Correlated Equilibria in Macroeconomics and Finance

May 15, 2013

Introduction

- Multiple equilibria in macroeconomics (RBC and DSGE) can arise as coordination problems that result from externalities, market distortions, increasing returns...Then sunspots can randomize across them.
- Multiple equilibria also arise in Neo-Keynesian models with money.
- These are reasonably well-understood.
- We will focus instead on REE multiplicities in macro models generated by Correlated Equilibria (in the sense of Aumann), that are induced by informational frictions.

- If players' strategies could be correlated with a "correlation device", we know from Aumann (1977) that new equilibria with equilibrium payoff vectors lie in the convex hull of the ordinary Nash equilibrium payoffs. With imperfect signals the new equilibria can lie outside the convex hull of Nash equilibria (Maskin and Tirole (1997).
- Correlation devices are often introduced deus ex machina. We will focus on models with unique fundamental equilibria, and sentiments will act as a correlation device to create new stochastic equilibria. These will not be the result of randomizations across multiple fundamental equilibria.
- Our first model is inspired by Angeletos and Lao (2011), where sentiments can drive output, and by the Lucas Island model.
- It is related to Cass and Shell (1983) and by Maskin and Tirole (1987).

- We try to capture the Keynesian notion that animal spirits or sentiments, unconnected to fundamentals, can drive employment and output fluctuations under rational expectations.
- The key Keynesian feature of our model is that employment and production decisions are based on expectations of aggregate demand driven by consumer sentiments, while realized demand follows from the production and employment decisions of firms.
- Nevertheless, in equilibrium all agents know correct distributions, all prices are flexible, all markets clear and consumer expectations about aggregate consumption, employment and real wages are correct each period.
- So we have rational expectations equilibria.

A Model of Sentiments and Fluctuations

- Consumers make consumption and labor supply plans based on their "sentiments," or expectations about aggregate demand, and real wages. Nominal wages are normalized to one.
- Each firm must make a production decision on the basis of **signals** about what its demand will be, before demand is realized.
- The signals are based on firms' market research about their demand, early orders, initial inquiries, as well as public signals of aggregate demand/consumer sentiments.
- Since the real wages and employment have not yet been determined, and production has not yet taken place, these signals capture consumer sentiment.

Signals

- In the first simplest benchmark model, the signal is a weighted sum of the firm's idiosyncratic demand shock and a shock to aggregate demand/sentiments, both of which enter the firm's demand curve.
- We can also add an *iid* firm-specific noise to each firm's signal.
- Later we also introduce a second noisy but public signal of aggregate demand. This signal may represent public forecasts of aggregate demand/sentiments.
- A second model replaces the idiosyncratic firm specific demand shocks with an aggregate shock. Producers can observe aggregate demand, but they cannot separately identify the sentiment shock from the fundamental aggregate shock.
- At the end, we introduce heterogeneous but correlated consumer sentiments as well. Surveying a subset of consumers yields a noisy signal on the common component of sentiments.

Model cont'd

- The informational structure is simple: trades take place in centralized markets (in contrast to Angeletos and La'o where trades are bilateral through random matching) and at the end of each period all history can become public knowledge.
- Firms optimally decide on how much to produce on the basis of their private but correlated signals about demand.
- Firms act on the signal to maximize profits: they hire labor at nominal wages, and produce. Only then aggregate output is realized and prices clear all markets.
- In equilibrium all agents "know" the correct distribution of the idiosyncratic and aggregate demand shocks.
- The realized real wage is equal the wage that the consumers and firms expected, given their sentiments.
- Aggregate output equals to the households' planed consumption. So households can in fact implement their consumption plans.
- Thus we have rational expectations equilibria.

Model cont'd

- We show that in the simple benchmark model, there can be two distinct rational expectations equilibria: one with constant output and one with stochastic output driven by self-fulfilling sentiments.
- But note that the self-fulfilling stochastic equilibrium **is not** a randomization over multiple equilibria.
- Signals to firms contain sentiments which act as a correlation device.

The Basic Benchmark Model: Household

$$\max E_0 \sum \beta^t [\log(C_t) - \psi N_t]$$

subject to

$$C_t \leq \frac{W_t}{P_t} N_t + \frac{\Pi_t}{P_t},$$

where W_t denotes nominal wage and Π_t aggregate profit income from firms, all measured in final goods. The first-order conditions for labor imply

$$\begin{aligned} \frac{1}{C_t} &= \psi \frac{P_t}{W_t} \\ \frac{W_t}{P_t} &= \psi C_t \qquad (1) \end{aligned}$$

Households have (so far) common point expectations/sentiments about aggregate output C_t, so given a nominal wage $W_t = 1$, they can infer a price P_t and a real wage $\frac{W_t}{P_t}$, consistent with (1). *IF they are right!*

Final Good Aggregator

- The final-good firms (or a representative consumer) produce a final good according to

$$C_t = Y_t = \left[\int \epsilon_{jt}^{\frac{1}{\theta}} Y_{jt}^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}}$$

where $\theta > 1$ and $\log \epsilon_{jt}$ are *iid* zero mean firm-specific shocks, and maximizes profit

$$\max P_t \left[\int \epsilon_{jt}^{\frac{1}{\theta}} Y_{jt}^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}} - \int p_{jt} Y_{jt} dj.$$

The demand function depends on both ϵ_{jt} and Y_t:

$$\frac{p_{jt}}{P_t} = Y_{jt}^{-\frac{1}{\theta}} (\epsilon_{jt} Y_t)^{\frac{1}{\theta}}$$

$$Y_{jt} = \left(\frac{P_t}{p_{jt}} \right)^{\theta} \epsilon_{jt} Y_t$$

Intermediate Goods

- Each intermediate firm produces good Y_{jt} without perfect knowledge either of ϵ_{jt} or of aggregate demand $\tilde{Y}_t$, which could be random and sentiment-driven. Instead, as in the Lucas island model, firms have a noisy indication of what their demand will be from a signal s_{jt},

$$s_{jt} = \lambda \log \epsilon_{jt} + (1 - \lambda) \log \tilde{Y}_t \equiv \lambda \log \epsilon_{jt} + (1 - \lambda) \log z_t \qquad (2)$$

  where the parameter λ reflects the weights of the idiosyncratic and aggregate components of demand. Based on the signal, the firm chooses to produce output and maximize profits.
- This signal in (2) is our simplest signal, to be generalized later.
- We will show that in a Rational Expectations Equilibrium the aggregate demand $\tilde{Y}_t$ that generates the signal will be equal to the actual aggregate output Y_t.

Intermediate Goods, Cont'd

An intermediate goods producer j has the production function

$$Y_{jt} = An_{jt}.$$

So the firm maximizes expected nominal profits $\Pi_{jt} = p_{jt} Y_{jt} - W_t \frac{Y_{jt}}{A}$ or

$$\max_{Y_{jt}} E_t \left[\left(P_t Y_{jt}^{-\frac{1}{\theta}} (\epsilon_{jt} Y_t)^{\frac{1}{\theta}} \right) Y_{jt} - W_t \frac{Y_{jt}}{A} | s_{jt}. \right]$$

The markup is constant (based on expected price and marginal cost):

$$\left(1 - \frac{1}{\theta}\right) = \frac{1}{A} \frac{E_t \left[W_t | s_{jt} \right]}{Y_{jt}^{-\frac{1}{\theta}} E_t \left[P_t (\epsilon_{jt} Y_t)^{\frac{1}{\theta}} | s_{jt} \right]}$$

After simplifications using $\frac{1}{C_t} = \frac{1}{Y_t} = \psi \frac{P_t}{W_t}$, and $W_t = 1:$

$$Y_{jt} = \left\{ \left(1 - \frac{1}{\theta}\right) \frac{A}{\psi} E_t \left[(\epsilon_{jt})^{\frac{1}{\theta}} \ Y_t^{\frac{1}{\theta}-1} \right] | s_{jt} \right\}^{\theta}.$$

Note that in equilibrium, since $\theta > 1$, Y_{jt} is negatively related to Y_t. If aggregate output increases, real wages rise, and firms reduce their output.

Certainty Equilibrium

There exists a fundamental certainty equilibrium with constant aggregate output $C_t = Y_t = Y^*$ and $P_t = P$. Information is perfect, consumers and firms expect $P_t = P$, $C_t = Y_t = Y^*$, and $\sigma_y^2 = 0$. The signal fully reveals the firm's own idiosyncratic demand ϵ_{jt}:

$$Y_{jt}^{\frac{1}{\theta}} = \left(1 - \frac{1}{\theta}\right) \frac{A}{\psi} \epsilon_{jt}^{\frac{1}{\theta}} Y_t^{\frac{1-\theta}{\theta}}, \tag{3}$$

Without loss of generality set $\left(1 - \frac{1}{\theta}\right) \frac{A}{\psi} = 1$. Final good output is:

$$Y_t = \left[\int \epsilon_{jt}^{\frac{1}{\theta}} Y_{jt}^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}} \tag{4}$$

$$Y_t^{\frac{\theta-1}{\theta}} = Y_t^{(1-\theta)\frac{\theta-1}{\theta}} \int \epsilon_{jt} dj,$$

or, if $\varepsilon_{jt} \equiv \log \epsilon_{jt}$ has zero mean and variance σ_ε^2,

$$\bar{\phi}_0 = \log Y_t = \frac{1}{\theta - 1} \log E \exp(\varepsilon_{jt}) = \frac{1}{2(\theta - 1)} \sigma_\varepsilon^2 \tag{5}$$

Self-Fulfilling Equilibrium

- We conjecture there exists an another equilibrium, such that aggregate output is not a constant. In particular all agents "know" output follows

$$\log Y_t = \phi_0 + z_t, \tag{6}$$

- The noisy signal received by each firm (now defined net of the constant term ϕ_0) is

$$s_{jt} = \lambda \varepsilon_{jt} + (1 - \lambda) z_t. \tag{7}$$

where $z_t \sim N\left(0, \sigma_z^2\right)$.

- With fluctuations in aggregate output, the signal is not fully revealing.

Sentiment-Driven REE

In the self-fulfilling equilibrium:

- a) Each period the sentiment z_t held by households in $\log Y_t = \phi_0 + z_t$ will be the realized z_t.
- b) So the distribution of the perceived sentiment $\{z\}$ will be consistent with the realized distribution of aggregate output $\{Y\}$.
- c) Prices will clear all markets each period.

Self-Fulfilling Equilibrium

Proposition

If $\lambda \in \left(0, \frac{1}{2}\right)$, *there exists a self-fulfilling rational expectations equilibrium with stochastic aggregate output* Y_t. *Furthermore* $\log Y_t$ *is normally distributed with mean*

$$\phi_0 = \frac{(1-\lambda) + (\theta - 1)\lambda}{\theta(1-\lambda)}\bar{\phi}_0 < \bar{\phi}_0$$

and variance

$$\sigma_z^2 = \frac{\lambda(1-2\lambda)}{(1-\lambda)^2\theta}\sigma_\varepsilon^2$$

- Welfare? Note that in this case (but not necessarily in models that follow) the mean of the constant output equilibrium $\bar{\phi}_0 > \phi_0$, so in this case it Pareto dominates the sentiment driven stochastic equilibrium.

Self-Fulfilling Equilibrium, Cont'd

- In the Certainty Equilibrium with $\sigma_z^2 = 0$, $Y_{jt} = \epsilon_{jt} Y_t^{1-\sigma}$, and since $\sigma > 1$, equilibrium firm-level outputs depend negatively on aggregate output as in the case of *strategic substitutability*. Hence, this fundamental equilibrium is unique.
- Given λ and the variance of the idiosyncratic shock σ_ε^2, for markets to clear for all possible realizations of the sentiment z_t, the variance σ_z^2 has to be precisely pinned down, as in the Proposition above.
- If σ_z^2 is too high, output is too low relative to aggregate demand by consumers, and vice-versa.
- In the next model, with aggregate as opposed to idiosyncratic fundamental shocks, we will get an interval for σ_z^2 for which RE equilibria obtain .

Imperfect signal with firm-specific noise (SKIP)

- So far we assumed that firms can get an initial signal for the overall demand for their product, but cannot disaggregate it into its components arising from idiosyncratic and from aggregate demand. They only observe their sum.
- Since the signals are based on early and initial demand indications for each of the firms, they may well contain additional firm-specific noise components. Suppose then that the signal takes the slightly more general form,

$$s_{jt} = v_{jt} + \lambda \varepsilon_{jt} + (1 - \lambda) z_t, \tag{8}$$

where v_{jt} is a pure firm-specific iid noise with zero mean and variance σ_v^2.

Multiple signals

- The government and public forecasting agencies as well as news media often release their own forecasts of the aggregate economy.
- Suppose firms receive two independent signals, s_{jt} and s_{pt}.
- The firm-specific signal s_{jt} is based on firm's own preliminary information about its demand:

$$s_{jt} = v_{jt} + \lambda \varepsilon_{jt} + (1 - \lambda) z_t$$

- The public signal is:

$$s_{pt} = z_t + e_t$$

  where $e_t \sim N\left(0, \sigma_e^2\right)$ is noise in the public forecast of aggregate demand.
- Later we will model the noisy public signal with heterogenous but correlated sentiments across consumers, so observing a subset of consumers will reveal a noisy signal of the average sentiment.

Multiple signals, Cont'd

Proposition

If $\lambda < \frac{1}{2}$, and $\sigma_v^2 < \lambda(1-2\lambda)\sigma_\varepsilon^2$, then there exists a self-fulfilling rational expectations equilibrium with stochastic aggregate output

$$\log Y_t = y_t = z_t + \eta e_t + \phi_0 = \hat{z}_t + \phi_0,$$

which has mean $\phi_0 = \frac{1}{2}\left(\frac{(1-\lambda+(\theta-1)\lambda)}{\theta(1-\lambda)}\frac{1}{(\theta-1)}\right)\sigma_\varepsilon^2 - \frac{(\theta-1)\sigma_v^2}{2\theta^2(1-\lambda)^2}$ *and variance*

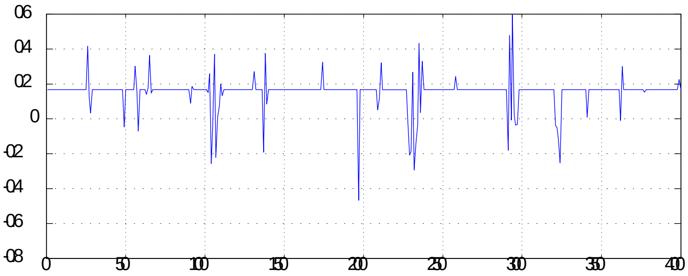
$$\sigma_{\hat{z}}^2 = \frac{\lambda(1-2\lambda)}{(1-\lambda)^2\theta}\sigma_\varepsilon^2 - \frac{1}{(1-\lambda)^2\theta}\sigma_v^2 > 0,$$

and where $\eta = -\frac{\sigma_z^2}{\sigma_e^2} = -\frac{1}{\gamma}$. In addition, with $\sigma_z^2 = \gamma\sigma_e^2 = 0$, there is a "certainty" equilibrium with constant output identical to the certainty equilibrium in the previous Proposition with a single signal.

- Note: the public forecast error e_t affects output.

- We now construct a persistent sunspot equilibrium with Markov transitions between the certainty and the stochastic self-fulfilling equilibrium.
- To construct such an equilibrium, we introduce a sunspot $S_t = 1$ or 0.
- We have the transition probabilities $\Pr(S_t = 1 | S_{t-1} = 1) = \rho$ and $\Pr(S_t = 0 | S_{t-1} = 0) = \xi\rho$.
- Then the stationary distribution is $\Pr(S_\infty = 1) = \frac{1-\xi\rho}{1-\rho+1-\xi\rho}$, $\Pr(S_\infty = 0) = \frac{1-\rho}{1-\rho+1-\xi\rho}$.
- The agents observe the sunspots first and if $S_t = 1$, they cooperate on the certainty equilibrium but if $S_t = 0$, then they cooperate on the uncertainty equilibrium.

- A simulated path of aggregate output is obtained in Figure 1 below. We set $\theta = 4,\ \sigma_\varepsilon^2 = 1,\ \rho = 0.95,\ \xi = 0.7$. For these parameters the certainty equilibrium is $\log Y_t = \frac{1}{2(\theta-1)}\sigma_\varepsilon^2 = \bar{\phi}_0 = 0.1667$.



Simulated Path of Output

- Since mean output is lower in the sentiment driven equilibrium, we have counter-cyclical volatility.

Persistence: Autoregressive productivity

- If productivity, A_t, is a stochastic process that firms can observe, then in parallel to our benchmark case we can express output as

$$\log Y_t = \phi_0 + z_t + \log A_t.$$

- Setting $\left(1 - \frac{1}{\theta}\right) \frac{1}{\psi}$ instead of $\left(1 - \frac{1}{\theta}\right) \frac{A_t}{\psi}$ to unity, as before market clearing would require the sum of log ouputs of firms to add to aggregate log output for every z_t.
- Later we will explore a model where A_t is stochastic, but cannot be observed by firms.

A Simple Abstract Model

- To set up the intuition consider the following simple model.
- Assume for simplicity that the economy is log-linear, so optimal log output of firms coming from a linear quadractic objective, is given by the rule

$$y_{jt} = E_t\{[\beta_0 \varepsilon_{jt} + \beta y_t] \,|s_{jt}\} \tag{9}$$

  where ε_{jt} is zero mean, *iid*.
- The coefficient β can be either negative or positive, so we can have either strategic substitutability or strategic complementarity in firms' actions. In the linearized version of the benchmark model it corresponds to $1-\theta$.
- Market clearing requires

$$y_t = \int y_{jt} dj. \tag{10}$$

Abstract Model Cont'd:

- We have, from a linear quadratic objective, for $\beta < 1$,

$$y_{jt} = E_t\{[\beta_0 \varepsilon_{jt} + \beta y_t] \,|s_{jt}\} \tag{11}$$

$$s_{jt} = v_{jt} + \lambda \varepsilon_{jt} + (1-\lambda)\, y_t \tag{12}$$

$$y_t = \int y_{jt} dj \tag{13}$$

- Assume that y_t is normally distributed with zero mean and variance σ_y^2. Based on (11), signal extraction:

$$y_{jt} = \frac{\lambda \beta_0 \sigma_\varepsilon^2 + (1-\lambda)\beta\sigma_y^2}{\sigma_v^2 + \lambda^2 \sigma_\varepsilon^2 + (1-\lambda)^2 \sigma_y^2} \left[v_{jt} + \lambda \varepsilon_{jt} + (1-\lambda)\, y_t\right]$$

- Market clearing implies:

$$y_t = \int y_{jt} dj = \frac{\lambda \beta_0 \sigma_\varepsilon^2 + (1-\lambda)\beta\sigma_y^2}{\sigma_v^2 + \lambda^2 \sigma_\varepsilon^2 + (1-\lambda)^2 \sigma_y^2}(1-\lambda) y_t . \tag{14}$$

Abstract Model Cont'd: Certainty and Self-Fulfilling Stochastic Equilibria

- Market clearing implies

$$y_t = \int y_{jt} dj = \frac{\lambda \beta_0 \sigma_\varepsilon^2 + (1-\lambda)\beta \sigma_y^2}{\sigma_v^2 + \lambda^2 \sigma_\varepsilon^2 + (1-\lambda)^2 \sigma_y^2}(1-\lambda) y_t. \qquad (15)$$

- The Certainty Equilibrium is $y_t \equiv 0$.
- But (15) holds for all y_t if

$$\sigma_y^2 = \frac{\lambda(\beta_0 - (1+\beta_0)\lambda)\sigma_\varepsilon^2 - \sigma_v^2}{(1-\lambda)^2(1-\beta)}$$

- Thus, σ_y^2 is pinned down uniquely and it defines the Self-Fulfilling Stochastic Equilibrium.
- Note that if $\beta < 1$, a necessary condition for σ_y^2 to be positive is $\lambda \in \left(0, \frac{\beta_0}{1+\beta_0}\right)$.

Stability Under Learning

- Our model is essentially static, but we can investigate whether the equilibria of the model are stable under adaptive learning.
- For simplicity we will confine our attention to the simplified abstract model of section with $\sigma_v^2 = 0$, where without loss of generality we set $\beta_0 = 1$. So the model is

$$s_{jt} = \lambda \varepsilon_{jt} + (1 - \lambda) z_t$$

$$y_{jt} = E_t \{ \varepsilon_{jt} + \beta z_t | s_{jt} \}$$

$$z_t = y_t = \int y_{jt} dj$$

Stability Under Learning Cont'd

- We can re-normalize our model so that the sentiment or sunspot shock z_t has unit variance by redefining output as $y_t = \log Y_t = \sigma_z z_t$. The variance of output y_t then is still σ_z^2.
- Suppose that agents understand that equilibrium y_t is proportional to z_t and they try to learn σ_z.
- Agents conjecture at the beginning of the period t that the constant of proportionality is $\sigma_{zt} = \frac{y_t}{z_t}$, and the realized output is

$$y_t = \frac{\lambda\sigma_\varepsilon^2 + (1-\lambda)\beta\sigma_{zt}^2}{\lambda^2\sigma_\varepsilon^2 + (1-\lambda)^2\sigma_{zt}^2}(1-\lambda)\sigma_{zt} z_t.$$

- Under adaptive learning with constant gains $g = 1-\alpha > 0$, now agents update $\sigma_{zt} = \frac{y_t}{z_t}$:

$$\sigma_{zt+1} = \sigma_{zt} + (1-\alpha)\left(\frac{y_t}{z_t} - \sigma_{zt}\right) \equiv h(\sigma_{zt})$$

Stability Under Learning Cont'd

- We have the adaptive learning updating rule

$$\sigma_{zt+1} = \alpha\sigma_{zt} + (1-\alpha)\left(\frac{y_t}{z_t}\right) = \sigma_{zt} + (1-\alpha)\left(\frac{y_t}{z_t} - \sigma_{zt}\right) \equiv h(\sigma_{zt})$$

- For any initial $\sigma_{zt} > 0$, we can show that σ_{zt} does not converge to 0, the fundamental equilibrium.
- By contrast the sentiment-driven sunspot equilibrium is locally stable under learning provided the gain $g = 1 - \alpha$ is not too large.
- In particular $h'(0) \; > 1$ and $|h(\sigma_z)| < 1$ at the sentiment driven equilibrium $\sigma_z = \left(\frac{\lambda(1-2\lambda)\sigma_\varepsilon^2}{(1-\lambda)^2(1-\beta)}\right)^{0.5}$.

Model with Aggregate Shocks and Signals on Aggregate Demand

- The households are subject to aggregate "preference" shocks A_t and sentiment shocks z_t in each period.
- In equilibrium, conditional on the aggregate shock and their sentiments, in equilibrium households will perfectly forecast the price level and therefore the real wage.
- The **aggregate** consumption plans made by the households are the source of "possibly" noisy **aggregate** demand signals for the intermediate goods producers. Based on their signal, intermediate goods producers decide how much to produce.
- Prices of each intermediate good adjusts to equalize demand and supply. These prices then determine the price of the final good. In equilibrium this realized price coincides with the price expected by households based on their sentiments.
- The results extend to the case where consumer sentiments are heterogenous but correlated.

More precisely,

- Firms get a "signal" that reveals aggregate demand $c_t\left(A_t, z_t\right)$, possibly with some *iid* noise v_{jt}.
- There is still a signal extraction problem since their optimal output responds differentially to A_t and z_t.
- For example in equilibrium, they might like to decrease output in response to higher z_t, but the optimal response of output to A_t is positive.
- There is a constant output equilibrium $c_t = \bar{c}$.
- In addition, there are a continuum of sunspot equilibria parametrized by σ_z^2.

Households

$$U_t = A_t \frac{C_t^{1-\gamma}}{1-\gamma} - N_t, \tag{16}$$

where C_t is consumption of the final good, A_t is the preference shock (or could be a productivity shock to the final goods producer) and N_t is labor. Household's budget constraint as $P_t C_t \leq N_t + \Pi_t$.

- Here P_t is the conjectured price of the final goods given productivity A_t, and sentiments about aggregate output Z_t, specified later for each equilibria.
- Π_t is the profit collected from all intermediate firms and the wage rate is normalized to 1. The first order condition for C_t is

$$A_t C_t^{-\gamma} = P_t. \tag{17}$$

Final good aggregation:

$$\max_{C_{jt}} P_t C_t - \int P_{jt} C_{jt} dj, \tag{18}$$

where C_t is produced by a continuum of intermediate goods according to the Dixit-Stiglitz production function, with $\theta > 1$,

$$C_t = \left[\int_0^1 C_{jt}^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}} . \tag{19}$$

The final goods producer (could be the consumer heself) maximizes profit or utility which yields the inverse demand curve for each individual intermediate goods,

$$\frac{P_{jt}}{P_t} = C_{jt}^{-\frac{1}{\theta}} C_t^{\frac{1}{\theta}}, \quad \textit{where} \quad P_t = \left[\int_0^1 P_{jt}^{1-\sigma} \right]^{\frac{1}{1-\sigma}} \tag{20}$$

Firms

The Intermediate goods firms: The intermediate goods production function is

$$C_{jt} = N_{jt}. \tag{21}$$

A intermediate goods producer j solves the following problem.

$$\max E[(P_{jt} C_{jt} - C_{jt})|S_{jt}] \tag{22}$$

Substituting out P_{jt}, and solving we have

$$(1 - \frac{1}{\theta}) C_{jt}^{-\frac{1}{\theta}} E[P_t C_t^{\frac{1}{\theta}} | S_{jt}] = 1 \tag{23}$$

So we have

$$C_{jt} = \left\{ E[P_t C_t^{\frac{1}{\theta}} | S_{jt}](1 - \frac{1}{\theta}) \right\}^{\theta} \tag{24}$$

$$C_{jt} = \left\{ (1 - \frac{1}{\theta}) E[A_t C_t^{\frac{1}{\theta} - \gamma} | S_{jt}] \right\}^{\theta}. \tag{25}$$

The Signal

$$\log S_{jt} = s_{jt} = \log C_t + v_{jt} = c_t + v_{jt}.$$

- In what follows, the noise v_{jt} will not be essential for our results: we could have set $\sigma_v^2 = 0$.
- In that case the signal s_{jt} would fully reveal aggregate consumption c_t to the intermediate goods firms.
- But then where is signal extraction?
- In the sentiment driven equilibria household expectations of aggregate output and price will be

$$\log C_t - \bar{c} = c_t = \phi a_t + \sigma_z z_t, \quad \log P_t - \bar{p} = p_t = \frac{a_t}{\theta} - \gamma\phi a_t - \gamma\sigma_z z_t$$

- Since firms do not observe a_t and z_t independently, they also form conditional expectations of them based on their signal $s_{jt} = c_t$.
- Note: There is no firm-specific shock so no λ weight.
- In equilibrium both firm and household expectations of aggregate output and price will be correct.

Equilibrium Again

(a) Based on the preference shock A_t and sentiments Z_t, households conjecture that the aggregate price is given by $P_t = P(A_t, Z_t)$;
(b) Based on the conjectured price P_t given A_t, Z_t, the households tentatively choose their consumption plan $C_t = C(A_t, Z_t)$ to maximize their utility;
(c) The consumption plan generates signals to firms as $\log S_{jt} = c_t + v_{jt}$;
(d) Based on the signal S_{jt}, firm j produces C_{jt} to maximize its expected profit;
(e) Given the production of C_{jt}, price P_{jt} adjusts to equalize demand and supply;
(f) The total production of final goods C_t, equals the households' planed consumption. Hence the realized price is also equal to the conjectured price P_t.

Equilibrium in logs

Without loss of generality assume $A_t = \exp(a_t/\theta)\left(\frac{\theta}{\theta-1}\right)^{\frac{1}{\theta}}$, where a_t is normally distributed with mean 0 and variance σ_a^2. Aggregate consumption is given by

$$C_t = \left[\int_0^1 C_{jt}^{\frac{\theta-1}{\theta}} dj\right]^{\frac{\theta}{\theta-1}}. \tag{26}$$

Then we can write down the system as

$$\begin{aligned} c_{jt} &= \log C_{jt} - \tilde{c} = E\{[a_t + (1-\gamma\theta)c_t]|s_{jt}\} && (27) \\ s_{jt} &= c_t + v_{jt} && (28) \\ c_t &= \log C_t - \bar{c} = \int_0^1 c_{jt} dj. && (29) \end{aligned}$$

where $\bar{c}$ are $\tilde{c}$ are constants to be determined. Note that in equation (27) $\beta = 1 - \gamma\theta < 1$ and can be negative. If $\beta < 0$,we are in the case of gross substitutes.

Constant Fundamental Equilibrium

Proposition

If the consumers conjecture that the aggregate price is given by

$$p_t = \log P_t - \bar{p} = \frac{a_t}{\theta} \tag{30}$$

where $\bar{p} = \frac{1}{\theta} \log\left(\frac{\theta}{\theta-1}\right) - \gamma \bar{c}$, *and* $\bar{c} = \tilde{c} = \frac{1}{2}\frac{1}{\gamma}\frac{1}{\theta^2}\sigma_a^2$. *Then*

$$c_{jt} = c_t = 0. \tag{31}$$

is always an equilibrium with constant consumption.

Note that prices act as a shock absorber: an increase in a_t is exactly offset by a rise in p_t and a fall in the real wage so consumers plan not to change their consumptions and labor supply.

Let $\mu = \frac{\sigma_v^2}{\sigma_a^2}$, and assume $0 < \mu < \frac{1}{4(1-\beta)}$ where $\beta = 1 - \gamma\theta < 1$ where $\gamma > 0$ is the curvature of utility of consumption and $\theta > 0$ is the elasticity of substitution in final good production.

Define the constant terms:

$$\bar{c} = \frac{1}{2}\frac{1}{\theta^2\gamma}[(\theta-1)\sigma_v^2 + (1+\beta\phi)(1-(1-\beta)\phi)\sigma_a^2], \tag{32}$$

$$\bar{p} = \frac{1}{\theta}\log\left(\frac{\theta}{\theta-1}\right) - \gamma\bar{c} \tag{33}$$

$$\tilde{c} = (1-\theta\gamma)\bar{c} + \frac{1}{2}\frac{(1+\beta\phi)(1-(1-\beta)\phi)\sigma_a^2}{\theta} \tag{34}$$

Stochastic Fundamental Equilibria, cont'd

Proposition

Let consumers conjecture that the aggregate price and consumption follow:

$$\log P_t - \bar{p} = \frac{a_t}{\theta} - \gamma\phi a_t \tag{35}$$

$$\log C_t - \bar{c} = \phi a_t \tag{36}$$

$$\phi = \frac{1}{2(1-\beta)} \pm \sqrt{\frac{1}{4(1-\beta)^2} - \frac{\mu}{1-\beta}} \geq 0. \tag{37}$$

where $\mu = \frac{\sigma_v^2}{\sigma_a^2}$. *Given the consumption expenditures of the households, in a rational expectations equilibrium each firm* j *produces*

$$\log C_{jt} - \tilde{c} = c_{jt} = \phi a_t + v_{jt}.$$

For these fundamental equilibria, prices do not fully absorb the a_t shock so output fluctuates.

Let output expectations be driven by sentiment shocks as well as a_t :

$$\log C_t - \bar{c} = c_t = \phi a_t + \sigma_z z_t. \tag{38}$$

The firm's signal now becomes

$$s_{jt} = c_t + v_{jt} = \phi a_t + \sigma_z z_t + v_{jt}$$

Given aggregate consumption, production of the individual firm j is

$$\begin{aligned} c_{jt} &= E\left(a_t + \beta c_t \middle| (c_t + v_{jt})\right) & (39) \\ &= E\left(a_t + \beta \phi a_t + \beta \sigma_z z_t \middle| (\phi a_t + \sigma_z z_t + v_{jt})\right) & (40) \\ &= \frac{(\phi + \beta \phi^2)\sigma_a^2 + \beta \sigma_z^2}{\phi^2 \sigma_a^2 + \sigma_v^2 + \sigma_z^2} (\phi a_t + \sigma_z z_t + v_{jt}). \end{aligned}$$

SKIP: Define the constant terms $\bar{c}$, $\bar{p}$ be

$$\bar{c} = \frac{1}{2\gamma}\left[\frac{\theta-1}{\theta^2}\sigma_v^2 + \frac{(1+\beta\phi)(1-(1-\beta)\phi)\sigma_a^2 - \beta\sigma_z^2(1-\beta)}{\theta^2}\right] \tag{41}$$

$$\bar{p} = \frac{1}{\theta}\log\left(\frac{\theta}{\theta-1}\right) - \gamma\bar{c} \tag{42}$$

Proposition

Suppose that $0 \leq \sigma_v^2 < \frac{1}{4(1-\beta)}\sigma_a^2$ *and* $\tilde{\mu} = \frac{\sigma_v^2 + \sigma_z^2(1-\beta)}{\sigma_a^2} < \frac{1}{4(1-\beta)}$. *There exists a continuum of sentiment-driven equilibra indexed by* σ_z^2,

$$0 < \sigma_z^2 < \frac{1}{4(1-\beta)^2}\sigma_a^2 - \frac{\sigma_v^2}{1-\beta}$$

where prices and optimal consumption expenditures follow:

$$\log P_t - \bar{p} = p_t = \frac{a_t}{\theta} - \gamma\phi a_t - \gamma\sigma_z z_t$$

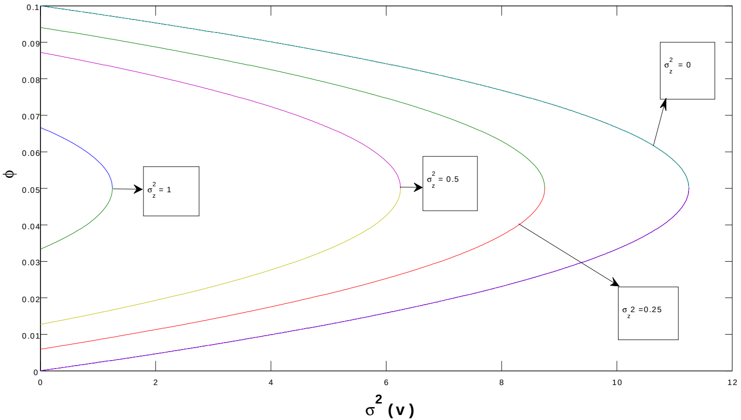
$$\log C_t - \bar{c} = c_t = \phi a_t + \sigma_z z_t$$

where ϕ *is given by*

$$\phi = \frac{1}{2(1-\beta)} \pm \sqrt{\frac{1}{4(1-\beta)^2} - \frac{\tilde{\mu}}{1-\beta}} \tag{43}$$

The two ϕ correspond to strong and weak price responses to A_t shocks.

We can plot the coefficients ϕ for the fundamental equilibria ($\sigma_z^2 = 0$) and the corresponding coefficients ϕ for the sentiment-driven equilibria against variance of the noise σ_v^2. We calibrate $\theta = 10$, $\gamma = 1$, the variance of $\log\ A_t$ at 4.5, and plot ϕ against feasible σ_v^2 for various variances of sentiments $\sigma_z^2 = (0.25,\ 0.5,\ 1)$ and:



Extensions: Heterogenous Sentiments

Households only observe A_t and the noisy signal $s_{it}^h = z_t + e_{it}$ and conjecture

$$\begin{aligned}\log C_t - \bar{c} &= c_t = \phi a_t + \sigma_z z_t \\ \log P_t - \bar{p} &= p_t = \phi_a^p a_t + \phi_z^p z_t\end{aligned}$$

They choose labor supply to maximize

$$E_i\{A_t \frac{C_{it}^{1-\gamma}}{1-\gamma} - N_t\}|[A_t, s_{it}^h]. \tag{44}$$

$$subject\ to\ \ P_t C_{it} \leq N_{it} + \Pi_t, \tag{45}$$

where Π_t is the share of total profits accruing from all the firms. The first order condition for consumers now changes to

$$C_{it} = \left\{ \frac{1}{E(P_t|s_{it}^h)} \left(\exp(a_t/\theta) \left(\frac{\theta}{\theta-1} \right) \right) \right\}^{\frac{1}{\gamma}}. \tag{46}$$

Aggregating across consumers, $c_t = \log C_t = \log(\int_0^1 C_{it} di)$.

As before, we assume that each firm receives a noisy signal $\log S_{jt} = c_t + v_{jt}$. The production decision by the firms is given by

$$C_{jt} = \left\{ E[P_t C_t^{\frac{1}{\theta}} | S_{jt}](1 - \frac{1}{\theta}) \right\}^{\theta}. \quad (47)$$

(Note $\sigma_v^2 \geq 0$, can be 0.) We have the following Proposition:

Extensions Cont'd

Proposition

Suppose $\sigma_v^2 < \frac{1}{4(1-\beta)}\sigma_a^2$ *and let* $\kappa = \frac{1}{1+\sigma_e^2}$. *There exists a continuum of sentiment-driven equilibria indexed by* $\sigma_z^2 \in \left(0, \frac{\kappa}{4(1-\beta)^2}\sigma_a^2 - \frac{\kappa\sigma_v^2}{1-\beta}\right)$. *At each equilibrium consumers "correctly" conjecture that aggregate price will be*

$$p_t = \log P_t - \bar{p} = \phi_a^p a_t + \phi_z^p z_t \equiv \left(\frac{1}{\theta} - \gamma\phi\right) a_t - \frac{\gamma}{\kappa}\sigma_z z_t \qquad (48)$$

and that aggregate consumption (output) is

$$\log C_t - \bar{c} = c_t = \phi a_t + \sigma_z z_t$$

$$\phi = \frac{1}{2(1-\beta)} \pm \sqrt{\frac{1}{4(1-\beta)^2} - \frac{\tilde{\mu}}{1-\beta}}$$

where $\tilde{\mu} = \frac{\sigma_v^2 + \sigma_z^2(1-\beta)/\kappa}{\sigma_a^2}$.

Proposition

Consumers' demand for each intermediate good is:

$$\log C^c_{it} - \tilde{c} = c_{it} = \phi a_t + \sigma_z (z_t + e_{it})$$

But each individual firm's optimal production is:

$$\log C^f_{jt} - \hat{c} = c_{jt} = \phi a_t + \sigma_z z_t + v_{jt}$$

The constant terms are given by

$$\bar{p} = \log\left(\frac{\theta}{\theta - 1}\right) - \frac{\theta - 1}{2\theta^2}\sigma_v^2 - \frac{1}{2}\Omega_s \tag{49}$$

$$\bar{c} = \frac{1}{\gamma}\left[\frac{\theta - 1}{2\theta^2}\sigma_v^2 + \frac{1}{2}\Omega_s\right] - \frac{\gamma}{2}\left(\frac{1}{\kappa}\sigma_z\right)^2 (1 - \kappa) + \frac{1}{2}\sigma_z^2 \frac{1 - \kappa}{\kappa} \tag{50}$$

$$\hat{c} = \bar{c} - \frac{1}{2}\sigma_z^2\sigma_e^2, \qquad \tilde{c} = \bar{c} - \frac{1}{2}\frac{\theta - 1}{\theta}\sigma_v^2. \tag{51}$$

Conclusion for the first two models

- "Even if economic fundamentals were certain, economic outcomes would still be random. . . . Each economic actor is uncertain about the strategies of the others. Business people, for example, are uncertain about the plans of their customers. . . . This type of economic randomness is generated by the market economy: it is thus endogenous to the economy, but extrinsic to the economic fundamentals."

  Aumann, Peck and Shell (1988)

Private Information and Sunspots in Sequential Asset Markets

Grossman-Stiglitz without noise traders

- The efficient markets hypothesis states that prices on traded assets reflect all publicly available information.
- In their classic work Grossman and Stiglitz (1980) discussed a model where some agents can obtain private information about asset returns, and can trade on the basis of that information.
- If however the rational expectations equilibrium price reveals the information about the asset, and information collection is costly, then agents have no incentive to collect the information before they observe the price and trade.
- But then prices no longer reflect the information about the asset, and markets are no longer efficient.
- A large empirical and theoretical literature has since then explored the informational efficiency of markets under private information.

Grossman-Stiglitz without noise traders

- We study the possibility of rational expectations sunspot equilibria driven by non-fundamentals in asset markets with private information.
- We have no noise traders: all agents in our model are Bayesian optimizers.
- We combine this approach with more recent approches based on informational frictions that arise naturally in our context.
- We show that sunspots can drive equilibria and market prices do not perfectly reveal the asset returns to uninformed traders.
- Additional related literature: Allen, Morris and Shin (2006), Angeletos, Lorenzoni and Pavan (2010), Vives (2008, 2014), Cespa and Vives (2014), Golosov, Lorenzoni, Tysvinski (2014).

Grossman-Stiglitz without noise traders

- In our simplest benchmark model short term traders have noisy information about the return or dividend yield of the asset, but hold and trade the asset before its return is realized at maturity.
- The returns to short-term traders consist of capital gains.
- Investors, on the other hand, who may not have private information about the returns or dividend yields but observe past and current prices, purchase and hold the asset for its final dividend return.
- We show that under such a market structure, in addition to equilibria where equilibrium prices fully reveal asset returns as in Grossman and Stiglitz (1980), there also exists a continuum of equilibria with prices driven by sunspot shocks.
- Furthermore the sunspot or sentiment shocks generate persistent fluctuations in the price of the risky asset that look to the econometrician like a random walk in an efficient market driven by fundamentals.

Benchmark Model

- We start with a three period benchmark model with a continuum of short-term traders and long-term investors.
- We index the short-term trader by j and the long-term investor by i.
- In period 0 there is a continuum of short-term traders of unit mass endowed with 1 unit of an asset, a Lucas tree. This tree yields a dividend D in period 2. We assume that

$$\log D = \theta. \tag{52}$$

  where $\theta \sim N\left(-\frac{1}{2}\sigma_\theta^2, \sigma_\theta^2\right)$ so $E(D) = 1$.
- In period 1 each short-term trader sells the asset and receives utility before D is realized in period 2.
- This short-term trader, maybe because he is involved in creating and structuring the asset, receives a signal s_j in period 0

$$s_j = \theta + e_j \tag{53}$$

  where $e_j \sim N\left(0, \sigma_e^2\right)$, independent of θ.

Short-Term traders

- So the short-term trader j in period 0 solves

$$\max_{x_{j0,}B_{j0}} \mathbb{E}[C_{j1}|P_0, s_j] \tag{54}$$

with the budget constraints

$$P_0 x_{j0} + B_{j0} = P_0 + w \tag{55}$$

$$C_{j1} = P_1 x_{j0} + B_{j0.} \tag{56}$$

where w is his endowment or labor income, x_{j0}, is the quantity of the asset and B_{j0} is a safe bond that he carries over to period 1.

- We assume, at the moment, that there is no restriction on B_{j0}. Therefore using the budget constraint we can rewrite the short-term trader j's problem as

$$\max_{x_{j0}\in(-\infty,+\infty)} \mathbb{E}[P_0 + w + (P_1 - P_0)x_{j0}|P_0, s_j] \tag{57}$$

Investors

- There is a continuum of investors of unit mass in period 1, each endowed with w, who trade with the short-term traders and enjoy consumption in period 2 when the dividend D is realized.
- These investors solve a similar problem, but have no direct information about the dividend of the Lucas tree, except through the prices they observe. Hence an investor i in period 1, solves

$$\max_{x_{i1}, B_{i1}} \mathbb{E}[C_{i2} | P_0, P_1] \tag{58}$$

$$P_1 x_{i1} + B_{i1} = w \tag{59}$$

$$C_{i2} = D x_{i1} + B_{i1.} \tag{60}$$

where w is his endowment, x_{i1} is their asset purchase, and B_{i1} is his bond holdings carried over to period 2.

- The objective function (58) can be written as

$$\max_{x_{i1} \in (-\infty, +\infty)} \mathbb{E}[w + (D - P_1) x_{i1} | P_0, P_1], \tag{61}$$

after substituting out B_{i1} from the budget constraints.

Equilibrium

An equilibrium is a pair of prices $\{P_0, P_1\}$ such that x_{j0} solves problem (57) and x_{i1} solves problem (61), and markets clear. Formally we define our equilibrium as follows:

Definition

An equilibrium is individual portfolio choices $x_{j0} = x(P_0, s_j)$ for the short-term traders in period 0, $x_{i1} = y(P_0, P_1)$ for the long-term investors in period 1, and two price functions $\{P_0 = P_0(\theta), P_1 = P_1(\theta)\}$ that jointly satisfy market clearing and individual optimization,

$$\int x_{j0} dj = 1 = \int x_{i1} di, \tag{62}$$

$$P_0 = \mathbb{E}[P_1 | P_0, s_j] \tag{63}$$

for all $s_j = \theta + e_j$, and

$$P_1 = \mathbb{E}\left[D | P_0, P_1\right], \tag{64}$$

where expectations are Bayesian optimal.

Equilbrium Cont'd

- Equation $\int x_{j0} dj = 1 = \int x_{i1} di$ gives market clearing.
- $P_0 = \mathbb{E}[P_1 | P_0, s_j]$ gives the first order conditions for an interior optimum for the short term trader, so he is indifferent to buying or selling the asset.
- Equation $P_1 = \mathbb{E}\left[D | P_0, P_1\right]$, the first order condition for the long term investors, says that the price that the long term investor is willing to pay is equal to their Bayesian updating of the dividend.
- Under these interior first order conditions our risk-neutral agents are indifferent about the amount of the asset they carry over, so for simplicity we may assume a symmetric equilibrium with

$$x_{i0} = x = 1, \text{ and } x_{i1} = x = 1$$

- Hence the market clearing condition holds automatically. In what follows, we only need to check equations (63) and (64) to verify an equilibrium

Revealing Equilibrium

Proposition: $P_0 = P_1 = \exp(\theta)$ is always an equilibrium.
Proof: The proof is straightforward. It is easy to check that both (63) and (64) are satisfied.

- In this case, the market price fully reveals the fundamental values.
- Whatever their individual signal, traders in period 0 will be happy to trade at $P_0 = \exp(\theta)$, which reveals the dividend to investors in period 1.
- Even though each trader j in period 0 gets a noisy private signal s_j about θ, which may be high or low, these traders ignore their signal because in maximizing their utility they only care about the price at which they can sell next period.
- If each short term trader believes the price in the next period depends on θ, competition in period 0 will then drive the market price exactly to $\exp(\theta)$. (How? See Implementability Section.)

Revealing Equilibrium Cont'd

- For a given market price, the expected payoff of holding one additional asset will be $\mathbb{E}\left\{[\exp(\theta) - P_0] \,|P_0, s_j\right\}$.
- As long as $\log P_0 \neq \theta$, traders with low signals would want to short the risky asset while other traders with high signals would want to go long on the risky asset.
- An equilibrium can only be reached when the market price has efficiently aggregated all private information in such a way that idiosyncratic signals cannot provide any additional profits based on private information: namely $\log P_0 = \int s_j dj = \theta$.
- Since price fully reveals the dividend, the long term investors will be happy to pay $\log P_1 = \theta$ in the next period.

Non-Revealing Equilibrium

There is however a second equilibrium where the market price reveals no information about dividends.

- **Proposition:** $P_0 = P_1 = 1$ is always an equilibrium.
- **Proof:** Both first order conditions (63) and (64) are satisfied. It is clear that with $P_0 = P_1 = 1$, investors in period 1 obtain no information about the dividend as the prices simply reflect the unconditional expectation of the dividends in period 2.
- Again in the above equilibrium, the short-term traders "optimally" ignore their private signals. If the short-term traders believe that the price in the next period is independent from θ, then their private signal s_j is no long relevant for their payoff, and these signals become irrelevant.

Sentiment-Driven Equilibria, Cont'd

- We now assume the traders in period 0 also receive some sentiment or sunspot shock $z \sim N(0,1)$ which they believe will drive prices.

Definition

An sentiment-driven equilibrium is given by optimal portfolio choices $x_{j0} = x(P_0, s_j, z)$ for the short-term trader in period 0, $x_{i1} = y(P_0, P_1)$ for the long-term investors in period 1, and two price functions $\{P_0 = P_0(\theta, z), P_1 = P_1(\theta, z)\}$ that jointly satisfy market clearing and individual optimization,

$$\int x_{j0}\,dj = 1 = \int x_{i1}\,di, \tag{65}$$

$$P_0 = \mathbb{E}[P_1 | P_0, s_j, z] \tag{66}$$

for all $s_j = \theta + e_j$ and z, and

$$P_1 = \mathbb{E}\left[D | P_0, P_1\right], \tag{67}$$

Proposition: There exists an continuum of sentiment driven equilibria indexed by $0 \leq \sigma_z \leq \frac{1}{4}\sigma_\theta^2$, with $x_{i0} = 1 = x_{j1}$ and the prices in two periods given by

$$\log P_1 = \log P_0 = \phi\theta + \sigma_z z, \tag{68}$$

where

$$0 \leq \phi = \frac{1}{2} \pm \sqrt{\frac{1}{4} - \frac{\sigma_z^2}{\sigma_\theta^2}} \leq 1. \tag{69}$$

Proof: Note that since the prices are the same in both periods, $P_0 = \mathbb{E}[P_1|P_0, s_j, z]$ is satisfied automatically. We only need to check if equation $P_1 = \mathbb{E}\left[D|P_0, P_1\right]$ is satisfied. Taking logs :

$$\begin{aligned} \log P_1 &= \phi\theta + \sigma_z z \\ &= \log \mathbb{E}\{\exp(\theta|\phi\theta + \sigma_z z)\}, \\ &= \mathbb{E}\left[\theta|\phi\theta + \sigma_z z\right] + \frac{1}{2} var(\theta|\phi\theta + \sigma_z z) \\ &= -\frac{1}{2}\sigma_\theta^2 + \frac{\phi\sigma_\theta^2}{\phi^2\sigma_\theta^2 + \sigma_z^2}\left[\phi\theta + \sigma_z z + \frac{\phi}{2}\sigma_\theta^2\right] \\ &\quad + \frac{1}{2}\left[\sigma_\theta^2 - \frac{\left(\phi\sigma_\theta^2\right)^2}{\phi^2\sigma_\theta^2 + \sigma_z^2}\right]. \end{aligned} \tag{70}$$

which follows from the property of the normal distribution. Comparing terms, coefficients of $\phi\theta + \sigma_z z$ yields

$$\frac{\phi\sigma_\theta^2}{\phi^2\sigma_\theta^2 + \sigma_z^2} = 1. \tag{71}$$

Solving equation (71) yields the expression of ϕ in the Proposition above.

Intuition

- In this case traders in period 0 get a common sunspot shock z.
- The investors in period 1, in forming their expectation of D conditional on the prices, believe prices are affected by the sunspot z.
- But they have a signal extraction problem distinguishing θ from z.
- For example, a low z will induce pessimistic expectations for the period 0 traders, who will pay a low price for the asset and expect a low price next period.
- The investor in period 1 will observe the period 0 price and infer that in part, this must be due to a low dividend yield, which will lead him to also pay a low price in period 1, thus confirming the expectations of the period 0 trader.
- For equilibria to be possible for every realization of the sunspot z, the variance of z that enters the signal extraction problem of the investor in period 1 must lie in the interval given in the above Proposition.
- Investors' first order conditions will then be satisfied in equilibrium, generating a continuum of sunspot equilibria indexed by σ_z^2.

Heterogenous but Correlated Sentiments, Market Signals on the Dividend and on Sunspots

- We relax the assumption that only the short-term investor receives information about the dividend through a private signal,and allow heterogenous signals
- We allow both the short term trader and the investor to receive private information on the dividend θ. The information set is

$$\Omega_0 = \{P_0, \theta_0 + e_j, z + \varepsilon_j\} \text{ and } \Omega_1 = \{P_0, P_1, \theta_1 + v_i\}$$

- ε_j are drawn from a normal distribution with mean of 0 and variance of σ_ε^2 and $cov(e_j, \varepsilon_j) = 0$.
- $s_{j0} = \theta_0 + e_j$ is the private signal on the dividend received by a trader j in the first period, and $s_{i1} = \theta_1 + v_i$ is the signal of the investor i in the second period.
- We assume that $cov(\theta_0, \theta) > 0$ and $cov(\theta_1, \theta) > 0$, but $cov(\theta_0, \theta_1) = 0$. For example, $\theta = \alpha\theta_0 + (1-\alpha)\theta_1$ with $0 < \alpha < 1$, but $cov(\theta_0, \theta_1) = 0$ satisfies these assumptions.
- W/O loss of generality can assume $\theta = \theta_0 + \theta_1$, $\theta_0 \sim N\left(-\frac{1}{2}\sigma_{\theta_0}^2, \sigma_{\theta_0}^2\right)$, $\theta_1 \sim N\left(-\frac{1}{2}\sigma_{\theta_0}^2, \sigma_{\theta_0}^2\right)$ and $v_i \sim N\left(0, \sigma_v^2\right)$.

Conclusion

- We study a market where sequential short term traders have private information and earn capital gains by trading a risky asset before it yields dividends, while uninformed investors purchase the asset for its dividend yield, forming expectations based on observed prices. No noise traders
- In a rational expectation equilibrium, prices based on fundamentals can reveal the information of private traders.
- We show that there are also rational expectations equilibria driven by sunspots that do not fully reveal private information.
- We show that our results on sunspot equilibria are robust to a wide range of on informational assumptions and market structures.
- If an econometrician studies the asset data generated by these sunspot equilibria, they will find that the asset prices follow a random walk that look as if they are generated by an efficient market reflecting fundamental values.