Galor Zeira

OLG with two sectors, two periods:

\[ Y_t^s = F(K_t, L_t^s) \text{ skilled (CRS)} \]
\[ Y_t^n = w_n L_t^n \text{ Unskilled, } w_n > 0 \]

Unskilled agents earn wages \( w_n \) in both period of life. Skilled agents are those who do not work in the first period but go to school by paying the fixed cost \( h \), to acquire human capital.

Utility from second period consumption \( c \) and from bequest \( b \):

\[ u = \alpha \log c + (1 - \alpha) \log b, \quad 0 < \alpha < 1 \]

Capital is mobile, with world interest rate \( r \). Lenders can spend time and cost \( z \) to prevent individual borrowers from evading payment, but borrower can still evade if he pays the cost \( \beta z \), \( \beta > 1 \). This creates a market imperfection. Firms, due to immobility and reputation etc, cannot
evade payment.
Given $L_t^s$, capital in each sector adjusts such that there is a constant capital labor ratio.

$$F_k(K_t, L_t^s) = r$$

Also, $w_s$ is the wage of skilled labor equal to its marginal product, and therefore depends on $r$. 
Wealth Distribution:
Individuals who borrow amount \( d \) must pay to cover costs of monitoring:
\[
i_d d = r d + z
\]
Incentive compatibility requires choice of \( z \) to prevent evasion:
\[
(1 + i_d) d \leq \beta z
\]
Combining:
\[
i_d = i = \frac{1 + \beta r}{\beta - 1} > r
\]
Individual Decisions. If decision is to remain unskilled, without investing in human capital \( h \), and with inheritance \( x \):
\[
U_n(x) = \log((x + w_n)(1 + r) + w_n) + \varepsilon
\]
\[
\varepsilon = \alpha \log \alpha + (1 - \alpha) \log(1 - \alpha)
\]
To see this, NOTE: Optimal consumption $c_n(x)$ and bequest $b_n(x)$ are
\[ \alpha ((x + w_n)(1 + r) + w_n) \] and
\[ (1 - \alpha)((x + w_n)(1 + r) + w_n), \] from utility maximization
\[ \begin{align*}
Max_b \ u &= \alpha \log \left[ (x + w_n)(1 + r) + w_n - b_n \right] \\
&\quad + (1 - \alpha) \log b_n
\end{align*} \]

FOC:
\[ \begin{align*}
\frac{1 - \alpha}{b_n} &= \frac{\alpha}{[(x + w_n)(1 + r) + w_n - b]}
\end{align*} \]
\[ \begin{align*}
\alpha b_n &= (1 - \alpha)[(x + w_n)(1 + r) + w_n - b_n] \\
\alpha b_n + b_n(1 - \alpha) &= b_n = (1 - \alpha)[(x + w_n)(1 + r) + w_n] \\
c_n &= (x + w_n)(1 + r) + w_n - b_n \\
&= \alpha((x + w_n)(1 + r) + w_n) \]
Skilled worker works only in second period of life, but accumulates $h$ in first period of life. If he has $x < h$:

$$U_s(x) = \log[w_s + (x - h)(1 + i)] + \varepsilon$$

Since $x - h < 0$, this agent is borrowing at rate $i$. His bequest is

$$b^s(x) = (1 - \alpha)(w_s + (x - h)(1 + i))$$

If the skilled agent saves $x - h > 0$,

$$U_s(x) = \log[w_s + (x - h)(1 + r)] + \varepsilon$$

Since $x - h > 0$, this agent is lending at rate $r$. His bequest is

$$b^s(x) = (1 - \alpha)(w_s + (x - h)(1 + r))$$
Comparing utilities $U_s(x)$ and $U_n(x)$, note that everyone prefers to remain unskilled irrespective of $x$ (that is even if they have a large $x$ and can afford to invest without borrowing because $x > h$) if

$$w_s + (x - h)(1 + r) < (x + w_n)(1 + r) + w_n$$

$$[w_s - h(1 + r)] < w_n(2 + r)$$

So assume this does not hold. Now with $x > h$, you want to invest. (Note that if this was not true there would be a worldwide excess supply of loans because everyone wants to lend their wages and no one borrows, so $r$ would have to fall.)
What is the condition to invest in $h$? It depends on $x$. Comparing $U_s(x)$ and $U_n(x)$, that is $w_s + (x - h)(1 + i)$ and $(x + w_n)(1 + r) + w_n$, you invest if

$$x \geq f = \frac{1}{i-r} [w_n(2+r) + h(1+i) - w_s]$$

Let the distribution of inheritences be given by the distribution $D_t(x)$:

$$\int_{0}^{\infty} dD(x) = L$$

Then

$$\int_{f}^{\infty} dD(x) = L_t^s$$

$$\int_{0}^{f} dD(x) = L_t^n$$

So the initial wealth distribution determines the labor allocation and output composition, which in turn determines the next period distribution $D_{t+1}(x)$. 
\[
x_{t+1} = (1 - \alpha)[(x_t + w_n)(1 + r) + w_n] \quad \text{if } x_t < f
\]
\[
x_{t+1} = (1 - \alpha)[(w_s) + (x_t - h)(1 + i)] \quad \text{if } f \leq x_t < h
\]
\[
x_{t+1} = (1 - \alpha)[(w_s) + (x_t - h)(1 + r)] \quad \text{if } h \leq x_t
\]

\(f\) is determined by the intersection of \(b_n = (1 - \alpha)[(x_t + w_n)(1 + r) + w_n]\) and \(b_s = (1 - \alpha)[(w_s) + (x_t - h)(1 + i)]\). Note the different slopes wrt \(x\).

So steady state for dynasty of individuals for which \(x < f\):

\[
\bar{x}_n = \frac{1 - \alpha}{1 - (1 - \alpha)(1 + r)} w_n(2 + r)
\]

assuming \(1 > (1 - \alpha)(1 + r)\) to make sure the bequest dynamics do not explode.

Steady state for individuals for which \(f \leq x\): There is a critical point \(g\):

\[
g = \frac{(1 - \alpha)[h(1 + i) - w_s]}{(1 + i)(1 - \alpha) - 1}
\]

so that:

\[
\bar{x}_s = \frac{1 - \alpha}{1 - (1 - \alpha)(1 + r)} [w_s - h(1 + r)]
\]
is the stable steady state if $x > g$ and $x_n$ is the stable steady state if $x < g$.

Also implicit in the Figure and discussion is the assumption that

\[ 1 < \left( \frac{\beta}{\beta - 1} \right)(1 - \alpha)(1 + r) = (1 - \alpha)(1 + i) \]

So that the slope $(1 - \alpha)(1 + i)$ in the middle range for $f \leq x_t < h$ is steep enough to create additional steady states. The
economy gets polarized into two groups: rich and poor.
In the long-run the number of unskilled are:

$$\int_0^g dD_t(x_t) = L^g_t$$

and long run average wealth, as:

$$\frac{x_s(L - L^g_t) + L^g_i x_n}{L} = x^s - \frac{L^g_t}{L}(x_s - x_n)$$

which is decreasing in \( \frac{L^g_t}{L} \). Note that the average wealth depends on initial \( \frac{L^g_t}{L} \), since the number of unskilled workers in the long run \( L^\infty_n \), and their fraction \( \frac{L^n\infty}{L} \) is equal to \( L^g_t \). Note that a rich economy where wealth is held by the few may end up poor on the average as the majority in \( L^g_t \) converge to \( L^\infty_n \) and wealth \( x_n \).
Discussion

What drives the model is 1. Credit market imperfections, which does not allow some to borrow at the international rate \( r \), because of default risks, and non-convexity in education costs of \( h \) : you cannot buy less than \( h \). If you could buy less, your wages would rise partially, and you may attain the level \( h \) eventually rather than getting stuck at the bottom. Here you can either buy \( h \) or zero.
Exercise (What is wrong with the argument below?) This could have been an exam question.

WELFARE. Consider a tax on the skilled to subsidize education, the acquisition of $h$. Will it be Pareto improving?

ANSWER (?) Provide the subsidy on education to the young who seek an education, to be financed by a tax on the skilled next period. Thus we tax the those who are skilled, to pay for their PREVIOUS education. This acts like a government loan that circumvents the inefficiency in the market for borrowing, and according to Galor/Zeira, can be Pareto improving if the costs of tax collection are lower than monitoring costs. If monitoring costs are avoided essentially we have a cheaper way of borrowing against future income. Is this correct, in the sense of being Pareto improving?
More precisely Galor and Zeira claim the following: If the government subsidizes education of the young, by taxing the skilled when they are old, this is a Pareto improvement. Is it? What is wrong with the argument?

1. Assume the tax rate is $t$, and that the government fully subsidizes education. If everyone gets an education (will they?), and there are $L$ people, then, for a balanced budget you need $tw_sL = Lh$, or $t = \frac{h}{w_s}$.

2. The income of those who get an education is $(1 - t)w_s + x(1 + r)$, since they do not pay for their education. Their utility is

$$U(x) = \log((1 - t)w_s + x(1 + r)) + \varepsilon$$

Given their bequest

$$x_{t+1} = (1 - \alpha)((1 - t)w_s + x_t(1 + r))$$

with long run steady state.
\[ x_s = \frac{(1 - \alpha)(1 - t)w_s}{1 - (1 - \alpha)(1 + r)} \]
The utility of the unskilled would be

$$U_n(x) = \log((x + w_n)(1 + r) + w_n) + \varepsilon$$

with long run steady state

$$x_n = \frac{(1 - \alpha)[w_n(2 + r)]}{1 - (1 - \alpha)(1 + r)}$$

Assume $1 - (1 - \alpha)(1 + r) > 0$, and

$$(1 - t)w_s + x(1 + r) > (x + w_n)(1 + r) + w_n$$

$$\left(1 - \frac{h}{w_s}\right)w_s = w_s - h > (w_n)(2 + r)$$

This says the value of an education is worth foregone wages. Now, under the tax system, everyone prefers to get an education.

The skilled with $x > h$ receive higher utility under the government tax system because
\[(1 - t)w_s + x(1 + r) > w_s + (x - h)(1 + r)\]

\[-tw_s > -h(1 + r)\]

\[-h > -h(1 + r)\]

\[1 < 1 + r\]
The skilled with $h \geq x$ had prior utility

$$U_s(x) = \log[w_s + (x - h)(1 + i)] + \varepsilon$$

so for them to be better off

$$(1 - t)w_s + x(1 + r) > w_s + (x - h)(1 + i)$$

$$(h - x)(1 + i) - tw_s + x(1 + r) > 0$$

$$(h - x)(1 + i) - h + x(1 + r) > 0$$

$$(h - x)i + rx > 0$$

So everyone is better off. What is wrong with this argument?