Endogenous information acquisition

ECON 101

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The Baseline Mode

I

The economy is populated by a large representative household that has a continuum of identical workers and a continuum of entrepreneurs, with a unit measure of each. The household derives utility from leisure and from consumption of a composite final good produced with a continuum of differentiated intermediate goods. Workers supply labor to entrepreneurs in a competitive labor market. Entrepreneur j is the monopolist of differentiated intermediate good j. The demand for each intermediate good j is affected by an idiosyncratic demand shock ϵ_{jt} and by aggregate demand driven by an aggregate productivity shock A_t. At the beginning of each period, after observing the aggregate productivity shock A_t, entrepreneur j decides whether to acquire information regarding ϵ_{jt}, and then produces accordingly. At the end of each period, the workers and entrepreneurs pool their wage and profit income for the household.

The Representative Household The household maximizes its utility

$$\max E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{C_t^{1-\gamma}}{1-\gamma} - \psi N_t - \psi N_{et} \right] \quad \text{for } \gamma < 1, \tag{1}$$

where C_t is the consumption of the household, N_t is the total working hours of workers, and N_{et} is the total working hours entrepreneurs spend on information acquisition (to be specified later). Note that in our model without capital, a reasonable parameter specification requires setting γ below 1.

The budget constraint for the household is

$$P_t C_t \leq W_t N_t + \Pi_t, \tag{2}$$

where P_t is the price of the consumption good, W_t is the nominal wage, and Π_t denotes total profit income earned by entrepreneurs. For the representative household there is no transfer of resources between periods, so the infinite-horizon maximization problem becomes the repeated one-period maximization problem.

The first-order condition of the maximization problem of (1) yields

$$\psi C_t^{\gamma} = \frac{W_t}{P_t}. \tag{3}$$

When making its consumption decision (or labor supply decision) according to (3), the representative household sees the nominal wage, W_t, and it forms (rational) expectations of the equilibrium aggregate price P_t and thus of the real wage, $\frac{W_t}{P_t}$.

In our baseline model, there is no aggregate uncertainty, so P_t is deterministic and is perfectly foreseen under rational expectations.

The Final Good Producer The consumption good is produced by a competitive final good firm facing competitive factor markets according to the Dixit-Stiglitz aggregate production function:

$$Y_t = \left[\int_0^1 \epsilon_{jt}^{\frac{1}{\theta}} y_{jt}^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}} \quad \text{for } \theta > 1. \tag{4}$$

The final good producer maximizes its profit:

$$\max_{y_{jt}} P_t \left[\int_0^1 \epsilon_{jt}^{\frac{1}{\theta}} y_{jt}^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}} - \int_0^1 p_{jt} y_{jt} dj,$$

where p_{jt} is the price of intermediate good j. The first-order condition with respect to input y_{jt} is

$$y_{jt} = \left(\frac{p_{jt}}{P_t} \right)^{-\theta} \epsilon_{jt} Y_t, \tag{5}$$

which shows that the demand for intermediate good j is affected by both idiosyncratic shock ϵ_{jt} and aggregate demand Y_t. We assume that ϵ_{jt} is independent across time and goods and $\log \epsilon_{jt} \equiv \varepsilon_{jt} \sim \mathcal{N}(-\frac{1}{2}\sigma_\varepsilon^2, \sigma_\varepsilon^2)$, so the mean of ϵ_{jt} is $\mathbb{E}\epsilon_{jt} = 1$. Denote $\tau_\varepsilon = 1/\sigma_\varepsilon^2$.

Intermediate Goods Producers Entrepreneurs are the producers of intermediate goods. Entrepreneur j is the monopolist of intermediate good j with production function

$$y_{jt} = A_t n_{jt}. \tag{6}$$

Entrepreneur j produces y_{jt} to maximize his profit under demand uncertainty driven by ϵ_{jt}. To reduce the uncertainty before production, entrepreneur j can spend m working hours to acquire some information about ϵ_{jt} (for example, via a market survey). If he chooses to do so, he receives a signal given by

$$s_{jt} = \varepsilon_{jt} + e_{jt},$$

where $e_{jt} \sim \mathcal{N}(0, \sigma_e^2)$, so the precision of the signal is $\tau_e = 1/\sigma_e^2$. If the entrepreneur does not acquire information, he knows only the prior, unconditional distribution of ε_{jt}; equivalently, he receives a useless signal s_{jt} with $\sigma_e^2 = \infty$.

Entrepreneurs hire workers based on nominal wage before the actual production and trades take place. Entrepreneurs of course have to form expectations of the equilibrium aggregate price P_t and hence the real wage when they make their hiring decision.
An informed entrepreneur j chooses y_{jt} to maximize his expected profit

$$y_{jt} = y(s_{jt}) = \arg\max_{y_{jt}} \mathbb{E}\left[p_{jt} y_{jt} - W_t n_{jt} | s_{jt}\right] \tag{7}$$

with constraints (5) and (6).[1] Here $\mathbb{E}(\cdot|s_{jt})$ is the conditional expectation operator over ϵ_{jt}. Denote the realized profit for an informed entrepreneur by $\pi(\epsilon_{jt}, s_{jt}) = p_{jt}\left(\epsilon_{jt}, y_{jt}\right) y_{jt} - W_t n_{jt}\left(y_{jt}\right)$.

[1] The profit in terms of utility units is the amount of the profit multiplied by ψ. Maximizing profits is equivalent to maximizing the "shareholder's value".

Likewise, an uninformed entrepreneur j solves

$$\tilde{y}_{jt} = \arg\max_{\tilde{y}_{jt}} \mathbb{E}\left[p_{jt}\tilde{y}_{jt} - W_t n_{jt}\right] \tag{8}$$

with constraints $\tilde{y}_{jt} = \left(\frac{p_{jt}}{P_t}\right)^{-\theta} \epsilon_{jt} Y_t$ and $\tilde{y}_{jt} = A_t n_{jt}$. Here $\mathbb{E}$ is simply the unconditional expectation operator over ϵ_{jt}. Denote the realized profit for an uninformed entrepreneur by $\tilde{\pi}_t(\epsilon_{jt}) = p_{jt}\left(\epsilon_{jt}, \tilde{y}_{jt}\right)\tilde{y}_{jt} - W_t n_{jt}\left(\tilde{y}_{jt}\right)$. Throughout the paper, we normalize the wage as the numeraire price:

$$W_t = 1.$$

Information Acquisition In order to acquire a signal s_{jt}, an entrepreneur needs to spend a fixed amount of time m. The ex ante expected profit for a firm acquiring information is

$$\pi_t^I = \mathbb{E}_{\epsilon_{jt}, s_{jt}} [\pi(\epsilon_{jt}, s_{jt})] = \mathbb{E}_{s_{jt}} \mathbb{E}_{\epsilon_{jt}|s_{jt}} [\pi(\epsilon_{jt}, s_{jt})|s_{jt}] .$$

Throughout the paper, $\mathbb{E}_x(\cdot)$ denotes the unconditional expectation operator over x and $\mathbb{E}_{x|y}(\cdot|y)$ denotes the conditional expectation operator over x conditional on y. The ex ante expected profit for an entrepreneur not acquiring information is

$$\pi_t^U = \mathbb{E}_{\epsilon_{jt}} [\tilde{\pi}_t(\epsilon_{jt})] .$$

As entrepreneurs are identical ex ante, all of them will acquire information if $\pi_t^I - m > \pi_t^U$ and none will acquire information if $\pi_t^I - m < \pi_t^U$. If $\pi_t^I - m = \pi_t^U$, entrepreneurs are indifferent in acquiring information or not. Denote by λ_t the fraction of entrepreneurs who acquire information. We must have

$$\begin{cases} \pi_t^I - m > \pi_t^U & \text{if } \lambda_t = 1 \\ \pi_t^I - m = \pi_t^U & \text{if } \lambda_t \in (0,1) \\ \pi_t^I - m < \pi_t^U & \text{if } \lambda_t = 0 \end{cases} . \tag{9}$$

Timeline We summarize the sequence of actions by consumers and firms, the information structure, and the rational expectations equilibria of our baseline model.

1. At the beginning of each period, after observing A_t, an entrepreneur makes his decision on whether to acquire a signal about ϵ_{jt}. Signal s_{jt} is obtained if he pays a constant cost m in terms of working hours; otherwise no signal (or a useless signal) is obtained.
2. Based on signal s_{jt}, nominal wage $W_t \equiv 1$ and rational expectations of P_t (or Y_t), an informed entrepreneur decides how much labor n_{jt} to hire to produce his intermediate good. An uninformed entrepreneur chooses n_{jt} based on the prior of ϵ_{jt}.
3. Given the production of y_{jt} and $\tilde{y}_{jt}$, price p_{jt} adjusts to equate demand and supply according to equation (5).
4. Goods markets open. Goods are exchanged at market clearing prices. The final consumption is realized.

The formal definition of equilibrium in our baseline model is as follows.

Definition

An REE is a sequence of aggregate allocations $\{C(A_t), Y(A_t), N(A_t), \Pi(A_t), \lambda(A_t)\}$, individual productions $y_{jt} = y(A_t, s_{jt})$ for informed entrepreneurs and $y_{jt} = \tilde{y}(A_t)$ for uninformed entrepreneurs, and prices $\{P(A_t), p(s_{jt}, \epsilon_{jt})\}$, such that for each realization of A_t, (i) $C(A_t)$ and $N(A_t)$ maximize households' utility given the equilibrium price $P_t = P(A_t)$ and aggregate profit $\Pi(A_t)$; (ii) equation (5) maximizes the final good firm's profit given shocks ϵ_{jt} and equilibrium prices $p(s_{jt}, \epsilon_{jt})$; (iii) given P_t and signal s_{jt}, $y(A_t, s_{jt})$ maximizes the expected profit of an informed entrepreneur and $\tilde{y}(A_t)$ maximizes the expected profit of an uninformed entrepreneur; (iv) A $\lambda(A_t)$ fraction of entrepreneurs acquire information about their ϵ_{jt}, so $\Pi(A_t) = \lambda_t \pi_t^I + (1 - \lambda_t)\pi_t^U$; and (v) all markets clear, namely, $C(A_t) = Y(A_t)$ and $N_t = \int_0^1 \frac{y_{jt}}{A_t} dj$.

Characterization of Equilibrium

- First, we work out a firm's optimal production given its information acquisition decision.
- Next, we aggregate all firms' production to obtain the aggregate output Y_t as a function of λ_t and A_t.
- Then, we compare π_t^I and π_t^U to solve firms' information acquisition problem, which yields another function involving λ_t and Y_t.
- Finally, we use these two functions to determine Y_t and λ_t simultaneously as functions of A_t.

Equilibrium Y_t for a Given λ_t Substituting (5) and (6) into (7), we have

$$y_{jt}(s_{jt}) = \arg\max_{y_{jt}} \mathbb{E}\left[\left(P_t \cdot y_{jt}^{1-\frac{1}{\theta}}(\epsilon_{jt} Y_t)^{\frac{1}{\theta}} - \frac{1}{A_t} y_{jt}\right) | s_{jt}\right] \tag{10}$$

for an informed entrepreneur. This yields

$$y_{jt} = y(A_t, s_{jt}) = \left(1 - \frac{1}{\theta}\right)^{\theta} (P_t A_t)^{\theta} \, Y_t \left[\mathbb{E}(\epsilon_{jt}^{\frac{1}{\theta}} | s_{jt})\right]^{\theta}, \tag{11}$$

where $\left[\mathbb{E}(\epsilon_{jt}^{\frac{1}{\theta}} | s_{jt})\right]^{\theta} = \exp\left[\frac{\tau_e}{\tau_\varepsilon + \tau_e} s_{jt} + \frac{1}{2} \frac{1-\theta}{\theta} \frac{1}{\tau_\varepsilon + \tau_e}\right]$.
Substituting from the labor market eq. $\psi C_t^{\gamma} = \frac{W_t}{P_t}$ with $W = 1$, we get

$$y_{jt} = y(A_t, s_{jt}) = \left(1 - \frac{1}{\theta}\right)^{\theta} (\psi^{\gamma} A_t)^{\theta} \, Y_t^{1-\theta\gamma} \left[\mathbb{E}(\epsilon_{jt}^{\frac{1}{\theta}} | s_{jt})\right]^{\theta}$$

- If $1 - \theta\gamma < 0$, y_{jt} and Y_t are strategic substitutes in production.

Similarly, we obtain the production for an uninformed entrepreneur

$$\tilde{y}_{jt} = \tilde{y}(A_t) = \left(1 - \frac{1}{\theta}\right)^{\theta} (P_t A_t)^{\theta}\, Y_t \left[\mathbb{E}(\epsilon_{jt}^{\frac{1}{\theta}})\right]^{\theta}, \tag{12}$$

where $\left[\mathbb{E}(\epsilon_{jt}^{\frac{1}{\theta}})\right]^{\theta} = \exp(\frac{1}{2}\frac{1-\theta}{\theta}\frac{1}{\tau_\varepsilon})$. Since a λ_t fraction of firms produce according to (11) and $1 - \lambda_t$ of them produce according to (12), the aggregate production defined in equation (4) becomes

$$Y_t = \left(1 - \frac{1}{\theta}\right)^{\theta} (P_t A_t)^{\theta}\, Y_t \left[\begin{array}{c} \int_0^{\lambda_t} \epsilon_{jt}^{\frac{1}{\theta}} \left(\mathbb{E}\left[\epsilon_{jt}^{\frac{1}{\theta}} | s_{jt}\right]\right)^{\theta-1} dj \\ + \int_{\lambda_t}^{1} \epsilon_{jt}^{\frac{1}{\theta}} \left(\mathbb{E}\left[\epsilon_{jt}^{\frac{1}{\theta}}\right]\right)^{\theta-1} dj \end{array}\right]^{\frac{\theta}{\theta-1}}. \tag{13}$$

The labor demand is simply given by $n_{jt} = y_{jt}/A_t$. Hence labor market clearing gives

$$N_t = \frac{1}{A_t}\left(1 - \frac{1}{\theta}\right)^{\theta} (P_t A_t)^{\theta} \, Y_t \left[\begin{array}{c} \int_0^{\lambda_t} \left(\mathbb{E}\left[\epsilon_{jt}^{\frac{1}{\theta}} | s_{jt} \right] \right)^{\theta} dj \\ + \int_{\lambda_t}^{1} \left(\mathbb{E}\left[\epsilon_{jt}^{\frac{1}{\theta}} \right] \right)^{\theta} dj \end{array} \right]. \tag{14}$$

Exploiting the law of iterated expectations (Tedious! See the proof of Proposition 1 in Appendix A of paper), we find

$$\begin{aligned} & \int_0^{\lambda_t} \epsilon_{jt}^{\frac{1}{\theta}} \left(\mathbb{E}\left[\epsilon_{jt}^{\frac{1}{\theta}} | s_{jt} \right] \right)^{\theta-1} dj + \int_{\lambda_t}^1 \epsilon_{jt}^{\frac{1}{\theta}} \left(\mathbb{E}\left[\epsilon_{jt}^{\frac{1}{\theta}} \right] \right)^{\theta-1} dj \\ = & \int_0^{\lambda_t} \left(\mathbb{E}\left[\epsilon_{jt}^{\frac{1}{\theta}} | s_{jt} \right] \right)^{\theta} dj + \int_{\lambda_t}^1 \left(\mathbb{E}\left[\epsilon_{jt}^{\frac{1}{\theta}} \right] \right)^{\theta} dj \end{aligned}$$

Hence, (13) can be transformed into

$$\frac{1}{P_t} = (1 - \frac{1}{\theta})\,(A_t z_t) \tag{15}$$

(13) and (14) together yield

$$Y_t = N_t\,(A_t z_t)\,, \tag{16}$$

where $z_t = z(\lambda_t)$ is given by

$$\begin{aligned} z(\lambda_t) &= \left[\int_0^{\lambda_t} \left(\mathbb{E}\left[\epsilon_{jt}^{\frac{1}{\theta}}|s_{jt}\right]\right)^{\theta} dj + \int_{\lambda_t}^{1} \left(\mathbb{E}\left[\epsilon_{jt}^{\frac{1}{\theta}}\right]\right)^{\theta} dj\right]^{\frac{1}{\theta-1}} \\ &= \left[\lambda_t \exp\left(-\frac{1}{2}\frac{\theta-1}{\theta}\frac{1}{\tau_\varepsilon+\tau_e}\right) + (1-\lambda_t)\exp\left(-\frac{1}{2}\frac{\theta-1}{\theta}\frac{1}{\tau_\varepsilon}\right)\right]^{\frac{1}{\theta-1}} \end{aligned} \tag{17}$$

which is the **endogenous TFP**.

Denoting $\overline{z} = \exp\left(-\frac{1}{2}\frac{1}{\theta}\frac{1}{\tau_\varepsilon + \tau_e}\right)$ and $\underline{z} = \exp\left(-\frac{1}{2}\frac{1}{\theta}\frac{1}{\tau_\varepsilon}\right)$, we have **endogenous TFP:**

$$z(\lambda_t) = \left[\lambda_t \overline{z}^{\theta-1} + (1-\lambda_t)\,\underline{z}^{\theta-1}\right]^{\frac{1}{\theta-1}}. \tag{18}$$

where $z(\lambda_t = 1) = \overline{z}$ and $z(\lambda_t = 0) = \underline{z}$.

- It is easy to see that $z'(\lambda_t) > 0$
- That is, if more firms acquire information, the aggregate production becomes more efficient. In fact, efficient allocation requires more resources to be allocated to firms with higher realized ϵ_{jt}, that is, efficient production should be contingent on realized ϵ_{jt}.
- So, more precise information about ϵ_{jt} achieved through information acquisition helps improve allocative efficiency.
- Equations (15), $\frac{1}{P_t} = (1 - \frac{1}{\theta})(A_t z_t)$ and (16) $Y_t = N_t (A_t z_t)$ are intuitive.
- (16) implies that despite heterogeneity among firms originating in idiosyncratic demand shocks, our economy works as if there existed a representative firm with productivity $A_t z(\lambda_t)$.
- (15) means that the real wage, $\frac{1}{P_t}$, is proportional to labor productivity $A_t z(\lambda_t)$ where the proportion $1 - \frac{1}{\theta}$ is the share of labor cost in aggregate GDP (i.e., the average profit-to-revenue ratio in the economy is $\frac{1}{\theta}$).

In equilibrium, (3), $\psi C_t^\gamma = \frac{W_t}{P_t}$, becomes $P_t = \frac{1}{\psi} Y_t^{-\gamma}$, which together with (15), $\frac{1}{P_t} = \left(1 - \frac{1}{\theta}\right)(A_t z_t)$, yields aggregate output as a function of λ_t and A_t:

$$Y_t\left(A_t, \lambda\right) = \left(1 - \frac{1}{\theta}\right)^{\frac{1}{\gamma}} \left(\frac{A_t z(\lambda_t)}{\psi}\right)^{\frac{1}{\gamma}}. \tag{19}$$

This implies that the aggregate production increases with λ_t.

Equilibrium λ_t under Information Acquisition We now turn to firms' information acquisition problem for another relationship between λ_t and Y_t. Exploiting the law of iterated expectations (see the proof of Proposition 1 in Appendix A), we obtain the ex ante expected profit for an informed entrepreneur,

$$\pi_t^I = \frac{1}{\theta - 1}\frac{1}{A_t} Y_t z_t^{-\theta} \overline{z}^{\theta-1},$$

and the expected profit for an uninformed entrepreneur,

$$\pi_t^U = \frac{1}{\theta - 1}\frac{1}{A_t} Y_t z_t^{-\theta} \underline{z}^{\theta-1}.$$

In fact, by using (15), $\frac{1}{P_t} = \left(1 - \frac{1}{\theta}\right)(A_t z_t)$, the following relationship holds:

$$\frac{1}{\theta} = \frac{\lambda_t \pi_t^I + (1 - \lambda_t)\,\pi_t^U}{Y_t P_t};$$

namely, the average profit-to-revenue ratio in the economy is $\frac{1}{\theta}$.

It is easy to see that $\pi_t^I > \pi_t^U$. So informed entrepreneurs always enjoy a higher expected profit. In other words, information is valuable to firms. However, acquiring information is costly. If $\lambda_t \in (0,1)$, (9) implies

$$\pi_t^I - \pi_t^U = \frac{1}{\theta-1}\frac{1}{A_t} Y_t z_t^{-\theta}\left(\overline{z}^{\theta-1} - \underline{z}^{\theta-1}\right) = m \tag{20}$$

in equilibrium. Substituting $A_t = \frac{\theta}{\theta-1}\frac{\psi Y_t^{\gamma}}{z_t}$ from (19), $Y_t = \left(1-\frac{1}{\theta}\right)^{\frac{1}{\gamma}}\left(\frac{A_t z(\lambda_t)}{\psi}\right)^{\frac{1}{\gamma}}$, into (20), we obtain the second equilibrium relationship between Y_t and λ_t:

$$\frac{1}{\theta}\left(z(\lambda_t)\right)^{1-\theta} Y_t^{1-\gamma}\left(\overline{z}^{\theta-1} - \underline{z}^{\theta-1}\right) = \psi m. \tag{21}$$

Under $\gamma < 1$, equation (21) defines λ_t as an increasing function of Y_t. The LHS of equation (21) is the benefit of acquiring information in *utility units*, and the RHS is the utility loss from foregoing leisure. When Y_t increases, the benefit increases (under $\gamma < 1$), leading to stronger incentives to acquire information. Conversely, under $\gamma > 1$, (21) defines λ_t as a decreasing function of Y_t.

Put slightly differently, we can rewrite (21) as

$$\frac{1}{\theta} Y_t \left(z(\lambda_t)\right)^{1-\theta} \left(\overline{z}^{\theta-1} - \underline{z}^{\theta-1}\right) = \frac{m}{\frac{1}{\psi} Y_t^{-\gamma}}. \tag{22}$$

The LHS of (22) is the benefit of information acquisition in *consumption units*, and the RHS is the cost in terms of real wage (by noting $P_t = \frac{1}{\psi} Y_t^{-\gamma}$). Under $\gamma < 1$, the real wage does not increase as fast as aggregate output Y_t. Hence, when Y_t goes up, the increase in the benefit outruns the increase in the cost, so an individual entrepreneur has incentives to switch from being uninformed to being informed by paying the cost. As λ_t goes up, the equilibrium (equation (22)) will be restored. Under $\gamma > 1$, the opposite applies.

Full Equilibrium Equations (19) and (21) jointly determine Y_t and λ_t. Substituting the expression of Y_t in (19) into the LHS of (21), we have

$$\frac{1}{\theta}\left[\left(1-\frac{1}{\theta}\right)\frac{1}{\psi}\right]^{\frac{1-\gamma}{\gamma}} A_t^{\frac{1-\gamma}{\gamma}} (z(\lambda_t))^{\frac{1-\theta\gamma}{\gamma}} \left(\overline{z}^{\theta-1} - \underline{z}^{\theta-1}\right) = \psi m. \tag{23}$$

When $\theta\gamma > 1$, the LHS of (23) is decreasing in λ_t; that is, when more other firms acquire information, the benefit of information acquisition for a particular individual firm is decreasing. In other words, when $\gamma \in \left(\frac{1}{\theta}, \infty\right)$, information acquisition is a strategic substitute across intermediate goods firms; when $\gamma \in \left(0, \frac{1}{\theta}\right)$, it is a strategic complement. Lemma 1 follows.

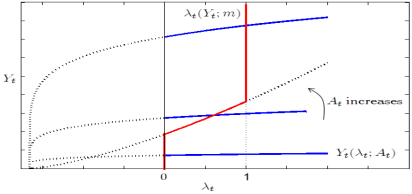
Lemma

Strategic complementarity (substitutability) in information acquisition coincides with strategic complementarity (substitutability) in production.

By (11) together with (3), y_{jt} is decreasing in Y_t iff $\theta\gamma > 1$. In fact, from (5) and (3) together with (11), an increase in aggregate output Y_t has two opposite effects on a particular individual firm in produciton: the demand curve shifts upward and the production cost (in terms of real wage) also goes up due to the general equilibrium effect. Under $1 - \theta\gamma > 0$, the first effect is stronger than the second effect, so production of intermediate goods firms exhibits strategic substitutability. We will show that when information acquisition is with strategic complementarity (i.e., $\gamma \in \left(0, \frac{1}{\theta}\right)$), equilibrium multiplicity can arise. Based on the above analysis, we can partition γ into three regions: $\gamma \in \left(0, \frac{1}{\theta}\right)$, $\gamma \in \left(\frac{1}{\theta}, 1\right)$, and $\gamma \in (1, \infty)$. We discuss three case of γ in order.

i) Case of $\gamma \in \left(\frac{1}{\theta}, 1\right)$

We mainly focus on this case because of the empirical relevance of the parameters. In this case, equation (21) defines λ_t as an increasing function of Y_t and information acquisition is with strategic substitutability. Figure 1 gives a diagrammatic analysis of the full equilibrium for this case.



In Figure 1, $Y_t(\lambda_t; A_t)$ is given by (19) while $\lambda_t(Y_t; m)$ is given by (21).The vertical lines $\lambda_t = 0$ and $\lambda_t = 1$ correspond to corner solutions in (9). When output Y_t is below a threshold, no entrepreneur will acquire information (at $\lambda_t = 0$). If output Y_t is very large, all entrepreneurs acquire information (at $\lambda_t = 1$). When output is in an intermediate range, an increase in output enhances the incentive to acquire information (under $\gamma < 1$), so $\lambda_t(Y_t; m)$ is upward sloping. By (19), $Y_t(\lambda_t; A_t)$ is also upward sloping. Under $\gamma > \frac{1}{\theta}$ (strategic substitutability in information acquisition), the slope of the curve given by (19) is smaller than that given by (21) at their interior intersection (i.e., $\frac{1}{\theta-1}\frac{1}{\gamma} < \frac{1}{1-\gamma}$), so the equilibrium is always unique. When A_t increases, the unique equilibrium shifts toward the upper-right corner, which means that both Y_t and λ_t increase (weakly).

Once we have λ_t and Y_t, it is straightforward to obtain the rest of the variables. Based on (16) and (19), we have the aggregate labor:

$$N_t = \left(1 - \frac{1}{\theta}\right)\left(\frac{1}{\psi}\right) Y_t^{1-\gamma}. \tag{24}$$

We can apply equation (15) to obtain the aggregate price P_t, equations (11) and (12) to get firms' production y_{jt}, and equation (5) to get price p_{jt}. The following proposition summarizes the equilibrium.

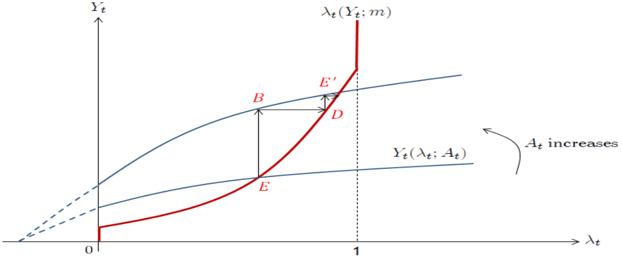
Proposition

If $\gamma \in \left(\frac{1}{\theta}, 1\right)$, the equilibrium with endogenous information acquisition is unique; $\log Y_t(A_t; m)$ is continuous and increasing in A_t, and $\lambda_t(A_t; m)$ is continuous and increasing in A_t.

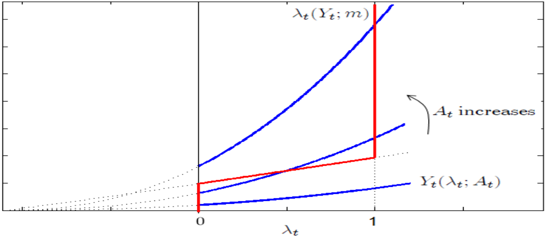
Proof.

See Appendix A. □

Under the endogenous information acquisition, there is an amplification effect. A higher initial TFP shock increases aggregate output, and a higher aggregate output increases incentives for firms to acquire information, which increases aggregate output further (through the endogenous TFP $z(\lambda_t)$), and so on. That is, there is an upward spiral (multiplier effect).



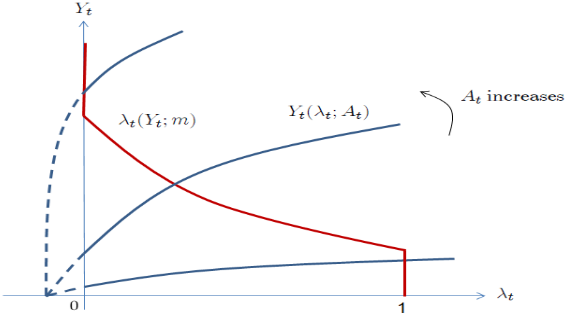
ii) Case of $\gamma \in \left(0, \frac{1}{\theta}\right)$,strategic complementarity.



In the Figure when A_t is sufficiently high or sufficiently low, there is a unique equilibrium (corresponding to the corner solution in (9)). When A_t is in the intermediate range, there are multiple (three) equilibria, corresponding to $\lambda_t = 0$, $\lambda_t = 1$ and the interior (unstable) solution. Note that under $\gamma < \frac{1}{\theta}$ (strategic complementarity in information acquisition), the slope of the curve given by (19) is higher than that given by (21) at their interior intersection (i.e., $\frac{1}{\theta-1}\frac{1}{\gamma} < \frac{1}{1-\gamma}$). The equilibrium of $\lambda_t = 1$ is most efficient and the equilibrium of $\lambda_t = 0$ is east efficient. In the less inefficient equilibrium, firms acquire less information and face higher uncertainty, and the aggregate output is lower.

iii) Case of $\gamma \in (1, \infty)$

For completeness, the case of $\gamma \in (1, \infty)$ is below. In this case, information acquisition is with strategic substitutability and the equilibrium is always unique. Nevertheless, equation (21) defines λ_t as a decreasing function of Y_t. Figure 3 presents the equilibrium in this case. It is clear from Figure 3 that an increase in A_t will lead to less information acquisition (i.e., a lower λ_t) in equilibrium.



Case of $\gamma \in (1, \infty)$

After obtaining the equilibrium Y_t and λ_t, we now discuss the implications of the baseline model.

Idiosyncratic Uncertainty Let $\tau_e = 1/\sigma_e^2$ and $\tau_\varepsilon = 1/\sigma_\varepsilon^2$ In equilibrium, the residual idiosyncratic uncertainty faced by an informed firm is

$$SD\left(\varepsilon_{jt}|s_{jt}\right) = \sqrt{\frac{1}{\tau_e + \tau_\varepsilon}}$$

which is lower than $\sqrt{\frac{1}{\tau_\varepsilon}}$, the residual idiosyncratic uncertainty faced by an uninformed firm. Considering that a higher λ_t is accompanied by a higher Y_t in equilibrium, Corollary 1 follows immediately.

Corollary

In the economy of the baseline model, information acquisition is endogenously procyclical and the idiosyncratic uncertainty faced by the firms is countercyclical (under $\gamma \in \left(\frac{1}{\theta}, 1\right)$).

Corollary 1 is a key result of our paper. It highlights that information acquisition in our model is endogenous and procyclical, which has implications for countercyclical uncertainty faced by firms.

Firm-level Dispersion Empirical literature often uses firm-level dispersion as a proxy for economic uncertainty. We examine two measures of firm-level dispersion. First, we calculate the standard deviation of production (in logs) at the firm level for given A_t and m. As $\log y_{jt} = \log n_{jt} + \log A_t$, employment at the firm level has the same standard deviation as production. We obtain

$$SD\left(\log y_{jt} | A_t\right) \tag{25}$$

$$= \begin{cases} 0 & \text{if } \log A_t < \log \underline{A} \\ \left[\lambda_t \left(\frac{\tau_e}{\tau_\varepsilon + \tau_e} \frac{1}{\tau_\varepsilon}\right) + \frac{\lambda_t - \lambda_t^2}{4\theta^2} \frac{1}{\tau_\varepsilon} \left(\frac{\tau_e}{\tau_\varepsilon + \tau_e}\right)^2\right]^{\frac{1}{2}} \sigma_\varepsilon & \text{if } \log \underline{A} \leq \log A_t \leq \log \overline{A} \\ \left(\frac{\tau_e}{\tau_\varepsilon + \tau_e}\right)^{\frac{1}{2}} \sigma_\varepsilon & \text{if } \log A_t > \log \overline{A} \end{cases}$$

where we calculate volatility as $SD(x) = \sqrt{\mathbb{E}^2(x) - [\mathbb{E}(x)]^2}$. The second line in (25) is increasing in $\lambda_t \in [0, 1]$ under parameter condition $\frac{\tau_e}{\tau_\varepsilon + \tau_e} \frac{1}{\tau_\varepsilon} \leq 4\theta^2$.

Second, we calculate the standard deviation of revenue-based TFP (or productivity) in logs at the firm level, defined as $SD\left(\log\left[\left(y_{jt}p_{jt}\right)/n_{jt}\right]\right)$. As $\log y_{jt} = \log n_{jt} + \log A_t$, the standard deviation of $\log p_{jt}$ at the firm level is the same as that of $\log\left[\left(y_{jt}p_{jt}\right)/n_{jt}\right]$. We obtain

$$SD\left(\log\left[\left(y_{jt}p_{jt}\right)/n_{jt}\right]|A_t\right) \tag{26}$$

$$= \begin{cases} \frac{1}{\theta}\sigma_{\varepsilon} & \text{if } \log A_t < \log \underline{A} \\ \left\{ \begin{array}{l} \lambda_t \left(\frac{1}{\theta}\right)^2 \left(\frac{1}{\tau_{\varepsilon}+\tau_e}\right) + \left(1-\lambda_t\right)\left(\frac{1}{\theta}\right)^2 \frac{1}{\tau_{\varepsilon}} \\ +\lambda_t\left(1-\lambda_t\right)\left[\frac{1}{2\theta^2}\frac{1}{\tau_{\varepsilon}+\tau_e} - \frac{1}{2\theta^2}\frac{1}{\tau_{\varepsilon}}\right]^2 \end{array} \right\}^{\frac{1}{2}} & \text{if } \log \underline{A} \le \log A_t \le \log \overline{A} \\ \frac{1}{\theta}\left(\frac{1}{\tau_{\varepsilon}+\tau_e}\right)^{\frac{1}{2}} & \text{if } \log A_t > \log \overline{A} \end{cases}$$

The second line in (26) is decreasing in $\lambda_t \in [0,1]$ under parameter condition $\frac{\tau_e}{\tau_{\varepsilon}+\tau_e}\frac{1}{\tau_{\varepsilon}} \le 4\theta^2$.

Corollary

Suppose $\frac{\tau_e}{\tau_\varepsilon+\tau_e}\frac{1}{\tau_\varepsilon} \leq 4\theta^2$*. In the economy of the baseline model, the firm-level dispersion in production or employment is procyclical, while the firm-level dispersion in productivity or sale price is countercyclical (under* $\gamma \in \left(\frac{1}{\theta}, 1\right)$*).*

Proof.

See Appendix A. □

- The intuition behind Corollary 2 is the following. In a boom with a higher aggregate output, a larger fraction of firms acquire information and thus their production is more responsive to their true demand shocks, which leads to higher dispersion of production and lower dispersion of productivity across firms.
- We can verify that the dispersion of sales across firms is procyclical.
- Note that more precise information about idiosyncratic shocks ϵ_{jt} leads to firms' sale prices being more similar; in the extreme case where firms have perfect information about idiosyncratic shocks (i.e., $\tau_e = \infty$), for example, their sale prices would be the same (i.e., they achieve the same optimal markup).

- Corollaries 1 and 2 together might give a theoretical clarification for two concepts: economic uncertainty faced by the firms and firm-level dispersion as a measure of uncertainty.
- In our model, firms face a decrease in economic uncertainty in a boom, which is also the case in the recent work of Fajgelbaum, Schaal and Taschereau-Dumouchel (2014) and Straub and Ulbricht (2015).
- However, the uncertainty measured by firm-level dispersion (in production) can increase in a boom. **In other words, firm-level dispersion as a proxy of uncertainty does not necessarily covary with uncertainty.**

Implications for Total Factor Productivity (TFP)

Since the endogenous total factor productivity $z(\lambda_t)$ in equation (17) increases in λ_t, we have the following corollary.

Corollary

Under $\gamma \in \left(\frac{1}{\theta}, 1\right)$, the measured endogenous total factor productivity (TFP) is procyclical.

- The procyclicality of TFP is a well-documented (see, e.g., Rotemberg and Summers (1990), Basu and Fernald (2001)) but it is also a long-standing difficulty for business cycle theories based on demand shocks.
- One traditional explanation is cyclical capital utilization (e.g., Burnside, Eichenbaum and Rebelo (1995), Bai, Rios-Rull and Storesletten (2012)): firms use resources more intensively in booms, so the measured TFP increases.
- The information acquisition mechanism in our model provides an alternative explanation.

Information Structure and Information Acquisition

Each entrepreneur receives two signals: x_{jt} and s_{jt}. First, following Angeletos and La'O (2013a), an entrepreneur receives a sentiment-related "public" signal:

$$x_{jt} = \varepsilon_{jt} + \Delta_t, \text{ where } \Delta_t \sim \mathcal{N}(0, \sigma_\Delta^2), \tag{27}$$

where the noise term, Δ_t, is the economy-wide common sentiment shock about *aggregate demand* (denote $\tau_\Delta = 1/\sigma_\Delta^2$). Second, as in the baseline model, s_{jt} is a "private" signal about the idiosyncratic demand shock, i.e.,

$$s_{jt} = \varepsilon_{jt} + e_{jt}, \text{ where } e_{jt} \sim \mathcal{N}(0, \sigma_e^2). \tag{28}$$

As in the baseline model, we consider discrete information acquisition; namely, if entrepreneur j spends m working hours to acquire information, his signal becomes more precise.

Timeline In the presence of aggregate shock Δ_t (which is imperfect information for entrepreneurs), the aggregate output Y_t and hence the aggregate price P_t are not deterministic. So when making decisions, entrepreneurs have to form expectations about Y_t (or Δ_t). The timing of events in this extension model is as follows:

1. At the beginning of each period, A_t and Δ_t are realized. The representative household has full information regarding Δ_t.
2. After observing A_t, an entrepreneur makes his decision on whether to acquire information or not.
3. Based on signals, x_{jt} and s_{jt}, and nominal wage $W \equiv 1$, an entrepreneur decides how much labor n_{jt} to hire in producing his intermediate good. An entrepreneur has to optimally forecast the real wage W_t/P_t based on A_t and his signals.
4. Given the production y_{jt}, price p_{jt} adjusts to equate demand and supply according to equation (5).
5. Goods markets open. Goods are exchanged at market clearing prices. The final consumption is realized.

Equilibrium with Exogenous Information

Before we turn to the case of endogenous information acquisition, we first analyze the equilibrium under exogenous information. That is, in this subsection we assume precision $\tau_e = 1/\sigma_e^2$ is exogenously given and symmetric (same) for *all* entrepreneurs.

Since the representative household has perfect information about Δ_t, its consumption problem (or labor supply decision) is still given by (3).[2] An entrepreneur's production decision is still given by (10) with the information set being changed to $\{x_{jt}, s_{jt}\}$; that is,

$$y_{jt} = y(A_t, x_{jt}, s_{jt}) = \left(1 - \frac{1}{\theta}\right)^{\theta} \left(\frac{A_t}{\psi}\right)^{\theta} \left\{\mathbb{E}_t \left[\left(Y_t^{\frac{1}{\theta}-\gamma} \epsilon_{jt}^{\frac{1}{\theta}}\right) | x_{jt}, s_{jt}\right]\right\}^{\theta}. \tag{29}$$

Note that Y_t is a function of Δ_t and thus an entrepreneur has uncertainty about Y_t and has to form expectations about it, which is different from the case in the baseline model.

[2] In the presence of aggregate shock Δ_t, the real wage depends on Y_t or Δ_t (but not on idiosyncratic shocks $\{\varepsilon_{jt}\}$).

The aggregate output is hence given by

$$Y_t = \left[\int_0^1 \epsilon_{jt}^{\frac{1}{\theta}} y_{jt}^{\frac{\theta-1}{\theta}} dj\right]^{\frac{\theta}{\theta-1}} = \left(1 - \frac{1}{\theta}\right)^{\theta} \left(\frac{A_t}{\psi}\right)^{\theta} \left[\int_0^1 \epsilon_{jt}^{\frac{1}{\theta}} \left(\mathbb{E}\left[Y_t^{\frac{1}{\theta}-\gamma} \epsilon_{jt}^{\frac{1}{\theta}} | x_{jt}, s_{jt}\right]\right) \tag{30}$$

Equations (29) and (30) jointly determine the aggregate and individual equilibrium outputs. Once we obtain Y_t and y_{jt}, we can first use (3) to compute the aggregate price P_t and then use (5) to compute the individual price p_{jt}. We use the guess-and-verify strategy to obtain Y_t and y_{jt}. We have the following proposition.

Proposition:Under information structure of (27) and (28), aggregate production is given by

$$\log Y_t = \log \bar{Y} + \frac{1}{\gamma}\log A_t + \kappa\Delta_t, \tag{31}$$

where $\bar{Y}$ depends on θ, ψ, γ, σ_ε^2, σ_e^2, and σ_Δ^2, and

$$\kappa = \frac{\tau_\Delta}{\theta\gamma\left(\tau_\varepsilon + \tau_e\right) + \tau_\Delta}. \tag{32}$$

The individual production is given by

$$\begin{aligned} & \log y_{jt} \qquad (33) \\ = \; & \log \bar{y} + \frac{1}{\gamma}\log A_t + \left[\frac{\tau_\Delta}{\tau_e + \tau_\Delta + \tau_\varepsilon} - (\theta\gamma - 1)\,\kappa\frac{\tau_e + \tau_\varepsilon}{\tau_e + \tau_\Delta + \tau_\varepsilon}\right] x_{jt} \\ & + \left[1 + (\theta\gamma - 1)\,\kappa\right]\frac{\tau_e}{\tau_e + \tau_\Delta + \tau_\varepsilon} s_{jt}, \end{aligned}$$

where $\bar{y}$ depends on θ, ψ, γ, σ_ε^2, σ_e^2, and σ_Δ^2.

Endogenous Information

We assume that A_t only takes one of two values, $A_t \in \{A_H, A_L\}$, where $A_H > A_L$. We construct A_H and A_L such that the information acquisition of the firms is symmetric. Specifically, all firms acquire information in equilibrium after observing $A_t = A_H$ but none of them do so after observing $A_t = A_L$. That is, we specify parameters to highlight the mechanism that information precision chosen by entrepreneurs is endogenous and is increasing in A_t. To save space, we provide the details of this subsection in the appendix.

We also prove that if $A_H >> A_L$, then $\mathbb{E}[\log Y_t | A_t = A_H] > \mathbb{E}[\log Y_t | A_t = A_L]$. Thus, we have the following Proposition.

Proposition

The economy exhibits procyclical information acquisition (under $\gamma < 1$).

- The intuition for Proposition 4 is similar to that for the baseline model.
- Under $\gamma < 1$, the real wage does not increase as fast as aggregate output Y_t.
- Hence, when Y_t increases, the increase in the benefit of information acquisition (in terms of consumption units) outruns the increase in the cost in terms of real wage, so the incentives to acquire information become stronger.

- Proposition 11 and Proposition 7 together also imply that the sentiment shock Δ_t is more important in recessions than in booms.
- In fact, the coefficient κ of term Δ_t in (31) is decreasing in τ_e or A_t.
- We can assume that information acquisition is symmetric across firms and it is a *continuous* function of A_t. We show that our results in this section (Proposition 7 and Corollary 8 below) are robust to this alternative setup.

Implications for Measured Uncertainty

First, it is straightforward to show that in equilibrium the residual idiosyncratic uncertainty faced by a firm is decreasing with information acquisition. In fact,

$$SD\left(\varepsilon_{jt} | x_{jt}, s_{jt}\right) = \sqrt{\frac{1}{\tau_e + \tau_\Delta + \tau_\varepsilon}},$$

that is, $SD\left(\varepsilon_{jt} | x_{jt}, s_{jt}\right)$ is decreasing in τ_e. The residual aggregate uncertainty (or forecast error) faced by a firm is

$$SD\left(\log Y_t | A_t, x_{jt}, s_{jt}\right) = \frac{\tau_\Delta}{\theta\gamma\left(\tau_\varepsilon + \tau_e\right) + \tau_\Delta}\sqrt{\frac{1}{\tau_e + \tau_\Delta + \tau_\varepsilon}}, \tag{34}$$

which is decreasing in τ_e.

Next, we calculate aggregate volatility, a common measure for economic uncertainty. We measure aggregate volatility as the conditional standard deviation of aggregate output (in logs). Based on (31), it is given by

$$SD(\log Y_t | A_t) = \sqrt{\left[\frac{\tau_\Delta}{\theta\gamma\left(\tau_\varepsilon + \tau_e\right) + \tau_\Delta}\right]^2 \frac{1}{\tau_\Delta}}.$$

Clearly, $SD(\log Y_t | A_t)$ is decreasing in τ_e. That is, aggregate volatility is decreasing under more precise information.

- Third, we calculate firm-level dispersion.
- For given realization of A_t and Δ_t, heterogeneity of ε_{jt} and e_{jt} across firms generates firm-level dispersion. Based on (33), the standard deviation of production or employment is given by

$$
\begin{aligned}
& SD\left(\log y_{jt} | A_t, \Delta_t\right) \\
= & \sqrt{\left[1-\frac{\theta\gamma\tau_{\varepsilon}}{\theta\gamma\left(\tau_{\varepsilon}+\tau_e\right)+\tau_{\Delta}}\right]^2 \frac{1}{\tau_{\varepsilon}}+\left[\frac{\theta\gamma\tau_e}{\theta\gamma\left(\tau_{\varepsilon}+\tau_e\right)+\tau_{\Delta}}\right]^2 \frac{1}{\tau_e}},
\end{aligned}
$$

which is increasing in τ_e.

- The standard deviation of revenue-based TFP (productivity) or sale price is given by

$$SD\left(\log\left[\left(y_{jt}p_{jt}\right)/n_{jt}\right]|A_t,\Delta_t\right)=\frac{1}{\theta}\sqrt{\left[\frac{\theta\gamma}{\theta\gamma\left(\tau_\varepsilon+\tau_e\right)+\tau_\Delta}\right]^2\left(\tau_\varepsilon+\tau_e\right)}, \tag{35}$$

which is decreasing in τ_e in the interval $\tau_e\in\left[\frac{\tau_\Delta}{\theta\gamma}-\tau_\varepsilon,\infty\right)$.

- Note that parameter condition $\frac{\tau_\Delta}{\theta\gamma}-\tau_\varepsilon<0$ or $\tau_\varepsilon>\frac{\tau_\Delta}{\theta\gamma}$ are easy to satisfy and we assume such a parameter condition, so (35) is always decreasing in τ_e. Corollary 4 follows.

Corollary

The economy exhibits i) countercyclical idiosyncratic uncertainty and countercyclical aggregate uncertainty, ii) countercyclical aggregate volatility, and iii) procyclical firm-level dispersion in production and countercyclical firm-level dispersion in productivity.

- We discuss the intuition behind Corollary 4. In our model, aggregate volatility comes from the common sentiment shock Δ_t.
- When firms acquire information about signal s_{jt} and become more informed of their idiosyncratic demand shock ε_{jt}, they are less responsive to signal x_{jt} and thereby to the "common demand shock" Δ_t, which decreases the aggregate volatility.
- In the extreme case of $\sigma_e^2 = 0$, for example, intermediate goods firms become perfectly informed of their idiosyncratic demand ε_{jt} for $A_t = A_H$ and the aggregate volatility becomes zero.
- When an individual firm uses signals x_{jt} and s_{jt} to forecast Y_t, the forecast error is also countercyclical due to two joint forces — countercyclical unconditional aggregate volatility and procyclical information precision (as seen in (34)).

- As for firm-level dispersion, as in the baseline model, when firms are more informed of their ε_{jt}, their production is more aligned with their ε_{jt}, increasing firm-level dispersion in production and decreasing firm-level dispersion in productivity.
- Corollary 4 implies that our model's predictions are consistent with countercyclical idiosyncratic uncertainty, countercyclical aggregate uncertainty, and countercyclical aggregate volatility, but not countercyclical firm-level dispersion in production. In Appendix C, we consider an extension, which can also explain countercyclical cross-sectional dispersion in production.

Conclusion

In the large and growing recent literature on economic uncertainty, one important issue seems to have received particular attention: the causality between economic uncertainty and macroeconomic activity. While some researchers propose that the direction of causality runs from the second moment (uncertainty) to the first moment (macroeconomic activity), through mechanisms such as the traditional "wait-and-see" effect and the rise in the cost of capital due to the concave-payoffs of debt contracts, some empirical findings suggest that the direction of causality might go the other way round.

- In this paper, we develop a third approach, suggesting that shocks such as a TFP shock can simultaneously drive uncertainty movements and business cycles and trigger two-way feedback effects between them.
- We introduce endogenous information acquisition for firms facing demand shocks in an otherwise standard monopolistically competitive model. The precision of the information about demand shocks optimally acquired by firms varies across business cycles.
- Pro-cyclical information acquisition arises naturally in our model with standard preference and technology specifications. The endogenous information acquisition affects not only the residual uncertainty faced by the firms in equilibrium but also the efficiency of resource allocation — the endogenous TFP.
- The prediction of our model is consistent with the observed countercyclical aggregate volatility — the macro-level measured economic uncertainty. Our framework can be also extended to explain countercyclical cross-sectional dispersion — the micro-level measured uncertainty..