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- We have a production economy where output y_t at time t is produced by competitive firms using capital k_t and labor l_t according to the production function

$$y_t = F(k_t, l_t),$$

with F linearly homogeneous.
- There is a continuum of agents of unit measure, with agents indexed by i. In the initial period 0, they each own capital stock, k_0^i.
- In each period, the government levies nonnegative proportional taxes on labor income ν_t and capital income τ_t, subject to an exogenous upper bound $\bar{\tau}$.
- The government uses tax receipts and one-period debt to pay for spending on a public good at an exogenous rate $\{g_t\}_{t=0}^{\infty}$, and to finance a lump-sum transfer T_t that is used for redistribution.

- Household preferences are given by

$$\sum_{t=0}^{\infty} \beta^t u\left(c_t^i\right)$$

where c_t^i is period t consumption by agent i. We will assume that household utilities are restricted to satisfy Gorman aggregation, that is to have linear Engel curves. These preferences are exhausted by the exponential and power classes (see Polak, 1971) which coincide with the HARA class of utility functions.

- Household preferences are given by

$$u(c) = \frac{\sigma}{1-\sigma}\left(\frac{A}{\sigma}c + B\right)^{1-\sigma}$$

.

- The HARA class reduces to logarithmic utility for $\sigma = 1$, to quadratic utility for $\sigma = -1$, to CRRA utility for $\sigma = 0$ and $B = 0$, to linear utility for $\sigma = 0$, and to exponential utility for $\sigma \to \infty$ and $b = 0$.

Each household is endowed with 1 unit of labor, which it supplies inelastically. W_t^i is the wealth of household i at time t, consisting of capital and maturing bonds. The period-by-period household budget constraint is

$$y_t^i = (1 - \tau_t)\, r_t W_t^i + (1 - \nu_t)\, w_t + T_t \geq c_t^i + W_{t+1}^i, \tag{1}$$

where r_t is the gross return to capital, w_t is the wage at time t . We have imposed the no arbitrage condition by stipulating that the net return on bonds and capital are equal. We also assume that households cannot run Ponzi schemes:

$$\lim_{t\to\infty} (\prod_{s=1}^{t+1} (r_s(1 - \tau_s))^{-1}) W_{t+1}^i \geq 0 \tag{2}$$

- The household budget constraint in present value form is:

$$
\begin{aligned}
& \sum_{t=0}^{\infty} c_t^i \prod_{s=0}^{t} \left(r_s(1-\tau_s)\right)^{-1} && (3) \\
= \quad & W_0^i + \sum_{t=0}^{\infty} \left(T_t + w_t\left(1-\nu_t\right)\right) \prod_{s=0}^{t} \left(r_s(1-\tau_s)\right)^{-1} && (4)
\end{aligned}
$$

- For convenience we define prices

$$q_t : q_t = \beta^{-t} \prod_{i=1}^{t} (r_i)^{-1}$$

- Rather than working with the sequence $\{\tau_t\}_{t=0}^{\infty}$, it is more convenient to work with the following transformation, which is one to one over the relevant domain:

$$1 + \theta_t := \prod_{s=0}^{t} (1 - \tau_s)^{-1} \tag{5}$$

We can write (3) as:

$$\sum_{t=0}^{\infty} \beta^t c_t^i q_t (1 + \theta_t) = W_0^i + \sum_{t=0}^{\infty} (T_t + w_t (1 - \nu_t)) \beta^t q_t (1 + \theta_t)$$

Definition

Let c_s and b_s represent the aggregate levels of consumption and bonds at time s.A competitive equilibrium is $\left\{c_s, k_s, b_s, \tau_s, \nu_s.T_s, r_s, w_s, \left\{c_s^i, W_s^i\right\}_{i\in(0,1)}\right\}_{s=0}^{\infty}$ that satisfies

1. $\left\{\left\{c_s^i, W_s^i\right\}_{i\in(0,1)}\right\}_{s=0}^{\infty}$ maximizes household utilities ST: (1) and (2).
2. Factor prices equal their marginal products and makets clear:

$$r_t = F_k\left(k_t, 1\right),\ w_t = F_l\left(k_t, 1\right)$$

$$\int c_t^i di = c_t,\ \int W_t^i di = k_t + b_t$$

3. The government budget satisfies:

$$\begin{aligned} a)\ \ \tau_t r_t\left(k_t + b_t\right) + \nu_t w_t + b_{t+1} &= r_t b_t + g_t + T_t \quad (6) \\ b)\ \ \lim_{t\to\infty}\left(\prod_{s=1}^{t+1}\left(r_s(1-\tau_s)\right)^{-1}\right) b_{t+1} &= 0 \end{aligned}$$

Let the sequence $\{g_s\}_{s=0}^{\infty}$ represent the exogenous government expenditures. A competitive equilibrium will exist provided that $\{g_s\}_{s=0}^{\infty}$ is not too high. First, we characterize properties of the competitive equilibria that will be used in proving the subsequent theorems.

Theorem

For any sequence $\{c_s, k_s\}_{s=0}^{\infty}$ *that satisfies*

1. $$k_{t+1} + c_t + g_t = F(k_t, 1) \tag{7}$$

2. $$\left(F_k(k_{t+1}, 1)(1 - \bar{\tau})\right)^{-1} \leq \frac{\beta^t u'(c_{t+1})}{u'(c_t)} \leq \left(F_k(k_{t+1}, 1)\right)^{-1} \tag{8}$$

there exists a competitive equilibrium

$$\left\{c_s, k_s, b_s, \tau_s, \nu_s, T_s, r_s, w_s, \left\{c_s^i, W_s^i\right\}_{i \in (0,1)}\right\}_{s=0}^{\infty}.$$

Note: $0 \leq \tau_s \leq \bar{\tau}$.

The Median Voter Theorem

We assume that the sequence of taxes and transfers is set by voting at time 0. Since agents vote over an infinite sequence of tax rates, the usual single-peakedness assumption required for the median voter theorem cannot be used. Our proof instead relies on the two observations:

1. Households differ from each other along a single dimension, i.e., their initial capital holdings.
2. Any change in tax rates affects households in two ways: by redistribution of wealth that it implies, and by distortion in after-tax prices that it generates. Gorman aggregation and the fact that all households have the same discount factor imply that the distortion in after-tax prices has a proportional effect on all households, independent of wealth. Thus all households trade off a single, common measure of distortions against the degree of redistribution engineered by the distortion. Households of different wealth disagree on the optimal point along this trade-off, but their disagreement will naturally be ordered according to their initial wealth level.

To prove the theorem we first establish the following Lemma:

Lemma

For each household i there exists a function $G : R^4 \to R$ such that the utility of the household in a competitive equilibrium is $G\left(V, c_0, \tau_0, W_0^i - W_0\right)$ where V, the utility of the agent with average wealth, is $\sum_{t=0}^{\infty} \beta^t u\left(c_t\right)$. Also,

$$sign\left(\frac{\partial G\left(V, c_0, \tau_0, W_0^i - W_0\right)}{\partial c_0}\right) = sign\left(\frac{\partial G\left(V, c_0, \tau_0, W_0^i - W_0\right)}{\partial \tau_0}\right) = sign\left(W_0 - W_0^i\right).$$

The utility attained by household i is:

$$G\left(V, c_0, \tau_0, W_0^i - W_0\right) = \left(\alpha^i\right)^{1-\sigma} V = \left[1 + \frac{A r_0\left(1-\tau_0\right)\left(W_0^i - W_0\right)\left(\frac{A}{\sigma} c_0 + B\right)^{-\sigma}}{V\left(1-\sigma\right)}\right]^{1-\sigma} V \quad (9)$$

Theorem

The tax sequence preferred by the household with median wealth is a Condorcet winner

Proof.

We will use an order restriction to prove the theorem. Consider two competitive equilibrium sequences $\{c_t\}_{t=0}^{\infty}$ and $\{\hat{c}_t\}_{t=0}^{\infty}$, and two initial tax rates θ_0 and $\hat{\theta}_0$. Define $V := \sum_{t=0}^{\infty} \frac{\sigma}{1-\sigma} \left(\frac{A}{\sigma} c + B \right)^{1-\sigma}$ and $\hat{V} := \sum_{t=0}^{\infty} \frac{\sigma}{1-\sigma} \left(\frac{A}{\sigma} \hat{c} + B \right)^{1-\sigma}$. Construct the set of households that (weakly) prefer the competitive equilibrium associated with $(\{c_t\}_{t=0}^{\infty}, \tau_0)$ to the one associated with $(\{\hat{c}_t\}_{t=0}^{\infty}, \hat{\tau}_0)$: □

$$H := \left\{ W_0^i : \left[1 + \frac{Ar_0\left(1-\tau_0\right)\left(W_0^i - W_0\right)\left(\frac{A}{\sigma}c_0 + B\right)^{-\sigma}}{V\left(1-\sigma\right)}\right]^{1-\sigma} V \geq \left[1 + \frac{Ar_0\left(1-\hat{\tau}_0\right)\left(W_0^i - W_0\right)\left(\frac{A}{\sigma}\hat{c}_0 + B\right)^{-\sigma}}{\hat{V}\left(1-\sigma\right)}\right]^{1-\sigma} \hat{V} \right\} \tag{10}$$

Conversely, let $\hat{H}$ be the set of households that (weakly) prefer the other equilibrium:

$$\hat{H} := \left\{ W_0^i : \left[1 + \frac{Ar_0\left(1-\tau_0\right)\left(W_0^i - W_0\right)\left(\frac{A}{\sigma}c_0 + B\right)^{-\sigma}}{V\left(1-\sigma\right)}\right]^{1-\sigma} V \leq \left[1 + \frac{Ar_0\left(1-\hat{\tau}_0\right)\left(W_0^i - W_0\right)\left(\frac{A}{\sigma}\hat{c}_0 + B\right)^{-\sigma}}{\hat{V}\left(1-\sigma\right)}\right]^{1-\sigma} \hat{V} \right\} \tag{11}$$

We need to prove that both H and $\hat{H}$ are convex, independently of the choice of sequences. Similar to single crossing property.

Theorem

The capital tax sequence $\{\tau_t\}_0^\infty$ *preferred by the median voter has the bang-bang property: if* $\tau_t < \bar{\tau}$*, then* $\tau_s = 0$ *for* $s > t$*.*

Case 1: At the allocation preferred by the median voter $G\left(V, c_0, \tau_0, W_0^m - W_0\right)$ is increasing in V. Let $\{c_t^*, k_t^*\}_{t=0}^\infty$ be the preferred sequences of AGGREGATE consumption and capital by a household with median wealth within those that satisfy (7) and (8). This allocation will be implemented by a sequence of capital income taxes $\{\tau_t^*\}_1^\infty$ .An equivalent statement for the theorem is , if $u'\left(c_{t+1}^*\right)\left[\beta F_k\left(k_{t+1},1\right)\left(1-\bar{\tau}\right)\right] < u'\left(c_t^*\right)$, then $u'\left(c_s^*\right)\beta F_k\left(k_{s+1},1\right) = u'\left(c_{s+1}^*\right)$ for all $s > t$. Suppose this were not true. Then $\{c^{*i}\}_{s=t+1}^\infty$ does not satisfy the first-order conditions for solving

$$\max_{\{c_s\}_{s=t+1}^\infty} \sum_{s=t+1}^\infty \beta^s \frac{\sigma}{1-\sigma}\left(\frac{A}{\sigma}c_t + B\right)^{1-\sigma} \qquad (12)$$

subject to (7) and (8) from period $t+1$ on, given $_{t+1}$.

It is thus possible to find an alternative sequence $\{c_s^{**}\}_{s=t+1}^{\infty}$ such that $(\{c^*\}_{s=0}^{t}, \{c_s^{**}\}_{s=t+1}^{\infty})$ satisfies (7) and (8), but such that, for sufficiently small $\varepsilon > 0$,

$$\sum_{s=t+1}^{\infty} \beta^s \frac{\sigma}{1-\sigma} \left(\frac{A}{\sigma} c_t^{**} + B \right)^{1-\sigma} = \sum_{s=t+1}^{\infty} \beta^s \frac{\sigma}{1-\sigma} \left(\frac{A}{\sigma} c_t^{*} + B \right)^{1-\sigma} + \varepsilon \Longrightarrow$$
$$\sum_{s=0}^{t} \beta^s \frac{\sigma}{1-\sigma} \left(\frac{A}{\sigma} c_t^{*} + B \right)^{1-\sigma} + \sum_{s=t+1}^{\infty} \frac{\sigma}{1-\sigma} \left(\frac{A}{\sigma} c_t^{**} + B \right)^{1-\sigma}$$
$$= \sum_{s=0}^{\infty} \beta^s \frac{\sigma}{1-\sigma} \left(\frac{A}{\sigma} c_t^{*} + B \right)^{1-\sigma} + \varepsilon \tag{13}$$

The new sequence has the same initial consumption, but a higher value for the utility aggregate V. By hypothesis we have $G\left(V, c_0, \tau_0, W_0^m - W_0\right)$ increasing in V. As a consequence, the new sequence would be preferred by the median household, which is a contradiction.

Case 2: At the allocation **preferred by the median voter** $G(V, c_0, \tau_0, W_0^m - W_0)$ is decreasing in V. Then we can prove by contradiction that capital income taxes must be at their upper bound at all periods. Suppose this were not true. Then pick the first period in which the growth rate of marginal utility for the average (aggregate) agent is below its lower bound:

$$[\beta (1 - \bar{\tau}) F_k (k_N, 1)] < \frac{u'(c_{N-1})}{u'(c_N)}$$

Then increase $u'(c_{N-1})$ (this entails raising the capital tax rate in period N towards $\bar{\tau}$). Since this decreases k_N adjust future consumption(s) appropriately, preserving feasibility and optimality of the path from k_N. Since the average (aggregate) agent prefers no distortions, $\tau_t = 0$ for all t, increasing taxes at N and other future periods, coupled with he ecrease in k_N will decrease V. As a consequence, the new sequence would be preferred by the median household, which is a contradiction. (See proof in paper).

Corollary

If preferences are CRRA ($B = 0$ and $\sigma > 0$) and production is linear ($y = rk$), the capital tax preferred by the median voter is $\bar{\tau}$ forever if $1 + \frac{\sigma r(1-\bar{\tau})(R-1)}{\left(1-\beta^{\frac{1}{\sigma}} r^{\frac{1-\sigma}{\sigma}} (1-\tau)^{\frac{1}{\sigma}}\right) r \left(1-\beta^{\frac{1}{\sigma}} (r(1-\tau))^{\frac{1-\sigma}{\sigma}}\right)^{-1}} \leq 0$, *which can only happen if $\sigma > 1$.*

Proof.

Under CRRA preferences and linear technology $G\left(V, c_0, \tau_0, W_0^m - W_0\right)$ is (weakly) decreasing in V if

$$\left[1+\sigma\frac{Ar_0\left(1-\tau_0\right)\left(W_0^i-W_0\right)\left(\frac{A}{\sigma}c_0+B\right)^{-\sigma}}{\left(1-\sigma\right)V}\right]\leq 0 \qquad (14)$$

It is straightforward to prove that when $\sigma > 1$, $\partial^2 G/\partial V^2 < 0$, $\partial^2 G/\partial c_0 \partial V > 0$, and $\partial^2 G/\partial V \partial \tau_0 > 0$. The equilibrium with $\tau_t = \bar{\tau}$, $t \geq 0$ has both the lowest V and the highest values of c_0 and τ_0 among all competitive equilibria. So→ it is sufficient to check that $\partial G/\partial V$ is strictly negative at this equilibrium to ensure that it is negative at all possible equilibria. Theorem 5 then implies the result: Set taxes at $\tau_t = \bar{\tau}, t \geq 0$ so the discounted utility and initial consumption of the agent with average wealth W_0 are $V = \sigma\left(1-\sigma\right)^{-1}\left(c_0\right)^{1-\sigma}\left(1-\beta^{\frac{1}{\sigma}}\left(r_0\left(1-\bar{\tau}\right)\right)^{\frac{1-\sigma}{\sigma}}\right)^{-1}$ and $c_0 = \left(\left(1-\beta^{\frac{1}{\sigma}}\left(r\left(1-\bar{\tau}\right)\right)^{\frac{1-\sigma}{\sigma}}\right)\left(1-\bar{\tau}\right)+\bar{\tau}\right)rW_0$. Substituting these into (14) we obtain the Corollary. □

As an example of the Corollary, consider the case $r = 1/\beta$, $\sigma = 2$, $B = 0$ and $\beta = .06$, and no government spending. A can be normalized to any positive value, Take $A = \sigma$. If more than 50% of the population has no capital, maximal taxes for ever will be the political outcome whenever $\bar{\tau} < 2.63\%$. If the tax rate applies only to capital income net of depreciation, rather than to both capital and its income, i.e., only to $(r-1)k$ rather than to rk, the corresponding tax rate is 66%. Another example with $k, \ c$ growing, redefine $r = 1.065$.