and increases with Y.*

Lemma 10 therefore establishes that the endogenous TFP is procyclical. Notice that the procyclicality of endogenous TFP holds generally for continuous distributions. Hence without loss of generality, a Power distribution $F(q) = q^\eta$ is chosen for tractability. In turn, firm-level measured productivity $\frac{1}{q}$ follows a Pareto distribution with the shape parameter η, which is consistent with the findings of a large literature (see, e.g., Melitz (2003) and references therein). Under the assumption of a Power distribution, combining equations (35) and (37) yields the aggregate output

$$Y_t = \left(\frac{\eta}{\eta - \zeta + 1}\right) a_{\min} S^{-\frac{1-\zeta}{\eta}} \left(A_t u_t^\alpha K_t^\alpha N_t^{1-\alpha}\right)^{1+\frac{1-\zeta}{\eta}}. \tag{39}$$

Note that the aggregate output again exhibits increasing returns to scale. When capital and labor increases, the supply of intermediate goods increases. This increases the cutoff q_t^* by equation (36). Since firms with a higher q are also more productive in production on average, the increased efficiency in reallocating credit implies that resources are better allocated across firms. As a result, output increases more than proportionally than capital and labor, exhibiting an effective increasing returns to scale. Notice that equation (39) reveals that the degree of increasing returns to scale clearly depends on the adverse selection problem and decreases with ζ and η. When $\eta = \infty$, the firms produce a product of homogeneous quality. Hence there is no asymmetric information or adverse selection. If $\zeta = 1$, firms are equally productive in the sense that their expected productivity is the same. It therefore does not matters how credit is allocated among firms. Given $\zeta < 1$, a smaller η implies that firms are more heterogenous, creating greater information asymmetry. Similarly, given η, a smaller ζ implies that the productivity of firms deteriorates more quickly with respect to their default risk, making adverse selection more damaging to resource allocation. The following proposition formally state this result.

Proposition 11 *The reduced-form aggregate production in our model exhibits increasing returns to scale if and only if adverse selection exists, i.e.,* $\zeta < 1$ *and* $\eta < \infty$*.*

In an important contribution, Basu and Fernald (1997) document increasing returns to scale in aggregate production but not at the micro level. In a recent paper, Liu and Wang (2014) show how credit constraints can generate endogenous variation in TFP, and hence aggregate increasing returns. In their model, the less productive firms are driven out of production. Differently from Liu and Wang (2014), our model relies on adverse selection and equilibrium default to generate increasing returns. The credit spread, $R_{ft} - 1$, and the expected default risk, $1 - E\left(q|q \leq q_t^*\right)$, are both countercyclical. These predictions are consistent with the empirical regularities found by Gilchrist and Zakrajšek (2012) and many others.

Since the aggregate increasing returns to scale are established, the indeterminacy in the steady state follows naturally. Interestingly, the steady state is unique. The following proposition summarizes these results.

Proposition 12 *Given a Power distribution, i.e.,* $F(q) = q^\eta$ *(or equivalently, firm productivity conforms to a Pareto distribution), the steady state is unique. Moreover, the model is indeterminate if and only if* $\sigma_{\min} < \sigma < \sigma_{\max}$*, where* $\sigma \equiv \frac{1-\zeta}{\eta}$*,* $\sigma_{\min} \equiv \frac{1}{\frac{1-\alpha}{1+\gamma} + \frac{\alpha}{1+\theta}} - 1$ *and* $\sigma_{\max} \equiv \frac{1}{\alpha} - 1$*.*

The restriction $\sigma > \sigma_{\min}$ requires increasing returns to scale to be large enough to overcome the income effect of labor supply so that an autonomous change in consumption leads to an increase in output. The restriction $\sigma < \sigma_{\max}$ is typically automatically satisfied. The restriction $\sigma < \frac{1}{\alpha} - 1$ simply requires that $\alpha(1 + \sigma) < 1$, which is the condition needed to rule out explosive growth in the model.

4. Conclusion

This paper shows that in a realistically calibrated dynamic general equilibrium model, adverse selection in credit markets can generate a continuum of equilibria in the form of indeterminacy, either through endogenous markups or endogenous TFP. Adverse selection can therefore potentially explain high output volatility as well as the emergence of probabilistic confidence and credit crises, or boom and bust cycles with jumps in output, consumption and investment, all under fully rational expectations, and in the absence of fundamental shocks. While the standard RBC model with a negative TFP shock cannot fully explain the increase in labor productivity during the Great Recession (see Ohanian (2010)), this feature of the Great Recession is consistent with the prediction of our baseline model in section 2, and is driven by pessimistic beliefs about aggregate output. The pessimistic beliefs reduce aggregate demand and increase markups, leading to a lower real wage and a lower labor supply. Labor productivity, however, rises due to decreasing returns to labor. Furthermore, our multiple equilibria results are robust to the introduction of reputational effects for borrowers, and to allowing heterogenous productivities.

To keep our analysis simple, we abstracted from certain important features of credit markets, for example, runs on financial intermediaries that may amplify the initial adverse selection problem as in the subprime mortgages during the Great Recession. Future research may examine the effects of adverse selection among financial intermediaries.

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Table and Figures for "Adverse Selection and Self-fulfilling Business Cycles"

December, 2017

Parameter	Value	Description
A	1	Aggregate Productivity
ρ	0.01	Discount factor
θ	0.3	Utilization elasticity of depreciation
δ	0.033	Depreciation rate
α	0.33	Capital income share
γ	0	Inverse Frisch elasticity of labor supply
ψ	1.75	Coefficient of labor disutility
π	0.13	Proportion of firms that produce lemons

Table 1: Calibration

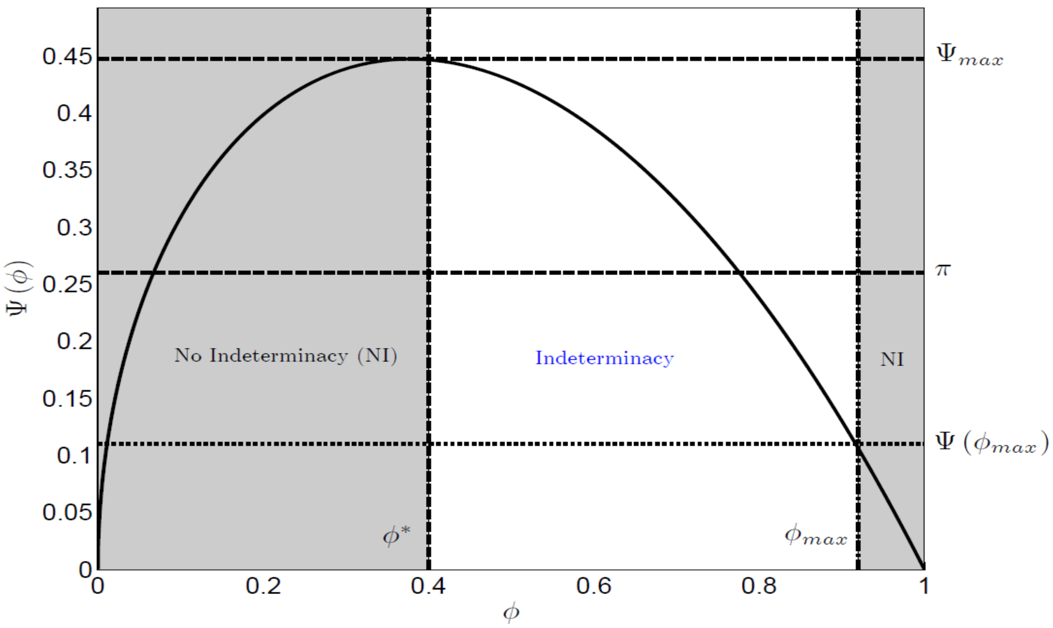

Figure 1: **Multiple Steady States and the Indeterminacy Region**

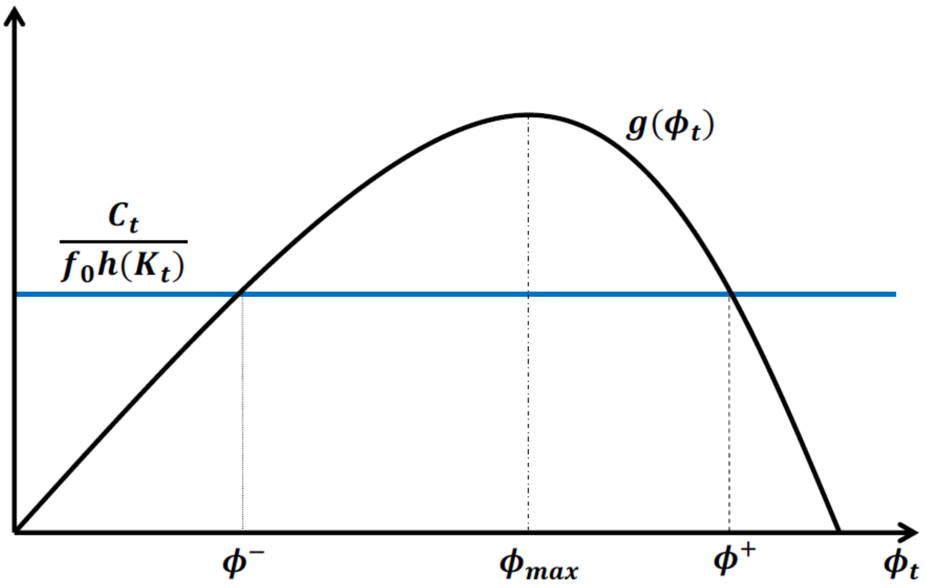

Figure 2: **Illustration of** (ϕ_t^+, ϕ_t^-)

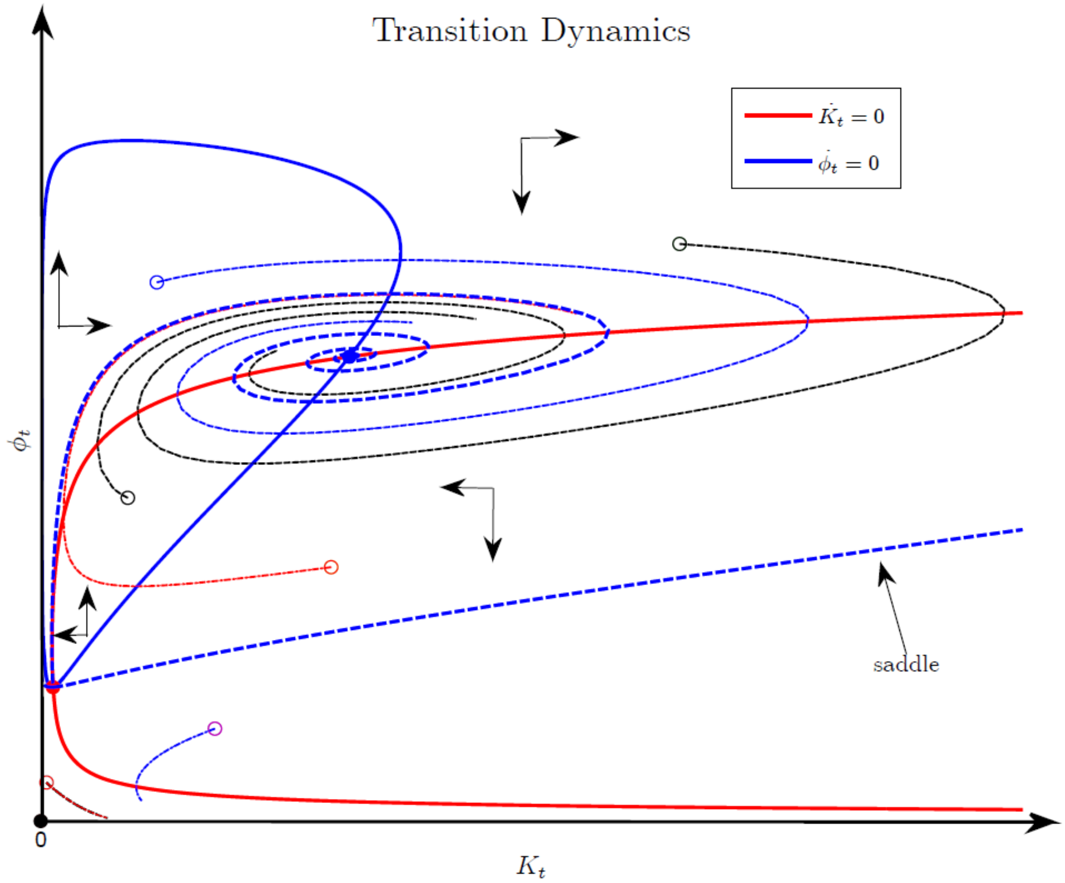

Figure 3: **Global Dynamics with One Saddle**

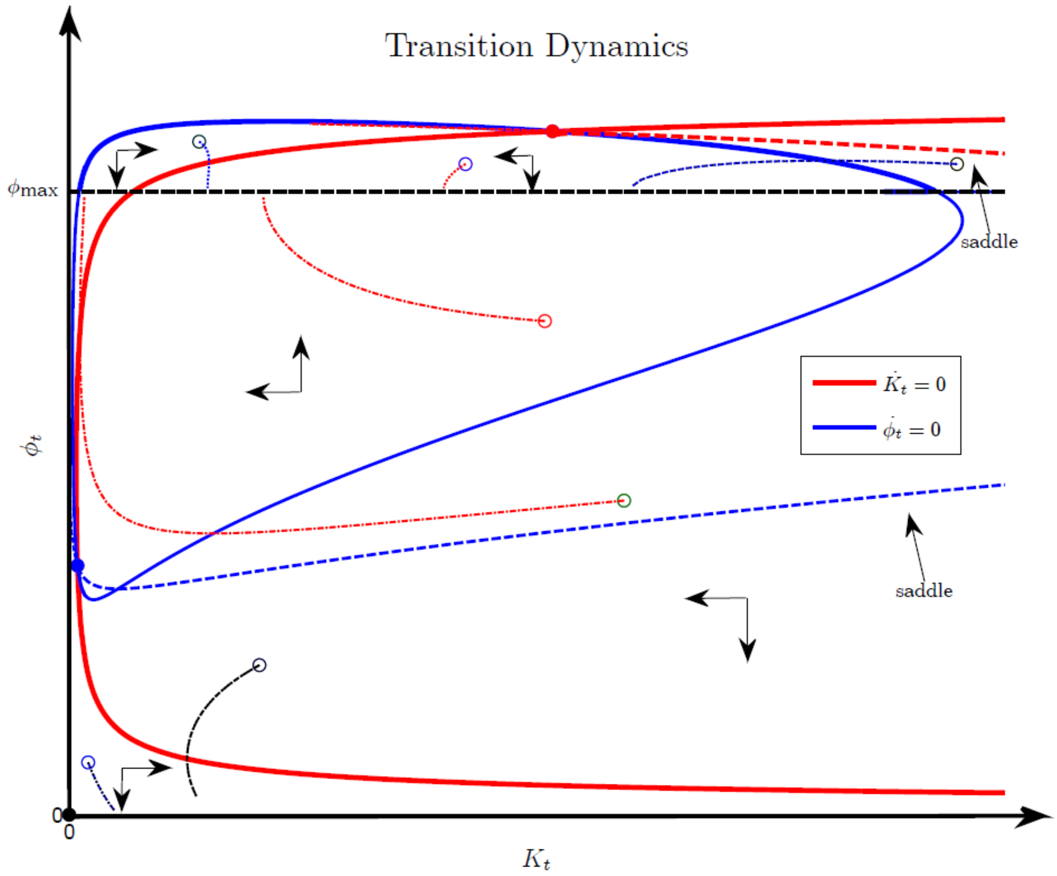

Figure 4: **Global Dynamics with Two Saddles**

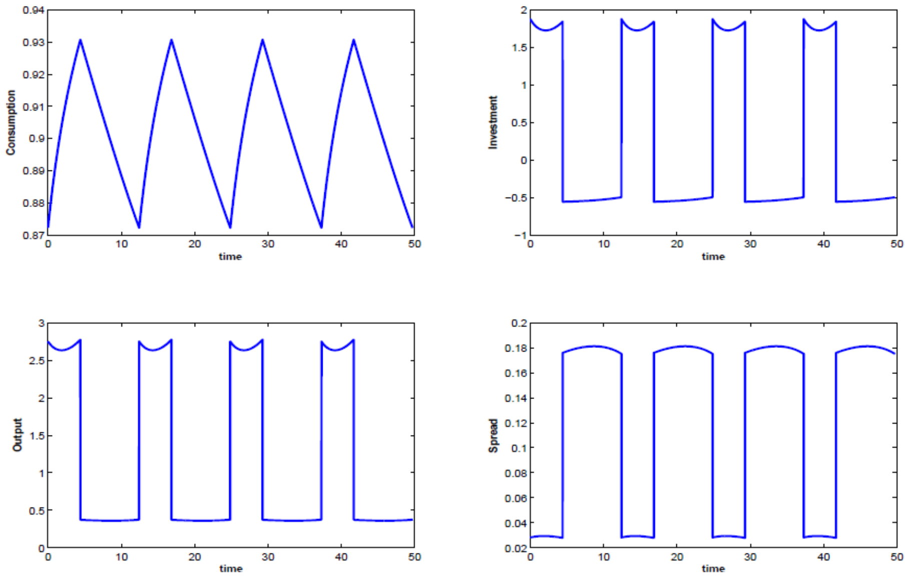

Figure 5: **Deterministic Cycles**

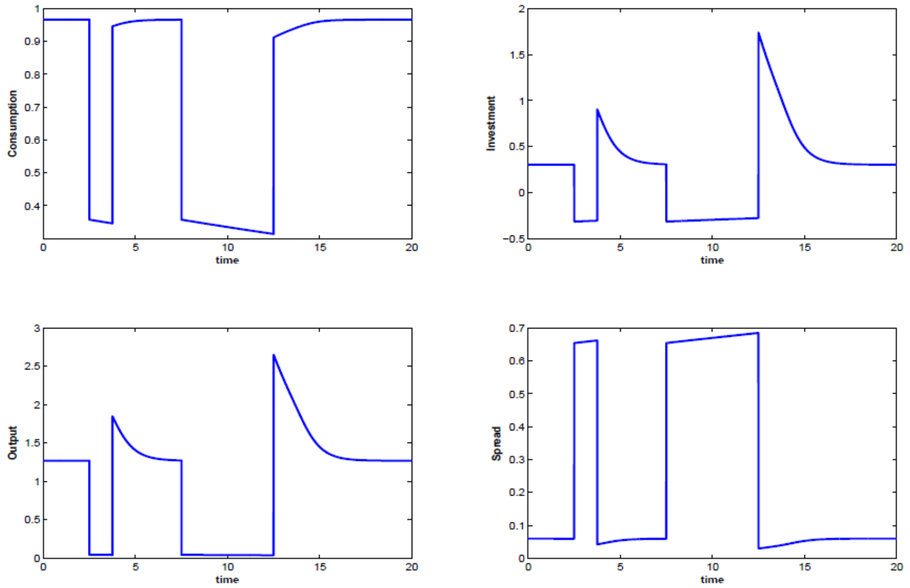

Figure 6: **Stochastic Switches between Branches**

Supplementary Materials for

Adverse Selection and Self-fulfilling Business Cycles

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JEL classification: E44, G01, G20.

A Online Appendix

Proof of Lemma 2: The proof is straightforward. First, from the explicit form of $Y(\phi)$, we can easily prove that $\Psi(\phi) \equiv \left(\frac{1-\phi}{\phi}\right) \cdot Y(\phi)$ strictly increases with ϕ when $\phi \in (0, \phi^*)$ but strictly decreases with ϕ when $\phi \in (\phi^*, 1)$. Second, since $\Psi(0) < \pi < \Psi^* = \Psi(\phi^*)$, there exists a unique solution between zero and ϕ^*, denoted by $\bar{\phi}_L$, that solves $\Psi(\phi) = \pi$. Likewise, there also exists a unique solution between ϕ^* and 1, denoted by $\bar{\phi}_H$, that solves $\Psi(\phi) = \pi$.

Proof of Proposition 3: To prove this proposition, we first characterize the local dynamics by the following system of linear differential equations:

$$\begin{bmatrix} \dot{k}_t \\ \dot{c}_t \end{bmatrix} = J \cdot \begin{bmatrix} \hat{k}_t \\ \hat{c}_t \end{bmatrix},$$

where the coefficient matrix

$$J \equiv \delta \begin{bmatrix} \left(\frac{1+\theta}{\alpha\phi}\right)\lambda_1 - (1+\tau)\lambda_1 & \left(\frac{1+\theta}{\alpha\phi}\right)(\lambda_2 - 1) + 1 - (1+\tau)\lambda_2 \\ \theta\left[(1+\tau)\lambda_1 - 1\right] & \theta(1+\tau)\lambda_2 \end{bmatrix},$$

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and $\lambda_1 \equiv \frac{a(1+\gamma)}{1+\gamma-b(1+\tau)}$, $\lambda_2 \equiv -\frac{b}{1+\gamma-b(1+\tau)}$, and $\delta = \rho/\theta$ is the steady state depreciation rate. The local dynamics around the steady state is determined by the roots of J. The model economy exhibits local indeterminacy if both roots of J are negative. Note that the sum of the roots equals the trace of J, and the product of the roots equals the determinant of J. Thus the sign of the roots of J can be observed from the sign of its trace and determinant. The trace and determinant are given by

$$\begin{aligned}\frac{\text{Trace}\,(J)}{\delta} &= \left(\frac{1+\theta}{\alpha\phi}\right)\lambda_1 - (1+\tau)\,\lambda_1 + \theta\,(1+\tau)\,\lambda_2, \\ \frac{\text{Det}\,(J)}{\delta^2\theta} &= \left[(1+\tau)\,\lambda_1 - 1 + \lambda_2\right]\left(\frac{1+\theta}{\alpha\phi} - 1\right) - \tau\lambda_2.\end{aligned}$$

Substituting out λ_1 and λ_2 we obtain

$$\frac{\text{Trace}\,(J)}{\delta} = \left[\left(\frac{\theta}{\phi}\right)\left(\frac{\alpha\,(1+\gamma)+(1+\theta)\,(1-\alpha)}{1+\theta-(1+\tau)\alpha}\right)\right]\cdot\left[\frac{\frac{(1+\gamma)(1+\theta)}{\alpha(1+\gamma)+(1+\theta)(1-\alpha)} - 1 + \tau^2}{\gamma+1-(1+\tau)b}\right]$$

Notice that $\gamma + 1 - (1+\tau)b < 0$ is equivalent to $\tau > \tau_{\min} \equiv \frac{(1+\gamma)(1+\theta)}{\alpha(1+\gamma)+(1+\theta)(1-\alpha)} - 1$. Since $\tau_{\min} > 0$, the term in the second square bracket is positive. Therefore Trace$(J) < 0$ if and only if $\tau > \tau_{\min}$. Or $\phi < \phi_{\max} = 1 - \tau_{\min}$.

Next, we determine the condition under which Det$(J) > 0$. Note that Det(J) can be rewritten as

$$\frac{\text{Det}\,(J)}{\delta^2\theta} = \frac{1+\theta}{(1+\tau)b-(\gamma+1)}\left\{(1+\gamma)(1-\alpha) - \left[\frac{(1-\alpha)(1+\theta)}{(1+\theta-\alpha\phi)} + (1+\gamma)\alpha\right]\tau\right\}.$$

If $\tau < \tau_{\min}$ or $\phi > \phi_{\max}$, then Det$(J) < 0$. Thus to guarantee Det$(J) > 0$, we must have $\phi < \phi_{\max}$, which implies that the first term on the right hand side is positive. As a result, given that $\phi < \phi_{\max}$, Det$(J) > 0$ if and only if the second term in the curly brackets is positive, which can be further simplified to

$$F\,(\tau) \equiv \alpha^2\tau^2 + \left[\alpha\theta + \frac{(1-\alpha)\,(1+\theta)}{(1+\gamma)}\right]\tau - (1-\alpha)\,(1+\theta-\alpha) < 0.$$

Recall $\phi^* = \arg\max_{0\le\phi\le1} \Psi(\phi)$, so $\Psi'(\phi)$ must be zero at ϕ^*. The first order condition then implies that ϕ^* solves

$$\Gamma\left(\phi\right) \equiv \alpha^2\phi^2 - \left[\frac{\left(1-\alpha\right)\left(1+\theta\right)}{1+\gamma} + \alpha\theta + 2\alpha^2\right]\phi + \left[\frac{\left(1-\alpha\right)\left(1+\theta\right)}{1+\gamma} + \left(2\alpha-1\right)\left(1+\theta\right)\right] = 0.$$

Notice $F\left(1-\phi\right) = \Gamma\left(\phi\right)$. Since $\Gamma\left(\phi^*\right) = 0$, it follows that $F\left(1-\phi^*\right) = 0$. The quadratic expression hence implies that $F\left(\tau\right) < 0$ if $\tau < 1-\phi^*$ or $\phi > \phi^*$. We have hence proved that $\mathrm{Det}(J) > 0$ if and only if $\phi^* < \phi < \phi_{\max}$.

Under the condition $\phi \in (\phi^*, \phi_{\max})$, $\mathrm{Trace}(J) < 0$ and $\mathrm{Det}(J) > 0$. Then the roots of J must both be negative. Hence the steady state is a sink and indeterminate.

Proof of Corollary 4: First, when adverse selection is severe enough, *i.e.*, $\pi \geq \Psi_{\max}$, the economy collapses. The only equilibrium is the trivial case with $\phi = 0$. Given that $\pi < \Psi_{\max}$, Lemma 2 implies that there are two solutions, which are denoted by $\left(\bar{\phi}_H, \bar{\phi}_L\right)$. It is always true that $\bar{\phi}_L < \phi^* < \bar{\phi}_H$. Then Proposition 3 immediately suggests that the steady state $\bar{\phi}_L$ is always a saddle. Since $\Psi(\phi)$ decreases with ϕ when $\phi > \phi^*$, as shown in Proposition 1, indeterminacy emerges if and only if $\phi \in (\phi^*, \phi_{\max})$. Therefore the local dynamics around the steady state $\phi = \bar{\phi}_H$ exhibits indeterminacy if and only if $\Psi(\phi_{\max}) < \pi < \Psi_{\max}$.

Proof of Proposition 5: As shown in section 2, the dynamical system in (C_t, K_t) is given by

$$\frac{\dot{C}_t}{C_t} = \left(\frac{\theta}{1+\theta}\right)\alpha\phi_t\frac{Y_t}{K_t} - \rho, \tag{A.1}$$

$$\dot{K}_t = Y_t - \left(\delta^0\frac{u_t^{1+\theta}}{1+\theta}\right)K_t - C_t, \tag{A.2}$$

Substituting u_t and N_t yields

$$C_t^{1-\alpha} = A^{1+\gamma}\left(\frac{\alpha}{\delta^0}\right)^{\frac{\alpha(1+\gamma)}{1+\theta}}\left(\frac{1-\alpha}{\psi}\right)^{(1-\alpha)}\phi_t^{1-\alpha+\frac{\alpha(1+\gamma)}{1+\theta}}Y_t^{1-\alpha-\left(1-\frac{\alpha}{1+\theta}\right)(1+\gamma)}K_t^{\frac{\alpha\theta(1+\gamma)}{1+\theta}}. \tag{A.3}$$

Then combining equations (16) and (A.3) yields

$$C_t = C\left(\phi_t, K_t\right) = f_0 \cdot g\left(\phi_t\right) \cdot h(K_t), \tag{A.4}$$

where $f_0 \equiv A^{\frac{1+\gamma}{1-\alpha}} \left(\frac{\alpha}{\delta^0}\right)^{\frac{\alpha(1+\gamma)}{(1+\theta)(1-\alpha)}} \left(\frac{1-\alpha}{\psi}\right)$, $h(K_t) \equiv K_t^{\frac{\alpha\theta(1+\gamma)}{(1+\theta)(1-\alpha)}}$, and

$$g\left(\phi_t\right) \equiv \left[\phi_t^{1-\alpha+\frac{\alpha(1+\gamma)}{1+\theta}} Y\left(\phi_t\right)^{1-\alpha-\left(1-\frac{\alpha}{1+\theta}\right)(1+\gamma)}\right]^{\frac{1}{1-\alpha}}.$$

In turn, differentiating both sides of equation (A.4) yields

$$\begin{aligned}
(1-\alpha)\frac{\dot{C}_t}{C_t} &= \left(1-\alpha+\frac{\alpha(1+\gamma)}{1+\theta}\right)\frac{\dot{\phi}_t}{\phi_t} + \left(1-\alpha-\left(1-\frac{\alpha}{1+\theta}\right)(1+\gamma)\right)\frac{\dot{Y}_t}{Y_t} + \left(\frac{\alpha\theta(1+\gamma)}{1+\theta}\right)\frac{\dot{K}_t}{K_t} \\
&= \left(1-\alpha+\frac{\alpha(1+\gamma)}{1+\theta}\right)\left(\frac{\phi_{\max}-\phi_t}{1-\phi_t}\right)\frac{\dot{\phi}_t}{\phi_t} + \left(\frac{\alpha\theta(1+\gamma)}{1+\theta}\right)\frac{\dot{K}_t}{K_t}
\end{aligned}$$

Since

$$u_t = \left(\frac{\alpha}{\delta^0}\frac{\phi_t Y\left(\phi_t\right)}{K_t}\right)^{\frac{1}{1+\theta}} \equiv u\left(K_t, \phi_t\right). \tag{A.5}$$

Finally, substituting equations (A.4) and (A.5) into (A.1) and (A.2) yields

$$\begin{aligned}
\left(1-\alpha+\frac{\alpha(1+\gamma)}{1+\theta}\right)\left(\frac{\phi_{\max}-\phi_t}{1-\phi_t}\right)\frac{\dot{\phi}_t}{\phi_t} + \left(\frac{\alpha\theta(1+\gamma)}{1+\theta}\right)\frac{\dot{K}_t}{K_t} &= (1-\alpha)\left(\frac{\alpha\theta}{1+\theta}\phi_t\frac{Y\left(\phi_t\right)}{K_t}-\rho\right), \\
\dot{K}_t &= \left(1-\frac{\alpha\phi_t}{1+\theta}\right)Y\left(\phi_t\right) - C\left(\phi_t, K_t\right),
\end{aligned}$$

the desired autonomous dynamical system in Proposition 5.

Proof of Corollary 6: We can easily verify that $g(0) = g(1) = 0$, $g''(\phi) < 0$, and $g'(\phi_{\max}) = 0$, where $\phi_{\max} = 1 - \tau_{\min}$, and $\tau_{\min} \equiv \frac{(1+\theta)(1+\gamma)}{(1+\theta)(1-\alpha)+\alpha(1+\gamma)} - 1$. Therefore $\phi_{\max} = \arg\max g(\phi)$. It then follows from equation (24) that C_t is a hump-shaped function of ϕ_t for a given level of K_t. Then the results in Corollary 6 follow.

Proof of Lemma 7: Notice that $\Psi(\phi) = \left(\frac{1-\phi}{\phi}\right) \cdot Y(\phi) \propto (1-\phi)\phi^{\frac{2\alpha-1}{1-\alpha}}$. When $\alpha < \frac{1}{2}$, we know that $(1-\phi)\phi^{\frac{2\alpha-1}{1-\alpha}}$ is decreasing in ϕ. It is easy to check that $\lim_{\phi\to 0}\Psi(\phi) = \infty$ and $\lim_{\phi\to 1}\Psi(\phi) = 0$. Hence equation (32) uniquely pins down the steady-state ϕ for any $\pi > 0$.

Proof of Proposition 8: Similar to the derivation of equation (19), we can easily prove that $Y = A^{\frac{1}{1-\alpha}} (k_y)^{\frac{\alpha}{1-\alpha}} N$, where $k_y = \frac{K}{Y} = \frac{\alpha\phi\theta}{\rho(1+\theta)}$, and $N = \left(\frac{1-\alpha}{1-\frac{\alpha}{1+\theta}} \frac{1}{\psi}\right)^{\frac{1}{1+\gamma}}$ so that

$$Y = A^{\frac{1}{1-\alpha}} \left(\frac{\alpha\phi\theta}{\rho(1+\theta)}\right)^{\frac{\alpha}{1-\alpha}} \left(\frac{1-\alpha}{1-\frac{\alpha}{1+\theta}} \frac{1}{\psi}\right)^{\frac{1}{1+\gamma}}.$$

Equation (32) can be used to solve for the steady-state ϕ and use equation (19) to obtain the steady state Y. Consumption and capital can then be computed from $C = c_y Y$ and $K = k_y Y$ respectively.

The log-linearized of the system of equilibrium equations can be simplified , again defining $\tau = 1 - \phi$ for convenience, to a system of two linear differential equations:

$$\begin{bmatrix} \dot{k}_t \\ \dot{c}_t \end{bmatrix} = \delta \begin{bmatrix} \left(\frac{1+\theta}{\alpha\phi}\right) s\phi (1+\tau) \lambda_1 & \left(\frac{1+\theta}{\alpha\phi}\right) [s\phi (1+\tau) \lambda_2 - (1-s\phi)] \\ \theta [(1+\tau)\lambda_1 - 1] & \theta(1+\tau)\lambda_2 \end{bmatrix} \begin{bmatrix} \hat{k}_t \\ \hat{c}_t \end{bmatrix} \equiv J \begin{bmatrix} \hat{k}_t \\ \hat{c}_t \end{bmatrix}.$$

Similarly, λ_1 and λ_2 are the elasticity of output to capital and consumption, respectively. They are given by

$$\hat{y}_t = \frac{a(1+\gamma)}{1+\gamma-b(1+\tau)} \hat{k}_t - \frac{b}{1+\gamma-b(1+\tau)} \hat{c}_t \equiv \lambda_1 \hat{k}_t + \lambda_2 \hat{c}_t,$$

Consequently the local dynamics is characterized by the log-linearized differential equations above where $s \equiv 1 - \frac{\alpha}{1+\theta}$, $c_y = s\phi$, and $\delta = \rho/\theta$. The local dynamics around the steady state is determined by the roots of the matrix of coefficients J. Notice that the trace and determinant of J are

$$\begin{aligned} \frac{\text{Trace}\,(J)}{\delta} &= \left(\frac{1+\theta}{\alpha}\right) s (1+\tau) \lambda_1 + \theta(1+\tau)\lambda_2 < 0, \\ \frac{\text{Det}\,(J)}{\delta^2\theta\left(\frac{1+\theta}{\alpha\phi}\right)} &= s\phi (1+\tau) \lambda_2 + (1-s\phi)(1+\tau)\lambda_1 - (1-s\phi) > 0. \end{aligned}$$

Similar to the analysis of the indeterminacy for our baseline model, here Trace$(J) < 0$ if and only if $\tau > \tau_{\min} \equiv \frac{(1+\theta)(1+\gamma)}{(1+\theta)(1-\alpha)+\alpha(1+\gamma)} - 1$. Given that $\tau > \tau_{\min}$, some algebraic manipulation shows that $\text{Det}(J) > 0$ if and only if $\tau < \frac{1+\theta}{\alpha} - 1$, and

$$A_1\tau^2 - A_2\tau - A_3 < 0,$$

where

$$\begin{aligned}
A_1 &\equiv s(1+\theta)(2+\alpha+\alpha\gamma) > 0 \\
A_2 &\equiv (1+\theta)(1+\alpha\gamma) - s\left[(1+\theta)(1-\alpha)(1-\gamma) + (1+\gamma)\alpha\right] \\
A_3 &\equiv (1+\theta)(1-\alpha)\left[s + +(1-s)\gamma\right] > 0.
\end{aligned}$$

Therefore $A_1\tau^2 - A_2\tau - A_3 < 0$ if and only if $\tau < \tau_H$, where τ_H is the positive solution to $A_1\tau^2 - A_2\tau - A_3 = 0$.

Proof of Corollary 9: Combining equation (32) and Proposition 8 immediately yields the results.

Proof of Lemma 10: First, using the implicit function theorem, equation (37) suggests that $\partial q^*/\partial Y > 0$. Second, since $TFP = \Gamma(q^*)A$, it is obvious that $\partial TFP/\partial q^* > 0$. Then using the chain rule gives

$$\frac{\partial TFP}{\partial Y} = \frac{\partial TFP}{\partial q^*} \cdot \frac{\partial q^*}{\partial Y} > 0.$$

Proof of Proposition 11: The proposition is immediately obtained by using equation (39).

Proof of Proposition 12: First, given the Power distribution, i.e., $F(q) = q^\eta$, we can analytically obtain the dynamical system, and then easily verify the uniqueness of the steady state. It remains for us to pin down the indeterminacy region. To establish the conditions for indeterminacy, we first log-linearize the equilibrium equations. Substituting out $\hat{u}_t$ from the log-linearized equation (5),

$$\hat{y}_t = a\hat{k}_t + b\hat{n}_t,$$

where $a \equiv \frac{\theta\alpha(1+\sigma)}{1+\theta-\alpha(1+\sigma)}$ and $b \equiv \frac{(1+\theta)(1-\alpha)(1+\sigma)}{1+\theta-\alpha(1+\sigma)}$. Finally, expressing $\hat{n}_t$ from the log-linearized equation (3),

$$\hat{y}_t = \lambda_1 \hat{k}_t + \lambda_2 \hat{c}_t,$$

where $\lambda_1 \equiv \frac{a(1+\gamma)}{1+\gamma-b}$ and $\lambda_2 \equiv -\frac{a}{1+\gamma-b}$. Therefore the dynamics can be expressed as a two-dimensional system of differential equations,

$$\begin{bmatrix} \dot{k}_t \\ \dot{c}_t \end{bmatrix} = \delta \begin{bmatrix} \left(\frac{1+\theta}{\alpha\phi} - 1\right)\lambda_1 & \left(\frac{1+\theta}{\alpha\phi}\right)(\lambda_2 - 1) + 1 - \lambda_2 \\ \theta(\lambda_1 - 1) & \theta\lambda_2 \end{bmatrix} \begin{bmatrix} \hat{k}_t \\ \hat{c}_t \end{bmatrix},$$

where $\delta = \rho/\theta$. Let J be the matrix of coefficients. The trace and determinant of J are

$$\begin{aligned} \frac{\mathrm{Trace}(J)}{\delta} &= \left(\frac{1+\theta}{\alpha\phi} - 1\right)\lambda_1 + \theta\lambda_2 = \frac{\left(\frac{1+\theta}{\alpha\phi} - 1\right)(1+\gamma)a - \theta b}{1+\gamma-b}, \\ \frac{\mathrm{Det}(J)}{\delta^2\theta} &= \left(\frac{1+\theta}{\alpha\phi} - 1\right)(\lambda_1 - 1 + \lambda_2) = \left(\frac{1+\theta}{\alpha\phi} - 1\right)\frac{(1+\gamma)(a-1)}{1+\gamma-b}. \end{aligned}$$

Indeterminacy arises if Trace$(J) < 0$ and Det$(J) > 0$. Under the assumption $a < 1$, or $\alpha(1+\sigma) < 1$, Det$(J) > 0$ is equivalent to $1+\gamma-b$, or $\sigma > \sigma_{\min} \equiv \left(\frac{1}{\frac{1-\alpha}{1+\gamma} + \frac{\alpha}{1+\theta}}\right) - 1$. Then Trace$(J) < 0$ requires $\left(\frac{1+\theta}{\alpha\phi} - 1\right)(1+\gamma)a > \theta b$. Rearranging terms yields the requirement $\frac{(1+\sigma)\eta}{1+\eta} < \frac{1}{\frac{1-\alpha}{1+\gamma} + \frac{\alpha}{1+\theta}}$. Recall that $\sigma = \frac{1-\zeta}{\eta}$, and thus $\frac{(1+\sigma)\eta}{1+\eta} = \frac{1+\eta-\zeta}{1+\eta} < 1 < \frac{1}{\frac{1-\alpha}{1+\gamma} + \frac{\alpha}{1+\theta}}$. Therefore the above requirement is automatically satisfied.