

1 Financial Constraints, Endogenous Markups, and Self-fulfilling Equilibria

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3 **Abstract**

4 We show that self-fulfilling equilibria and indeterminacy can easily arise in a simple financial accelerator model with reasonable
5 parameter calibrations and without increasing returns in production. A key feature for generating indeterminacy in our model is
6 the countercyclical markup due to the procyclical loan to output ratio. We illustrate, via simulations, that our financial accelerator
7 model can generate rich business cycle dynamics, including hump-shaped output in response to demand shocks as well as serial
8 autocorrelation in output growth rates.

9 *Keywords:* Financial Constraints, Endogenous Markups, Self-fulfilling Equilibria, Indeterminacy.

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11 *JEL codes:* E2, E44

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FINANCIAL CONSTRAINTS, ENDOGENOUS MARKUPS, AND SELF-FULFILLING

1. Introduction

Since the seminal work of Bernanke and Gertler (1990)¹ the financial accelerator model with collateral constraints has been widely used to explain the amplification of shocks to business cycles. Since firms and businesses face borrowing costs to finance their working capital which depends on the collateral value of their assets and output, any downturn or negative shock that depresses the value of their collateral will curtail their ability to finance investments and increase their operating costs. This, in turn, will amplify the downturn. Conversely, any positive shock that appreciates the value of a firm's collateral will decrease the cost of external finance, increase profitability, and amplify the effect of the initial shock. This mechanism, however, suggests the possibility of self-fulfilling multiple equilibria: Optimistic expectations of higher output may well lead to increased lending to financially constrained firms. Even though our model has no increasing returns in production, the relaxation of the borrowing constraint implies that unit marginal costs can increase with output as firms compete for more labor and capital. In such a case markups can become countercyclical and factor returns can increase sufficiently so that the expectation of higher output can become self-fulfilling. The purpose of this paper is to show that multiple equilibria and indeterminacy can easily arise in a simple financial accelerator model with realistic parameter calibrations, and that the model can reasonably match some of the quantitative features of economic data.

Our paper is related to several other papers on financial constraints and business cycle fluctuations that ascribe a significant part of such fluctuations to financial shocks. Examples include the work of Jermann and Quadrini (2011), Liu, Wang and Zha (2011), Gertler and Kiyotaki (2011), among many others. But where do financial shocks come from? A growing literature links financial constraints to asset bubbles as a source of financial shocks. For example, Farhi and Tirole (2011), Miao and Wang (2011), Wang and Wen (2012), Miao, Wang and Xu (2012), and Ventura and Martin (2012) study asset bubbles in economies with borrowing constraints. They show that the growth and burst of asset bubbles can generate endogenous fluctuations in borrowing limits which result in booms and busts in the real economy. A shortcoming of such asset bubble models is that the bubbleless steady state is a sink, and cannot explain the recurrent fluctuations in the borrowing limits unless bubbles arrive exogenously.

Our paper is also related to the recent paper of Liu and Wang (2010). They show that financial constraints can generate indeterminacy through an endogenous TFP channel as a result of resource reallocation across firms. In their model, firms differ in productivity and in the absence of credit constraints, only the most productive firms survive while the unproductive firms with high costs perish. Some unproductive firms however continue to produce as the more productive firms are financially constrained by the value of their assets. An expected increase in aggregate output increases the value of the assets of all firms, and relaxes their borrowing constraints. This relaxation in the borrowing constraints allows more productive firms to expand production. This in turn pushes up the factor prices and increases the cost of production for the unproductive firms. As some of the unproductive firms stop producing the resource reallocation towards more productive firms generates endogenous increasing return to scale. Liu and Wang (2010) show that their model is isomorphic to the Benhabib-Farmer (1994) model after aggregation.

This paper provides an alternative and complementary mechanism for indeterminacy to Liu and Wang (2010). While

¹See also Bernanke, Gertler and Gilchrist (1995, 1996, 1999), and Kiyotaki and Moore (1997).

FINANCIAL CONSTRAINTS, ENDOGENOUS MARKUPS, AND SELF-FULFILLING

1 they emphasize reallocation effects of financial constraints, we focus on an endogenous markup channel. For this purpose,
2 we introduce borrowing constraints into an otherwise standard Dixit-Stiglitz monopolistic competition model. Firms
3 rent capital and hire labor in the competitive markets to produce differentiated intermediate goods. The firms however
4 may default on their promise or contract to repay their debt. We assume therefore that firms face borrowing constraints
5 when financing their working capital, determined by the fraction of firm revenues and assets that the creditors can
6 recover, minus some fixed collection costs. This constrains the output as well as the unit marginal costs of firms. Given
7 the fixed collection costs however, if households expect a higher equilibrium output, they will be willing to increase
8 their lending to firms, even if the marginal costs of firms rise and their markups decline as they compete for additional
9 labor and capital.² At the new equilibrium both output and factor returns will be higher. Despite the income effects
10 on labor supply, the increase in wages associated with lower markups will allow employment and output to increase, so
11 the optimistic expectations of higher output will be fulfilled.

12 We describe the baseline model in the next section. Section 2.6. provides the main results characterizing the parameter
13 ranges where equilibrium is indeterminate, as well as examples and graphical illustrations. Section 2.7. discusses the
14 calibration of the parameters of our model based on estimates of these parameters in the literature. Section 3. offers
15 extensions that relaxes the fixed cost component of the borrowing constraint, eliminates capacity utilization, and provides
16 examples of indeterminacy under these extensions. The section 4. introduces sunspots into the discrete-time version of
17 the model, calibrates it to match the moments of US data, and generates impulse responses to technology and sunspot
18 shocks. Section 5. concludes.

19 2. A baseline model

20 2.1. Firms

To illustrate the driving features of our model, we start with a simple benchmark model of monopolistic competition
and borrowing constraints for intermediate goods producers. The production side is a standard Dixit and Stiglitz model
of monopolistic competition. There is a competitive final goods producer that combines a continuum of intermediate
goods $Y_t(i)$ **of unit mass** to produce final goods Y_t according to the technology

$$Y_t = \left[\int Y_t^{\frac{\sigma-1}{\sigma}}(i) di \right]^{\frac{\sigma}{\sigma-1}}, \quad (1)$$

where $\sigma \geq 1$. The final goods producer solves

$$\max_{y_t(i)} \left[\int Y_t^{\frac{\sigma-1}{\sigma}}(i) di \right]^{\frac{\sigma}{\sigma-1}} - \int P_t(i) Y_t(i) di. \quad (2)$$

where $P_t(i)$ the price of the i -th type of intermediate goods. The first-order conditions lead to the following inverse
demand functions for intermediate goods:

$$P_t(i) = Y_t^{-\frac{1}{\sigma}}(i) Y_t^{\frac{1}{\sigma}}, \quad (3)$$

²The countercyclical markup is consistent with data (see Rotemberg and Woodford (1999)).

FINANCIAL CONSTRAINTS, ENDOGENOUS MARKUPS, AND SELF-FULFILLING

where the aggregate price index is

$$1 = \left[\int P_t^{1-\sigma}(i) di \right]^{\frac{1}{1-\sigma}}. \quad (4)$$

Intermediate goods producers. The technology for producing intermediate goods is given by

$$Y_t(i) = AK_t^\alpha(i)N_t^{1-\alpha}(i), \quad (5)$$

where $A > 0$, $0 < \alpha < 1$. We assume symmetry: the technology for producing intermediate goods is the same for all i . The profit for i 'th intermediate good producer is

$$\Pi_t(i) = P_t(i)Y_t(i) - w_tN_t(i) - r_tK_t(i). \quad (6)$$

Denote by $\phi_t = \frac{1}{A} \left(\frac{r_t}{\alpha} \right)^\alpha \left(\frac{w_t}{1-\alpha} \right)^{1-\alpha}$ the unit cost for the intermediate goods firms. Then their profit is:

$$\Pi_t(i) = P_t(i)Y_t(i) - \phi_t Y_t(i). \quad (7)$$

The Financial Constraints. Unlike the final goods producer, that producers of intermediate goods are assumed to face financial constraints due to limited enforcement. In our simple benchmark model we assume that in the beginning of each period, the i 'th intermediate goods firm decides to rent capital $K_t(i)$ from the households and hire labor $N_t(i)$. The firm promises to pay $w_tN_t(i) + r_tK_t(i) \equiv b_t(i)$ at the end of the period. In this sense, the households are effectively providing credit to finance the firm's working capital. However the firm may default on its contract or promise. We assume that if the firm does not pay its debt $b_t(i)$, the households can recover a fraction $\xi < 1$ of the firm's revenue $P_t(i)Y_t(i)$ by incurring a liquidation cost f .³ One possibility is that the firm must pay the labor wages as production takes place, and that creditors can always redeem the physical capital, but that the interest on borrowing may not be fully recoverable. So if the household can recover $\xi P_t(i)Y_t(i) - f$, they will lend to the firm only if $\xi P_t(i)Y_t(i) - f$ can at least cover the wage bill plus principal and interest. Knowing that the household cannot recover more than $\xi P_t(i)Y_t(i) - f$, the firm will have no incentive to repay more than $\xi P_t(i)Y_t(i) - f$. The incentive-compatibility constraint for the firm then is:

$$P_t(i)Y_t(i) - [w_tN_t(i) + r_tK_t(i)] \geq P_t(i)Y_t(i) - [\xi P_t(i)Y_t(i) - f], \quad (8)$$

or

$$w_tN_t(i) + r_tK_t(i) \leq \xi P_t(i)Y_t(i) - f. \quad (9)$$

After substituting $P_t(i)$ from equation (3) into equation (8), the profit maximization for the i 'th firm becomes

$$\max_{Y_t(i)} Y_t^{1-\frac{1}{\sigma}}(i) Y_t^{\frac{1}{\sigma}} - \phi_t Y_t(i), \quad (10)$$

subject to

$$\phi_t Y_t(i) + f \leq \xi Y_t^{1-\frac{1}{\sigma}}(i) Y_t^{\frac{1}{\sigma}}. \quad (11)$$

³ Given w_t, r_t , final output Y_t , and the borrowing constraint (11), the feasible choices of $Y_t(i)$ are represented by the

³To calibrate $\xi < 1$ we note that outstanding credit market debt for domestic non-financial business corporate and non-corporate sectors in the US in 2012 stood at 12 trillion, or about 77% of GDP. See the Federal Reserve "Flow of Funds report", June 07, 2012, table D.3 in particular, at <http://www.federalreserve.gov/releases/z1/Current/z1.pdf>

FINANCIAL CONSTRAINTS, ENDOGENOUS MARKUPS, AND SELF-FULFILLING

1 shaded area in Figure 1.⁴

If we denote by $\mu_t(i)$ the Lagrangian multiplier of constraint (11), the first-order conditions for the profit maximization are

$$r_t K_t(i) = \alpha \phi_t Y_t(i), \quad (12)$$

$$w_t N_t = (1 - \alpha) \phi_t Y_t(i), \quad (13)$$

and

$$\left(1 - \frac{1}{\sigma}\right) P_t(i) - \phi_t + \mu_t(i) \left[\xi \left(1 - \frac{1}{\sigma}\right) P_t(i) - \phi_t\right] = 0, \quad (14)$$

with the slackness condition

$$\left[\xi Y_t^{1-\frac{1}{\sigma}}(i) Y_t^{\frac{1}{\sigma}} - \phi_t Y_t(i) - f\right] \mu_t(i) = 0. \quad (15)$$



2
3 Figure 1. The Credit Constraints and Feasible Output Choice.

4 *2.2. Households*

We now turn to the intertemporal optimization problem faced by a representative household. To facilitate the stability analysis of a steady state, we set our model in continuous time. Note that since firms have identical production functions, in a symmetric equilibrium we have $K_t(i) = e_t K_t$, $N_t(i) = N_t$, $P_t(i) = 1$, $Y_t(i) = Y_t$ and $\Pi_t(i) = \Pi_t = Y_t - w_t N_t - r_t e_t K_t$. The instantaneous utility of the representative household is given by

$$\log C_t - \psi \frac{N_t^{1+\chi}}{1+\chi}, \quad (16)$$

⁴In an alternative formulation the firm also borrows enough to directly purchase its capital stock in addition to its needs for operating costs: $w_t N_t(i) + (1 + r_t) K_t(i)$. If for simplicity we assume that lenders can always recover the capital stock in case of default but only a fraction of the output, the constraint becomes $w_t N_t(i) + (1 + r_t) K_t(i) \leq \xi P_t(i) Y_t(i) + K_t(i) - f$. After cancelling $K_t(i)$ from both sides, we again have the constraint (9). In this case however debt would exceed GDP, since it would include the borrowed capital stock. Of course some part of the capital stock may represent business equity that also yielding a competitive return of r_t , rather than debt. In such a case however equity returns and principal may be subordinated to debt, but for simplicity we may abstract from these considerations.

FINANCIAL CONSTRAINTS, ENDOGENOUS MARKUPS, AND SELF-FULFILLING

where C is consumption, N is labor supply, and $\chi \geq 0$. Taking the market interest rate r_t and wage w_t as given, the representative household maximizes

$$\int_0^\infty [\log C_t - \psi \frac{N_t^{1+\chi}}{1+\chi}] e^{-\rho t} dt, \quad (17)$$

subject to

$$\dot{K}_t = r_t e_t K_t - \delta(e_t) K_t + w_t N_t - C_t + \Pi_t, \quad (18)$$

1 where K_t is the capital stock and K_0 is given. We model endogenous capacity utilization along the lines of Greenwood,
2 Hercowitz and Huffman (1988). For simplicity we assume that the households choose the capacity utilization rate e_t . A
3 higher e_t implies that the capital is more intensively utilized, at the cost of faster depreciation, so that $\delta(e_t)$ is a convex
4 increasing function. The parameter ρ represents the discount rate, and Π_t is total profit of all firms.

5 We note at this point that the indeterminacy results that follow will hold even in the absence of variable capacity
6 utilization, but we include it in our model to improve calibration results in section 2.7..

The first-order conditions for the household's optimization problem are given by

$$\frac{\dot{C}_t}{C_t} = r_t e_t - \rho - \delta(e_t), \quad (19)$$

$$r_t = \delta'(e_t), \quad (20)$$

and

$$\psi N_t^\chi = \frac{1}{C_t} w_t. \quad (21)$$

7 2.3. Equilibrium

An equilibrium of our economy is a collection of prices $\{w_t, r_t, P_t(i)\}$ and quantities $\{K_t(i), N_t(i), Y_t(i), Y_t, K_t, N_t, e_t, \Pi_t\}$ such that a) given the prices and the aggregate Π_t , the households choose K_t , e_t and N_t to maximize their utility; b) given $P_t(i)$, the final good firm chooses $\{Y_t(i)\}$ to maximize its profits defined in (2); c) given w_t, r_t , and the financial constraint (11), the intermediate goods producers maximizes its profit by choosing $K_t(i)$ and $N_t(i)$; and all markets clear. The budget constraint becomes

$$\dot{K}_t = Y_t - C_t - \delta(e_t) K_t. \quad (22)$$

The wage w_t and the interest rate r_t are

$$w_t = (1 - \alpha) \phi_t \frac{Y_t}{N_t}, \quad (23)$$

and

$$r_t = \alpha \phi_t \frac{Y_t}{e_t K_t}. \quad (24)$$

Equation (14) becomes

$$(1 - \frac{1}{\sigma}) - \phi_t + \mu_t (\xi (1 - \frac{1}{\sigma}) - \phi_t) = 0. \quad (25)$$

8 We then have the following lemma regarding the financial constraint (11).

FINANCIAL CONSTRAINTS, ENDOGENOUS MARKUPS, AND SELF-FULFILLING

Lemma 1 *If $\xi(1 - \frac{1}{\sigma}) < \phi_t < 1 - \frac{1}{\sigma}$, then the financial constraint binds; that is*

$$\phi_t Y_t(i) + f = \xi Y_t^{1 - \frac{1}{\sigma}}(i) Y_t^{\frac{1}{\sigma}}. \quad (26)$$

Based on fact that $Y_t(i) = Y_t$, we obtain

$$\phi_t = \xi - \frac{f}{Y_t}. \quad (27)$$

1 The intuition for Lemma 1 is as follows. The firms' profit function is $\Pi_t(i) = Y_t^{1 - \frac{1}{\sigma}}(i) Y_t^{\frac{1}{\sigma}} - \phi_t Y_t(i)$ and if the profit
2 for the marginal unit evaluated at equilibrium is $(1 - \frac{1}{\sigma}) - \phi_t > 0$, the firms would have the incentive to increase their
3 output. The revenue that households can recover in case of default exceeds the cost on the marginal unit if $\phi_t \leq \xi(1 - \frac{1}{\sigma})$.
4 It then suggests that the original output level cannot be optimal because firms would be able to borrow and produce
5 more to increase their production and their profits. If $\xi(1 - \frac{1}{\sigma}) < \phi_t < 1 - \frac{1}{\sigma}$ however, firms would not be able to
6 increase their production since the borrowing constraint binds: an additional unit of output would allow the firms to
7 borrow only an additional $\xi(1 - \frac{1}{\sigma})$, which is not enough to cover the marginal unit production cost ϕ_t .⁵

8 We will focus on the parameters that make financial constraint (11) always binding in equilibrium. To summarize,
9 the following system of equations fully characterize the equilibrium

$$\frac{\dot{C}_t}{C_t} = \phi_t \frac{\alpha Y_t}{K_t} - \rho - \delta(e_t), \quad (28)$$

$$\dot{K}_t = Y_t - \delta(e_t) K_t - C_t, \quad (29)$$

$$\psi N_t^X = \frac{1}{C_t} \phi_t \frac{(1 - \alpha) Y_t}{N_t}, \quad (30)$$

$$Y_t = A(e_t K_t)^\alpha N_t^{1 - \alpha}, \quad (31)$$

$$\phi_t \frac{\alpha Y_t}{e_t K_t} = \delta'(e_t) \quad (32)$$

$$\phi_t = \xi - \frac{f}{Y_t}, \quad (33)$$

subject to the constraint $\xi(1 - \frac{1}{\sigma}) < \phi_t < 1 - \frac{1}{\sigma}$. Following Greenwood, Hercowitz and Huffman (1988) let the depreciation
function be given by

$$\delta(e_t) = \delta_0 \frac{e_t^{1 + \nu}}{1 + \nu}. \quad (34)$$

We then have

$$\phi_t \frac{\alpha Y_t}{e_t K_t} = \delta'(e_t) = \delta_0 e_t^\nu. \quad (35)$$

⁵Formally,

$$\begin{aligned} (1 - \frac{1}{\sigma}) - \phi_t &= -\mu_t (\xi(1 - \frac{1}{\sigma}) - \phi_t) \\ 0 &> \frac{1}{-\mu_t} = \frac{(\xi(1 - \frac{1}{\sigma}) - \phi_t)}{(1 - \frac{1}{\sigma}) - \phi_t} \\ \text{sign} \left((1 - \frac{1}{\sigma}) - \phi_t \right) &= -\text{sign}(\xi(1 - \frac{1}{\sigma}) - \phi_t) \\ \xi(1 - \frac{1}{\sigma}) - \phi_t &< (1 - \frac{1}{\sigma}) - \phi_t \text{ if } \xi < 1 \\ \xi(1 - \frac{1}{\sigma}) &< \phi_t < (1 - \frac{1}{\sigma}) \end{aligned}$$

FINANCIAL CONSTRAINTS, ENDOGENOUS MARKUPS, AND SELF-FULFILLING

1 2.4. Steady state

2 We first solve for the deterministic steady state. Denote by X the steady state value of X_t . Unfortunately, the model
 3 does not have a full analytical solution for the steady state with the fixed financial cost. In the following, we describe
 4 the major steps for solving for steady state $\{Y, K, N, e, c, \phi, r, w\}$. We first express the other variables as a function of
 5 the steady state ϕ recursively. We normalize A to 1.

1. Using the first-order condition $\phi \frac{\alpha Y}{K} = \delta_0 e^{\nu+1}$ from equation (35), we obtain

$$\delta(e) = \frac{1}{1+\nu} \phi \frac{\alpha Y}{K}.$$

2. Equation (28) then implies $\frac{\nu}{1+\nu} \phi \frac{\alpha Y}{K} = \rho$, so the ratio of capital to output is

$$\frac{K}{Y} = \frac{\nu}{1+\nu} \frac{\phi \alpha}{\rho}.$$

3. Equation (35) then implies $\delta(e) = \delta_0 \frac{e^{1+\nu}}{1+\nu} = \frac{1}{1+\nu} \phi \frac{\alpha Y}{K} = \frac{\rho}{\nu}$, or

$$e = \left[\frac{(1+\nu)\rho}{\nu\delta_0} \right]^{\frac{1}{1+\nu}}.$$

6 We set $\delta_0 = (1+\nu)\rho/\nu$ to normalize e to 1.

7 4. To solve for N , we first use equation (29) to obtain the ratio of consumption to output

$$\begin{aligned} \frac{C}{Y} &= 1 - \delta(e) \frac{K}{Y} \\ &= 1 - \frac{\rho}{\nu} \frac{\nu}{1+\nu} \frac{\phi \alpha}{\rho} \\ &= 1 - \frac{\phi \alpha}{1+\nu}, \end{aligned}$$

where the second line uses the fact $\delta(e) = \rho/\nu$ and $\frac{K}{Y} = \frac{\nu}{1+\nu} \frac{\phi \alpha}{\rho}$. We then use equation (30) to express

$$N = \left[\phi \frac{(1-\alpha)}{\frac{C}{Y}} \frac{1}{\psi} \right]^{\frac{1}{1+\alpha}} = \left[\phi \frac{(1-\alpha)}{1 - \frac{\phi \alpha}{1+\nu}} \frac{1}{\psi} \right]^{\frac{1}{1+\alpha}}.$$

5. Given N, K and $e = 1$, we can then obtain output using equation (31) as

$$Y = K^\alpha N^{1-\alpha} = \left(\frac{\nu}{1+\nu} \frac{\phi \alpha}{\rho} Y \right)^\alpha \left[\phi \frac{(1-\alpha)}{1 - \frac{\phi \alpha}{1+\nu}} \frac{1}{\psi} \right]^{\frac{1-\alpha}{1+\alpha}},$$

or

$$Y = \left(\frac{\nu}{1+\nu} \frac{\phi \alpha}{\rho} \right)^{\frac{\alpha}{1-\alpha}} \left[\phi \frac{(1-\alpha)}{1 - \frac{\phi \alpha}{1+\nu}} \frac{1}{\psi} \right]^{\frac{1}{1+\alpha}} \equiv Y(\phi).$$

6. Finally from the definition of $\phi = \xi - \frac{f}{Y}$, we have

$$f = (\xi - \phi)Y(\phi) \equiv \Psi(\phi), \tag{36}$$

FINANCIAL CONSTRAINTS, ENDOGENOUS MARKUPS, AND SELF-FULFILLING

1 which determines the steady-state value of ϕ . In what follows we can treat the steady state value of marginal cost
 2 ϕ as a parameter and allow f to adjust.

7. For the existence of a steady state however we will need to assume $\xi(1 - \frac{1}{\sigma}) < \phi < 1 - \frac{1}{\sigma}$. Define $\bar{\Psi} = \max_{0 \leq \phi \leq \xi} \Psi(\phi)$. Notice also that

$$\Psi(\xi) = \Psi(0) = 0. \quad (37)$$

Lemma 2 *If $0 < f < \bar{\Psi}$, then equation (36) has at least two solutions such that*

$$\Psi(\phi) - f = 0. \quad (38)$$

3 **Lemma 3** *$0 < f < \Psi(\xi(1 - \frac{1}{\sigma}))$ there is a steady state ϕ such that $\xi(1 - \frac{1}{\sigma}) < \phi < \xi^6$.*

4 **Proof.** Since $\Psi(\xi) - f < 0$ and $\Psi(\xi(1 - \frac{1}{\sigma})) - f > 0$, by the intermediate value theorem there is a steady state ϕ that
 5 lies between $\xi(1 - \frac{1}{\sigma})$ and ξ such that $\Psi(\phi) - f = 0$. ■

6 2.5. Log-linearization

7 After obtaining the steady state (Y, K, N, C, ϕ) , we log-linearize the system of equations around the steady state value.
 8 We denote by \hat{X}_t the percentage deviation of variable X_t from its steady state value X_t , that is, $\hat{X}_t = \log X_t - \log X$.
 9 Let $\dot{c}_t = \frac{d(\log C_t - \log C)}{dt}$ and $\dot{k}_t = \frac{d(\log K_t - \log K)}{dt}$. Then the log-linearized system of equations is

$$\dot{c}_t = \rho[\hat{Y}_t - \hat{K}_t + \hat{\phi}_t] \quad (39)$$

$$\dot{k}_t = \frac{(1 + \nu)\delta}{\alpha\phi}(\hat{Y}_t - \hat{K}_t) - \left(\frac{(1 + \nu)\delta}{\alpha\phi} - \delta \right) (\hat{C}_t - \hat{K}_t) - \delta(\hat{Y}_t + \hat{\phi}_t - \hat{K}_t) \quad (40)$$

$$\chi\hat{N}_t = \hat{\phi}_t + \hat{Y}_t - \hat{N}_t - \hat{C}_t \quad (41)$$

$$\hat{Y}_t = \alpha(\hat{K}_t + \hat{e}_t) + (1 - \alpha)\hat{N}_t \quad (42)$$

$$\hat{e}_t = \frac{1}{1 + \nu}(\hat{\phi}_t + \hat{Y}_t - \hat{K}_t) \quad (43)$$

$$\hat{\phi}_t = \frac{f/Y}{\xi - f/Y}\hat{Y}_t \equiv \gamma\hat{Y}_t \quad (44)$$

10 where $\delta = \frac{\rho}{\nu}$. Note that γ can also be defined by the steady state value of ϕ as $\gamma = \frac{\xi - \phi}{\phi}$. Eliminating \hat{N}_t from equation
 11 (41), $\hat{\phi}_t$ from equation (44), and \hat{e}_t from equation (43) allows us to obtain the expression for \hat{Y}_t in terms of capital and
 12 consumption. We first substitute \hat{e}_t out of the production function to obtain

$$\begin{aligned} \hat{Y}_t &= \frac{1}{1 + \nu - (1 + \gamma)\alpha} [(1 + \nu)(1 - \alpha)\hat{N}_t + \alpha\nu\hat{K}_t] \\ &\equiv \omega_1\hat{N}_t + \omega_2\hat{K}_t \end{aligned} \quad (45)$$

⁶We discuss the conditions that give a steady state with $\xi(1 - \frac{1}{\sigma}) < \phi < (1 - \frac{1}{\sigma})$ in section 2.7..

FINANCIAL CONSTRAINTS, ENDOGENOUS MARKUPS, AND SELF-FULFILLING

where $\omega_1 = \frac{(1+\nu)(1-\alpha)}{1+\nu-(1+\gamma)\alpha}$ and $\omega_2 = \frac{\alpha\nu}{1+\nu-(1+\gamma)\alpha}$. It is easy to check that $\omega_1 + \omega_2 > 1$ if $\gamma > 0$. Finally we need to substitute out \hat{N}_t . Combining the labor demand and labor supply curves we have

$$\hat{Y}_t = \lambda_1 \hat{K}_t + \lambda_2 \hat{C}_t, \quad (46)$$

1 where $\lambda_1 = \frac{\omega_2(1+\chi)}{\chi+1-(1+\gamma)\omega_1}$, $\lambda_2 = \frac{-\omega_1}{\chi+1-(1+\gamma)\omega_1}$.

Using the factor $\hat{\phi}_t = \gamma \hat{Y}_t$ from (44), the log-linearized Euler condition becomes:

$$\dot{c}_t = \rho[(1+\gamma)(\lambda_1 \hat{K}_t + \lambda_2 \hat{C}_t) - \hat{K}_t]. \quad (47)$$

2 Then equation (40) yields

$$\begin{aligned} \dot{k}_t &= \left\{ \frac{(1+\nu)\delta}{\alpha\phi} \lambda_1 - \delta(1+\gamma)\lambda_1 \right\} \hat{K}_t \\ &+ \left\{ \frac{(1+\nu)\delta}{\alpha\phi} (\lambda_2 - 1) + \delta - \delta(1+\gamma)\lambda_2 \right\} \hat{C}_t \end{aligned} \quad (48)$$

In a matrix form

$$\begin{bmatrix} \dot{k}_t \\ \dot{c}_t \end{bmatrix} = J \begin{bmatrix} \hat{K}_t \\ \hat{C}_t \end{bmatrix} \quad (49)$$

where

$$J = \delta \begin{bmatrix} \frac{(1+\nu)}{\alpha\phi} \lambda_1 - (1+\gamma)\lambda_1 & \frac{(1+\nu)}{\alpha\phi} (\lambda_2 - 1) + 1 - (1+\gamma)\lambda_2 \\ \nu[(1+\gamma)\lambda_1 - 1] & \nu(1+\gamma)\lambda_2 \end{bmatrix}. \quad (50)$$

3 Notice that we have used the factor $\rho = \delta\nu$ to substitute δ out in J .

4 2.6. Dynamics around the steady state

The local dynamics around the steady state is determined by the roots of J . The trace of the J is

$$\text{Trace}(J) = \delta \left[\frac{(1+\nu)}{\alpha\phi} \lambda_1 - (1+\gamma)\lambda_1 + \nu(1+\gamma)\lambda_2 \right], \quad (51)$$

and the determinant of J is

$$\det(J) = \left\{ [(1+\gamma)\lambda_1 - 1 + \lambda_2] \left(\frac{1+\nu}{\alpha\phi} - 1 \right) - \gamma\lambda_2 \right\} \delta^2 \nu. \quad (52)$$

The roots of J , x_1 and x_2 satisfy the following constraints

$$x_1 + x_2 = \text{Trace}(J), \quad (53)$$

and

$$x_1 x_2 = \det(J). \quad (54)$$

5 If $\det(J) > 0$ and $\text{Trace}(J) < 0$, then the roots x_1 and x_2 will both be negative, and the model will have local
6 indeterminacy around the steady state. Since given other parameters the trace and determinant are functions of γ and
7 ϕ , we will first examine the possibility of indeterminacy in the parameter space of γ and ϕ . We will then use the mapping
8 between (γ, ϕ) and (f, ξ) to establish the possibility of indeterminacy supported by the deep parameters of the model.

FINANCIAL CONSTRAINTS, ENDOGENOUS MARKUPS, AND SELF-FULFILLING

Proposition 4 *Let γ and ϕ satisfy the following two constraints*

$$\gamma > \frac{(1+\nu)(1+\chi)}{\alpha(1+\chi) + (1+\nu)(1-\alpha)} - 1 \equiv \gamma_{\min}, \quad (55)$$

1 *and*

$$\gamma < \min\left(\frac{1+\nu-\alpha}{\alpha}, \frac{\left(\frac{1+\nu}{\phi} - \alpha\right)(1+\chi) - (1+\nu)(1-\alpha)}{\alpha(1+\chi) + (1+\nu)(1-\alpha)}, \frac{(1-\alpha)(1+\chi)}{(1+\nu)(1-\alpha)\frac{1}{1+\nu-\alpha\phi} + (1+\chi)\alpha}\right) \equiv \gamma_{\max}. \quad (56)$$

Then

$$\text{Trace}(J) < 0, \det(J) > 0 \quad (57)$$

2 **Proof.** See Appendix A1. ■

To gain intuition for self-fulfilling expectations of higher output and higher factor rewards, we first focus on labor demand and supply curves incorporating the equilibrium effects of the borrowing constraint on marginal costs and markups. The labor demand curve is given by

$$\hat{w}_t = (1+\gamma)\hat{Y}_t - \hat{N}_t = \frac{\alpha\nu}{1+\nu - (1+\gamma)\alpha} \hat{K}_t + \left[\frac{(1+\gamma)(1+\nu)(1-\alpha)}{1+\nu - (1+\gamma)\alpha} - 1 \right] \hat{N}_t, \quad (58)$$

and the labor supply curve in the economy is

$$\hat{w}_t = \hat{C}_t + \chi\hat{N}_t. \quad (59)$$

3 The slope of the labor market demand curve is positive and steeper than that of the labor supply curve under the
4 condition $(1+\gamma) > \frac{(1+\nu)(1+\chi)}{\alpha(1+\chi) + (1+\nu)(1-\alpha)}$ of the Proposition above. The indeterminacy result then parallels the results
5 in Benhabib and Farmer (1994) and Wen (1998). However unlike their works, our model has no increasing returns in
6 the production technology. Instead indeterminacy arises from the borrowing constraints and their indirect effects on
7 marginal costs through wages and the rental rate on capital. If households expect a higher equilibrium output, they
8 will be willing to increase their lending to firms. Given positive fixed collection costs f , an expected increase in output
9 levels relaxes the borrowing constraint disproportionately more so that the unit marginal costs of firms, $\phi_t = \xi - \frac{f}{Y_t}$,
10 can rise and markups can fall. This implies that as firms compete for inputs, factor rewards will also increase with Y_t .
11 The labor demand curve incorporating these general equilibrium effects on marginal costs will then be positively sloped
12 and steeper than the labor supply curve. Normally, higher output levels increase the demand for leisure, so barring
13 inferiority in preferences, the higher demand for labor will be contained by the income effect on labor supply. However
14 if the labor demand slopes up more steeply than labor supply, employment will increase robustly as the labor supply
15 curve shifts to the left with income effects. The rise in labor hours as well as the accumulation of capital will raise
16 output, so that the optimistic output expectations of households will be self-fulfilling.

Before turning to calibrations, we formally state the indeterminacy result of the paper. We define the set

$$\Omega = \{(\gamma, \phi) | \text{constraint}(55), \text{constraint}(56) \text{ are satisfied}\}.$$

FINANCIAL CONSTRAINTS, ENDOGENOUS MARKUPS, AND SELF-FULFILLING

1 **Proposition 5** For $(\gamma, \phi) \in \Omega$, $\phi < 1 - \frac{1}{\sigma}$ and $\gamma = \frac{\xi - \phi}{\phi} < \frac{1}{\sigma - 1}$ (that is $\xi(1 - \frac{1}{\sigma}) < \phi$), construct the set $\Theta = \{(\xi, f) : |$
 2 such that $\gamma = \frac{\xi - \phi}{\phi}$, $f = (\xi - \phi)Y(\phi)$ and $(\gamma, \phi) \in \Omega\}$. For $(\xi, f) \in \Theta$, the model is indeterminate.

Proof. For $(\xi, f) \in \Theta$, by construction we can find a solution for ϕ and γ such that

$$\gamma = \frac{\xi - \phi}{\phi}; f = (\xi - \phi)Y(\phi). \quad (60)$$

3 And since $(\gamma, \phi) \in \Omega$, we have $Trace < 0$, $\det(J) > 0$ by Proposition 1. So the model is indeterminate around the steady
 4 state. ■

5 2.7. Calibrations

6 To calibrate our model we use some standard parameter values in the literature. We set the quarterly discount rate
 7 $\rho = 0.01$, so the quarterly discount factor is $\beta = \frac{1}{1+\rho} = 0.99$. We make the labor supply fully elastic, so $\chi = 0$. The
 8 factor share of capital is, $\alpha = \frac{1}{3}$. The steady state depreciation rate is given by equation (34), where the normalized
 9 the steady state utilization rate is $e = 1$. This gives $\delta_0 = 0.04333$. If we assume that the lifetime of new equipment
 10 averages 30 quarters, or that $\delta(e) = 0.0333$, then we can solve for $\nu = 0.3$. The markup in our model is the inverse of the
 11 steady state marginal cost $\frac{1}{\phi} - 1$. We set the steady state markup to 12%, which implies a steady state marginal cost
 12 $\phi = \xi - \frac{f}{Y(\phi)} = 0.88$. This also represents the steady state ratio of non-financial private business debt to GDP. As noted
 13 earlier, this ratio in the data is a little lower at 0.77, and excludes all private debt issued by financial institutions, mostly
 14 to consumers, as well as state and federal debt. (See footnote 3 or the Federal Reserve "Flow of Funds Report", June
 15 07, 2012, table D.3 in particular, at <http://www.federalreserve.gov/releases/z1/Current/z1.pdf>.) Note that choosing
 16 the steady state markup and therefore ϕ constrains the parameters ξ and f through equation (36): $\phi = \xi - \frac{f}{Y(\phi)}$.
 17 Furthermore the steady state marginal cost ϕ together with parameters ξ and σ must satisfy the inequality constraints
 18 of Proposition 5, $\xi(1 - \frac{1}{\sigma}) < \phi < 1 - \frac{1}{\sigma}$. In models with constant markups, the steady state Dixit-Stiglitz elasticity σ
 19 is usually calibrated to immediately obtain a markup of 10 – 15%, and often σ is set to 10 (see for example Dotsey and
 20 King (2005) or Sbordone (2008), p. 20). We set $\sigma = 10$ as well, even though σ does not exactly determine ϕ in our
 21 model but constrains it. Finally we can calibrate the financial constraint parameters to assure that the markup is indeed
 22 12%. If we set $\xi = 0.9768$ then equation (36) implies a steady state value of the fixed liquidation cost $f = 0.1908$ in
 23 case of a default. Below in Figure 2 we illustrate the regions of indeterminacy in the $\xi - f$ plane. Finally for the above
 24 calibration we can easily check that the constraints $\xi(1 - \frac{1}{\sigma}) < \phi < 1 - \frac{1}{\sigma}$ are satisfied, so by Proposition 5 the steady
 25 state is indeed indeterminate, with the value of output at this steady state equal to 1.9711.

26 Note that for our calibrated parameters, the implied liquidation costs amount to 12% of the firm's total sale revenue⁷.
 27 For 88 firms that reorganized during 1982-1993, Alderson and Betker (1995) find that the liquidation costs measured
 28 as the percent of loss in going-concern value is large. For example, only 25% of firms have liquidation costs less than
 29 12.8%. The mean and median liquidation costs among all the firms are 36.5% and 34.7%, respectively. Using a very
 30 comprehensive sample of corporate bankruptcies, Bris, Welch and Zhu (2006) find the direct expenses alone accounts

⁷The liquidation costs include the fixed cost f and the loss in output $(1 - \xi)Y$. So in the steady state they account for the fraction $f/Y + (1 - \xi)$ of total output (sales revenue).

FINANCIAL CONSTRAINTS, ENDOGENOUS MARKUPS, AND SELF-FULFILLING

1 for 8.1% of pre-bankruptcy assets for firms who filed for Bankruptcy in the United States under Chapter 7 of the U.S.
 2 Bankruptcy Code, and 16.9% for firms who filed for Bankruptcy Under Chapter 11⁸ (see Bris, Welch and Zhu (2006),
 3 table X, p. 1281). Similarly for international data Thorburn (2000) reports that direct expenses on average account
 4 for 19.1% (with medium 13.2%) of the market value of assets in Sweden. So the implied liquidation costs in our model
 5 are in line with the data. In addition, both Thorburn (2000) and Bris, Welch and Zhu (2006) find that the expenses to
 6 assets ratio declines significantly with firm's scale, suggesting an important fixed cost component in the direct liquidation
 7 expenses.

Figure 2 illustrates the combinations of f and ξ that yield indeterminacy with the other parameters set to $\nu = 0.3$, $\alpha = \frac{1}{3}$, $\rho = 0.01$, $\chi = 0$. The feasible parameter values for f and ξ are graphed in these two shaded areas. Consider the borrowing constraint $f = (\xi - \phi)Y(\phi)$. For a given ξ there exist a minimum f and a maximum f consistent with the steady state equilibrium such that $\frac{\sigma-1}{\sigma}\xi < \phi < \frac{\sigma-1}{\sigma}$. Notice that if $f = 0$ and $\phi = \xi$, as long as $\xi < \frac{\sigma-1}{\sigma}$, the condition $\frac{\sigma-1}{\sigma}\xi < \phi < \frac{\sigma-1}{\sigma}$ is automatically satisfied. This implies that for $\xi < \frac{\sigma-1}{\sigma}$ the minimum f is zero. But if $\xi \geq \frac{\sigma-1}{\sigma}$ then $f = 0$ (hence $\phi = \xi$) is no longer consistent with the equilibrium. If f is too small, then ϕ will be larger than $\frac{\sigma-1}{\sigma}$. Since $f = (\xi - \phi)Y(\phi)$ is decreasing in ϕ , the lower bound for f is $f_{\min}(\xi) = (\xi - \frac{\sigma-1}{\sigma})Y(\frac{\sigma-1}{\sigma})$ for $\xi \geq \frac{\sigma-1}{\sigma}$. We can write it as $f_{\min}(\xi) = \max((\xi - \frac{\sigma-1}{\sigma})Y(\frac{\sigma-1}{\sigma}), 0)$ for $0 < \xi \leq 1$. On the other hand if f is too large, then the marginal cost will fall below $\frac{\sigma-1}{\sigma}\xi$. Now maximizing f over $\frac{\sigma-1}{\sigma}\xi < \phi < \frac{\sigma-1}{\sigma}$ the upper bound for f for a given ξ is $f_{\max}(\xi) = \frac{1}{\sigma}\xi Y(\frac{\sigma-1}{\sigma}\xi)$. For these feasible parameters, if f is greater than some cut-off level, then the implied γ will be bigger than γ_{\min} . It turns out that the condition $\gamma < \gamma_{\max}$ is automatically satisfied. The cut-off f can be determined by

$$f_{cut}(\xi) = \max\left\{\frac{\gamma_{\min}}{1 + \gamma_{\min}}\xi Y\left(\frac{\xi}{1 + \gamma_{\min}}\right), f_{\min}(\xi)\right\}.$$

8 For any f such that $f_{cut}(\xi) \leq f < f_{\max}(\xi)$, we have $\gamma > \gamma_{\min}$, so the model is locally indeterminate around the steady
 9 state. In Figure 2, the indeterminacy region is shown in red.

⁸Chapter 7 expenses mainly include expenses on debtor's attorney, accountant, and trustee. Chapter 11 expenses mainly include debtor expenses and unsecured creditors' expenses.



Figure 2. Parameter Spaces for Indeterminacy.

The shaded areas (the red areas together with the green areas) are the feasible ξ and f . The upper shaded areas with red color yield indeterminacy around the steady state.

3. Discussion and extensions of the model

3.1. The role of fixed costs

In this section we argue that it is not the fixed liquidation costs per se that generates indeterminacy. It is the procyclical leverage generated by fixed liquidity costs that is the source of indeterminacy. Note that with fixed costs the debt to GDP ratio $b_t/Y_t = \xi - f/Y_t$ is procyclical. In what follows, we construct an example in which the firm's borrowing constraint is

$$\phi_t Y_t(i) \leq \xi\left(\frac{Y_t}{Y}\right) Y_t^{1-\frac{1}{\sigma}}(i) Y_t^{\frac{1}{\sigma}}, \tag{61}$$

where $\xi_t = \xi\left(\frac{Y_t}{Y}\right) < \frac{\sigma-1}{\sigma}$ is an increasing function of $\frac{Y_t}{Y}$ with $\xi(1) = \phi < \frac{\sigma-1}{\sigma}$ and $\xi'(1) = \gamma\phi$. In this case, the marginal cost is $\phi_t = \xi_t$. The condition $\xi_t \frac{\sigma-1}{\sigma} < \phi_t < \frac{\sigma-1}{\sigma}$ is automatically satisfied, so the borrowing constraint is binding. The equilibrium can be characterized by a system of nonlinear equations similar to equations (28) to (33), except that equation (33) is now replaced by $\phi_t = \xi_t = \xi\left(\frac{Y_t}{Y}\right)$. The log-linearized system of equations, however, is exactly the same as equations (39) to (44). So we can directly invoke Proposition 2 to show indeterminacy if

$$\gamma_{\min}(\phi) < \gamma < \gamma_{\max}(\phi) \tag{62}$$

for $0 < \phi < \frac{\sigma-1}{\sigma}$. The more general function $\xi\left(\frac{Y_t}{Y}\right)$ which replaces the fixed liquidation cost eliminates the constraint relating γ and ϕ in the benchmark model, and provides more flexibility in generating a range of parameters such that indeterminacy holds.

FINANCIAL CONSTRAINTS, ENDOGENOUS MARKUPS, AND SELF-FULFILLING

1 3.2. The role of capacity utilization

2 With the more general function $\xi(\frac{Y_t}{Y})$, we now show that endogenous capacity utilization is not essential for indeter-
 3 minacy even though it makes indeterminacy possible for a wider range of parameters, as demonstrated by Wen (1998).
 4 The equilibrium is characterized by:

$$\frac{\dot{C}_t}{C_t} = \phi_t \frac{\alpha Y_t}{K_t} - \rho - \delta, \quad (63)$$

$$\dot{K}_t = Y_t - \delta(e_t)K_t - C_t, \quad (64)$$

$$\psi N_t^X = \frac{1}{C_t} \phi_t \frac{(1-\alpha) Y_t}{N_t}, \quad (65)$$

$$Y_t = AK_t^\alpha N_t^{1-\alpha}, \quad (66)$$

$$\phi_t = \xi_t = \xi\left(\frac{Y_t}{Y}\right). \quad (67)$$

The local dynamics around the steady state is:

$$\begin{bmatrix} \dot{k}_t \\ \dot{c}_t \end{bmatrix} = \begin{bmatrix} \frac{\rho+\delta}{\alpha\phi} \lambda_1 - \delta & \frac{\rho+\delta}{\alpha\phi} \lambda_2 - \frac{\rho+\delta}{\alpha\phi} + \delta \\ (\rho+\delta)[(1+\gamma)\lambda_1 - 1] & (\rho+\delta)(1+\gamma)\lambda_2 \end{bmatrix} \begin{bmatrix} \hat{K}_t \\ \hat{C}_t \end{bmatrix} \quad (68)$$

5 where $\lambda_1 = \frac{\alpha(1+\chi)}{\chi+1-(1+\gamma)(1-\alpha)}$ and $\lambda_2 = \frac{-(1-\alpha)}{\chi+1-(1+\gamma)(1-\alpha)}$.⁹

Proposition 6 *The model is indeterminate if*

$$\frac{(1+\chi)}{1-\alpha} - 1 < \gamma < \frac{1}{1-\alpha} \frac{\frac{1}{\phi} - \frac{\delta}{\rho+\delta}}{1 - \frac{\delta}{\rho+\delta}} (1+\chi) - 1, \quad (69)$$

and

$$\gamma < \frac{(1-\alpha)(1+\chi)}{(1+\alpha\chi) + (1-\alpha)\frac{\delta}{\frac{\rho+\delta}{\alpha\phi} - \delta}}. \quad (70)$$

6 **Proof.** See Appendix A2. ■

7 **Example 7** *Suppose $\chi = 0$, $\sigma = 10$, $\rho = 0.01$, $\alpha = \frac{1}{3}$, $\sigma = 10$, and $\delta = 0.0333$. We assume $\xi(1) = 0.88$ and*
 8 *$\xi'(1) = 0.88\gamma$. This implies $\phi = 0.88$ in the steady state. The model is indeterminate if $0.5 < \gamma < 0.5582$.*

9 3.3. Credit Constraints and Firms' Savings

10 In our model, credit constraints gives firms incentives to build up a large amount of savings from retained earnings.
 11 This would allow them to eventually overcome the borrowing constraints. The possibility that the constrained agents
 12 will ultimately accumulate enough wealth to become fully self-financing is well-known in the extensive literature using
 13 models of credit constraints. To explain why firms nonetheless hold large amounts of debt despite credit constraints
 14 (see footnote 2.1.), the literature has introduced several additional features into models of credit constraints. A standard
 15 approach is to assume that the borrower has a finite lifetime (see e.g., Bernanke, Gertler, and Gilchrist (1999), Song,
 16 Storesletten and Zilibotti (2011) among many others). Another common approach is to assume the borrowers discount
 17 future more than the lenders do (see e.g., Kiyotaki and Moore (1997), Liu, Wang and Zha (2011)). A large corporate

⁹With minor modifications and reinterpretations, it is possible to transform our model so that its local dynamics around the steady states associated with (68) are isomorphic to the dynamics in Wen (1998).

FINANCIAL CONSTRAINTS, ENDOGENOUS MARKUPS, AND SELF-FULFILLING

1 finance literature assumes that there are tax benefits of debt. Finally, as pointed by Jensen (1986), firms typically
 2 prefer debt financing because of the agency problem between the manager and the shareholders. The idea is that the
 3 managers may use retained earnings to benefit their own private interests, for example by having large expense accounts
 4 and management perks. Arellano, Bai and Kehoe (2012) have recently incorporated the ideas of Jensen (1986) into their
 5 model to prevent firms to become fully self-financing.

6 We use a slightly modified approach of Arellano, Bai and Kehoe (2012) to model the agency problem introduced
 7 by Jensen (1986). We focus on the conditions under which firms have no incentive to save. For this purpose, we use a
 8 discrete time model to illustrate the intuition. When the time interval between two periods t and $t+1$ approaches zero,
 9 the discrete model will converge to its counterpart in continuous time.

Denote the value of a firm with retained earnings $S_t(i)$ in period t as $V_t(S_t(i))$. The firm then solves

$$V_t(S_t(i)) = \max_{Y_t(i), S_{t+1}(i)} Y_t^{1-\frac{1}{\sigma}}(i)Y_t - \phi_t Y_t(i) + S_t(i) - \frac{S_{t+1}(i)}{(1-\tau)R_{ft}} + \beta E_t \frac{C_t}{C_{t+1}} V_{t+1}(S_{t+1}(i)), \quad (71)$$

with a credit constraint to be specified below. We assume that the return on the retained earnings is the risk-free rate, and satisfies $1 = R_{ft}\beta E_t C_t/C_{t+1}$. The parameter $1 > \tau > 0$ captures the agency problem between the managers and the shareholders as in Jensen (1986) if the managers can use a fraction τ of the retained earnings for their own non-verifiable private benefit. In addition, we assume that the firm makes factor payment paid after the production is completed (maybe to assure that households do not walk away after receiving their factor's payment). Since firms' savings (retained earnings) are liquid they can also easily divert them if they default. Specifically, we assume the firm can divert a fraction $1 - \omega$ of its retained earnings in default, as in Gertler and Kiyotaki (2012). Instead of assuming a fixed liquidity cost, we assume, as in the case of default in section 3.1., that the creditors can re-claim a fraction $\xi(\frac{Y_t}{Y})$ of the firm's total sales revenue. As discussed in section 3.1., the general function $\xi(\frac{Y_t}{Y})$ allows more flexibility. The borrowing constraint (61) then changes to

$$\phi_t Y_t(i) \leq \xi\left(\frac{Y_t}{Y}\right) Y_t^{1-\frac{1}{\sigma}}(i) Y_t^{\frac{1}{\sigma}} + \omega S_t(i), \quad (72)$$

Finally we assume that $S_{t+1}(i) \geq 0$. We are interested in obtaining parameters such that in the steady state $S_{t+1}(i) = 0$. Denote μ_t as the Lagrangian multiplier of the constraint (72). In a symmetric equilibrium, we have

$$\mu_t = \frac{(1 - \frac{1}{\sigma}) - \phi_t}{\phi_t - \xi(1 - \frac{1}{\sigma})}, \quad (73)$$

and

$$\frac{\partial V_t(S_t(i))}{\partial S_t(i)} = 1 + \mu_t \omega = 1 + \omega \frac{(1 - \frac{1}{\sigma}) - \phi_t}{\phi_t - \xi(1 - \frac{1}{\sigma})}, \quad (74)$$

10 The intuition for this equation is as follows. Here $\omega \left((1 - \frac{1}{\sigma}) - \phi_t \right) / (\phi_t - \xi(1 - \frac{1}{\sigma}))$ is external financial premium. Because
 11 of the borrowing constraint ($\mu_t > 0$), the firm values a dollar of internal saving more than a dollar: one dollar allows the
 12 firm to relax its credit constraint by ω dollars to produce an additional $1/(\phi_t - \xi(1 - \frac{1}{\sigma}))$ units of output. Each unit of

1 output yields a profit of $(1 - \frac{1}{\sigma}) - \phi_t$. The firm will have no incentive to save if and only if

$$\begin{aligned} \frac{1}{1-\tau} &> \beta R_{ft} E_t \frac{C_t}{C_{t+1}} \frac{\partial V_{t+1}(S_{t+1}(i))}{\partial S_{t+1}(i)} \\ &= \beta R_{ft} E_t \frac{C_t}{C_{t+1}} \left[1 + \omega \frac{(1 - \frac{1}{\sigma}) - \phi_{t+1}}{\phi_{t+1} - \xi(1 - \frac{1}{\sigma})} \right], \end{aligned} \quad (75)$$

So in the steady state with $S = 0$ we have $\phi = \xi$ by equation (72). Then the above restriction implies

$$1 > (1 - \tau) \left[1 + \omega \frac{(1 - \frac{1}{\sigma}) - \phi}{\phi - \phi(1 - \frac{1}{\sigma})} \right]. \quad (76)$$

2 Clearly either sufficiently large τ or sufficiently small ω will satisfy the above constraint so that $S_{t+1}(i) = 0$ in the steady
3 state. For small fluctuations around the steady state we will have $S_{t+1}(i) = 0$ and the equilibrium condition will then
4 be the same to these in the benchmark model.

Alternatively we can assume that firms face stochastic death as in the model of Bernanke, Gertler, and Gilchrist (1999). Suppose a firm has a probability θ of exit and a measure θ of new firms enter in each period. So the number of firms is stationary. We focus on the conditions under which all firms have no incentive to save. This then requires

$$1 > \beta(1 - \theta) R_{ft} E_t \frac{C_t}{C_{t+1}} \left[1 + \omega \frac{(1 - \frac{1}{\sigma}) - \phi_{t+1}}{\phi_{t+1} - \xi(1 - \frac{1}{\sigma})} \right]. \quad (77)$$

5 In the steady state, this becomes $1 > (1 - \theta) \left[1 + \omega \frac{(1 - \frac{1}{\sigma}) - \phi}{\phi - \phi(1 - \frac{1}{\sigma})} \right]$.

6 Typically, the estimated the external financial premium is small. For example, Ferre De Graeve (2008) estimates it
7 to be 1.30% per annum. In the U.S. And Liu, Wang and Zha (2011) estimate it to be about 3.60% per annum. Then
8 if θ or τ exceeds 1% percent when the model is calibrated quarterly, firms will have no incentive to save. Note that we
9 can make the external financial premium arbitrarily small to fit these estimated value by letting ϕ approach $1 - \frac{1}{\sigma}$ even
10 with $\omega = 1$. The model is still indeterminate if condition (62) is satisfied.

11 4. Simulations and Impuse Responses

12 We now use our calibrated model to compute the implied moments and cross moments of consumption, investment,
13 hours and output, and study the impulse responses of our model. We first write the model in discrete time and solve it
14 by log-linearizing the equations that characterize the equilibrium around the steady state. We adopt a parameterization
15 that we used and discussed in calibrating our model in section 2.7.. The basic parameters are $\beta = \frac{1}{1+\rho} = 0.99$, $\alpha = 1/3$,
16 $\delta = 0.033$, $\sigma = 10$ and $\nu = 0.3$. We set $\xi = 0.9768$, $f = 0.1908$ and fix the productivity level to $A = 1$. These parameter
17 values imply steady state values $\phi = 0.88$ and $\gamma = \frac{f/Y}{\xi - f/Y} = 0.11$. As discussed in section 2.7., these parameters are not
18 only in accordance with various estimates in the literature, but put the model squarely within the indeterminacy range
19 with a continuum of equilibria. To coordinate the expectations of agents we therefore rely on sunspot shocks. Below we
20 study whether the various moments and impulse responses generated by our model with sunspots can match the data.

We begin without productivity shocks. The model's solution takes the form

$$\begin{pmatrix} \hat{K}_{t+1} \\ \hat{C}_{t+1} \end{pmatrix} = M \begin{pmatrix} \hat{K}_t \\ \hat{C}_t \end{pmatrix} + \begin{pmatrix} 0 \\ \varepsilon_{t+1} \end{pmatrix} \quad (78)$$

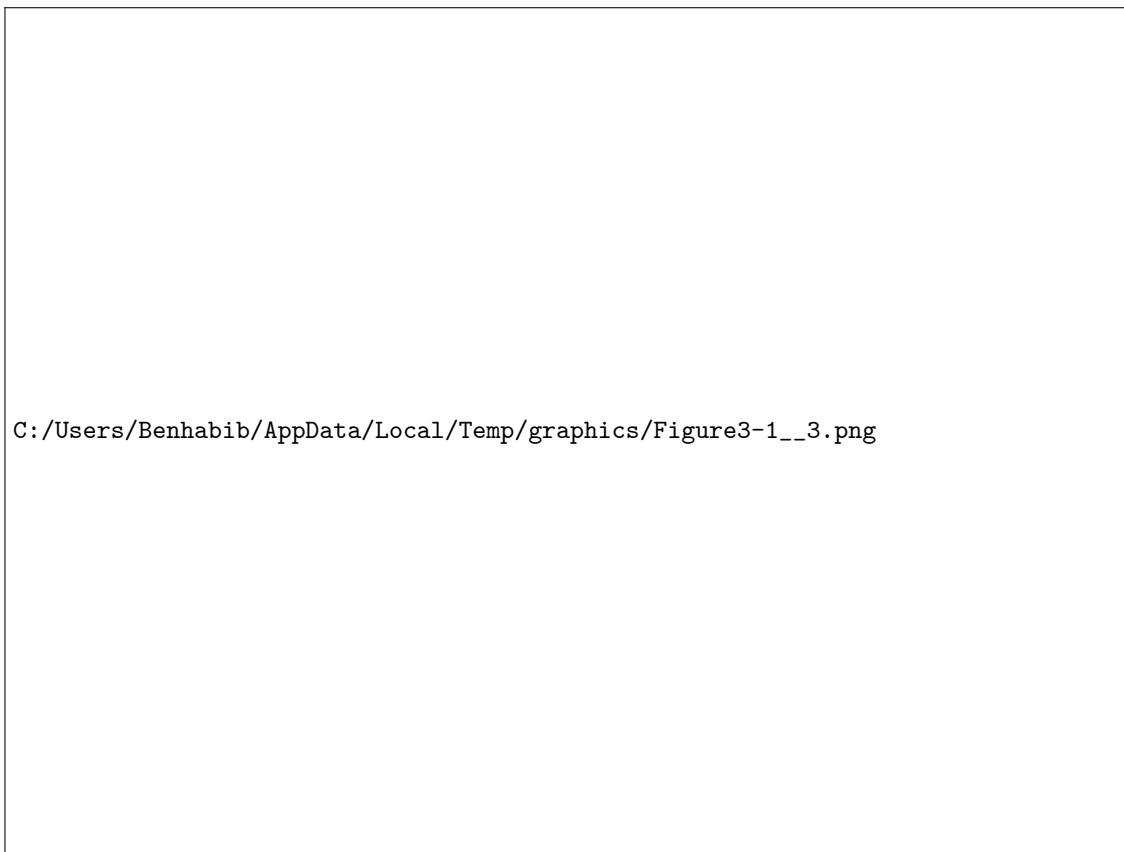
FINANCIAL CONSTRAINTS, ENDOGENOUS MARKUPS, AND SELF-FULFILLING

where M is a two-by-two matrix and $\varepsilon_{t+1} = \hat{C}_{t+1} - E_t \hat{C}_{t+1}$ is the zero mean *iid* sunspot shock. Since we work with a linearized model we can compute the relative moments and cross moments analytically.¹⁰ The remaining variables can be written as functions of \hat{K}_t and \hat{C}_t :

$$\begin{pmatrix} \hat{Y}_t \\ \hat{I}_t \\ \hat{N}_t \\ \hat{e}_t \end{pmatrix} = H \begin{pmatrix} \hat{K}_t \\ \hat{C}_t \end{pmatrix} \quad (79)$$

1 where H is a four-by-two matrix.

2 Figure 3 shows the impulse responses of output, investment, consumption and hours to an unexpected one percentage
3 increase in the initial consumption level induced by the agent's optimistic expectations about future income, under the
4 calibration of parameters given above.



5
6 Figure 3. Impulse Responses to a consumption (sunspot) Shock.

7 These impulse response functions resemble those obtained in the models with increasing returns to scale. From Figure 3
8 we see that output, investment, consumption and hours commove. The impulse responses also demonstrate that labor
9 is slightly more volatile than output, an important feature of the data that the standard RBC model has difficulty
10 explaining with a TFP shock. They also exhibit cycles in output, investment, consumption and hours, so the model
11 has the potential to explain the boom-bust patterns often observed in data. However, as in the models with increasing
12 returns to scale, the extremely large impact of autonomous consumption on output and investment seems empirically

¹⁰Of course if we can match the moments and cross moments relative to output, we can then choose the standard deviation of the sunspot shock, or later below the standard deviation of the sunspot and productivity shocks, so as to match the absolute volatilities of output and other variables to that of the data.

FINANCIAL CONSTRAINTS, ENDOGENOUS MARKUPS, AND SELF-FULFILLING

1 unjustified. In the impact period, one percentage increase in consumption leads to a 27 percent increase in output and
 2 a 116 percent increase in investment.

These volatile responses of output and investment can be understood by studying the effect of consumption on labor. Equating the labor demand (58) and labor supply (59) we have

$$\hat{N}_t = \frac{1}{\frac{(1+\gamma)(1+\nu)(1-\alpha)}{1+\nu-(1+\gamma)\alpha} - 1 - \chi} \hat{C}_t. \quad (80)$$

where $\frac{(1+\gamma)(1+\nu)(1-\alpha)}{1+\nu-(1+\gamma)\alpha} - 1$ is the slope of the labor demand curve and χ is the slope of the labor supply curve. When these two slopes are close, a one percentage increase in autonomous consumption increase can lead to huge increases in labor and hence output. Denote s as the ratio of steady state investment to income. Then from the resource constraint,

$$s\hat{I}_t + (1-s)\hat{C}_t = \hat{Y}_t, \quad (81)$$

3 so it is clear that the combination of smooth consumption and volatile income will make investment even more volatile
 4 as $s \ll 1$. In the current calibration $s = 0.23$. So the response of investment upon impact will be about 4.4 times that
 5 of output.

6 Table 1 reports some basic moments of the linearized model assuming that sunspots are the only driving force. All
 7 moments for the model are calculated analytically. The table shows that all variables are positively correlated with
 8 output. The correlations between them are also highly persistent. By our construction, the sunspots are i.i.d., so the
 9 persistence of the variables is not due to the persistence of exogenous shocks, but comes from the internal propagation
 10 mechanism of the model. Table 1 confirms that labor is slightly more volatile than output. The relative volatility of
 11 labor is 1.10 in the data and 1.08 in the model while the relative volatility of labor is 0.53 in the real business cycle
 12 model (see table 2).

13 Table 1: Sample and Model Moments

var	US Sample			Model		
	σ_X/σ_Y	$corr(X, Y)$	$corr(X_t, X_{t-1})$	σ_X/σ_Y	$corr(X, Y)$	$corr(X_t, X_{t-1})$
Y	1.00	1.00	0.87	1.00	1.00	0.91
N	1.01	0.88	0.92	1.08	0.99	0.91
C	0.52	0.83	0.90	0.07	0.48	0.97
I	3.33	0.92	0.92	4.31	0.99	0.91
ϕ	0.32	0.16	0.70	0.11	1.00	0.91

14 Note: Variables (Y, N, C, I, ϕ) denote output, labor (in hours), consumption, investment and marginal cost
 15 respectively. The marginal cost in the data can be computed via $\phi = \frac{\text{labor share}}{1-\alpha}$. σ_X/σ_Y is the standard
 16 deviation of variable X relative to output, $corr(X, Y)$ computes the correlation between X and output, and
 17 $corr(X_t, X_{t-1})$ computes the first-order autocorrelation of X_t .

To better match the relative volatilities of consumption and output we now introduce a TFP shock into the model. We assume that the technology level in the economy follows an AR(1) process

$$\hat{A}_{t+1} = \rho_a \hat{A}_t + \sigma_a \varepsilon_{at+1}. \quad (82)$$

FINANCIAL CONSTRAINTS, ENDOGENOUS MARKUPS, AND SELF-FULFILLING

1 Following Benhabib and Wen (2004), we assume the sunspots shocks and technology shocks are correlated. Following
 2 King and Rebelo (1999) , we assume $\rho_a = 0.98$. The technology shock ε_{at} and sunspot shocks ε_t are assumed to be
 3 perfectly correlated and the relative volatility of sunspot to technology shocks is set to $\sigma_s/\sigma_\varepsilon = 1.5$. This brings the
 4 relative volatility of consumption closer to data. The moments obtained using correlated TFP shocks and sunspots
 5 shocks are in Table 2.

6 Table 2: Moments with correlated TFP and Sunspot Shocks

var	Model with Correlated Shocks			The RBC Model		
	σ_X/σ_Y	$corr(X, Y)$	$corr(X_t, X_{t-1})$	σ_X/σ_Y	$corr(X, Y)$	$corr(X_t, X_{t-1})$
Y	1.00	1.00	0.98	1.00	1.00	0.95
N	0.96	0.95	0.99	0.53	0.73	0.90
C	0.37	0.55	0.99	0.62	0.86	0.99
I	3.38	0.96	0.98	2.65	0.88	0.91
ϕ	0.11	1.00	0.98	0	N.A	N.A


7 The RBC model refers to $f = 0$, so $\gamma = 0$, and $\phi = \xi$ is a constant. We select the parameter values such
 8 that the two models have the same steady state. For the RBC model, we use TFP shocks with $\rho_a = 0.98$ as
 9 the only driving force.

10 **Hump-Shaped output Dynamics.** The above simulation exercises show that our model with indeterminacy
 11 is similar in its ability to the RBC model to match some key moments of the data. In the simulation exercise that
 12 follows, we illustrate how our indeterminacy model can also predict some aspects of actual fluctuations that standard
 13 RBC models cannot explain, such as the hump-shaped, trend-reverting impulse response of output to transitory demand
 14 shocks, and the substantial serial correlation in output growth rates in the data (see Cogley and Nason(1995)). Since
 15 there is some significant empirical evidence favoring demand shocks as a main source of business cycles, (e.g., see
 16 Blanchard and Quah(1989), Watson (1993), Cogley and Nason (1995) and Benhabib and Wen (2004)), it is important
 17 to examine whether demand shocks can generate persistent business cycles. We consider two types of demand shocks as
 18 in Benhabib and Wen (2004): government spending shocks and preference shocks. With preference shocks the period-by-
 19 period utility function is now given by $U = \exp(\Delta_t) \log C_t - \psi \frac{N_t^{1+\chi}}{1+\chi}$. We assume that the preference shocks Δ_t follow an
 20 AR(1) process, namely $\Delta_t = \rho_\Delta \Delta_{t-1} + \varepsilon_{\Delta t}$. With government spending in period t, G_t , the resource constraint changes
 21 to $\dot{K}_t = Y_t - \delta(e_t)K_t - C_t - G_t$. We assume that $\log(G_t) = \rho_g \log(G_{t-1}) + \varepsilon_{gt}$. We choose $\rho_g = \rho_\Delta = 0.90$ as in Benhabib
 22 and Wen (2004). To highlight the effect of indeterminacy on the propagation mechanism of RBC models, we graph the
 23 impulse responses to a persistent government spending shock with and without indeterminacy in Figure 4. Figure 5
 24 graphs the impulse response of the model to a persistent preference shock. For the model without indeterminacy we set
 25 $f = 0$ and reset $\xi = 0.88$ so that the models with and without indeterminacy have the same steady state. We set the
 26 steady state government spending to GDP ratio to 0.2 as in Benhabib and Wen (2004).

27 Several features of Figure 4 deserve particular mention. First, in the case of $f = 0$, the marginal cost $\phi_t = \xi$ is
 28 a constant. Hence the impulse responses of our model with financial constraints resemble those of a standard RBC
 29 model. Figure 4 and Figure 5 show that the RBC model without indeterminacy has difficulty generating realistic business
 30 cycle fluctuations. For the determinate case, Figure 4 shows that consumption and investment move against each other

FINANCIAL CONSTRAINTS, ENDOGENOUS MARKUPS, AND SELF-FULFILLING

1 after a positive government spending shock. An increase in government spending generates a negative wealth effect,
2 which reduces both consumption and leisure. The decrease in leisure leads to an increase in output, and an increase
3 in output together with a decrease in consumption imply that investment has to increase. Second, even though the
4 model generates comovement without indeterminacy under persistent preference shocks, the responses of output to such
5 demand shocks are monotonic. Neither government spending shocks nor preference shocks can generate the hump-shaped
6 output dynamics observed in the data. And these monotonic and persistent output responses to demand shocks mostly
7 come from the persistence of shocks, not from an inner propagation mechanism of the model. If the persistence of the
8 shocks is reduced, the persistence of output responses will be reduced accordingly. When the model is indeterminate,
9 the responses of output to both the government spending shocks and the preference shocks are dramatically changed.
10 For the indeterminate case Figure 4 and Figure 5 clearly show persistent and hump-shaped responses of output to both
11 shocks. In addition, these persistent responses of output are not due to the persistence in shocks. As Figure 3 has already
12 demonstrated, the model with indeterminacy can generate persistent fluctuations even under i.i.d shocks. Figure 4 and
13 Figure 5 again highlight the similarity of our indeterminacy model to models with increasing returns to scale, so it
14 can explain other puzzles. For example, Benhabib and Wen (2004) demonstrate that their indeterminacy model based
15 on increasing returns to scale can explain the forcecastable movement puzzle pointed out by Rotemberg and Woodford
16 (1996). It is easy to show that our indeterminacy model can also replicate the highly forecastable comovements observed
17 in changes in output, hours, investment and consumption highlighted by Rotemberg and Woodford (1996). To avoid
18 repetition, we skip such a simulation exercise and refer readers to Benhabib and Wen (2004). In brief, these simulation
19 exercises illustrate the ability of our indeterminacy model to replicate rich business cycle dynamics observed in the data.



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Figure 4. Impulse responses to a government spending shock.

Solid lines are responses under determinacy ($f = 0$) and dashed lines are responses under indeterminacy.

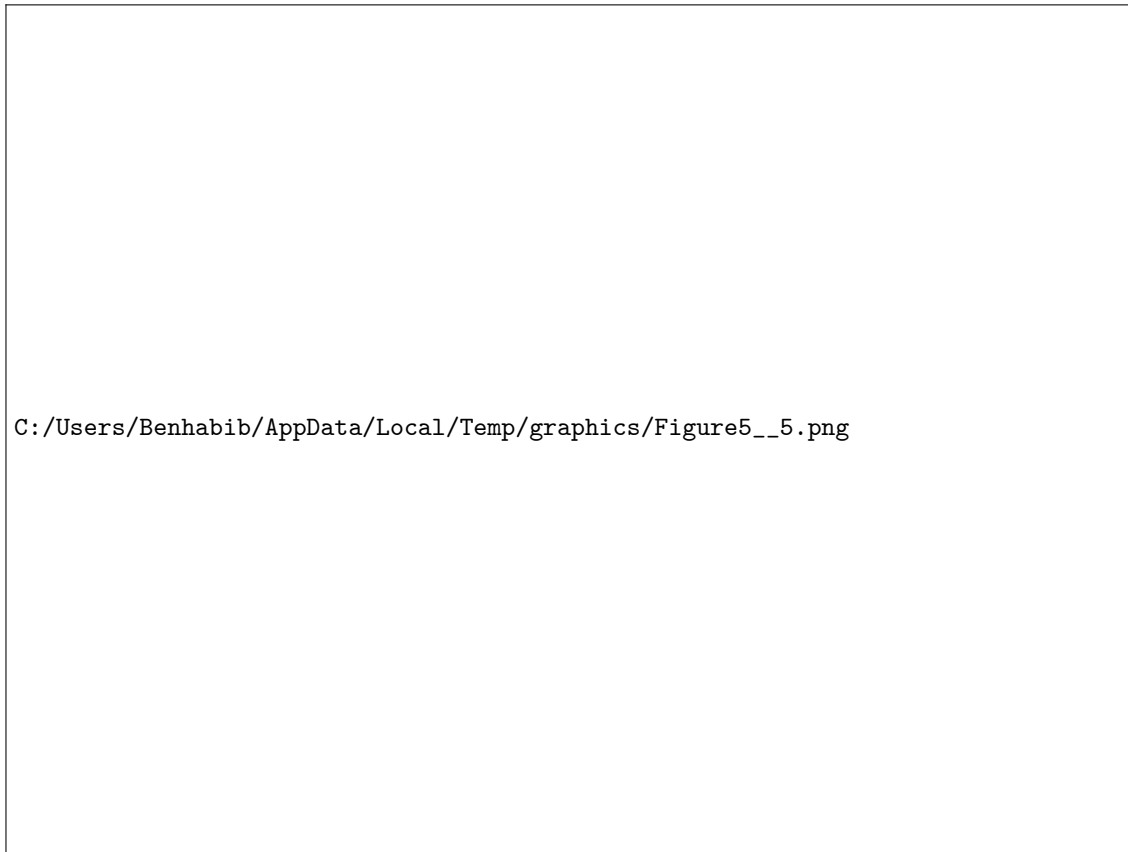


Figure 5. Impulse responses to a preference shock.

Solid lines are responses under determinacy ($f = 0$) and dashed lines are responses under indeterminacy.

5. Conclusion

We conclude that borrowing or collateral constraints can be a source of self-fulfilling fluctuations in economies that have no increasing returns to scale in production. Expectations of higher output can relax borrowing constraints, and firms can expand their output by bidding up factor prices and eliciting a labor supply response that allows the initial expectations to be fulfilled. The parameter ranges and markups that allow self-fulfilling expectations to occur are within realistic ranges and compatible with US macroeconomic data. Simulating our data we obtain moments and impulse responses that match the US macroeconomic data reasonably well.

1 **Appendix**

2 **A Proofs**

3 *A.1 The Proofs of Proposition 1*

First we substitute for $\lambda_1 = \frac{\omega_2(1+\chi)}{\chi+1-(1+\gamma)\omega_1}$ and $\lambda_2 = \frac{-\omega_1}{\chi+1-(1+\gamma)\omega_1}$, and we obtain

$$\frac{\text{Trace}(J)}{\delta} = \frac{1}{\chi+1-(1+\gamma)\omega_1} \left[\left(\frac{1+\nu}{\alpha\phi} - 1 - \gamma \right) \omega_2(1+\chi) - \nu(1+\gamma)\omega_1 \right]. \quad (\text{A.1})$$

The determinant of J is

$$\frac{\det(J)}{\delta^2\nu} = \left(\frac{1+\nu}{\alpha\phi} - 1 \right) \frac{(1+\chi)[\omega_2(1+\gamma) - 1] + \gamma\omega_1}{\chi+1-(1+\gamma)\omega_1} + \frac{\gamma\omega_1}{\chi+1-(1+\gamma)\omega_1}. \quad (\text{A.2})$$

4 If $(1+\gamma) > \frac{(1+\nu)(1+\chi)}{\alpha(1+\chi)+(1+\nu)(1-\alpha)}$, we have

$$\begin{aligned} (1+\gamma)\omega_1 - 1 &= \frac{(1+\gamma)(1+\nu)(1-\alpha)}{1+\nu-(1+\gamma)\alpha} - 1, \\ &> \frac{\frac{(1+\nu)(1+\chi)}{\alpha(1+\chi)+(1+\nu)(1-\alpha)}(1+\nu)(1-\alpha)}{1+\nu-\frac{(1+\nu)(1+\chi)}{\alpha(1+\chi)+(1+\nu)(1-\alpha)}\alpha} - 1, \\ &= \chi + 1 - 1 > \chi. \end{aligned}$$

It follows that

$$\frac{1}{\chi+1-(1+\gamma)\omega_1} < 0, \quad (\text{A.3})$$

5 and

$$\begin{aligned} &\left(\frac{1+\nu}{\alpha\phi} - 1 - \gamma \right) \omega_2(1+\chi) - \nu(1+\gamma)\omega_1 \\ &= \frac{(1+\nu) - \alpha\phi(1+\gamma)}{\alpha\phi} \frac{\alpha\nu(1+\chi)}{1+\nu-(1+\gamma)\alpha} - \nu(1+\gamma) \frac{(1+\nu)(1-\alpha)}{1+\nu-(1+\gamma)\alpha} \\ &= \frac{\nu}{1+\nu-(1+\gamma)\alpha} \left\{ \frac{1+\nu}{\phi}(1+\chi) - (1+\gamma)[\alpha(1+\chi) + (1+\nu)(1-\alpha)] \right\} \\ &> 0. \end{aligned} \quad (\text{A.4})$$

6 So the trace is negative. Finally we have

$$\begin{aligned} &\left(\frac{1+\nu}{\alpha\phi} - 1 \right) \frac{(1+\chi)[\omega_2(1+\gamma) - 1] + \gamma\omega_1}{\chi+1-(1+\gamma)\omega_1} + \frac{\gamma\omega_1}{\chi+1-(1+\gamma)\omega_1}, \\ &= \frac{1+\nu}{(1+\gamma)\omega_1 - \chi - 1} \left\{ (1+\chi)(1-\alpha) - [(1-\alpha)(1+\nu)/(1+\nu-\alpha\phi) + (1+\chi)\alpha] \gamma \right\} \\ &> 0 \end{aligned} \quad (\text{A.5})$$

7 so the determinant is positive. Q.E.D.

FINANCIAL CONSTRAINTS, ENDOGENOUS MARKUPS, AND SELF-FULFILLING

1 A. 2 The Proofs of Proposition 3

2 In the case without capacity utilization, the trace is

$$\begin{aligned} \text{Trace} &= \frac{\rho + \delta}{\alpha\phi} \lambda_1 + (\rho + \delta)(1 + \gamma)\lambda_2 - \delta, \\ &= \frac{\rho + \delta}{\chi + 1 - (1 + \gamma)(1 - \alpha)} \left(\frac{\alpha(1 + \chi)}{\alpha\phi} - (1 + \gamma)(1 - \alpha) \right) - \delta. \end{aligned}$$

3 The second line uses $\lambda_1 = \frac{\alpha(1 + \chi)}{\chi + 1 - (1 + \gamma)(1 - \alpha)}$ and $\lambda_2 = \frac{-(1 - \alpha)}{\chi + 1 - (1 + \gamma)(1 - \alpha)}$. The determinant is

$$\begin{aligned} \frac{\det(J)}{(\rho + \delta)} &= \frac{\rho + \delta}{\alpha\phi} [(1 + \gamma)\lambda_1 + \lambda_2 - 1] + \delta[1 - (1 + \gamma)(\lambda_1 + \lambda_2)] \\ &= \left[\frac{\rho + \delta}{\alpha\phi} - \delta \right] [(1 + \gamma)\lambda_1 + \lambda_2 - 1] - \gamma\lambda_2\delta \end{aligned} \quad (\text{A.6})$$

The necessary and sufficient condition for indeterminacy is that $\text{Trace}(J) < 0$ and $\frac{\det(J)}{(\rho + \delta)} > 0$. Under the condition $\chi + 1 < (1 + \gamma)(1 - \alpha)$ the trace is negative if

$$\frac{1}{\chi + 1 - (1 + \gamma)(1 - \alpha)} \left(\frac{1 + \chi}{\phi} - (1 + \gamma)(1 - \alpha) \right) < \frac{\delta}{\rho + \delta}. \quad (\text{A.7})$$

or

$$\frac{(1 + \chi)}{\phi} - (1 + \gamma)(1 - \alpha) > [\chi + 1 - (1 + \gamma)(1 - \alpha)] \frac{\delta}{\rho + \delta}. \quad (\text{A.8})$$

If we rearrange terms we have

$$\left(\frac{1}{\phi} - \frac{\delta}{\rho + \delta} \right) (1 + \chi) > (1 + \gamma)(1 - \alpha) \frac{\rho}{\rho + \delta}. \quad (\text{A.9})$$

So the necessary and sufficient condition for the trace to be negative is:

$$(1 + \chi) < (1 + \gamma)(1 - \alpha) < \frac{\frac{1}{\phi} - \frac{\delta}{\rho + \delta}}{1 - \frac{\delta}{\rho + \delta}} (1 + \chi). \quad (\text{A.10})$$

4 We now check the sign of the determinant. Substituting $\lambda_1 = \frac{\alpha(1 + \chi)}{\chi + 1 - (1 + \gamma)(1 - \alpha)}$, $\lambda_2 = \frac{-(1 - \alpha)}{\chi + 1 - (1 + \gamma)(1 - \alpha)}$, we obtain

$$\begin{aligned} \frac{\det(J)}{(\rho + \delta)} &= \left[\frac{\rho + \delta}{\alpha\phi} - \delta \right] \frac{(1 - \alpha)(1 + \chi) - \gamma(1 + \alpha\chi)}{(1 + \gamma)(1 - \alpha) - 1 - \chi} \\ &\quad - \frac{\gamma\delta(1 - \alpha)}{(1 + \gamma)(1 - \alpha) - 1 - \chi} \end{aligned} \quad (\text{A.11})$$

Under the condition $\chi + 1 - (1 + \gamma)(1 - \alpha) < 0$, $\det(J) > 0$ is equivalent to

$$(1 - \alpha)(1 + \chi) > \gamma(1 + \alpha\chi) + \gamma(1 - \alpha) \frac{\delta}{\frac{\rho + \delta}{\alpha\phi} - \delta} \quad (\text{A.12})$$

5 which implies that γ can not be too big. Q.E.D.

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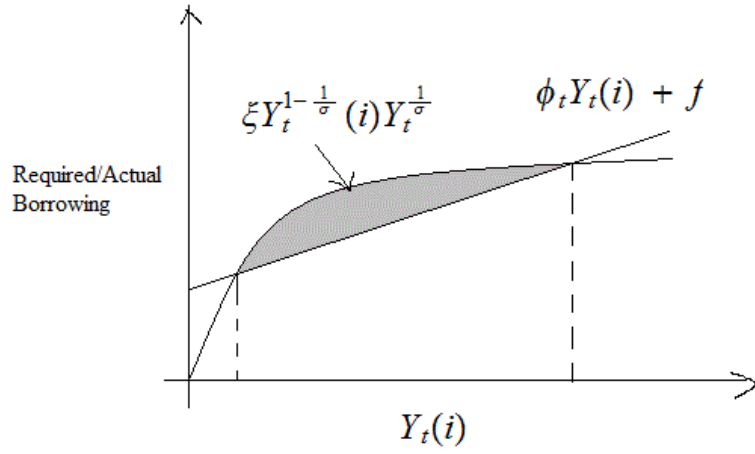


Figure 1. The Credit Constraints and Feasible Output Choice.

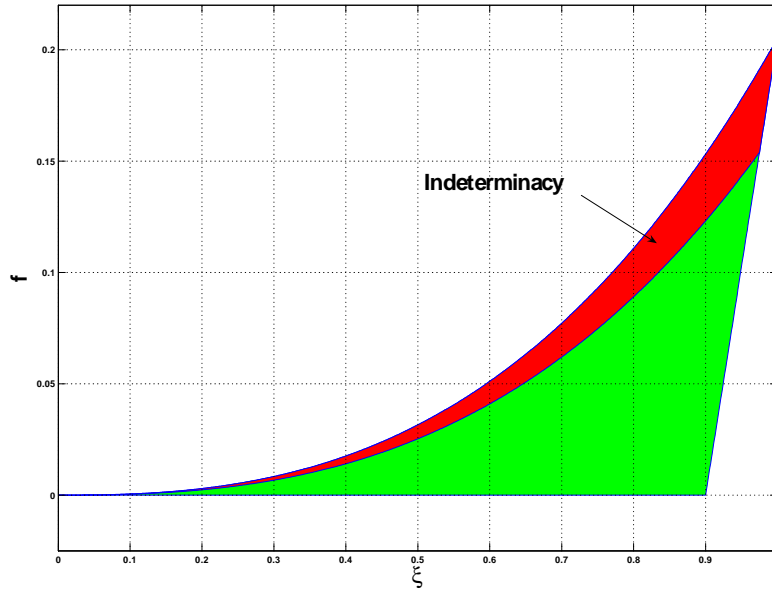


Figure 2. Parameter Spaces for Indeterminacy.

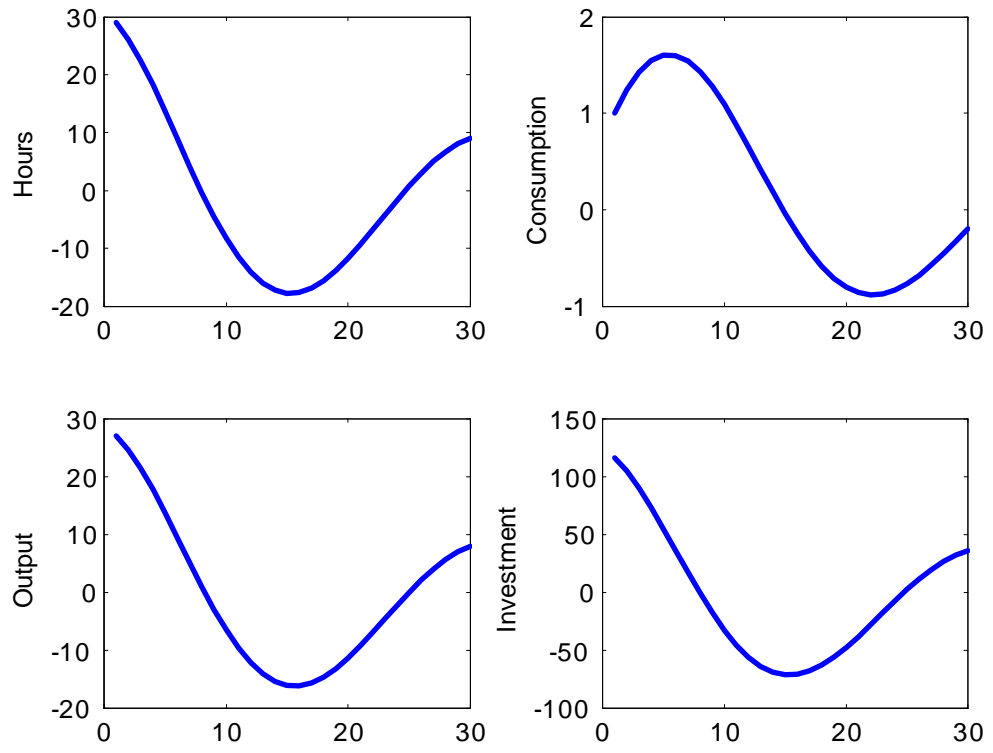


Figure 3. Impulse Responses to a consumption (sunspot) Shock.

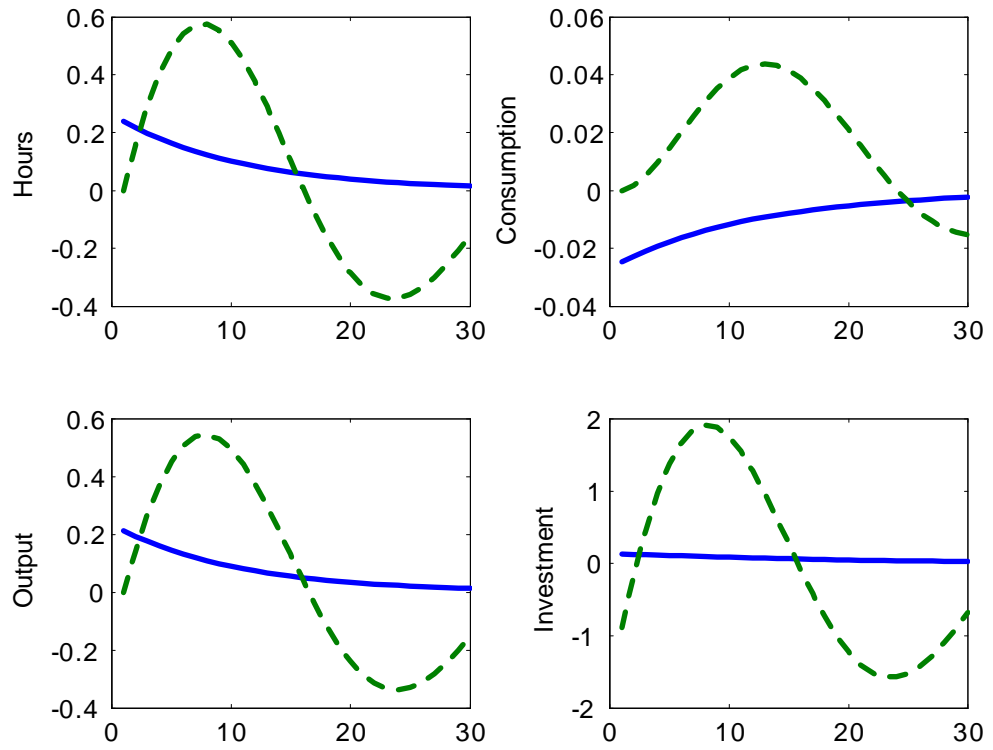


Figure 4. Impulse responses to a government spending shock.

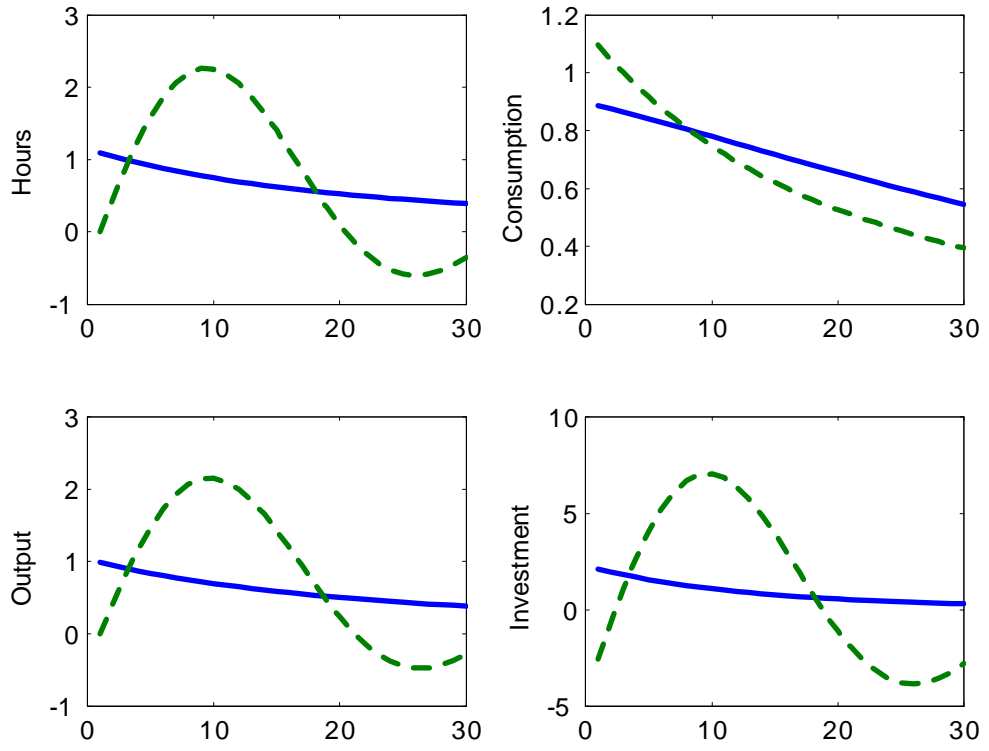


Figure 5. Impulse responses to a preference shock.