Galor Zeira

OLG with two sectors, two periods:

$$Y_t^s = F(K_t, L_t^s) \quad skilled \;\; (CRS))$$

$$Y_t^n = w_n L_t^n \qquad Unskilled, w_n > 0$$

Unskilled agents earn wages w_n in both period of life.Skilled agents are those who do not work in the first period but go to school by paying the fixed cost $h,$ to acquire human capital.

Utility from second period consumption c and from bequest b:

$$u = \alpha \log\, c + (1-\alpha)\log\, b, \qquad 0 < \alpha < 1$$

Capital is mobile, with world interest rate $r.$ Lenders can spend time and cost z to prevent individual borrowers from evading payment, but borrower can still evade if he pays the cost $\beta z,\ \beta > 1.$ This creates a market imperfection. Firms, due to immobility and reputation etc, cannot evade payment.

Given L_t^s, capital in each sector adjusts such that there is a constant capital labor ratio.

$$F_k(K_t, L_t^s) = r$$

Also, w_s is the wage of skilled labor equal to its marginal product, and therefore depends on r.

Wealth Distribution:

Individuals who borrow amount d must pay

to cover costs of monitoring:

$$i_d d = rd + z$$

Incentive compatibility requires choice of z to prevent evasion:

$$(1 + i_d)d \leq \beta z$$

Combining:

$$i_d = i = \frac{1 + \beta r}{\beta - 1} > r$$

Individual Decisions. If decision is to remain unskilled, without investing in human capital h, and with inheritence x:

$$U_n(x) = \log((x + w_n)(1 + r) + w_n) + \varepsilon$$

$$\varepsilon = \alpha \log \alpha + (1 - \alpha)\log(1 - \alpha)$$

To see this, NOTE: Optimal consumption $c_n(x)$ and bequest $b_n(x)$ are $\alpha((x+w_n)(1+r)+w_n)$ and $(1-\alpha)((x+w_n)(1+r)+w_n)$, from utility maximization

$$Max_b \; u = \alpha \log \left[(x+w_n)(1+r)+w_n-b_n\right] + (1-\alpha)\log b_n$$

FOC:

$$\frac{1-\alpha}{b_n} = \frac{\alpha}{\left[(x+w_n)(1+r)+w_n-b\right]}$$

$$\alpha b_n = (1-\alpha)\left[(x+w_n)(1+r)+w_n-b_n\right]$$

$$\alpha b_n + b_n(1-\alpha) = b_n = (1-\alpha)\left[(x+w_n)(1+r)+w_n\right.$$

$$c_n = (x+w_n)(1+r)+w_n-b_n$$

$$= \alpha((x+w_n)(1+r)+w_n)$$

Skilled worker works only in second period of life, but accumulates h in first perod of life. If he has $x < h$:

$$U_s(x) = \log[w_s + (x - h)(1 + i)] + \varepsilon$$

Since $x - h < 0$, This agent is borrowing at rate i. His bequest is

$$b^s(x) = (1 - \alpha)(w_s + (x - h)(1 + i))$$

If the skilled agent saves $x - h > 0$,

$$U_s(x) = \log[w_s + (x - h)(1 + r)] + \varepsilon$$

Since $x - h > 0$, this agent is lending at rate r. His bequest is

$$b^s(x) = (1 - \alpha)(w_s + (x - h)(1 + r))$$

Comparing utilities $U_s(x)$ and $U_n(x)$, note that everyone prefers to remain unskilled irrespective of x (that is even if they have a large x and can afford to invest without borrowing because $x > h$) if

$$w_s + (x - h)(1 + r) < (x + w_n)(1 + r) + w_n$$

$$[w_s - h(1 + r)] < w_n(2 + r)$$

So assume this does not hold. Now with $x > h,$ you want to invest. (Note that if this was not true there would be a worldwide excess supply of loans because everyone wants to lend their wages and no one borrows, so r would have to fall.)

What is the condition to invest in h? It depends on x. Comparing $U_s(x)$ and $U_n(x)$, that is $w_s + (x - h)(1 + i)$ and $(x + w_n)(1 + r) + w_n$, you invest if

$$x \geq f = \frac{1}{i - r}[w_n(2 + r) + h(1 + i) - w_s]$$

Let the distribution of inheritences be given by the distribution $D_t(x)$:

$$\int_0^\infty dD(x) = L$$

Then

$$\int_f^\infty dD(x) = L_t^s$$

$$\int_0^f dD(x) = L_t^n$$

So the initial wealth distribution determines the labor allocation and output composition, which in turn determines the next period distribution $D_{t+1}(x)$.

$$x_{t+1} = (1-\alpha)[(x_t + w_n)(1+r) + w_n] \quad if\ x_t < f$$

$$x_{t+1} = (1-\alpha)[(w_s) + (x_t - h)(1+i)] \quad if\ f \le x_t < h$$

$$x_{t+1} = (1-\alpha)[(w_s) + (x_t - h)(1+r)] \quad if\ h \le x_t$$

f is determined by the intersection of $b_n = (1-\alpha)[(x_t + w_n)(1+r) + w_n]$ and $b_s = (1-\alpha)[(w_s) + (x_t - h)(1+i)]$. Note the different slopes wrt x.

So steady state for dynasty of individuals for which $x < f$:

$$\bar{x}_n = \frac{1-\alpha}{1-(1-\alpha)(1+r)} w_n(2+r)$$

assuming $1 > (1-\alpha)(1+r)$ to make sure the bequest dynamics do not explode.

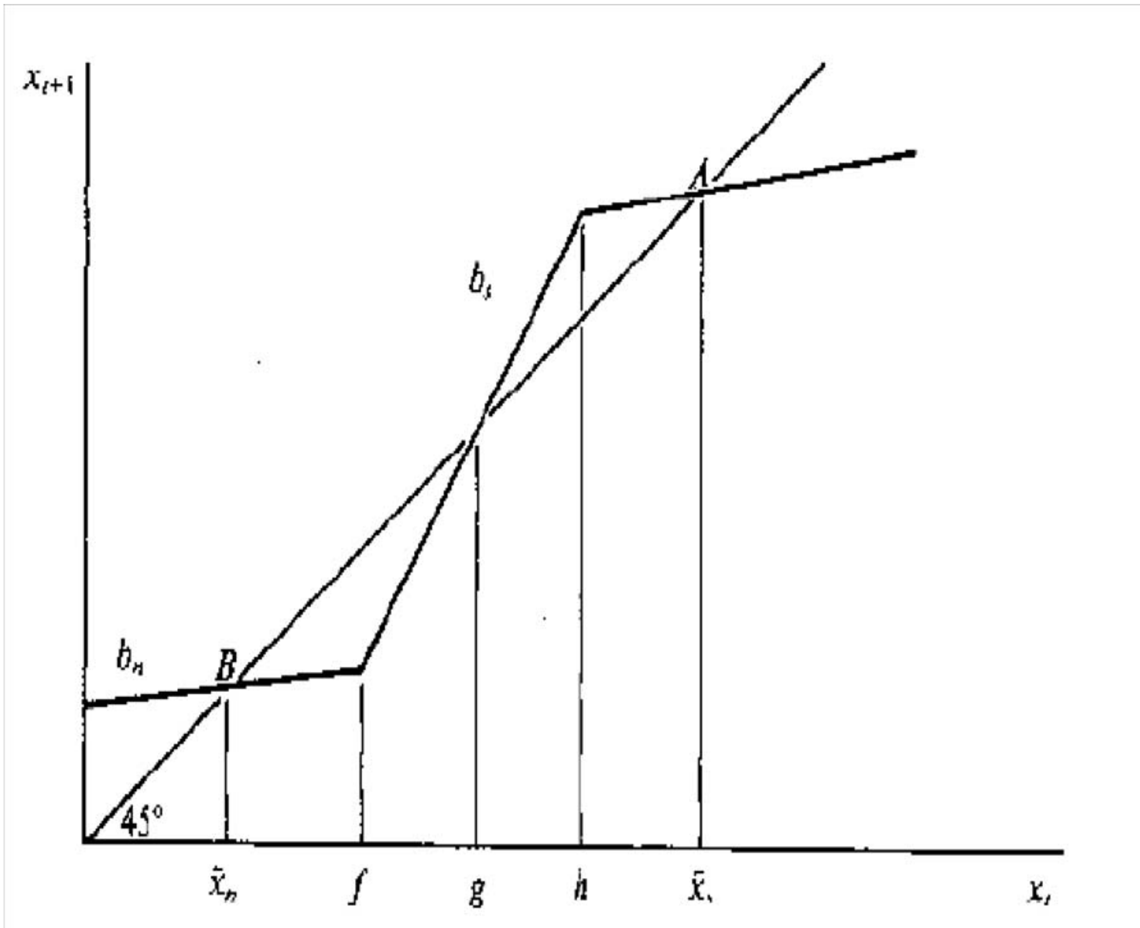
Steady state for individuals for which $f \le x$: There is a critical point g :

$$g = \frac{(1-\alpha)[h(1+i) - w_s]}{(1+i)(1-\alpha) - 1}$$

so that:

$$\bar{x}_s = \frac{1-\alpha}{1-(1-\alpha)(1+r)}[w_s - h(1+r)]$$

is the stable steady state if $x > g$ and x_n is the stable steady state if $x < g$.



Also implicit in the Figure and discussion is the assumption that

$$1 < \left(\frac{\beta}{\beta - 1}\right)(1 - \alpha)(1 + r) = (1 - \alpha)(1 + i)$$

So that the slope $(1 - \alpha)(1 + i)$ in the middle range for $f \leq x_t < h$ is steep enough to create additional steady states. The economy gets polarized into two groups: rich and poor.

In the long-run the number of unskilled are:

$$\int_0^g dD_t(x_t) = L_t^g$$

and long run average wealth, as:

$$\frac{x_s(L - L_t^g) + L_t^g x_n}{L} = x^s - \frac{L_t^g}{L}(x_s - x_n)$$

which is decreasing in $\frac{L_t^g}{L}$. Note that the average wealth depends on initial $\frac{L_t^g}{L}$, since the number of unskilled workers in the long run $L_{\infty,}^n$, and their fraction $\frac{L_{\infty,}^n}{L}$ is equal to L_t^g. Note that a rich economy where wealth is held by the few may end up poor on the average as the majority in L_t^g converge to L_∞^n and wealth x_n.

Discussion

What drives the model is 1. Credit market imperfections, which does not allow some to borrow at the international rate r, because of default risks, and non-convexity in education costs of h :you cannot buy less than h. If you could buy less, your wages would rise partially, and you may attain the level h eventually rather than getting stuck at the bottom. Here you can either buy h or zero.

Exercise (What is wrong with the argument below?) This could have been an exam question.

WELFARE. Consider a tax on the skilled to subsidize education, the acquisition of h. Will it be Pareto improving?

ANSWER (?) Provide the subsidy on education to the young who seek an education, to be financed by a tax on the skilled next period. Thus we tax the those who are skilled, to pay for their PREVIOUS education. This acts like a government loan that circumvents the inefficiency in the market for borrowing, and according to Galor/Zeira, can be Pareto improving if the costs of tax collection are lower than monitoring costs. If monitoring costs are avoided essentially we have a cheaper way of borrowing against future income. Is this correct, in the sense of being Pareto improving?

More precisely Galor and Zeira claim the following: If the government subsidizes education of the young, by taxing the skilled when they are old, this is a Pareto improvement. Is it? What is wrong with the argument?

1. Assume the tax rate is t, and that the goverment fully subsidizes education. If everyone gets an education (will they?), and there are L people, then, for a balanced budget you need $tw_sL = Lh$, or $t = \frac{h}{w_s}$.

2. The income of those who get an education is $(1-t)w_s + x(1+r)$, since they do not pay for their education. Their utility is

$$U(x) = \log((1-t)w_s + x(1+r)) + \varepsilon$$

Given their bequest

$$x_{t+1} = (1-\alpha)((1-t)w_s + x_t(1+r))$$

with long run steady state

$$x_s = \frac{(1-\alpha)(1-t)w_s}{1-(1-\alpha)(1+r)}$$

The utility of the unskilled would be

$$U_n(x) = \log((x + w_n)(1 + r) + w_n) + \varepsilon$$

with long run steady state

$$x_n = \frac{(1 - \alpha)[w_n(2 + r)]}{1 - (1 - \alpha)(1 + r)}$$

Assume $1 - (1 - \alpha)(1 + r) > 0,$ and

$$(1 - t)w_s + x(1 + r) > (x + w_n)(1 + r) + w_n$$

$$\left(1 - \frac{h}{w_s}\right)w_s = w_s - h > (w_n)(2 + r)$$

This says the value of an education is worth foregone wages. Now, under the tax system, everyone prefers to get an education.

The skilled with $x > h$ receive higher utility under the government tax system because

$$
\begin{aligned}
(1-t)w_s + x(1+r) &> w_s + (x-h)(1+r) \\
-tw_s &> -h(1+r) \\
-h &> -h(1+r) \\
1 &< 1+r
\end{aligned}
$$

The skilled with $h \geq x$ had prior utility

$$U_s(x) = \log[w_s + (x - h)(1 + i)] + \varepsilon$$

so for them to be better off

$$(1 - t)w_s + x(1 + r) > w_s + (x - h)(1 + i)$$

$$(h - x)(1 + i) - tw_s + x(1 + r) > 0$$

$$(h - x)(1 + i) - h + x(1 + r) > 0$$

$$(h - x)i + rx > 0$$

So everyone is better off. What is wrong with this argument?