

# BARRO and SALA-I-MARTIN: JOEG, 1997

## Production

$$Y_i = A_i(L_i)^{1-\alpha} \sum_{j=1}^{N_i} (X_{ij})^\alpha \quad i = 1, 2$$

Output = Expenditures

$$Y_i = C_i + \sum_{j=1}^{N_i} X_{ij} + RD$$

$C_i$  and  $X_{ij}$  require one unit of  $Y_i$ . Invention of new variety requires  $\eta_i$ .

Assume  $N_1(0) > N_2(0)$ , and new discoveries occur in country 1.

Country 1:

Monopoly profit flow to inventor of intermediate good  $j$ :

$$\pi_{1j} = (P_{1j} - 1)X_{1j}$$

From Prod. fn:

$$\frac{\partial Y_1}{\partial X_{1j}} = A_1 \alpha (L_1)^{1-\alpha} (X_{1j})^{\alpha-1}$$

If we set this marginal product to the price of the intermediate good,  $P_{1j}$ , and setting the price of  $Y_1 = 1$  :

$$X_{1j} = L_1 \left( \frac{A_1 \alpha}{P_{1j}} \right)^{\frac{1}{1-\alpha}}$$

Intermediate good profit function then:

$$\text{Max}_{P_{1j}} (P_{1j} - 1)L_1 \left( \frac{A_1 \alpha}{P_{1j}} \right)^{\frac{1}{1-\alpha}}$$

$$\text{Max}_{P_{1j}} L_1 (A_1 \alpha)^{\frac{1}{1-\alpha}} \left( (P_{1j})^{\frac{\alpha}{\alpha-1}} - (P_{1j})^{\frac{1}{\alpha-1}} \right)$$

$$\text{FOC} : \left( \frac{\alpha}{1-\alpha} \right) (P_{1j})^{\frac{1}{1-\alpha}} = \left( \frac{1}{1-\alpha} \right) (P_{1j})^{\frac{2-\alpha}{1-\alpha}}$$

$$0 = \left( \frac{1}{1-\alpha} \right) (P_{1j})^{\frac{1}{1-\alpha}} (\alpha - P_{1j})$$

$$P_{ij} = P_1 = \alpha^{-1}$$

Substituting into  $X_{1j}$  and then  $Y_1$  :

$$X_{1j} = X_1 = L_1(A_1)^{\frac{1}{1-\alpha}} (\alpha)^{\frac{2}{1-\alpha}}$$

$$Y_1 = (A_1)^{\frac{1}{1-\alpha}} \alpha^{\frac{2\alpha}{1-\alpha}} L_1 N_1$$

$$\frac{Y_1}{L_1} = (A_1)^{\frac{1}{1-\alpha}} \alpha^{\frac{2\alpha}{1-\alpha}} N_1$$

Output per worker rises with  $A_1$ ,  $N_1$ .

Monopoly profit flow:

$$\pi_{1j} = \pi_1 = (1 - \alpha)L_1(A_1)^{\frac{1}{1-\alpha}} \alpha^{\frac{1+\alpha}{1-\alpha}}$$

with present value:

$$V_1(t) = \pi_1 \int_t^\infty e^{-\int_t^s r_1(v)dv} ds$$

If there is free entry,  $V_1(t) = \eta_1$ , which implies  $r_1$  is constant:

$$r_1 = \frac{\pi_1}{\eta_1}$$

So rate of return is the ratio of profit flow over cost.

Consumers:

$$U = \int_0^{\infty} e^{-\rho t} (1 - \theta)^{-1} (c^{1-\theta} - 1) dt$$

FOC:

$$\frac{\dot{C}_1}{C_1} = \theta^{-1} (r_1 - \rho)$$

If  $C_1$ ,  $N_1$  and  $Y_1$  grow at the same rate, assuming  $\frac{\pi_1}{\eta_1} - \rho > 0$  (since  $N_1$  cannot decrease),

$$\begin{aligned} \gamma &= \frac{\dot{N}}{N} = \theta^{-1} (r_1 - \rho) = \theta^{-1} \left( \frac{\pi_1}{\eta_1} - \rho \right) \\ &= \theta^{-1} \left( \left( \frac{(1-\alpha)}{\alpha} \right) (\eta_1)^{-1} L_1 (A_1)^{\frac{1}{1-\alpha}} \alpha^{\frac{2}{1-\alpha}} - \rho \right) \end{aligned}$$

$C_1$ ,  $Y_1$  grow at the same rate as  $N_1$ :

$$Y_1 = (A_1)^{\frac{1}{1-\alpha}} \alpha^{\frac{2\alpha}{1-\alpha}} L_1 N_1$$

$$\begin{aligned} C_1 &= Y_1 - \eta \dot{N}_1 - N_1 X_1 = Y_1 - \eta \gamma N_1 - N_1 X_1 \\ &= (A_1)^{\frac{1}{1-\alpha}} \alpha^{\frac{2\alpha}{1-\alpha}} L_1 N_1 - \eta \gamma N_1 - N_1 L_1 (A_1)^{\frac{1}{1-\alpha}} (\alpha)^{\frac{2}{1-\alpha}} \end{aligned}$$

so  $C_1$ ,  $N_1$  grows as  $N_1$ .

## COUNTRY 2

Adapting intermediate goods cost

$V_2(t) = v_2\left(\frac{N_2}{N_1}\right) < \eta_2$  for  $\frac{N_2}{N_1} < 1$ , and  $n_2$  otherwise. Results are parallel to country 1:

$$P_{2j} = \alpha^{-1}$$

$$X_{2j} = X_2 = L_2(A_2)^{\frac{1}{1-\alpha}} (\alpha)^{\frac{2}{1-\alpha}}$$

$$Y_2 = (A_2)^{\frac{1}{1-\alpha}} \alpha^{\frac{2\alpha}{1-\alpha}} L_2 N_2$$

$$\frac{Y_2}{N_2} = (A_2)^{\frac{1}{1-\alpha}} \alpha^{\frac{2\alpha}{1-\alpha}} L_2$$

$$\pi_{2j} = \pi_2 = (1 - \alpha)L_2(A_2)^{\frac{1}{1-\alpha}} \alpha^{\frac{1+\alpha}{1-\alpha}}$$

So

$$\frac{Y_2}{Y_1} = \left(\frac{A_2}{A_1}\right)^{\frac{1}{1-\alpha}} \left(\frac{N_2}{N_1}\right)$$

Also,

$$v_2 \left( \frac{N_2}{N_1} \right) = \pi_2 \int_t^\infty e^{-\int_t^s r_2(v) dv} ds$$

implies

$$r_2 = \frac{\pi_2}{v_2} + \frac{\dot{v}_2}{v_2}$$

because, differentiating

$$v_2 \left( \frac{N_2}{N_1} \right) = \pi_2 \int_t^\infty e^{-\int_t^s r_2(v) dv} ds, :$$

$$\begin{aligned} \dot{v}_2(t) &= \pi_2(-1 + r_2(t)) \int_t^\infty e^{-\int_t^s r_2(v) dv} ds \\ &= -\pi_2 + r_2(t)v_2(t) \end{aligned}$$

Note that here  $r_2$  is not simply  $\frac{\pi_2}{v_2}$ , but includes the capital gain term  $\frac{\dot{v}_2}{v_2}$ , as it reflects the changing cost of entry that is reflected in the monopoly profits.

Also:

$$\frac{\dot{C}_2}{C_2} = \theta^{-1}(r_2 - \rho)$$

## STEADY STATE GROWTH

$$\frac{\dot{N}_i}{N_i} = \frac{\dot{C}_i}{C_i} = \frac{\dot{Y}_i}{Y_i} = \gamma_1 = \gamma_2 = \gamma; \left(\frac{N_2}{N_1}\right)^* = \text{const}$$

For the moment assume  $\left(\frac{N_2}{N_1}\right)^* < 1$ .

$$r_1 = r_2 = r = \frac{\pi_1}{\eta_1} = \frac{\pi_2}{v_2}$$

$$v_2 = \eta_1 \left(\frac{\pi_2}{\pi_1}\right) = \eta_1 \left(\frac{A_2}{A_1}\right)^{\frac{1}{1-\alpha}} \left(\frac{L_2}{L_1}\right)$$

So if

$$v_2 = \eta_1 \left(\frac{A_2}{A_1}\right)^{\frac{1}{1-\alpha}} \left(\frac{L_2}{L_1}\right) < \eta_2$$

$$\frac{\eta_1}{\eta_2} \left(\frac{A_2}{A_1}\right)^{\frac{1}{1-\alpha}} \left(\frac{L_2}{L_1}\right) < 1$$

Country 1 is always the leader with no incentive to imitate since  $N_1 > N_2$ , country 2 always imitates since  $v_2 < \eta_2$ . Growth rates in two countries are also equalized.

$$\gamma = \frac{\dot{C}_1}{C_1} = \frac{\dot{C}_2}{C_2} = \theta^{-1}(r - \rho)$$

## DYNAMICS

Now let

$$v_2 = \eta_2 \left( \frac{N_2}{N_1} \right)^\sigma$$

Then

$$\left( \frac{N_2}{N_1} \right)^* = \left( \frac{\eta_1}{\eta_2} \left( \frac{A_2}{A_1} \right)^{\frac{1}{1-\alpha}} \left( \frac{L_2}{L_1} \right) \right)^{\frac{1}{\sigma}}$$

Let

$$H = \frac{N_2}{N_1} \cdot \sigma = \frac{v_2' H}{v_2}$$

. Using  $r_2 = \frac{\pi_2}{v_2} + \frac{\dot{v}_2}{v_2}$ ,

$$\frac{\dot{C}_2}{C_2} = \theta^{-1} (r_2 - \rho) = \theta^{-1} \left( \frac{\pi_2}{v_2} + \frac{\dot{v}_2}{v_2} - \rho \right)$$

$$\frac{\dot{C}_2}{C_2} = \theta^{-1} \left( \frac{\pi_2}{v_2} + \sigma \frac{\dot{H}}{H} - \rho \right)$$

Now

$$v_2 \dot{N}_2 = Y_2 - C_2 - N_2 X_2$$

$$\dot{N}_2 = \frac{1}{v_2} (Y_2 - C_2 - N_2 X_2)$$

$$\frac{\dot{N}_2}{N_2} = \frac{1}{v_2} \left( \frac{Y_2}{N_2} - X_2 - \frac{C_2}{N_2} \right)$$

$$= \frac{1}{v_2} \left( L_2(A_2)^{\frac{1}{1-\alpha}} \left( \alpha^{\frac{2\alpha}{1-\alpha}} - \alpha^{\frac{2}{1-\alpha}} \right) - \frac{C_2}{N_2} \right)$$

$$= \frac{1}{v_2} \left( L_2(A_2)^{\frac{1}{1-\alpha}} \alpha^{\frac{2}{1-\alpha}} (\alpha^{-2} - 1) - \frac{C_2}{N_2} \right)$$

$$= \frac{1}{v_2} \left( L_2(A_2)^{\frac{1}{1-\alpha}} \alpha^{\frac{2}{1-\alpha}} \left( \frac{1-\alpha}{\alpha} \right) \left( \frac{1+\alpha}{\alpha} \right) - \frac{C_2}{N_2} \right)$$

**bBUT**

$$\begin{aligned}\pi_2 &= (1 - \alpha)L_2(A_2)^{\frac{1}{1-\alpha}} \alpha^{\frac{1+\alpha}{1-\alpha}} \\ &= \frac{(1 - \alpha)}{\alpha} L_2(A_2)^{\frac{1}{1-\alpha}} \alpha^{\frac{2}{1-\alpha}}\end{aligned}$$

$$\frac{(1 + \alpha)}{\alpha} \pi_2 = \frac{(1 + \alpha)}{\alpha} \frac{(1 - \alpha)}{\alpha} L_2(A_2)^{\frac{1}{1-\alpha}} \alpha^{\frac{2}{1-\alpha}}$$

**SO**

$$\frac{\dot{N}_2}{N_2} = \frac{1}{v_2} \left( L_2(A_2)^{\frac{1}{1-\alpha}} \alpha^{\frac{2}{1-\alpha}} \left( \frac{1 - \alpha}{\alpha} \right) \left( \frac{1 + \alpha}{\alpha} \right) - \frac{C_2}{N_2} \right)$$

$$\frac{\dot{N}_2}{N_2} = \frac{1}{v_2} \left( \frac{(1 + \alpha)}{\alpha} \pi_2 - \frac{C_2}{N_2} \right)$$

**Let**  $\chi_2 = \frac{C_2}{N_2}$ . **Then**

$$\frac{\dot{H}}{H} = \frac{\dot{N}_2}{N_2} - \frac{\dot{N}_1}{N_1} = \frac{1}{v_2} \left( \frac{(1 + \alpha)}{\alpha} \pi_2 - \chi_2 \right) - \gamma_1$$

Now

$$\begin{aligned}\frac{\dot{C}_2}{C_2} &= \theta^{-1} \left( \frac{\pi_2}{v_2} + \sigma \frac{\dot{H}}{H} - \rho \right) \\ &= \theta^{-1} \left( \frac{\pi_2}{v_2} + \sigma \left( \frac{1}{v_2} \left( \frac{(1+\alpha)}{\alpha} \pi_2 - \chi_2 \right) - \gamma_1 \right) - \rho \right) \\ &= \frac{\theta^{-1}}{v_2} \left( \pi_2 + \sigma \left( \left( \frac{(1+\alpha)}{\alpha} \pi_2 - \chi_2 \right) - \gamma_1 \right) - \rho \right)\end{aligned}$$

However

$$\begin{aligned}\frac{\dot{\chi}_2}{\chi_2} &= \frac{\dot{C}_2}{C_2} - \frac{\dot{N}_2}{N_2} \\ \frac{\dot{\chi}_2}{\chi_2} &= \frac{1}{\theta v_2} \left( \pi_2 + (\theta - \sigma) \left( \chi_2 - \frac{(1+\alpha)}{\alpha} \pi_2 \right) \right) \\ &\quad - \theta^{-1} (\rho + \sigma \gamma_1)\end{aligned}$$

So the two dynamic equations are:

$$\frac{\dot{H}}{H} = \frac{1}{v_2} \left( \frac{(1 + \alpha)}{\alpha} \pi_2 - \chi_2 \right) - \gamma_1$$

$$\frac{\dot{\chi}_2}{\chi_2} = \frac{1}{\theta v_2} \left( \pi_2 + (\theta - \sigma) \left( \chi_2 - \frac{(1 + \alpha)}{\alpha} \pi_2 \right) \right) - \theta^{-1}(\rho + \sigma \gamma_1)$$

where  $v_2$  depends on  $H = \frac{N_2}{N_1}$  since we had assumed

$$v_2 \left( \frac{N_2}{N_1} \right) = \eta_2 \left( \frac{N_2}{N_1} \right)^\sigma = \eta_2(H)^\sigma.$$

Upward sloping saddle path (for the two cases where  $\theta > \sigma$  and also for  $\theta < \sigma$ ) implies that  $N_2$  grows faster than  $N_1$  in approaching the steady state ratio  $\left( \frac{N_2}{N_1} \right)^*$ .

Since  $Y_i$  is proportional to  $N_i$ ,  $Y_2$  grows faster than  $Y_1$ . Eventually at steady state both countries grow at  $\gamma_1$ .

NOT in the article

(Skip to the Simplify the consumption  
Function section)

Evaluate Jacobian:

$$J_{11} = -v_2^{-2} v_2' \left( \frac{(1+\alpha)}{\alpha} \pi_2 - \chi_2 \right) H$$

$$J_{12} = -v_2^{-1} H$$

$$J_{21} = -v_2^{-2} \theta^{-1} v_2' \left( \pi_2 + (\theta - \sigma) \left( \chi_2 - \frac{(1+\alpha)}{\alpha} \pi_2 \right) \right) \chi_2 \\ - \left( \frac{\chi_2 - \frac{(1+\alpha)}{\alpha} \pi_2}{\theta v_2} \right) \sigma'$$

$$J_{22} = v_2^{-1} \theta^{-1} (\theta - \sigma) \chi_2$$

DET

$$\begin{aligned}
& -\chi_2 H v_2^{-2} v_2' \left( \frac{(1+\alpha)}{\alpha} \pi_2 - \chi_2 \right) v_2^{-1} \theta^{-1} (\theta - \sigma) \\
& + v_2' \chi_2 H v_2^{-2} \theta^{-1} \left( \pi_2 + (\theta - \sigma) \left( \chi_2 - \frac{(1+\alpha)}{\alpha} \pi_2 \right) \right) \\
& = -\chi_2 H v_2^{-3} \theta^{-1} v_2' \left( \begin{aligned} & \left( \frac{(1+\alpha)}{\alpha} \pi_2 - \chi_2 \right) (\theta - \sigma) \\ & + \left( \pi_2 + (\theta - \sigma) \left( \chi_2 - \frac{(1+\alpha)}{\alpha} \pi_2 \right) \right) \end{aligned} \right) \\
& + v_2^{-1} H \left( \frac{\chi_2 - \frac{(1+\alpha)}{\alpha} \pi_2}{\theta v_2} \right) \sigma' \\
& = -\chi_2 H v_2^{-3} \theta^{-1} v_2' \pi_2 - v_2^{-2} H \theta^{-1} \left( \chi_2 - \frac{(1+\alpha)}{\alpha} \pi_2 \right) \sigma'
\end{aligned}$$

TRACE

$$-\ v_2^{-2} \left( \frac{(1+\alpha)}{\alpha} \pi_2 - \chi_2 \right) H + v_2^{-1} \theta^{-1} (\theta - \sigma) \chi_2$$

## Steady State

$$\frac{(1 + \alpha)}{\alpha} \pi_2 - \gamma_1 v_2 = \chi_2 = \frac{C_2}{N_2}$$

$$\pi_2 + (\theta - \sigma) \left( \chi_2 - \frac{(1 + \alpha)}{\alpha} \pi_2 \right) = \theta^{-1} (\rho + \sigma \gamma_1) \theta v_2$$

$$(\pi_2 - (\theta - \sigma) (\gamma_1 v_2)) = \theta^{-1} (\rho + \sigma \gamma_1) \theta v_2$$

$$\frac{\pi_2}{((\theta - \sigma) \gamma_1 + (\rho + \sigma \gamma_1))} = v_2$$

$$\frac{\pi_2}{(\theta \gamma_1 + \rho)} = v_2 = \eta_2 (H)^\sigma$$

$$\pi_2 \left( \frac{(1 + \alpha)}{\alpha} - \frac{\gamma_1}{(\theta \gamma_1 + \rho)} \right) = \chi_2$$

What if

$$v_2 = \eta(1 - H)^{-1}$$

$$\frac{\dot{v}_2}{v_2} = \frac{\dot{H}}{1 - H}$$

$$\frac{\dot{H}}{H} = \frac{\dot{N}_2}{N_2} - \frac{\dot{N}_1}{N_1} = \left( \frac{1}{v_2} \right) \left( \frac{(1 + \alpha)}{\alpha} \pi_2 - \chi_2 \right) - \gamma_1$$

Now

$$\frac{\dot{C}_2}{C_2} = \theta^{-1} \left( \frac{\pi_2}{v_2} + \left( \frac{H}{1 - H} \right) \frac{\dot{H}}{H} - \rho \right)$$

$$= \theta^{-1} \left( \begin{array}{c} \frac{\pi_2}{v_2} + \left( \frac{H}{1 - H} \right) \\ \cdot \left( \frac{1}{v_2} \left( \frac{(1 + \alpha)}{\alpha} \pi_2 - \chi_2 \right) - \gamma_1 \right) - \rho \end{array} \right)$$

$$= \theta^{-1} \frac{1}{v_2} \left( \begin{array}{c} \pi_2 + \frac{H}{1 - H} \left( \left( \frac{(1 + \alpha)}{\alpha} \pi_2 - \chi_2 \right) - \gamma_1 \right) \\ - \rho \end{array} \right)$$

However

$$\frac{\dot{\chi}_2}{\chi_2} = \frac{\dot{C}_2}{C_2} - \frac{\dot{N}_2}{N_2}$$

$$\frac{\dot{\chi}_2}{\chi_2} = \frac{1}{\theta v_2} \left( \begin{array}{l} \pi_2 + \left( \theta - \left( \frac{H}{1-H} \right) \right) \\ \cdot \left( \chi_2 - \frac{(1+\alpha)}{\alpha} \pi_2 \right) \end{array} \right) \\ - \theta^{-1} \left( \rho + \left( \frac{H}{1-H} \right) \gamma_1 \right)$$

So the two dynamic equations are:

$$\frac{\dot{H}}{H} = \frac{1}{v_2} \left( \frac{(1 + \alpha)}{\alpha} \pi_2 - \chi_2 \right) - \gamma_1$$

$$\begin{aligned} \frac{\dot{\chi}_2}{\chi_2} = & \frac{1}{\theta v_2} \left( \pi_2 + \left( \theta - \left( \frac{H}{1 - H} \right) \right) \left( \chi_2 - \frac{1 + \alpha}{\alpha} \pi_2 \right) \right) \\ & - \theta^{-1} \left( \rho + \sigma \left( \frac{H}{1 - H} \right) \gamma_1 \right) \end{aligned}$$

$$\frac{\dot{H}}{H} = (\eta^{-1}(1 - H)) \left( \frac{(1 + \alpha)}{\alpha} \pi_2 - \chi_2 \right) - \gamma_1$$

$$\begin{aligned} \frac{\dot{\chi}_2}{\chi_2} = & (\theta^{-1} \eta^{-1} (1 - H)) \\ & \cdot \left( \pi_2 + \left( \theta - \left( \frac{H}{1 - H} \right) \right) \left( \chi_2 - \frac{(1 + \alpha)}{\alpha} \pi_2 \right) \right) \\ & - \theta^{-1} \left( \rho + \sigma \left( \frac{H}{1 - H} \right) \gamma_1 \right) \end{aligned}$$

## SIMPLIFY CONSUMPTION FUNCTION

$$\dot{N}_2 = \left( \frac{1}{v_2} \right) (Y_2 - C_2 - N_2 X_2)$$

For simplicity assume  $C_2 = \mu(Y_2 - N_2 X_2)$

$$\dot{N}_2 = \left( \frac{1}{v_2} \right) (Y_2 - N_2 X_2) (1 - \mu)$$

$$\frac{\dot{H}}{H} = \frac{\dot{N}_2}{N_2} - \frac{\dot{N}_1}{N_1} = (1 - \mu) \left( \frac{1}{v_2} \right) \left( \frac{Y_2}{N_2} - X_2 \right) - \gamma_1$$

But  $\frac{Y_2}{N_2} = (A_2)^{\frac{1}{1-\alpha}} \alpha^{\frac{2\alpha}{1-\alpha}} L_2$  and

$X_2 = L_2 (A_2)^{\frac{1}{1-\alpha}} (\alpha)^{\frac{2}{1-\alpha}}$  are constants, and

$$\frac{Y_2}{N_2} - X_2 = L_2 (A_2)^{\frac{1}{1-\alpha}} \alpha^{\frac{2\alpha}{1-\alpha}} (1 - \alpha^2)$$

$$\begin{aligned} \frac{\dot{H}}{H} &= \frac{\dot{N}_2}{N_2} - \frac{\dot{N}_1}{N_1} \\ &= (1 - \mu) \left( \frac{1}{v_2} \right) \left[ L_2 (A_2)^{\frac{1}{1-\alpha}} \alpha^{\frac{2\alpha}{1-\alpha}} (1 - \alpha^2) \right] - \gamma_1 \end{aligned}$$

LET  $v_2 = nH^\sigma \rightarrow$  exponential diffusion

$$\begin{aligned}\dot{H} &= \left[ (1 - \mu) \left( L_2(A_2)^{\frac{1}{1-\alpha}} \alpha^{\frac{2\alpha}{1-\alpha}} (1 - \alpha^2) \right) \right] \frac{H^{1-\sigma}}{n} \\ &\quad - \gamma H \\ &= H \left( \begin{array}{c} \left[ (1 - \mu) \left( L_2(A_2)^{\frac{1}{1-\alpha}} \alpha^{\frac{2\alpha}{1-\alpha}} (1 - \alpha^2) \right) \right] \frac{H^{-\sigma}}{n} \\ -\gamma \end{array} \right)\end{aligned}$$

with steady state at

$$H^\sigma = \gamma \left( \left[ (1 - \mu) \left( L_2(A_2)^{\frac{1}{1-\alpha}} \alpha^{\frac{2\alpha}{1-\alpha}} (1 - \alpha^2) \right) \right] n \right)^{\frac{1}{\sigma}}$$

which is stable:  $\frac{\partial \dot{B}}{\partial B} < 0$  at the steady state

LET  $v_2 = n(1 - H)^{-1} \rightarrow$  Logistic

$$\begin{aligned} \dot{H} &= (1 - \mu) \left( L_2(A_2)^{\frac{1}{1-\alpha}} \alpha^{\frac{2\alpha}{1-\alpha}} (1 - \alpha^2) \right) \frac{(1 - H)}{n} H \\ &\quad - \gamma_1 H \\ &= H \left[ \begin{array}{c} (1 - \mu) \left( L_2(A_2)^{\frac{1}{1-\alpha}} \alpha^{\frac{2\alpha}{1-\alpha}} (1 - \alpha^2) \right) \frac{(1-H)}{n} \\ -\gamma_1 \end{array} \right] \\ &= H(Q(1 - H) - \gamma) = HQ(1 - \gamma Q^{-1} - H) \end{aligned}$$

Stable Steady State at

$$H = 1 - \gamma Q^{-1}$$

only if  $1 - \gamma Q^{-1} > 0$ , otherwise for any  $H > 0$ ,  $\dot{H} < 0$ ,  $\frac{N_2}{N_1} \rightarrow 0$